Sequence-based LQG Control over Stochastic Networks with Linear Integral Constraints

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Abstract— **In this paper, we consider sequence-based LQG control of stochastic linear systems with linear integral state and input constraints over networks subject to stochastic packet delays and losses. For this scenario, we derive a novel closed-loop optimal control law that consists of a feedback term that depends linearly on the current state estimate and network acknowledgments, and a feedforward term that depends on the initial system state, the constraints, and network acknowledgments. The feedback term is given in closed-form, while the feedforward term demands the solution of a quadratic program. The number of decision variables corresponds to the number of constraints. The presented control law is evaluated by means of a Monte-Carlo simulation.**

I. Introduction

In modern control systems, control loops are often closed by packet-based networks. Such control systems are then referred to as Networked Control Systems (NCS). The employment of networks allows for flexible architectures and reduces installation and maintenance costs [1]. While in early days of NCS design, only networks dedicated to be used in control loops such as CAN or PROFIBUS, were employed, there is an upcoming trend towards designing NCSs using general purpose networks, e.g., Ethernet or WLAN. However, the usage of general purpose networks introduces effects into the control loop such as stochastic packet delays and losses, variable medium access times, quantization, and limited bandwidth [1], [2]. If not anticipated, these effects can degrade system performance or even destabilize the control loop. Consequently, for high-quality networked control, explicit consideration of network-induced effects is required. In this paper, we focus on stochastic packet delays and losses assuming that other effects are negligible or mitigated by other means.

Sequence-based control is a promising approach to compensate for stochastic packet delays and losses. The idea of this control method, first introduced in [3], is to let the controller transmit a sequence of predicted control inputs to the actuator instead of a single control input. By doing so, the actuator can apply control inputs from successfully transmitted data packets in case subsequent packets are delayed or lost. In literature, sequence-based control was investigated from Model Predictive Control (MPC) perspective, e.g., in [4], [5], or from the perspective of Linear Quadratic Gaussian (LQG) control, e.g., in [6], [7]. In [6], an optimal sequence-based control law was derived for NCS with stochastic packet losses.

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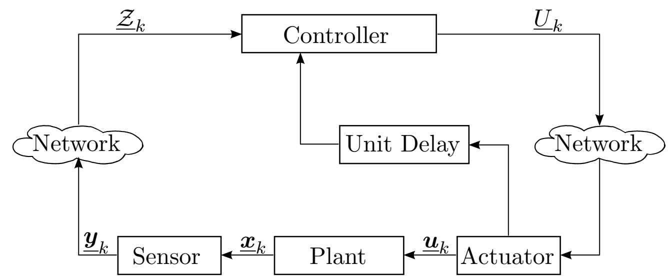

Fig. 1. Considered system setup. The connection between the actuator and the controller indicates the TCP-like nature of the controller-actuator network.

These results were generalized in [7] to NCS with stochastic packet delays and losses. Stability criteria for this optimal control law and the minimum required sequence length are given in [8].

In this paper, we extend the results presented in [7] to systems subject to state and input constraints. Since hard constraints cannot be satisfied if the process noise is unbounded [9], we will consider soft expectation-type integral constraints. In finite-horizon control, integral constraints are a generalization of constraints defined for a single horizon step only. LQG control of non-networked systems with this type of constraints was considered in [10], [11], [12], [13]. In [10] and [13], the authors developed an efficient interior-point technique for solving quadratic programming problems in Hilbert spaces and applied this technique to continuous-time LQ control of linearly constrained systems. In [11] and in a more thorough discussion in [12], these results were generalized to linearly constrained LQG control in discrete time. The authors further proved that Lagrange-based solution methods are applicable in the considered setup. We will use this result, to derive the optimal closed-loop control law for linear NCS with linear integral state and input constraints.

Outline The remainder of this paper is organized as follows. In Sec. II, we describe the considered setup and formulate the control problem. The proposed solution to this problem is presented in Sec. III and evaluated by means of a Monte-Carlo simulation in Sec. IV. Conclusion is given in Sec. V.

Notation Deterministic quantities are in normal letters (a), while random variables are in bold letters ($\boldsymbol{a}$), vector-valued quantities are underlined ($\underline{v}$), and matrices are in bold capital letters ($\mathbf{A}$). Time-variant quantities are indicated by an index, e.g., $\underline{v}_k$. Expression $\boldsymbol{a} \sim f(a)$ indicates that a random variable $\boldsymbol{a}$ has the probability density function $f(a)$. A matrix with all elements being zeros is denoted by $\mathbf{0}$ and the identity matrix by $\mathbf{I}$. For a matrix $\mathbf{A}$, $\mathbf{A}^\top$ denotes its transpose and $\mathbf{A}^\dagger$ its Moore-Penrose pseudoinverse.

II. Problem Formulation

In this paper, we consider the NCS depicted in Fig. 1 whose components are time-triggered and synchronized. The plant and the sensor are discrete-time linear systems described in state space by

$$\begin{aligned} \underline{\boldsymbol{x}}_{k+1} &= \mathbf{A}\underline{\boldsymbol{x}}_k + \mathbf{B}\underline{\boldsymbol{u}}_k + \underline{\boldsymbol{w}}_k \ , \\ \underline{\boldsymbol{y}}_k &= \mathbf{C}\underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}_k \ , \end{aligned} \tag{1}$$

where $\underline{\boldsymbol{x}}_k \in \mathbb{R}^n$ denotes the state of the system, $\underline{u}_k \in \mathbb{R}^m$ the control input, and $\underline{\boldsymbol{y}}_k \in \mathbb{R}^p$ the measurement at time step k. The matrices $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ can be time-variant. Both the plant and the sensor are subject to stationary independent additive zero-mean Gaussian noises $\underline{\boldsymbol{w}}_k \sim f(\underline{w}_k)$ and $\underline{\boldsymbol{v}}_k \sim f(\underline{v}_k)$ with covariances

$$\mathrm{E}\left\{\underline{\boldsymbol{w}}_k \underline{\boldsymbol{w}}_k^\top\right\} = \mathbf{W} \quad \text{and} \quad \mathrm{E}\left\{\underline{\boldsymbol{v}}_k \underline{\boldsymbol{v}}_k^\top\right\} = \mathbf{V} \ .$$

The initial system state $\underline{\boldsymbol{x}}_0$ is Gaussian with

$$\overline{\underline{x}}_0 = \mathrm{E}\left\{\underline{\boldsymbol{x}}_0\right\} \quad \text{and} \quad \mathbf{X}_0 = \mathrm{E}\left\{(\underline{\boldsymbol{x}}_0 - \overline{\underline{x}}_0)(\underline{\boldsymbol{x}}_0 - \overline{\underline{x}}_0)^\top\right\} \ .$$

Control inputs $\underline{\boldsymbol{u}}_k$ are applied to the plant by an actuator. The controller communicates with the actuator over a stochastic network (CA-link). Data packets transmitted over the network can experience i.i.d. stochastic delays or even get lost. Packet delays $\boldsymbol{\tau}_k^{CA} \in \mathbb{N}_0$ in the CA-link are governed by a stationary stochastic process with a given probability density function (pdf) $f(\tau_k^{CA})$. Packet losses are modeled as infinite delays. Further, the CA-link provides the controller with instantaneous acknowledgments on successful transmissions (TCP-like network[1]). Of course, the assumption of a TCP-like network is very limiting since in real world applications this channel also has to be modeled stochastically. However, if packet delays in the acknowledgment channel are bounded, it is possible to model this channel as a Markov chain and thus remain within the applied modeling framework.

