Nominal Bonds, Real Bonds, and Equity[*]

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Abstract

We decompose the term structure of expected equity returns into (1) the real short rate, (2) a premium for holding real long-term bonds, or the real duration premium, the excess returns of nominal long-term bonds over real bonds which reflects (3) expected inflation and (4) inflation risk, and (5) a real cashflow risk premium, which is the excess return of equity over nominal bonds. The shape of the nominal and real bond yield curves are upward sloping due to increasing duration and inflation risk premiums. The term structures of expected equity returns and equity risk premiums, in contrast, are downward sloping due to the decreasing effect of short-term expected inflation, or trend inflation, across horizons. Around 70% of the variation of expected equity returns at the 10-year horizon is due to variation in the output gap and trend inflation.

1 Introduction

While recent research has made considerable progress in understanding the term structure of nominal Treasury yields and real TIPS yields, the term structure of expected equity returns and their macroeconomic relation to the nominal and real yield curves is less well understood.[1] This is surprising because the difference between nominal bond and real yields, or inflation compensation, is the sum of expected inflation and the inflation risk premium and the equity price-dividend ratio is the expected present value of future real dividend growth discounted using risk-adjusted real bond yields. Thus, macroeconomic factors that are known to drive the nominal and real term structures should potentially also contain information about expected equity returns and equity risk premiums.

   We build a model that prices nominal bonds, real bonds, and equity in a unified framework and examine their combined term structures. We decompose the term structure of expected equity returns into (1) the real short rate, (2) a real duration premium for holding long-term real bonds, (3) expected inflation, (4) the inflation risk premium, where (3) and (4) are reflected in long-term nominal bonds, and (5) a real cashflow risk premium, which is the expected equity return in excess of a long-term nominal bond yield.[2] Each of these components have their own term structures. Using the model, we show how variations in economic growth, inflation, monetary policy, and real dividend growth affects each of these risk premium components over time and across holding periods.

   Pricing equity requires discounting future real cashflows using real interest rates and real risk premiums. While a complete term structure of nominal bonds is available over long periods, only long-term real bond prices are observed in data in the most recent sample. Real short rates and real risk premiums needed to discount future real cashflows are empirically unobserved

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[1] The modeling of equity and bonds has largely developed separately but has made considerable progress over the last two decades. For reduced-form bond models with latent and macro factors, see, among many others, Duffie and Kan (1996), Dai and Singleton (2000, 2002, 2003), and Ang and Piazzesi (2003). Equilibrium bond pricing models are developed by Cox, Ingersoll and Ross (1985), Buraschi and Jiltsov (2005), Wachter (2006), Piazzesi and Schneider (2006), Ulrich (2010, 2011a), and Bansal and Shaliastovich (2010), among others. Reduced-form equity valuation models are derived by Fama and French (1988), Ang and Liu (2001), Ang and Bekaert (2007), Lettau and Ludvigson (2005), Lettau and Wachter (2007), among others. A large equilibrium equity-pricing literature includes Mehra and Prescott (1987), Bansal and Yaron (2004), Campbell and Cochrane (1999), Menzly, Santos and Veronesi (2004), and Lettau, Ludvigson and Wachter (2008), among others.

[2] Our breakdown is similar to Ibbotson and Chen (2003), but unlike Ibbotson and Chen (2003) they are consistently derived in one overall model.

(see also comments by Lettau and Wachter (2010)). We endogenize the unobserved real pricing kernel by using the rich available data for nominal and real bonds, realized inflation, together with a specification for inflation risk premiums.

