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Nuclear Physics B 899 (2015) 617–630

NUCLEAR PHYSICS B

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Addendum

Addendum to "Leptogenesis in an SU(5) × A_5 golden ratio flavour model" [Nucl. Phys. B 896 (2015) 311–329]

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Received 20 July 2015; received in revised form 13 August 2015; accepted 21 August 2015

Available online 28 August 2015

Editor: Tommy Ohlsson

Abstract

We derive and discuss the solution of the Boltzmann equations for leptogenesis in a phenomenologically viable SU(5) × A_5 golden ratio flavour model proposed in [1,2]. The model employs, in particular, the seesaw mechanism of neutrino mass generation. We find that the results on the baryon asymmetry of the Universe, obtained earlier in [2] using approximate analytic expressions for the relevant CP violating asymmetry and efficiency factors, are correct, as was expected, up to 20–30%. The phenomenological predictions for the low energy neutrino observables, derived using values of the parameters of the model for which we reproduce the observed value of the baryon asymmetry, change little with respect to those presented in [2]. Among the many predictions of the model we find, for instance, that the neutrinoless double beta decay effective Majorana mass m_{ee} lies between 3.3 meV and 14.3 meV.

DOI of original article: http://dx.doi.org/10.1016/j.nuclphysb.2015.04.019.

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http://dx.doi.org/10.1016/j.nuclphysb.2015.08.019
0550-3213/

1. Introduction

The origin of flavour is one of the most challenging unresolved fundamental problems in particle physics. The questions of why there are three generations (not more and not less), of the origin of the hierarchies of the fermion masses and of the very different quark and neutrino mixing patterns are still far from having received a satisfactory explanation.

In recent years an approach to the problem of flavour based on discrete flavour symmetries became widely used especially in treating the flavour problem in the lepton sector, for a recent review see, e.g., [3]. A large number of models employing discrete flavour symmetries have been proposed. However, many of these models focus only on leptons and only reproduce the observed neutrino mixing angles with possibly a few additional predictions for the leptonic CP violation phases and/or the absolute neutrino mass scale.

Here we will focus on a particular model [1] which reproduces *all flavour information in the quark sector* and in addition to reproducing the mixing angles in the neutrino sector, provides predictions for the absolute neutrino mass scale and the leptonic CP violation phases. In [2] we discussed a slight modification of the original model, which allowed us to accommodate successfully the generation of the baryon asymmetry of the Universe within the leptogenesis scenario [4]. In that previous publication [2] we used analytic approximations to calculate the baryon asymmetry, which can be expected to be correct only up to 20–30%. In the present article we go beyond these approximations and calculate the baryon asymmetry by solving the relevant system of Boltzmann equations numerically. We show that using this more precise method of calculation of the baryon asymmetry one can still generate successfully the observed value of it in the model considered. We discuss also the impact of the new results on the baryon asymmetry on the predictions of the low energy observables of the model – on the correlation between the angles θ_{13} and θ_{23}, on the values of the leptonic CP violation phases, on the value of the effective Majorana mass in neutrinoless double beta decay, m_{ee}, etc.

The article is organised as follows. In Section 2 we review the model constructed in [1] and its modification proposed in [2]. In Section 3 we discuss the Boltzmann equations and the solutions for the baryon asymmetry we obtain. We update the results on the neutrino masses and mixing angles previously obtained in [1,2] in Section 4. Section 5 contains summary and conclusions.

2. The leptonic Yukawa and Majorana mass matrices

In this section we briefly recapitulate the Yukawa couplings and Majorana mass matrices in the lepton sector of the model of interest to fix notations. The structure of these matrices is justified by the flavour symmetries of the model and is discussed extensively in [1,2]. The interested reader is referred to these articles for details.

The right-handed neutrino Majorana mass matrix reads

$$M_{\mathrm{RR}} = y_2^n \begin{pmatrix} 2\sqrt{\frac{2}{3}}(v_2+v_3) & -\sqrt{3}v_2 & -\sqrt{3}v_2 \\ -\sqrt{3}v_2 & \sqrt{6}v_3 & -\sqrt{\frac{2}{3}}(v_2+v_3) \\ -\sqrt{3}v_2 & -\sqrt{\frac{2}{3}}(v_2+v_3) & \sqrt{6}v_3 \end{pmatrix} \tag{2.1}$$

where v_2 and v_3 are complex (vevs) of a flavon breaking the A_5 family symmetry. This matrix is of the golden ratio pattern type A [5], i.e., it is diagonalised by

$$U_{\rm GR} = \begin{pmatrix} \sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\phi_g\sqrt{5}}} & 0 \\ -\sqrt{\frac{1}{2\phi_g\sqrt{5}}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{1}{2\phi_g\sqrt{5}}} & -\sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} P_0 \,, \tag{2.2}$$

where $\phi_g = \frac{1+\sqrt{5}}{2}$ is the golden ratio and P_0 is diagonal matrix containing the two CP violation Majorana phases α_1 and α_2, $P_0 = {\rm Diag}(\exp(-\frac{{\rm i}\alpha_1}{2}), \exp(-\frac{{\rm i}\alpha_2}{2}), 1)$. The phases $\alpha_1/2$ and $\alpha_2/2$ are related to those in the convention used by the Particle Data Group [6], $\alpha_{21}/2$ and $\alpha_{31}/2$, as follows: $\alpha_{21} = \alpha_2 - \alpha_1$, $\alpha_{31} = -\alpha_1$.