To mitigate network-induced delays and losses, the controller transmits sequences of control inputs $\underline{U}_k$ to the actuator. These sequences consist not only of the current control input but also of $N-1, N \in \mathbb{N}$ predicted control inputs, i.e.,

$$\underline{U}_k = \begin{bmatrix} \underline{u}_{k|k}^\top & \underline{u}_{k+1|k}^\top & \cdots & \underline{u}_{k+N-1|k}^\top \end{bmatrix}^\top ,$$

where the index $k+i|k$ indicates that the control input was computed based on the information available to the controller at time step k and is intended to be applied to the plant at time step $k+i, \ i \in \{0, 1, \ldots, N-1\}$. The selection logic at the actuator buffers the control sequence that was computed based on the most recent information, i.e., the control sequence $\underline{U}_k$ with the largest index k, and applies appropriate control inputs from the buffer to the plant. In case the actuator buffer runs empty, the actuator applies the default control input $\underline{u}^d = \underline{0}$.

To compute the sequences $\underline{U}_k$, the controller not only uses acknowledgments on successful transmissions but also sensor measurements $\underline{\boldsymbol{y}}_k$. The measurements are transmitted from the sensor to the controller over – possibly another – stochastic network (SC-link). Each measurement can experience a delay $\boldsymbol{\tau}_k^{SC} \in \mathbb{N}_0$ described by the probability distribution $f(\tau^{SC})$. Losses in the SC-link are also modeled as infinite delays. Since the measurements can be delayed or lost, the controller can receive none, one, or several measurements at each time step. The set of measurements available to the controller at time step k will be denoted by $\mathcal{Z}_k$. Further, it is not required that the network acknowledges successful transmissions (UDP-like network) because the sensor will not retransmit any measurements.

The performance of the controlled system is measured by a cumulative linear quadratic function

$$J_0 = \frac{1}{2}\mathrm{E}\left\{\underline{\boldsymbol{x}}_K^\top \mathbf{Q}_K \underline{\boldsymbol{x}}_K + \sum_{k=0}^{K-1}\left[\underline{\boldsymbol{x}}_k^\top \mathbf{Q}_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{u}}_k^\top \mathbf{R}_k \underline{\boldsymbol{u}}_k\right] \middle| \mathcal{I}_0\right\}, \tag{2}$$

where $K \in \mathbb{N}$ is the length of the considered optimization horizon, and the matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$ are positive semidefinite and positive definite, respectively. The expected value in (2) is conditioned on the information set $\mathcal{I}_0$ available to the controller at time step 0. This information set will be defined later.

The control task is to minimize the cumulative costs (2) w.r.t. the control sequences $\underline{U}_{0:K-1} = \{\underline{U}_0, \ldots, \underline{U}_{K-1}\}$. However, the optimization is not unconstrained but subject to $M \in \mathbb{N}_0$ linear integral constraints

$$g_s(\underline{\boldsymbol{x}}_{0:K}, \underline{\boldsymbol{u}}_{0:K-1}) \leq c_s \ , \qquad c_s \in \mathbb{R}, \ s \in \{1, \ldots, M\} \ ,$$

with

$$\begin{aligned} &g_s(\underline{\boldsymbol{x}}_{0:K}, \underline{\boldsymbol{u}}_{0:K-1}) \\ &= \mathrm{E}\left\{(\underline{a}_K^s)^\top \underline{\boldsymbol{x}}_K + \sum_{k=0}^{K-1}\left[(\underline{a}_k^s)^\top \underline{\boldsymbol{x}}_k + (\underline{b}_k^s)^\top \underline{u}_k\right] \middle| \mathcal{I}_0\right\}, \end{aligned} \tag{3}$$

where $\underline{a}_k^s \in \mathbb{R}^n$ and $\underline{b}_k^s \in \mathbb{R}^m$ are state and input weighting parameters, respectively. The superscript s denotes the constraint number.

The considered control problem can be summarized as

$$\begin{aligned} &\min_{\underline{U}_{0:K-1}} \ J_0 \\ &\quad \text{s.t. } g_s(\underline{\boldsymbol{x}}_{0:K}, \underline{u}_{0:K-1}) \leq c_s, \quad s \in \{1, \ldots, M\} \ . \end{aligned} \tag{4}$$

The proposed solution to optimization problem (4) is described in the next section.

III. Control Law Derivation

In this section, we describe the derivation of the control law that solves (4). For this purpose, we first model the considered NCS as a Markov Jump Linear System (MJLS) using state augmentation and express the cost functional and the integral constraints by means of the augmented state. Finally, the

[1]The term TCP-like is used only to indicate that the network provides acknowledgments on successful transmission and does not refer to real TCP/IP.

optimization is described in Sec. III-B. To find the solution, we first convert the constrained optimization problem into an unconstrained problem using the Lagrange multiplier method. Then, it is possible to perform a closed-form optimization w.r.t. the control sequences $\underline{U}_{0:K-1}$. The optimization w.r.t. Lagrange multipliers has to be computed numerically.