We first model nominal bonds by building on a large macro-term structure literature. We follow Taylor (1993) and assume the Federal Reserve (Fed) sets the Fed Funds rate as a function of the output gap and trend inflation (or short-term expected inflation) as well as monetary policy shocks. Through no arbitrage, nominal yields are risk-adjusted expectations about future Fed Funds rates and reflect macro risk premiums. After defining the nominal pricing kernel, inflation, and inflation risk premiums, the model endogenously generates the real pricing kernel. The real short rate depends on the same macro variables that enter the nominal Taylor rule but has different loadings which reflect the covariance of inflation shocks with the macro variables and inflation risk. While the majority of affine term structure models rely on latent factors extracted from yields to obtain a close fit to data (see, for example, the summary by Piazzesi (2010)), we are able to price nominal and real bonds with only macro variables – the output gap, trend inflation, and monetary policy shocks.[3]

Equity is a bond perpetuity with stochastic real dividend cashflows. We price the real dividend stream using the implied real pricing kernel and assume the cashflow shock is priced, which gives rise to an equity cashflow risk premium. Since real dividend growth depends on the macro variables, the same factors that drive nominal and real bond yields also affect equity prices and equity risk premiums. As different macro variables have different degrees of persistence, expected equity returns contain temporary and near-permanent components, as Alvarez and Jermann (2005), Bansal and Yaron (2004), Hansen, Heaton and Li (2008), and other authors show play important roles in the dynamics of equity risk premiums.

We find the term structures of nominal and real bonds are upward sloping, on average, while the term structure of total expected equity returns and expected equity risk premiums, are generally downward sloping. Equity, therefore, is less risky with horizon. Long-term real bonds pay a positive duration premium which is mainly driven by monetary policy shocks, whereas long-term nominal bond yields pay a positive inflation premium that is mainly driven by shocks to trend inflation.[4] The downward-sloping term structure of expected equity returns is consistent with Lettau and Wachter (2010), Binsbergen, Brandt and Koijen (2011), Binsbergen

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[3] Ulrich (2011a) also prices real and nominal bonds using only observable factors in an equilibrium model, but does not price equity.

[4] Ang and Piazzesi (2003), Buraschi and Jiltsov (2005), Piazzesi and Schneider (2006), Ulrich (2010), and Joslin, Priebsch and Singleton (2010), among many others, find evidence of a positive risk premium.

et al. (2011), but this literature does not identify the macro determinants of the term structure of equity risk premiums.[5] The decreasing effect of trend inflation as horizon increases explains the downward-sloping term structures of the real cashflow risk premium and the total expected equity return. In the long run, equity is a real security.

Our model exactly fits the very high correlation between dividend yields and 10-year nominal bond yields, which is 0.87 in our sample. This important stylized fact is labeled the "Fed model" and it is puzzling because bond yields are driven largely by inflation compensation, but equity premiums are a real concept (see Bekaert and Engstrom (2010)). Increases in trend inflation lead to increases in Fed Funds rates, according to the Taylor policy rule, and hence higher real and nominal discount rates. At the same time, increases in trend inflation signal bad times ahead for future expected real cashflows. Expected real cashflows fall, while at the same time the real cashflow premium increases. Both effects lower equity valuations and increase dividend yields. This causes dividend yields to strongly comove with nominal bond yields.

Our model falls into a growing literature that jointly prices equities and bonds. Recent papers in this literature include Bekaert, Engstrom and Xing (2009), Baele, Bekaert and Inghelbrecht (2010), Bekaert and Engstrom (2010), Bekaert, Engstrom and Grenadier (2010), Lettau and Wachter (2010), and Koijen, Lustig and Van Nieuwerburgh (2011). None of these papers start with fundamental macro drivers of nominal and real yield curves, as argued by Taylor-style policy rules of Fed actions. In many of these papers, the drivers of the real short rate and real risk premiums are entirely latent, while in our model they are observable. Importantly, we explain how this underlying macro risk can account for upward-sloping nominal and real bond curves, but downward-sloping equity risk premiums. The methodology of our paper is most similar to Lemke and Werner (2009), who also work in a no-arbitrage, affine model and price bonds and equity. The most important differences are that we endogenize the real pricing kernel and work with only observable macro factors. Lemke and Werner specify latent real interest rate factors and exogenously specify the dividend yield as a latent factor, rather than pricing the dividend yield consistently with nominal and real bonds as we do.

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[5] Downward-sloping equity premiums contradict the theoretical models of Campbell and Cochrane (1999), Bansal and Yaron (2004), and Gabaix (2009), as explained in Binsbergen et al. (2011). Croce, Lettau and Ludvigson (2009) show that a long-run risk model with investors who cannot distinguish between short-term and long-term shocks can explain the downward-sloping equity premium.