The matrix of charged lepton Yukawa couplings has the form:

$$Y_e = \begin{pmatrix} 0 & -1/2a_{21} & 0 \\ 6a_{12} & 6a_{22} & 6a_{32} \\ 0 & 0 & -3/2a_{33} \end{pmatrix} \,, \tag{2.3}$$

where the a_{ij} are complex parameters which are fixed by the quark sector (since it is a GUT model) and the charged lepton masses [1]. Note that we did not use here standard GUT relations but the relations proposed in [7] which are in good agreement with the current data on fermion masses and the Higgs mass results [8–10]. Since the a_{ij} depend on $\tan\beta$ and to redo the fit is very time consuming we have fixed this parameter here to 30 which is in good agreement with the aforementioned GUT relations.

The matrix of neutrino Yukawa couplings can be written as

$$Y_\nu = Y_\nu^{\rm LO} + \delta Y_\nu \,. \tag{2.4}$$

The matrix

$$Y_\nu^{\rm LO} = y_1^n \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \,, \tag{2.5}$$

appeared in the original model [1], while

$$\delta Y_\nu \equiv |y_1^n| c\, {\rm e}^{{\rm i}\gamma} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \,, \tag{2.6}$$

was introduced in the second study [2]. The phases can be chosen such that y_1^n is real and then it turned out that the baryon asymmetry is generated only by the small correction δY_ν (note that $c \ll 1$). For more details and analytical estimates the reader is referred to [2].

It proves convenient for our further discussion to define the parameters which will play an important role in the analysis we will perform:

$$M_1 = \frac{1}{\sqrt{6}}(X+Y) = \frac{1}{\sqrt{6}}|X||1+\rho\, {\rm e}^{{\rm i}\phi}|{\rm e}^{{\rm i}\phi_1}, \ \phi_1 = \arg(X+Y) \,, \tag{2.7}$$

$$M_2 = \frac{1}{\sqrt{6}}(X-Y) = \frac{1}{\sqrt{6}}|X||1-\rho\, {\rm e}^{{\rm i}\phi}|{\rm e}^{{\rm i}\phi_2}, \ \phi_2 = \arg(X-Y) \,, \tag{2.8}$$

$$M_3 = \sqrt{\frac{2}{3}}X = \sqrt{\frac{2}{3}}|X|\, {\rm e}^{{\rm i}\phi_3}, \ \phi_3 = \arg(X) \,, \tag{2.9}$$

where

$$X = (4v_3 + v_2) y_2^n \ , \tag{2.10}$$
$$Y = 3\sqrt{5} v_2 y_2^n \ , \tag{2.11}$$
$$\rho = \left| \frac{Y}{X} \right| \ , \tag{2.12}$$
$$\phi = \arg(Y) - \arg(X) \ . \tag{2.13}$$

One of the Majorana phases which we choose to be ϕ_1 can be set to zero by applying a redefinition of the heavy Majorana fields. The remaining two phases ϕ_2 and ϕ_3 can be expressed in terms of ρ and ϕ using the complex mass sum rule $M_1 + M_2 = M_3$

$$\cos\phi_2 = \frac{|M_3|^2 - |M_1|^2 - |M_2|^2}{2|M_1||M_2|} = \frac{1-\rho^2}{\sqrt{1-2\rho^2\cos 2\phi + \rho^4}} \ , \tag{2.14}$$
$$\cos\phi_3 = \frac{|M_1|^2 - |M_2|^2 + |M_3|^2}{2|M_1||M_3|} = \frac{1+\rho\cos\phi}{\sqrt{1+2\rho\cos\phi + \rho^2}} \ . \tag{2.15}$$

We note that only normal ordering is viable in the model considered [1,2], and the Yukawa couplings are degenerate in LO, so that we have $|M_3| < |M_2| < |M_1|$. The Majorana phases α_1, α_2 are employed in the renormalisation group evolution package REAP [11] we are going to use in our numerical analysis, and the phases ϕ_2 and ϕ_3 are related, up to corrections of order c^2, via

$$\alpha_1 = -\phi_3 \text{ and } \alpha_2 = \phi_2 - \phi_3 \, . \tag{2.16}$$

3. Boltzmann equations

In this section we discuss the Boltzmann equations for this model. Since we set the leptogenesis scale to be $M_S \approx 10^{13}$ GeV, we find with $\tan\beta = 30$, $10^9(1+\tan^2\beta)$ GeV $< M_S < 10^{12}(1+\tan^2\beta)$ GeV. Values of M_S in this interval correspond [12] to the two-flavour leptogenesis regime [13,14]. We will perform the analysis of the baryon asymmetry generation in this regime.

We use the set of Boltzmann equations in supersymmetric leptogenesis [15–17] which we briefly summarise below (for notational details and further explanations, see the original papers).

The baryon asymmetry generated in the two-flavour regime in leptogenesis is determined, in particular, by the evolution of the heavy Majorana neutrino and sneutrino number densities (abundances), Y_{N_i} and $Y_{\widetilde{N}_i}$, and of the lepton charge and CP violating asymmetries in the charges $L_e + L_\mu$ and L_τ, $\hat{Y}_{\Delta_2} \equiv \hat{Y}_{\Delta_e} + \hat{Y}_{\Delta_\mu}$ and $\hat{Y}_{\Delta_\tau}$ (where $\hat{Y}_{\Delta_\ell} \equiv Y_{\Delta_\ell} + Y_{\Delta_{\tilde{\ell}}}$, with $\Delta_{\ell(\tilde{\ell})} \equiv B/3 - L_{\ell(\tilde{\ell})}$), during the epoch of the evolution of the Universe when the abundances Y_{N_i} and $Y_{\widetilde{N}_i}$ start to deviate from their equilibrium values (the out-of-equilibrium Sakharov condition [18]). The evolution of the quantities of interest in the epoch of interest can be described by a system of coupled Boltzmann equations, which in the case of the MSSM and of the two-flavour regime we consider read [15–17]