A. System Modeling

If the control sequence $\underline{U}_t, t \in \{0, \dots, k\}$ is buffered by the actuator at time step k, we denote the age of this sequence by θ_k with

$$\theta_k = \begin{cases} k - t, & \text{for } t \geq k - N \ , \\ N, & \text{otherwise} \ . \end{cases}$$

If the actuator buffer runs empty, we set $\theta_k = N$. With this definition, the control input applied to the plant at time step k is given by

$$\underline{u}_k = \begin{cases} \underline{u}_{k|k-\theta_k}, & \text{for } \theta_k < N \ , \\ \underline{u}^d = \underline{0}, & \text{for } \theta_k = N \ . \end{cases} \tag{5}$$

We will refer to θ_k as the *mode* of the system. As demonstrated in [7], the mode θ_k can be modeled as the state of a Markov chain. The transition matrix $\mathbf{T}$ of this Markov chain can be obtained from the delay pdf $f(\tau_k^{CA})$. The elements $p_{ji} = \text{Prob}(\theta_k = i | \theta_{k-1} = j)$ of the transition matrix are given by

$$p_{ji} = \begin{cases} 0, & \text{for } i > j + 1 \ , \\ 1 - \sum\limits_{r=0}^{i} q_r, & \text{for } i = j + 1 \ , \\ q_r, & \text{for } i \leq j, \ j < N \ , \\ 1 - \sum\limits_{r=0}^{N} q_r, & \text{for } i = j = N \ , \end{cases}$$

where q_r denotes the probability that a data packet is delayed by $r \in \mathbb{N}_0$ time steps.

To consider control inputs that still could be applied to the plant at subsequent time steps, we augment the state $\underline{\boldsymbol{x}}_k$ by the vector $\underline{\eta}_k$ with

$$\underline{\eta}_k = \begin{bmatrix} \begin{bmatrix} \underline{u}_{k|k-1}^\top & \underline{u}_{k+1|k-1}^\top & \cdots & \underline{u}_{k+N-2|k-1}^\top \end{bmatrix}^\top \\ \begin{bmatrix} \underline{u}_{k|k-2}^\top & \underline{u}_{k+1|k-2}^\top & \cdots & \underline{u}_{k+N-3|k-2}^\top \end{bmatrix}^\top \\ \vdots \\ \underline{u}_{k|k-N+1} \end{bmatrix} .$$

With this definition of $\underline{\eta}_k$, it is possible to express (5) by means of a stochastic linear system according to

$$\begin{aligned} \underline{\eta}_{k+1} &= \mathbf{F}\underline{\eta}_k + \mathbf{G}\underline{U}_k \ , \\ \underline{u}_k &= \mathbf{H}_k\underline{\eta}_k + \mathbf{J}_k\underline{U}_k \ , \end{aligned} \tag{6}$$

with

$$\mathbf{G} = \begin{array}{l} \begin{array}{cc} \scriptstyle \#columns{:}\, m & \scriptstyle m(N-1) \end{array} \quad \scriptstyle \#rows{:} \\ \left[\begin{array}{cc} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \begin{array}{l} \}m(N-1) \\ \}m\frac{(N-1)(N-2)}{2} \end{array} \end{array} , \quad \mathbf{J}_k = \begin{array}{l} \begin{array}{cc} \scriptstyle \#columns{:}\, m & \scriptstyle m(N-1) \end{array} \quad \scriptstyle \#rows{:} \\ \left[\begin{array}{cc} \delta_{\theta_k,0}\mathbf{I} & \mathbf{0} \end{array}\right] \}m \end{array} ,$$

$$\mathbf{F} = \begin{array}{l} \begin{array}{ccccccc} \scriptstyle \#columns{:}\, m & \scriptstyle m(N-1) & \scriptstyle m & \scriptstyle m(N-2) & & \scriptstyle m & \scriptstyle m \end{array} \quad \scriptstyle \#rows{:} \\ \left[\begin{array}{ccccccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{array}\right] \begin{array}{l} \}mN \\ \}m(N-1) \\ \}m(N-2) \\ \\ \}m \end{array} \end{array} ,$$

$$\mathbf{H}_k = \begin{array}{l} \begin{array}{cccccc} \scriptstyle \#columns{:}\, m & \scriptstyle m(N-2) & \scriptstyle m & \scriptstyle m(N-3) & & \scriptstyle m \end{array} \quad \scriptstyle \#rows{:} \\ \left[\begin{array}{cccccc} \delta_{\theta_k,1}\mathbf{I} & \mathbf{0} & \delta_{\theta_k,2}\mathbf{I} & \mathbf{0} & \dots & \delta_{\theta_k,N-1}\mathbf{I} \end{array}\right] \}m \end{array} ,$$

where $\delta_{\theta_k,i}$ denotes the Kronecker-delta. Please note that the stochastic matrices $\mathbf{H}_k$ and $\mathbf{J}_k$ are driven by the Markov chain which models the age of the control sequences buffered by the actuator.

Finally, introducing the augmented system state

$$\underline{\boldsymbol{\xi}}_k = \begin{bmatrix} \underline{\boldsymbol{x}}_k^\top & \underline{\eta}_k^\top \end{bmatrix}^\top$$

yields

$$\begin{aligned} \underline{\boldsymbol{\xi}}_{k+1} &= \begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{H}_k \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \underline{\boldsymbol{\xi}}_k + \begin{bmatrix} \mathbf{B}\mathbf{J}_k \\ \mathbf{G} \end{bmatrix} \underline{U}_k + \begin{bmatrix} \underline{\boldsymbol{w}}_k \\ \underline{0} \end{bmatrix} \\ &= \widehat{\mathbf{A}}_k \underline{\boldsymbol{\xi}}_k + \widehat{\mathbf{B}}_k \underline{U}_k + \underline{\widehat{\boldsymbol{w}}}_k \ . \end{aligned} \tag{7}$$

Using this state augmentation, we can express the cost function J_0 and the constraint functions g_s as

$$J_0 = \frac{1}{2}\mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_K^\top \widehat{\mathbf{Q}}_K \underline{\boldsymbol{\xi}}_K + \sum_{k=0}^{K-1} \left[\underline{\boldsymbol{\xi}}_k^\top \widehat{\mathbf{Q}}_k \underline{\boldsymbol{\xi}}_k + \underline{U}_k^\top \widehat{\mathbf{R}}_k \underline{U}_k \right] \middle| \mathcal{I}_0 \right\}$$

with

$$\begin{aligned} &\widehat{\mathbf{Q}}_K = \begin{bmatrix} \mathbf{Q}_K & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \widehat{\mathbf{Q}}_k = \begin{bmatrix} \mathbf{Q}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_k^\top \mathbf{R}_k \mathbf{H}_k \end{bmatrix}, \\ &\widehat{\mathbf{R}}_k = \mathbf{J}_k^\top \mathbf{R}_k \mathbf{J}_k \ , \end{aligned}$$

and

$$g_s = \mathrm{E}\left\{ (\underline{\hat{\boldsymbol{a}}}_K^s)^\top \underline{\boldsymbol{\xi}}_K + \sum_{k=0}^{K-1} \left[(\underline{\hat{\boldsymbol{a}}}_k^s)^\top \underline{\boldsymbol{\xi}}_k + (\underline{\hat{\boldsymbol{b}}}_k^s)^\top \underline{U}_k \right] \middle| \mathcal{I}_0 \right\}$$

with

$$\underline{\hat{\boldsymbol{a}}}_K^s = \begin{bmatrix} \underline{a}_K^s \\ \underline{0} \end{bmatrix}, \ \underline{\hat{\boldsymbol{a}}}_k^s = \begin{bmatrix} \underline{a}_k^s \\ \mathbf{H}_k^\top \underline{b}_k^s \end{bmatrix}, \ \underline{\hat{\boldsymbol{b}}}_k^s = \mathbf{J}_k^\top \underline{b}_k^s \ .$$

These results follow directly from inserting (6) into (2) and (3), and using definition (7).