$$\frac{dY_{N_i}}{dz} = -\frac{z}{sH(M_{\rm FLG})} \, 2\left(\gamma_D^i + \gamma_{S,\,\Delta L=1}^i\right)\left(\frac{Y_{N_i}}{Y_{N_i}^{\rm eq}} - 1\right) , \tag{3.1}$$
$$\frac{dY_{\widetilde{N}_i}}{dz} = -\frac{z}{sH(M_{\rm FLG})} \, 2\left(\gamma_D^{\tilde{i}} + \gamma_{S,\,\Delta L=1}^{\tilde{i}}\right)\left(\frac{Y_{\widetilde{N}_i}}{Y_{\widetilde{N}_i}^{\rm eq}} - 1\right) , \tag{3.2}$$

$$\frac{d\hat{Y}_{\Delta_2}}{dz} = -\frac{z}{sH(M_{\rm FLG})}\sum_{i=1}^{3}\Bigg[\left(\epsilon_i^e+\epsilon_i^{\tilde{e}}+\epsilon_i^{\mu}+\epsilon_i^{\tilde{\mu}}\right)\left(\gamma_D^i+\gamma_{S,\,\Delta L=1}^i\right)\left(\frac{Y_{N_i}}{Y_{N_i}^{\rm eq}}-1\right)$$
$$+\left(\epsilon_{\tilde{i}}^e+\epsilon_{\tilde{i}}^{\tilde{e}}+\epsilon_{\tilde{i}}^{\mu}+\epsilon_{\tilde{i}}^{\tilde{\mu}}\right)\left(\gamma_D^{\tilde{i}}+\gamma_{S,\,\Delta L=1}^{\tilde{i}}\right)\left(\frac{Y_{\widetilde{N}_i}}{Y_{\widetilde{N}_i}^{\rm eq}}-1\right)$$
$$-\Bigg(\frac{\gamma_D^{i,e}+\gamma_D^{i,\tilde{e}}+\gamma_D^{i,\mu}+\gamma_D^{i,\tilde{\mu}}}{2}+\gamma_{W,\,\Delta L=1}^{i,e}+\gamma_{W,\,\Delta L=1}^{i,\tilde{e}}$$
$$+\gamma_{W,\,\Delta L=1}^{i,\mu}+\gamma_{W,\,\Delta L=1}^{i,\tilde{\mu}}+\frac{\gamma_D^{\tilde{i},e}+\gamma_D^{\tilde{i},\tilde{e}}+\gamma_D^{\tilde{i},\mu}+\gamma_D^{\tilde{i},\tilde{\mu}}}{2}$$
$$+\gamma_{W,\,\Delta L=1}^{\tilde{i},e}+\gamma_{W,\,\Delta L=1}^{\tilde{i},\tilde{e}}+\gamma_{W,\,\Delta L=1}^{\tilde{i},\mu}+\gamma_{W,\,\Delta L=1}^{\tilde{i},\tilde{\mu}}\Bigg)$$
$$\times\frac{A_{22}\hat{Y}_{\Delta_2}+A_{2\tau}\hat{Y}_{\Delta_\tau}}{\hat{Y}_\ell^{\rm eq}}\Bigg], \tag{3.3}$$

$$\frac{d\hat{Y}_{\Delta_\tau}}{dz} = -\frac{z}{sH(M_{\rm FLG})}\sum_{i=1}^{3}\Bigg[\left(\epsilon_i^\tau+\epsilon_i^{\tilde{\tau}}\right)\left(\gamma_D^i+\gamma_{S,\,\Delta L=1}^i\right)\left(\frac{Y_{N_i}}{Y_{N_i}^{\rm eq}}-1\right)$$
$$+\left(\epsilon_{\tilde{i}}^\tau+\epsilon_{\tilde{i}}^{\tilde{\tau}}\right)\left(\gamma_D^{\tilde{i}}+\gamma_{S,\,\Delta L=1}^{\tilde{i}}\right)\left(\frac{Y_{\widetilde{N}_i}}{Y_{\widetilde{N}_i}^{\rm eq}}-1\right)$$
$$-\Bigg(\frac{\gamma_D^{i,\tau}+\gamma_D^{i,\tilde{\tau}}}{2}+\gamma_{W,\,\Delta L=1}^{i,\tau}+\gamma_{W,\,\Delta L=1}^{i,\tilde{\tau}}$$
$$+\frac{\gamma_D^{\tilde{i},\tau}+\gamma_D^{\tilde{i},\tilde{\tau}}}{2}+\gamma_{W,\,\Delta L=1}^{\tilde{i},\tau}+\gamma_{W,\,\Delta L=1}^{\tilde{i},\tilde{\tau}}\Bigg)$$
$$\times\frac{A_{\tau 2}\hat{Y}_{\Delta_2}+A_{\tau\tau}\hat{Y}_{\Delta_\tau}}{\hat{Y}_\ell^{\rm eq}}\Bigg]. \tag{3.4}$$

In these expressions, $Y_{N_i}^{(\rm eq)}$ is the N_i (equilibrium) abundance, $z \equiv M_{\rm FLG}/T$ with $M_{\rm FLG}$ being the mass scale of flavoured leptogenesis and T being the temperature of the thermal bath, $s = g_* 2\pi^2 T^3/45$ is the entropy density, $H(T) \simeq 1.66\sqrt{g_*}T^2/m_{\rm Pl}$ is the expansion rate of the Universe, $g_* = 228.75$ and the Planck mass $m_{\rm Pl} \simeq 1.22 \times 10^{19}$ GeV. γ_D^i is the thermally averaged N_i decay rate, $\gamma_{S,\Delta L=1}^i$ is the $\Delta L = 1$ scattering rate of N_i with leptons, quarks and gauge bosons, $\gamma_D^{i,l(\tilde{l})}$ is the flavour dependent N_i inverse decay rate, and $\gamma_{W,\Delta L=1}^{i,l(\tilde{l})}$ is the washout rate of the $\Delta L = 1$ scatterings. The A matrix that relates the $B/3 - L_l$ asymmetry Y_{Δ_l} and the lepton charge asymmetry in doublets, Y_l, defined in $Y_l = \sum_{l'} A_{ll'} Y_{\Delta_{l'}}$, in MSSM and the two-flavour regime reads [15]

$$A = \frac{1}{761}\begin{pmatrix} -541 & 152 \\ 46 & -494 \end{pmatrix}. \tag{3.5}$$

The expressions for the CP violating asymmetries generated in the heavy Majorana (s)neutrino decays are the same as those derived in [2]:

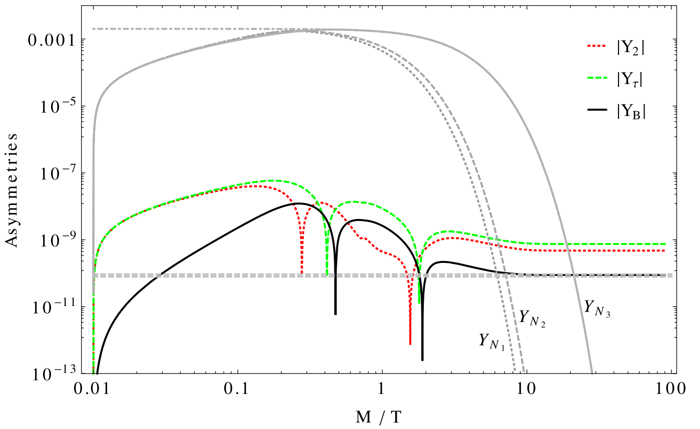

Fig. 1. Solutions for the Boltzmann equations using the example input set. The horizontal grey band represents the 3σ region for the observed baryon asymmetry $Y_B = (8.58 \pm 0.22) \times 10^{-11}$, where, for simplicity, we obtained the 3σ region by multiplying the 1σ error by a factor of three. The dot-dashed line is $Y_{N_3}^{\rm eq}$. See text for further details.

$$\epsilon_1^\tau = \frac{c\left(y_1^n\right)^2}{8\pi}\frac{1}{\sqrt{10}}\left(\sin\gamma\cos\phi_2 f(\tfrac{m_1}{m_2}) - \sin\gamma\frac{m_2^2}{m_2^2-m_1^2} + \cos\gamma\sin\phi_3 f(\tfrac{m_1}{m_3})\right), \tag{3.6}$$

$$\epsilon_2^\tau = \frac{c\left(y_1^n\right)^2}{8\pi}\frac{1}{\sqrt{10}}\left(-\sin\gamma\cos\phi_2 f(\tfrac{m_2}{m_1}) + \sin\gamma\frac{m_1^2}{m_1^2-m_2^2} - \cos\gamma\sin(\phi_3-\phi_2) f(\tfrac{m_2}{m_3})\right), \tag{3.7}$$

$$\epsilon_3^\tau = \frac{c\left(y_1^n\right)^2}{8\pi}\frac{1}{\sqrt{10}}\cos\gamma\left(-\sin\phi_3 f(\tfrac{m_3}{m_1}) + \sin(\phi_3-\phi_2) f(\tfrac{m_3}{m_2})\right). \tag{3.8}$$

Due to the fact that leading order neutrino Yukawa coupling is unitary (except for an overall factor $\left(y_1^n\right)^2$), we have $\epsilon_i^2 \equiv \epsilon_i^e + \epsilon_i^\mu = -\epsilon_i^\tau$ to leading order.

We do not include here the thermal corrections to the CP asymmetries since they give only negligible contributions within the low temperature regime [19,20]. We neglect also the $\Delta L = 2$ process for the same reason as stated in [2], i.e., given the value of y_1^n, the $\Delta L = 2$ processes do not have a significant impact at the leptogenesis scale of interest $M_S \cong 10^{13}$ GeV.

The final baryon asymmetry is

$$Y_B = \frac{10}{31}\left(\hat{Y}_{\Delta_2} + \hat{Y}_{\Delta_\tau}\right). \tag{3.9}$$

As an example we show the result of the Boltzmann equations in Fig. 1 for a single set of parameters chosen from the numerical scan in the next section. The parameters for this plot are

$$\tan\beta = 30,\ M_3 = 8.51\times 10^{12}\ \text{GeV},\ c = 0.053,\ \rho = 6.29,\ \gamma = 3.33\,. \tag{3.10}$$

And we have

$$Y_B = 8.86\times 10^{-11}\,, \tag{3.11}$$

$$Y_{\Delta_2} = -4.72\times 10^{-10}\,, \tag{3.12}$$

$$Y_{\Delta_\tau} = 7.47\times 10^{-10}\,. \tag{3.13}$$

Table 1
The best-fit values and the 3σ ranges for the parameters taken from [21]. The two minima for both θ_{13} and θ_{23} correspond to normal and inverted mass ordering, respectively.

Parameter	Best-fit ($\pm 1\sigma$)	3σ range
θ_{12} in $^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$
θ_{13} in $^\circ$	$8.50^{+0.20}_{-0.21} \oplus 8.51^{+0.20}_{-0.21}$	$7.85 \to 9.10 \oplus 7.87 \to 9.11$
θ_{23} in $^\circ$	$42.3^{+3.0}_{-1.6} \oplus 49.5^{+1.5}_{-2.2}$	$38.2 \to 53.3 \oplus 38.6 \to 53.3$
δ in $^\circ$	251^{+67}_{-59}	$0 \to 360$
Δm^2_{21} in 10^{-5} eV^2	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$
Δm^2_{31} in 10^{-3} eV^2 (NH)	$2.457^{+0.047}_{-0.047}$	$2.317 \to 2.607$
Δm^2_{32} in 10^{-3} eV^2 (IH)	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$

Note that at this point we had given two different example points in our previous publication [2]. But with the improved calculations none of them is in good agreement with the experimental result on Y_B anymore and hence we have chosen here a different example point. This shows furthermore the importance of the current study.