Finally, we summarize the information available to the controller at time step k as the set

$$\mathcal{I}_k = \left\{ \underline{\overline{x}}_0, \mathbf{X}_0, \underline{U}_{0:k-1}, \mathcal{Z}_{1:k}, \theta_{0:k-1} \right\} \ .$$

Having reformulated the considered NCS by means of a MJLS, we solve the control problem in the next section.

Remark 1: The results on constrained control of MJLS as, e.g., presented in [14], [15] for quadratically constrained MJLS, are not applicable in the considered setting because the mode θ_k is available to the controller only with a delay and full observation of the state $\underline{\boldsymbol{x}}_k$ is not possible.

Remark 2: As has been demonstrated in [7], the separation of control and estimation, i.e., the absence of the dual effect, and thus the optimality of linear policies holds for the unconstrained version of sequence-based LGQ control due to the acknowledgments provided by the CA-link. These properties are also preserved for the optimization problem considered in this paper.

B. Derivation of the Control Law

Because the properties mentioned in Remark 2 hold for the problem considered in this paper, we can apply Lagrangian relaxation to solve (4) [11]. For this purpose, we introduce the vector of Lagrange multipliers $\underline{\lambda} \geq \underline{0}$ with an entry for each of the M constraints, i.e., $\underline{\lambda} \in \mathbb{R}^{M}_{0+}$. Consequently, according to the Karush-Kuhn-Tucker (KKT) conditions [16], the considered constrained control problem (4) can be converted into an unconstrained dual optimization problem given by

$$\begin{aligned}&\max_{\underline{\lambda}}\left[\min_{\underline{U}_{0:K-1}}\left[J_0+\underline{\lambda}^\top\underline{\gamma}-\underline{\lambda}^\top\underline{\zeta}\right]\right]\\&=\max_{\underline{\lambda}}\left[\min_{\underline{U}_{0:K-1}}\left[J_0+\underline{\lambda}^\top\underline{\gamma}\right]-\underline{\lambda}^\top\underline{\zeta}\right]\ ,\end{aligned}\tag{8}$$

where $\underline{\gamma}$ and $\underline{\zeta}$ denote the concatenated constraint functions g_s and the corresponding bounds c_s, $s \in \{1, \ldots, M\}$ with

$$\underline{\gamma}=\begin{bmatrix}g_1(\underline{\boldsymbol{x}}_{0:K},\underline{u}_{0:K-1})\\ \vdots\\ g_M(\underline{\boldsymbol{x}}_{0:K},\underline{u}_{0:K-1})\end{bmatrix}\ \text{and}\ \ \underline{\zeta}=\begin{bmatrix}c_1\\ \vdots\\ c_M\end{bmatrix}.$$

To find a solution to (8), we first perform the minimization w.r.t. $\underline{U}_{0:K-1}$ applying dynamic programming. For this purpose, we express the function $J_0 + \underline{\lambda}^\top\underline{\gamma}$ by means of a recursion according to

$$\begin{aligned}\mathcal{C}_K =&\mathrm{E}\left\{\frac{1}{2}\underline{\boldsymbol{\xi}}_K^\top\widehat{\mathbf{Q}}_K\underline{\boldsymbol{\xi}}_K+\underline{\lambda}^\top\widetilde{\mathbf{Q}}_K^\top\underline{\boldsymbol{\xi}}_K\middle|\mathcal{I}_K\right\}\ ,\\ \mathcal{C}_k =&\mathrm{E}\left\{\frac{1}{2}\left(\underline{\boldsymbol{\xi}}_k^\top\widehat{\mathbf{Q}}_k\underline{\boldsymbol{\xi}}_k+\underline{U}_k^\top\widehat{\mathbf{R}}_k\underline{U}_k\right)\right.\\ &\left.+\underline{\lambda}^\top\widetilde{\mathbf{Q}}_k^\top\underline{\boldsymbol{\xi}}_k+\underline{\lambda}^\top\widetilde{\mathbf{R}}_k^\top\underline{U}_k+\mathcal{C}_{k+1}\middle|\mathcal{I}_k\right\}\ ,\end{aligned}$$

with

$$\widetilde{\mathbf{Q}}_k=\begin{bmatrix}\underline{\hat{\boldsymbol{a}}}_k^1 & \ldots & \underline{\hat{\boldsymbol{a}}}_k^M\end{bmatrix}\ \text{and}\ \widetilde{\mathbf{R}}_k=\begin{bmatrix}\underline{\hat{\boldsymbol{b}}}_k^1 & \ldots & \underline{\hat{\boldsymbol{b}}}_k^M\end{bmatrix}\ .$$

The terms $\mathcal{C}_k$ are often referred to as costs-to-go. The optimal solution of

$$\min_{\underline{U}_{0:K-1}} J_0+\underline{\lambda}^\top\underline{\gamma}$$

corresponds to minimal costs-to-go $\mathcal{C}_0^*$ at time step 0.

According to dynamic programming algorithm, we begin at time step K. The minimal costs-to-go are

$$\mathcal{C}_K^* = \mathcal{C}_K\ .$$

After substituting $\underline{\boldsymbol{\xi}}_K$ in $\mathcal{C}_K^*$ using (7) and a manipulation, the optimal costs-to-go at time step $K-1$ are given by