4. Results

In this section we show the results for the masses and the mixing angles as well as the results for Y_B in dependence on the parameters in our model. To obtain realistic values for Y_B coming from the numerical solution of the Boltzmann equations given in Section 3 we had to slightly increase the value of the neutrino Yukawa coupling from $y_1 = 0.1$ to $y_1 = 0.12$ which is a 20% change as anticipated.

4.1. Masses and mixing angles

For our numerical scan we use the method as described in [1]. For the parameters which characterise the charged lepton and quark sector we use the fit results given there with $\tan\beta = 30$ and $M_{\mathrm{SUSY}} = 1$ TeV. The renormalisation group evolution is done using the REAP package [11].

There are six free parameters left in our model, the moduli $|X|$ and $|Y|$, the phases ϕ and δ^e_{12} in the leading order matrices and the modulus $|c|$ and the phase γ, which come from the correction to the neutrino Yukawa matrix. We performed a random scan over these parameters and impose the experimental ranges for the mixing angles, mass squared differences (cf. Table 1) and Y_B as constraints. For Y_B we used [22,23]

$$Y_B = (8.58 \pm 0.22) \times 10^{-11}\,, \tag{4.1}$$

where the 3σ uncertainty is obtained, for simplicity, by multiplying the 1σ error by a factor of three. In order to calculate Y_B we solved the Boltzmann equations given in Section 3 numerically.

Before we present the results for the normal ordering of the neutrino masses we comment briefly on the case of inverted ordering. In the original model [1] the inverted ordering was not viable due to incompatible constraints for θ_{12} coming from the mass sum rule on one hand and from the angle sum rule in our model on the other hand. Due to the correction for the neutrino Yukawa matrix the mixing angles are modified with corrections of order c which nevertheless have to be of the order of $c \approx 0.4$ to save the inverted ordering as we have shown in an estimate given in [2]. Since c is associated with a higher dimensional operator such high values are not plausible.

Turning now to the normal ordering, we show the results for the masses and mixing angles in Fig. 2. We find all parameters in agreement with the experimental 1σ (3σ) ranges for the masses, mixing angles and Y_B.

The correlations between θ_{13} and the Majorana phases are weaker than in [2] which is due to the constraints on the phase ϕ and the value of $|Y|$ coming from the numerical solution of Y_B. Furthermore we need here a larger value of c which washes the correlations out and enlarges the ranges for the phases. Nevertheless we find the phases to be in similar ranges as in [2], namely we obtain

$$\delta \in [9^\circ, 119^\circ] \text{ or } [239^\circ, 344^\circ]\,, \tag{4.2}$$

$$\alpha_1 \in [0^\circ, 134^\circ] \text{ or } [220^\circ, 360^\circ]\,, \tag{4.3}$$

$$\alpha_2 \in [66^\circ, 134^\circ] \text{ or } [222^\circ, 282^\circ]\,. \tag{4.4}$$

For the rephasing invariant $J_{\rm CP}$ which determines the magnitude of CP violation effects in neutrino oscillations [26] we find values in the ranges $J_{\rm CP} = \pm(0.006, 0.036)$. Our predictions for neutrinoless double beta decay are shown in Fig. 3. For the lightest neutrino mass m_1 which is mostly determined by the mass sum rule we obtain values between 12 meV and 22 meV. For the observable in neutrinoless double beta decay m_{ee} we obtain values between 3.3 meV and 14.3 meV. For the sum of the neutrino masses we predict

$$\sum m_\nu \in 0.077\text{–}0.099 \text{ eV}\,, \tag{4.5}$$

which might be determined, e.g., from cosmology. So far there is only an upper bound [22]

$$\sum m_\nu < 0.23 \text{ eV}, \tag{4.6}$$

which is well in agreement with our prediction. The second observable is the kinematic mass m_β as measured in the KATRIN experiment [27] which is given as

$$m_\beta^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2\,. \tag{4.7}$$

Here we predict $m_\beta \approx 0.015$–0.023 eV which is below the projected reach of $m_\beta > 0.2$ eV of KATRIN.

4.2. Leptogenesis

Moving on to our predictions for Y_B, we show the results of our parameter scan in Figs. 4 and 5.

We find parameter points which are in agreement with the 1σ (3σ) range for Y_B. The ranges for the ϕ and ρ are as in our previous study between 4.7 and 7.6 for ρ and between 0.66 and 1.38 or -1.38 and -0.66 for ϕ since they are mostly determined by the masses and mixing angles. However the constraints coming from the numerical solution of Y_B now forbid values of ϕ between -0.66 and 0.66. Interesting is also the range for the mass of the lightest right-handed neutrino which is in the range from $0.778 \cdot 10^{13}$ GeV to $0.862 \cdot 10^{13}$ GeV which shows a clear correlation to c but no correlation to γ. The other correlations can be seen in Fig. 5. We furthermore want to note that the rather narrow range of M_3 is due to the fact that we fixed y_1^n to a certain value. As a rule of thumb the scale of light neutrino masses has to remain the same and hence a variation of 10% in y_1^n leads to a variation of 20% in M_3.

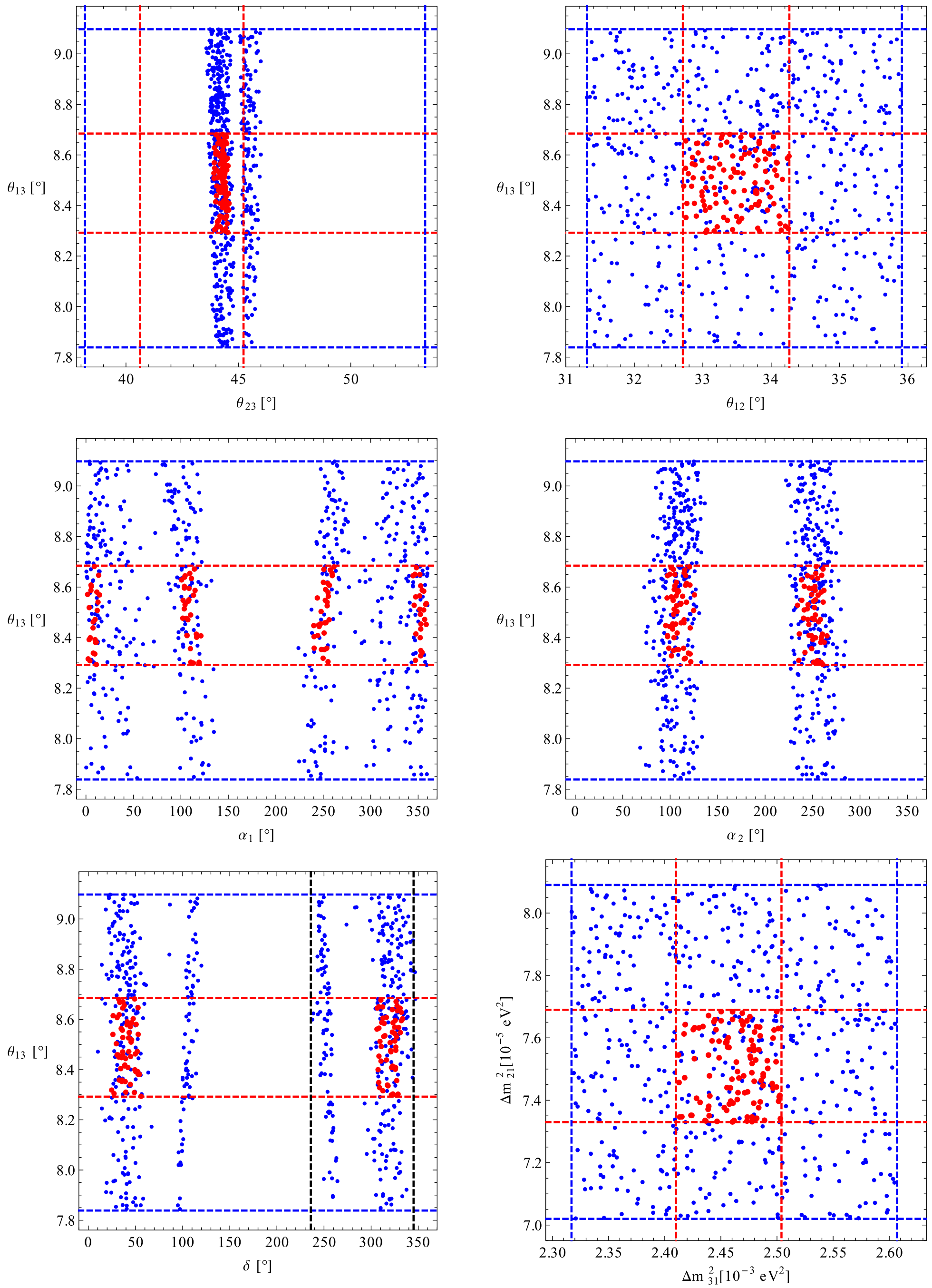

Fig. 2. Results of our numerical parameter scan. Blue (red) points are in agreement within 3σ (1σ) of the low energy neutrino masses and mixings and Y_B in our model. The allowed experimental 3σ (1σ) regions are limited by blue (red) dashed lines. The black dashed lines represent the 1σ range for the not directly measured CP phase δ from the global fit [21]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

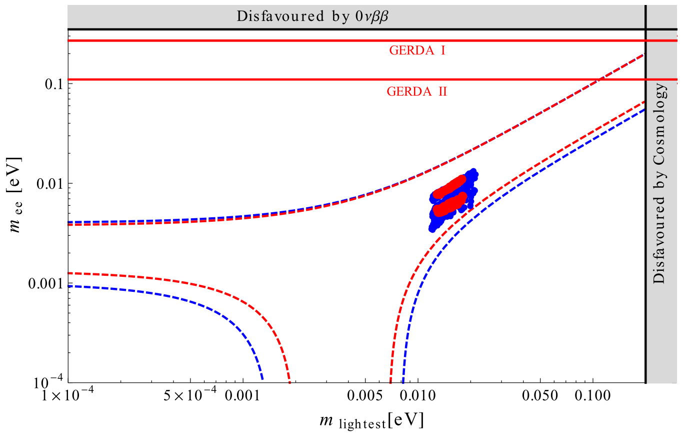

Fig. 3. Prediction for the effective neutrino mass m_{ee} accessible in neutrinoless double beta decay experiments as a function of the lightest neutrino mass m_1. The allowed experimental 3σ (1σ) regions for the masses and mixing angles in the case of normal ordering are limited by blue (red) dashed lines. Blue (red) points are in agreement within 3σ (1σ) of the low energy neutrino masses and mixings and Y_B in our model. The grey region on the right side shows the bounds on the lightest mass from cosmology [22] and the grey region in the upper part displays the upper bound on the effective mass from the EXO experiment [24]. The red, straight lines represent the sensitivity of GERDA phase I respectively GERDA phase II [25]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The size of the correction c has to be in the range between 0.035 and 0.085 and γ is in narrow ranges around 0 or π. This also explains the disconnected regions for the mixing angles and phases which did not appear so in our previous studies. As in our previous study the sign of Y_B is uniquely determined by γ. For the 1σ ranges of the parameters the values of γ have to be in the region around 0. This implies that we only find parameter points for the 1σ ranges compatible with a negative value of Y_B. Note that this is mainly driven by the 1σ range of θ_{23} which is not very well determined. Indeed, with the results from the Valencian fitting collaboration [28] where θ_{23} is allowed to be in the second quadrant, even at the 1σ level positive values for Y_B are possible.