$$\begin{aligned}\mathcal{C}_{K-1}^* =&\min_{\underline{U}_{K-1}}\mathrm{E}\left\{\frac{1}{2}\underline{\boldsymbol{\xi}}_{K-1}^\top\left(\widehat{\mathbf{Q}}_{K-1}+\widehat{\mathbf{A}}_{K-1}^\top\widehat{\mathbf{Q}}_K\widehat{\mathbf{A}}_{K-1}\right)\underline{\boldsymbol{\xi}}_{K-1}\right.\\ &+\underline{\lambda}^\top\left(\widetilde{\mathbf{Q}}_K^\top\widehat{\mathbf{A}}_{K-1}+\widetilde{\mathbf{Q}}_{K-1}^\top\right)\underline{\boldsymbol{\xi}}_{K-1}\\ &+\frac{1}{2}\underline{U}_{K-1}^\top\left(\widehat{\mathbf{R}}_{K-1}+\widehat{\mathbf{B}}_{K-1}^\top\widehat{\mathbf{Q}}_K\widehat{\mathbf{B}}_{K-1}\right)\underline{U}_{K-1}\\ &+\underline{\boldsymbol{\xi}}_{K-1}^\top\left(\widehat{\mathbf{A}}_{K-1}^\top\widehat{\mathbf{Q}}_K\widehat{\mathbf{B}}_{K-1}\right)\underline{U}_{K-1}\\ &\left.+\underline{\lambda}^\top\left(\widetilde{\mathbf{Q}}_K^\top\widehat{\mathbf{B}}_{K-1}+\widetilde{\mathbf{R}}_{K-1}^\top\right)\underline{U}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\\ &+\mathrm{E}\left\{\underline{\widehat{\boldsymbol{w}}}_{K-1}^\top\widehat{\mathbf{Q}}_K\underline{\widehat{\boldsymbol{w}}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\ .\end{aligned}\tag{9}$$

Differentiation of (9) w.r.t. $\underline{U}_{K-1}$ and setting the result to zero yields

$$\begin{aligned}\underline{U}_{K-1}=&-\left(\mathrm{E}\left\{\widehat{\mathbf{R}}_{K-1}+\widehat{\mathbf{B}}_{K-1}^\top\widehat{\mathbf{Q}}_K\widehat{\mathbf{B}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\right)^\dagger\\ &\times\left[\mathrm{E}\left\{\widehat{\mathbf{B}}_{K-1}^\top\widehat{\mathbf{Q}}_K\widehat{\mathbf{A}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\right.\\ &\left.+\mathrm{E}\left\{\widehat{\mathbf{B}}_{K-1}^\top\widetilde{\mathbf{R}}_K+\widetilde{\mathbf{R}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\underline{\lambda}\right].\end{aligned}$$

The pseudoinverse is used instead of the normal inverse because the entries of the control sequences past the end of the optimization horizon are not defined and thus the augmented matrices may become singular. For a more thorough discussion of this issue see [7]. Please note that $\underline{U}_{K-1}$ is a function of the estimate $\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}$, the mode θ_{K-2}, and the Lagrange multipliers $\underline{\lambda}$, i.e., the control sequence $\underline{U}_{K-1}$ is a policy.

Consequently, the minimum of (9) is given by

$$\begin{aligned}\mathcal{C}_{K-1}^*=&\frac{1}{2}\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_{K-1}^\top\mathbf{P}_{K-1}\underline{\boldsymbol{\xi}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\\ &+\underline{\lambda}^\top\mathrm{E}\left\{\widetilde{\mathbf{P}}_{K-1}^\top\underline{\boldsymbol{\xi}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}+\frac{1}{2}\underline{\lambda}^\top\mathbf{S}_{K-1}\underline{\lambda}\\ &+\frac{1}{2}\mathrm{E}\left\{\underline{\boldsymbol{\varepsilon}}_{K-1}^\top\overline{\mathbf{P}}_{K-1}\underline{\boldsymbol{\varepsilon}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\\ &+\frac{1}{2}\mathrm{E}\left\{\underline{\widehat{\boldsymbol{w}}}_{K-1}^\top\widehat{\mathbf{Q}}_K\underline{\widehat{\boldsymbol{w}}}_{K-1}\middle|\mathcal{I}_{K-1}\right\}\ ,\end{aligned}$$

with the estimation error

$$\underline{\boldsymbol{\varepsilon}}_k=\underline{\boldsymbol{\xi}}_k-\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k\middle|\mathcal{I}_k\right\}$$

and the matrices

$$\begin{aligned}&\mathbf{P}_K=\widehat{\mathbf{Q}}_K,\ \overline{\mathbf{P}}_K=\mathbf{0},\ \widetilde{\mathbf{P}}_K=\widetilde{\mathbf{Q}}_K,\ \mathbf{S}_K=\mathbf{0},\\ &\overline{\mathbf{P}}_k=\mathrm{E}\left\{\widehat{\mathbf{A}}_k^\top\mathbf{P}_{k+1}\widehat{\mathbf{B}}_k\middle|\mathcal{I}_k\right\}\left(\mathrm{E}\left\{\widehat{\mathbf{R}}_k+\widehat{\mathbf{B}}_k^\top\mathbf{P}_{k+1}\widehat{\mathbf{B}}_k\middle|\mathcal{I}_k\right\}\right)^\dagger\\ &\qquad\times\mathrm{E}\left\{\widehat{\mathbf{B}}_k^\top\mathbf{P}_{k+1}\widehat{\mathbf{A}}_k\middle|\mathcal{I}_k\right\},\\ &\mathbf{P}_k=\mathrm{E}\left\{\widehat{\mathbf{Q}}_k+\widehat{\mathbf{A}}_k^\top\mathbf{P}_{k+1}\widehat{\mathbf{A}}_k\middle|\mathcal{I}_k\right\}-\overline{\mathbf{P}}_k,\\ &\widetilde{\mathbf{P}}_k=\mathrm{E}\left\{\widehat{\mathbf{A}}_k^\top\widetilde{\mathbf{P}}_{k+1}+\widetilde{\mathbf{Q}}_k\middle|\mathcal{I}_k\right\}-\mathrm{E}\left\{\widehat{\mathbf{A}}_k^\top\mathbf{P}_{k+1}\widehat{\mathbf{B}}_k\middle|\mathcal{I}_k\right\}\\ &\qquad\times\left(\mathrm{E}\left\{\widehat{\mathbf{R}}_k+\widehat{\mathbf{B}}_k^\top\mathbf{P}_{k+1}\widehat{\mathbf{B}}_k\middle|\mathcal{I}_k\right\}\right)^\dagger\mathrm{E}\left\{\widehat{\mathbf{B}}_k^\top\widetilde{\mathbf{P}}_{k+1}+\widetilde{\mathbf{R}}_k\middle|\mathcal{I}_k\right\},\end{aligned}$$

$$\mathbf{S}_k = \mathrm{E}\left\{\mathbf{S}_{k+1} \middle| \mathcal{I}_k\right\} - \mathrm{E}\left\{\widetilde{\mathbf{P}}_{k+1}^\top \widehat{\mathbf{B}}_k + \widetilde{\mathbf{R}}_k^\top \middle| \mathcal{I}_k\right\} \times \left(\mathrm{E}\left\{\widehat{\mathbf{R}}_k + \widehat{\mathbf{B}}_k^\top \mathbf{P}_{k+1} \widehat{\mathbf{B}}_k \middle| \mathcal{I}_k\right\}\right)^\dagger \mathrm{E}\left\{\widehat{\mathbf{B}}_k^\top \widetilde{\mathbf{P}}_{k+1} + \widetilde{\mathbf{R}}_k \middle| \mathcal{I}_k\right\}. \tag{10}$$