As γ is in narrow ranges around the CP conserving values, the Majorana phases are the major sources for CP violation here, as it can be seen from Fig. 4.

5. Summary and conclusions

In this work we have revised predictions for leptogenesis in an SU(5) $\times$ A_5 golden ratio GUT flavour model from [2]. In that publication we had used approximations to calculate the baryon asymmetry which are known to be precise only up to 20–30%. Instead we have solved here the full set of Boltzmann equations numerically and could show that we can still successfully accommodate the experimental values of all mixing angles, fermion masses and the baryon asymmetry even at the 1σ level. Nevertheless, to do so we had to adjust some parameters. For instance, we have increased the neutrino Yukawa coupling by 20% from $y_1 = 0.1$ to $y_1 = 0.12$.

All of the main features of the original model are still valid. For instance, the neutrino mass sum rule is correct up to small corrections and the inverted ordering is still ruled out. Furthermore, the Yukawa coupling ratios $y_\tau / y_b \approx -3/2$ are unaffected by the modification of our model

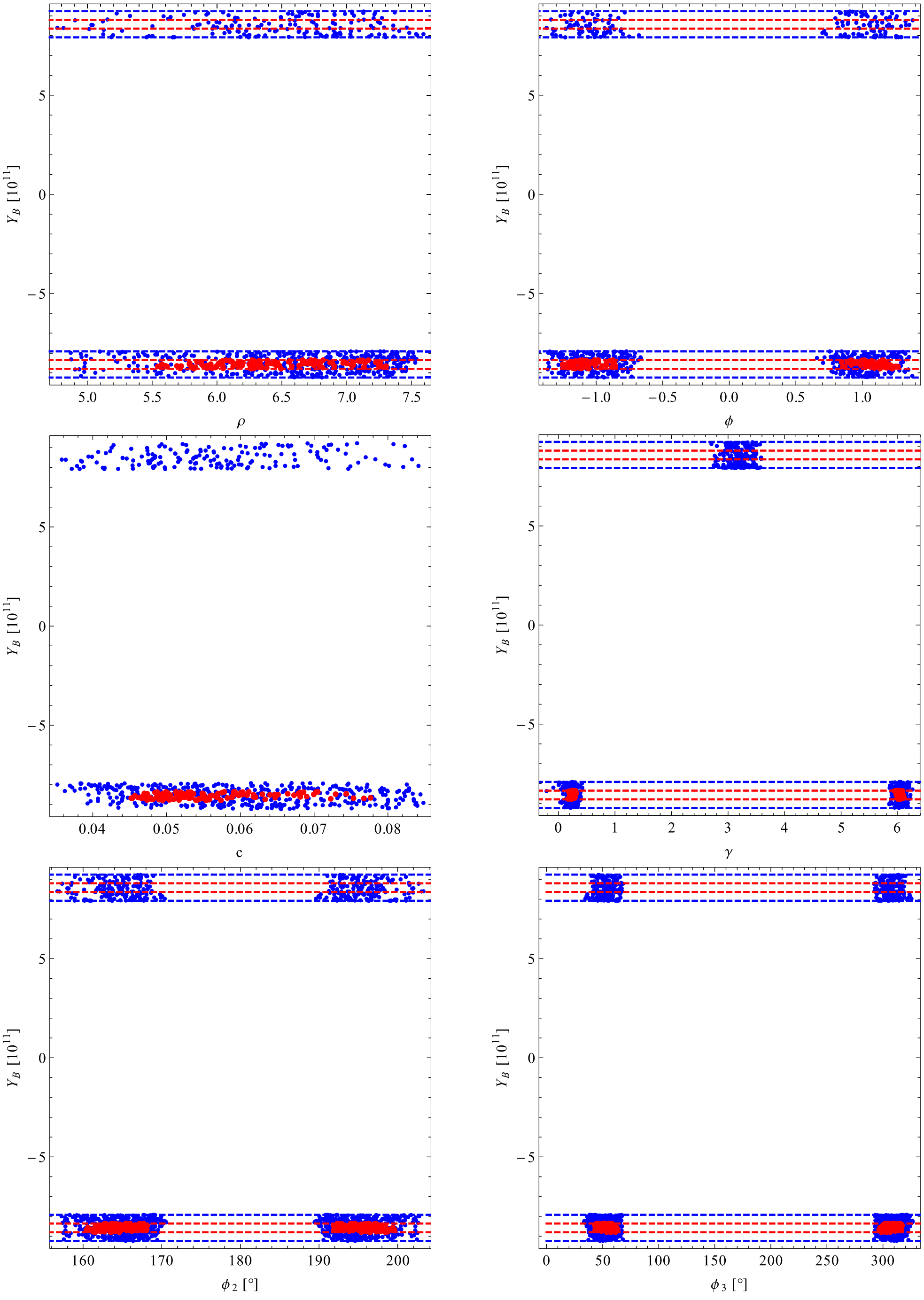

Fig. 4. Results of our numerical scan for the total baryon asymmetry Y_B in dependence of the six most relevant parameters. Blue (red) points are in agreement within 3σ (1σ) of the low energy neutrino masses and mixings and Y_B in our model. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