Applying dynamic programming up to time step $k = 0$ yields

$$\begin{aligned} \mathcal{C}_0^* &= \min_{\underline{U}_{0:K-1}} J_0 + \underline{\lambda}^\top \underline{\gamma} \\ &= \frac{1}{2} \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0^\top \mathbf{P}_0 \underline{\boldsymbol{\xi}}_0 \middle| \mathcal{I}_0\right\} + \underline{\lambda}^\top \mathrm{E}\left\{\widetilde{\mathbf{P}}_0^\top \middle| \mathcal{I}_0\right\} \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0 \middle| \mathcal{I}_0\right\} \\ &+ \frac{1}{2} \sum_{k=0}^{K-1} \mathrm{E}\left\{\underline{\boldsymbol{\varepsilon}}_k^\top \overline{\mathbf{P}}_k \underline{\boldsymbol{\varepsilon}}_k \middle| \mathcal{I}_0\right\} + \frac{1}{2} \underline{\lambda}^\top \mathbf{S}_0 \underline{\lambda} \\ &+ \frac{1}{2} \sum_{k=0}^{K-1} \mathrm{E}\left\{\underline{\widehat{\boldsymbol{w}}}_k^\top \mathbf{P}_{k+1} \underline{\widehat{\boldsymbol{w}}}_k \middle| \mathcal{I}_k\right\}. \end{aligned} \tag{11}$$

With these results, the unconstrained problem (8) reduces to

$$\begin{aligned} &\max_{\underline{\lambda}} \Bigg[\frac{1}{2} \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0^\top \mathbf{P}_0 \underline{\boldsymbol{\xi}}_0 \middle| \mathcal{I}_0\right\} + \underline{\lambda}^\top \mathrm{E}\left\{\widetilde{\mathbf{P}}_0^\top \middle| \mathcal{I}_0\right\} \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0 \middle| \mathcal{I}_0\right\} \\ &\quad + \frac{1}{2} \sum_{k=0}^{K-1} \mathrm{E}\left\{\underline{\boldsymbol{\varepsilon}}_k^\top \overline{\mathbf{P}}_k \underline{\boldsymbol{\varepsilon}}_k \middle| \mathcal{I}_0\right\} + \frac{1}{2} \underline{\lambda}^\top \mathbf{S}_0 \underline{\lambda} - \underline{\lambda}^\top \underline{\zeta} \\ &\quad + \frac{1}{2} \sum_{k=0}^{K-1} \mathrm{E}\left\{\underline{\widehat{\boldsymbol{w}}}_k^\top \mathbf{P}_{k+1} \underline{\widehat{\boldsymbol{w}}}_k \middle| \mathcal{I}_k\right\} \Bigg] \\ &= \max_{\underline{\lambda}} \left[\frac{1}{2} \underline{\lambda}^\top \mathbf{S}_0 \underline{\lambda} + \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0^\top \middle| \mathcal{I}_0\right\} \mathrm{E}\left\{\widetilde{\mathbf{P}}_0 \middle| \mathcal{I}_0\right\} \underline{\lambda} - \underline{\zeta}^\top \underline{\lambda} \right] \\ &\quad + \frac{1}{2} \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0^\top \mathbf{P}_0 \underline{\boldsymbol{\xi}}_0 \middle| \mathcal{I}_0\right\} + \frac{1}{2} \sum_{k=0}^{K-1} \mathrm{E}\left\{\underline{\boldsymbol{\varepsilon}}_k^\top \overline{\mathbf{P}}_k \underline{\boldsymbol{\varepsilon}}_k \middle| \mathcal{I}_0\right\} \\ &\quad + \frac{1}{2} \sum_{k=0}^{K-1} \mathrm{E}\left\{\underline{\widehat{\boldsymbol{w}}}_k^\top \mathbf{P}_{k+1} \underline{\widehat{\boldsymbol{w}}}_k \middle| \mathcal{I}_k\right\}. \end{aligned}$$

This quadratic optimization problem with the constraint $\underline{\lambda} \geq \underline{0}$ can be solved numerically using, e.g., interior point algorithms. The number of decision variables corresponds to the number of constraints.

Having computed the Lagrange multipliers, we obtain the control law

$$\underline{U}_k = \underline{U}_k^{fb} + \underline{U}_k^{ff},$$

with the feedback term $\underline{U}_k^{fb}$ given by

$$\underline{U}_k^{fb} = -\left(\mathrm{E}\left\{\widehat{\mathbf{R}}_k + \widehat{\mathbf{B}}_k^\top \mathbf{P}_{k+1} \widehat{\mathbf{B}}_k \middle| \mathcal{I}_k\right\}\right)^\dagger \times \mathrm{E}\left\{\widehat{\mathbf{B}}_k^\top \mathbf{P}_{k+1} \widehat{\mathbf{A}}_k \middle| \mathcal{I}_k\right\} \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k \middle| \mathcal{I}_k\right\}, \tag{12}$$

which depends linearly on the minimum mean square estimate (MMSE) $\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k \middle| \mathcal{I}_k\right\}$ of the augmented system state and the mode θ_{k-1}. The feedforward term $\underline{U}_k^{ff}$ is given by

$$\underline{U}_k^{ff} = -\left(\mathrm{E}\left\{\widehat{\mathbf{R}}_k + \widehat{\mathbf{B}}_k^\top \mathbf{P}_{k+1} \widehat{\mathbf{B}}_k \middle| \mathcal{I}_k\right\}\right)^\dagger \times \mathrm{E}\left\{\widehat{\mathbf{B}}_k^\top \widetilde{\mathbf{P}}_{k+1} + \widetilde{\mathbf{R}}_k \middle| \mathcal{I}_k\right\} \underline{\lambda} \tag{13}$$

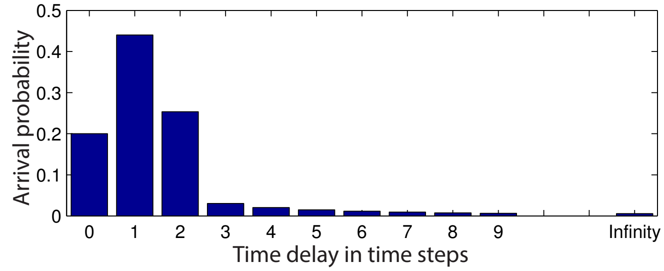

Fig. 2. Delay pdf of the networks used in the CA- and the SC-link.

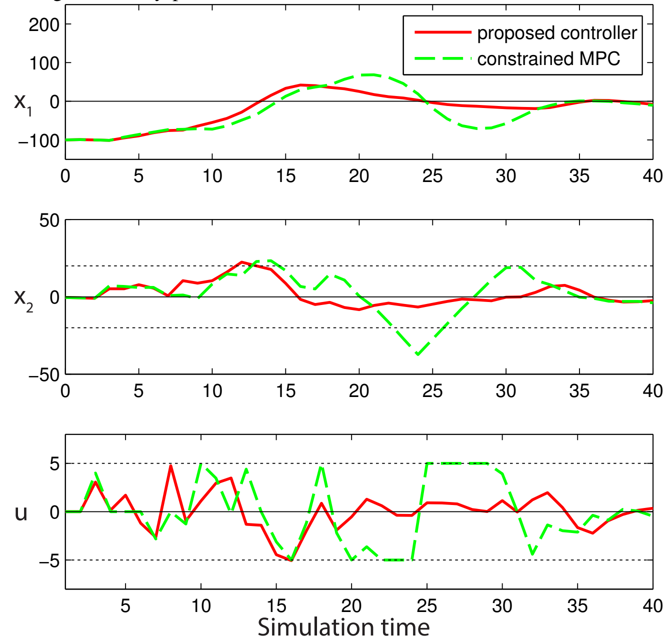

Fig. 3. Example run with control sequence length $N = 6$.

and depends on the initial system state $\overline{\underline{x}}_0$, the constraints $g_s(\underline{\boldsymbol{x}}_{0:K}, \underline{\boldsymbol{u}}_{0:K-1})$, and the mode θ_{k-1}. The estimation of the augmented system state corresponds to the estimation of the state $\underline{\boldsymbol{x}}_k$ of the initial system because other entries of $\underline{\boldsymbol{\xi}}_k$, i.e., $\underline{\eta}_k$, contain previously transmitted control sequences $\underline{U}_{k-N+1}, \ldots, \underline{U}_{k-1}$ that are known to the controller. Estimators that can be used to obtain a state estimate from the measurements transmitted by the sensor can be found in, e.g., [17], [18]. The computation of the expected values in (10), (12), and (13) is described in the appendix.

It is worth noting that the derived control law is closed-loop optimal. Furthermore, the control law can be precomputed offline. If the constraints or the initial state $\overline{\underline{x}}_0$ change during runtime, only the convex quadratic program

$$\max_{\underline{\lambda}} \left[\frac{1}{2} \underline{\lambda}^\top \mathbf{S}_0 \underline{\lambda} + \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_0^\top \middle| \mathcal{I}_0\right\} \mathrm{E}\left\{\widetilde{\mathbf{P}}_0 \middle| \mathcal{I}_0\right\} \underline{\lambda} - \underline{\zeta}^\top \underline{\lambda} \right]$$

has to be recomputed to obtain the Lagrange multiplier $\underline{\lambda}$. The feedback term (12) remains unchanged.

The implementation of the proposed controller is available at the CloudRunner project website [19].

IV. Evaluation

To evaluate the presented controller, we compared it with a model predictive control law adapted from [3]. The simulated plant was a double integrator with parameters

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ \mathbf{V} = 0.1^2,$$
$$\mathbf{W} = \begin{bmatrix} 0.2^2 & 0 \\ 0 & 0.2^2 \end{bmatrix}, \ \mathbf{Q}_k = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \ \mathbf{R}_k = 1,$$
$$\underline{\overline{x}}_0 = \begin{bmatrix} -100 \\ 0 \end{bmatrix}, \ \mathbf{X}_0 = \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix}.$$

The delay pdf employed in the CA-link as well as in the SC-link is depicted in Fig. 2. As an estimator, we implemented the filter proposed in [17] with buffer length $N^{buf} = 4$. The optimization horizon was chosen to $K = 40$. The constraints were set to

$$-20 \leq \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \mathrm{E}\left\{\underline{\boldsymbol{x}}_k\right\} \leq 20 \quad \text{and} \quad -5 < \underline{u}_k < 5 \ ,$$

for the MPC controller. These constraints can be converted into 160 integral constraints for the proposed controller with

$$\begin{aligned}
\underline{a}_i^i &= \begin{bmatrix} 0 & 1 \end{bmatrix}^\top, \ \text{for } i \leq 40, \\
\underline{a}_{i-40}^i &= \begin{bmatrix} 0 & -1 \end{bmatrix}^\top, \ \text{for } 41 \leq i \leq 80, \\
\underline{a}_k^i &= \begin{bmatrix} 0 & 0 \end{bmatrix}^\top, \ \text{otherwise}, \\
\underline{b}_{i-80}^i &= 1, \ \text{for } 81 \leq i \leq 120, \\
\underline{b}_{i-120}^i &= -1, \ \text{for } 121 \leq i \leq 160, \\
\underline{b}_k^i &= 0, \ \text{otherwise}, \\
c_i &= \begin{cases} 20, & \text{for } i \leq 80, \\ 5, & \text{for } 81 \leq i \leq 160. \end{cases}
\end{aligned}$$

Fig. 3 demonstrates an example run with control sequence length $N = 6$. Depicted are the trajectories of the state $\underline{\boldsymbol{x}}_k$ as well as the control inputs applied to the plant. It can be seen that both controllers are able to maintain input constraints. However, constraints on the second entry of the state are violated by the proposed controller at time step $k = 12$ and by the MPC controller for $k = \{13, 14, 23, 24, 25\}$.

Figs. 4, 5, and 6 demonstrate the evaluation results for the Monte-Carlo simulation with 1000 runs per simulated control sequence length. Fig. 4 depicts the average cumulated LQG costs for control sequence lengths $N = [1; 10]$. The costs of both controllers converge from control sequence length $N = 4$ on. The costs of the proposed controller are approx. 40% lower than the costs of the MPC controller. Fig. 5 demonstrates the average magnitude and Fig. 6 the average number of constraint violations over control sequence lengths $N = [1; 10]$. These quantities also converge from $N = 4$. The average constraint violation magnitude of the proposed controller is approximately half the violation magnitude of the MPC controller. Also, the proposed controller conducts approx. 35% less constraint violations than the MPC controller. These results allow to conclude that in the considered scenario, the proposed controller outperforms the model predictive control law adapted from [3].