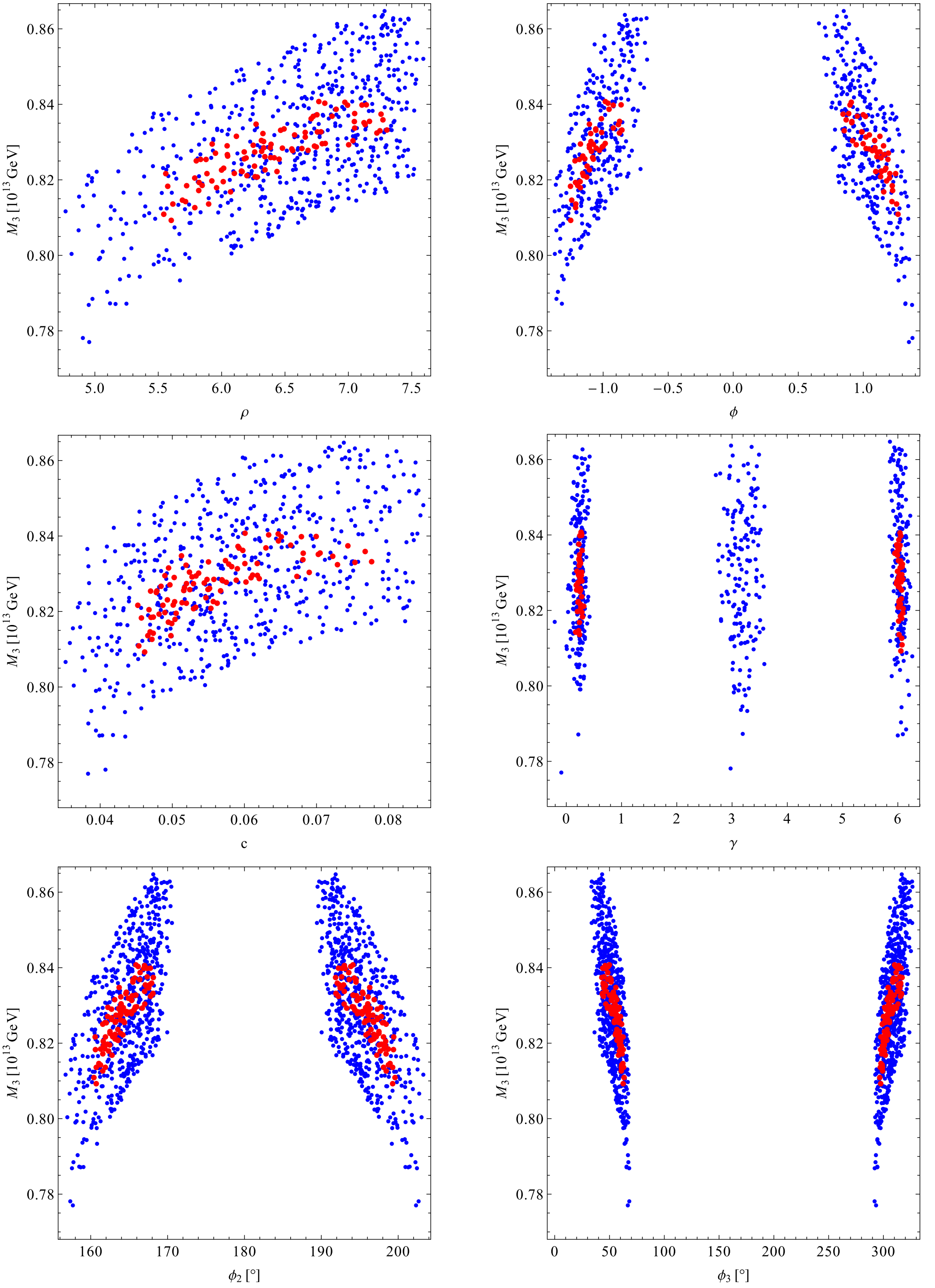

Fig. 5. Results of our numerical scan showing the correlations between the lightest right-handed neutrino mass and the six most relevant parameters for leptogenesis. Blue (red) points are in agreement within 3σ (1σ) of the low energy neutrino masses and mixings and Y_B in our model. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and hence imply non-trivial constraints on the spectrum of the supersymmetric partners of the Standard Model particles.

The phenomenological predictions are not changed significantly as well. For the lightest neutrino mass we find the range $m_1 \in [0.012, 0.022]$ eV, but the strong correlation between θ_{13} and the phases is somewhat weakened due to an increased value of c. For the phases we find $\delta \in [9°, 119°]$ or $[239°, 344°]$, $\alpha_1 \in [0°, 134°]$ or $[220°, 360°]$ and $\alpha_2 \in [66°, 134°]$ or $[222°, 282°]$. Related to the Majorana phases is the observable in neutrinoless double beta decay where we predict m_{ee} between 3.3 meV and 14.3 meV.

We have proposed the first SU(5) × A_5 model to our knowledge which can successfully accommodate for all parameters in the neutrino sector and for the baryon asymmetry of the Universe as calculated from the Boltzmann equations and for the not yet measured quantities in the neutrino sector we make testable predictions. For this our model needs comparatively few parameters which makes it very appealing and testable in future experiments.

Acknowledgements

X.Z. would like to thank E. Molinaro for sharing his code on solving Boltzmann equations. S.T.P. acknowledges very useful discussions with E. Molinaro. This work was supported in part by the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442-INVISIBLES), by the INFN program on Theoretical Astroparticle Physics (TASP), by the research grant 2012CPPYP7 (*Theoretical Astroparticle Physics*) under the program PRIN 2012 funded by the Italian MIUR and by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan (S.T.P.). J.G. acknowledges support by the DFG-funded research training group GRK 1694 "Elementarteilchenphysik bei höchster Energie und höchster Präzision".

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