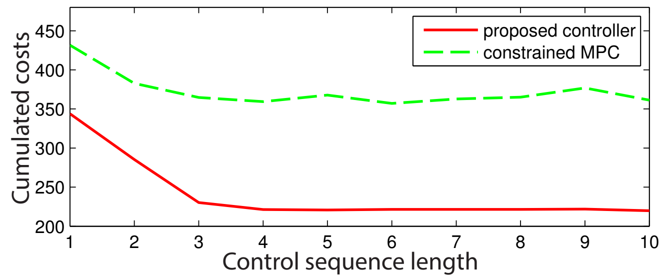

Fig. 4. Average cumulated costs over control sequence length.

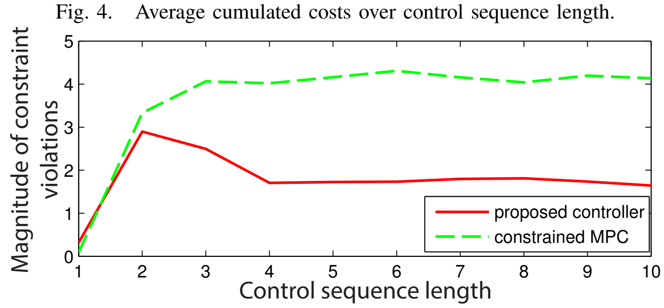

Fig. 5. Average magnitude of constraint violations over control sequence length.

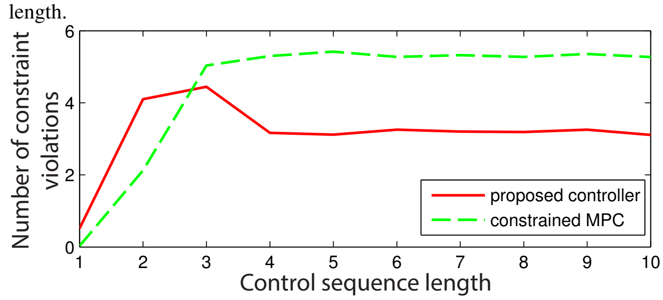

Fig. 6. Average number of constraint violations over control sequence length.

V. Conclusion

In this work, we considered control of stochastic linear systems subject to linear integral state and input constraints over networks with stochastic packet delays and losses. The derived control law is closed-loop optimal and possesses the advantage that it can be precomputed offline. The control law consists of a dynamic feedback term given in closed form and a static feedforward term that requires a numerical solution of a quadratic program. The number of decision variables corresponds to the number of constraints. In the simulated scenario, the presented control law was able to outperform the model predictive controller adapted from [3].

Future work will consist in deriving feasibility and stability conditions from system parameters, and considering quadratic expectation-type integral constraints and generalizing the control method to linear chance constraints.

Appendix

The expected values of the matrices in (10), (12), and (13) can be precomputed offline by conditioning the expectations at time step k on the Markov mode $\theta_{k-1} = i$, $i \in \{0, \ldots, M\}$. This is possible because at time step k this mode will be part of the information set $\mathcal{I}_k$ available to the controller.

With $\mathbf{X}_{t|k}^{(i)}$ denoting the realization of the matrix $\mathbf{X}_t$ conditioned on the mode $\theta_k = i$, we obtain

$$\begin{aligned}\underline{U}_{k|k-1}^{(j)} = &-\left(\sum_{i=0}^{N} p_{ji}\left[\widehat{\mathbf{R}}_{k|k}^{(i)} + \big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{B}}_{k|k}^{(i)}\right]\right)^{\dagger}\\ &\times\left[\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{A}}_{k|k}^{(i)}\right]\right)\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k\middle|\mathcal{I}_k\right\}\right.\\ &\left.+\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\widetilde{\mathbf{P}}_{k+1|k}^{(i)} + \widetilde{\mathbf{R}}_{k|k}^{(i)}\right]\right)\underline{\lambda}\right]\end{aligned}$$

with

$$\begin{aligned}\mathbf{P}_{k|k-1}^{(j)} = &\left(\sum_{i=0}^{N} p_{ji}\left[\widehat{\mathbf{Q}}_{k|k}^{(i)} + \big(\widehat{\mathbf{A}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{A}}_{k|k}^{(i)}\right]\right)\\ &-\overline{\mathbf{P}}_{k|k-1}^{(j)},\\ \overline{\mathbf{P}}_{k|k-1}^{(j)} = &\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{A}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{B}}_{k|k}^{(i)}\right]\right)\\ &\times\left(\sum_{i=0}^{N} p_{ji}\left[\widehat{\mathbf{R}}_{k|k}^{(i)} + \big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{B}}_{k|k}^{(i)}\right]\right)^{\dagger}\\ &\times\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{A}}_{k|k}^{(i)}\right]\right),\end{aligned}$$

and

$$\begin{aligned}\widetilde{\mathbf{P}}_{k|k-1}^{(j)} = &\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{A}}_{k|k}^{(i)}\big)^{\top}\widetilde{\mathbf{P}}_{k+1|k}^{(i)} + \widetilde{\mathbf{Q}}_{k|k}^{(i)}\right]\right)\\ &-\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{A}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{B}}_{k|k}^{(i)}\right]\right)\\ &\times\left(\sum_{i=0}^{N} p_{ji}\left[\widehat{\mathbf{R}}_{k|k}^{(i)} + \big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{B}}_{k|k}^{(i)}\right]\right)^{\dagger}\\ &\times\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\widetilde{\mathbf{P}}_{k+1|k}^{(i)} + \widetilde{\mathbf{R}}_{k|k}^{(i)}\right]\right)\end{aligned}$$

$$\begin{aligned}\mathbf{S}_{k|k-1}^{(j)} = &\left(\sum_{i=0}^{N} p_{ji}\mathbf{S}_{k+1|k}^{(i)}\right)\\ &-\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widetilde{\mathbf{P}}_{k+1|k}^{(i)}\big)^{\top}\widehat{\mathbf{B}}_{k|k}^{(i)} + \big(\widetilde{\mathbf{R}}_{k|k}^{(i)}\big)^{\top}\right]\right)\\ &\times\left(\sum_{i=0}^{N} p_{ji}\left[\widehat{\mathbf{R}}_{k|k}^{(i)} + \big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{B}}_{k|k}^{(i)}\right]\right)^{\dagger}\\ &\times\left(\sum_{i=0}^{N} p_{ji}\left[\big(\widehat{\mathbf{B}}_{k|k}^{(i)}\big)^{\top}\mathbf{P}_{k+1|k}^{(i)}\widehat{\mathbf{A}}_{k|k}^{(i)}\right]\right).\end{aligned}$$

VI. ACKNOWLEDGMENTS

This work was supported by the German Science Foundation (DFG) within the Research Training Group RTG 1194 “Self-organizing Sensor-Actuator-Networks”.

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