2 Model

We build the model in stages starting from nominal bonds, progressing to real bonds, and then to equity. This progression is natural and we motivate it as follows. First, the dynamics of nominal bonds reflect economic growth, inflation dynamics, and the actions of monetary policy, as shown by a large macro-finance term structure literature beginning with Ang and Piazzesi (2003). Federal Reserve (Fed) interventions in the Fed Funds market are well described by a Taylor (1993) policy rule, where the Fed Funds rate is a function of economic growth, inflation, and monetary actions. The Taylor rule is pervasively used as both a descriptive and prescriptive tool for monetary policy (see, for example, Asso, Kahn and Leeson (2010)). Through no arbitrage, policy actions on the Fed Funds rate are reflected at all maturities in the term structure of nominal bond yields. Since the payoffs of nominal bonds are fixed in nominal terms, however, the nominal yield curve does not characterize the risk of stochastic real cashflows–which are needed to price equity.

Equity is a real claim, not in the sense that it always moves one-to-one with inflation, but it represents ownership of physical plant and property, and is a claim to a stream of production activities generated by firms. To derive real discount rates, we need to characterize the term structure of real bonds. This is done by specifying the dynamics of inflation and inflation risk, which allows us to link the nominal and real term structures. Note that the real short rate needed to discount real cashflows is unobserved in data, but it is implied by our model given the Taylor rule for the nominal short rate, inflation, and the prices of risk of macro factors. Using the real discount rate curve, we can price equity by specifying the perpetuity of real dividend cashflows and real dividend risk.

2.1 Nominal Short Rates

Following Taylor (1993), Clarida, Galí and Gertler (2000), and others, we specify that the Fed sets the Fed Funds rate, $r_t^\$$, as a linear function of the current output gap, inflation expectations, and a monetary policy shock:

$$r_t^\$ = c + ag_t + b\pi_t^e + f_t, \tag{1}$$

where g_t is the output gap, π_t^e is a measure of inflation expectations, and f_t is a monetary policy shock. Following Cogley and Sbordone (2006), Ascari and Ropele (2007), Coibin and Gorodnichenko (2011), and others, we refer to π_t^e as trend inflation to contrast it with expected inflation over multiple periods. The loadings a and b represent the constant response of the Fed

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to changes in the output gap and trend inflation, respectively. In our empirical work, we demean our state variables, so the constant c coincides with the mean of the Fed Funds rate in data.

We collect the factors in the vector $X_t^o = (g_t \, \pi_t^e \, f_t)'$, where the superscript "o" denotes that these state variables are observable. Thus, we can express the policy rule of the Fed as

$$r_t^\$ = \delta_0^\$ + \delta_1^{\$'} X_t^o,$$

where $\delta_0^\$ = c$ and $\delta_1^\$ = (a \, b \, 1)'$.

The state vector evolves as a VAR(1):

$$X_t^o = \mu + \Phi X_{t-1}^o + \Sigma \varepsilon_t, \tag{2}$$

where the residuals $\varepsilon_t \sim$ i.i.d. $N(0, I)$ and the companion form, Φ, and conditional covariance, $\Sigma \Sigma'$, are given by

$$\Phi = \begin{pmatrix} \Phi_{gg} & \Phi_{g\pi^e} & 0 \\ \Phi_{\pi^e g} & \Phi_{\pi^e \pi^e} & 0 \\ 0 & 0 & \Phi_{ff} \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \Sigma_{gg} & 0 & 0 \\ \Sigma_{\pi^e g} & \Sigma_{\pi^e \pi^e} & 0 \\ 0 & 0 & \Sigma_{ff} \end{pmatrix}.$$

In this specification, we assume that the monetary policy shocks, f_t, are orthogonal to the macro variables, g_t and π_t^e, similar to Ang and Piazzesi (2003). Econometrically, f_t is the residual of the Fed Funds rate after controlling for the current output gap and trend inflation. Although correlated monetary policy shocks can be identified (see, for example, Ang, Dong and Piazzesi (2007)), we work with uncorrelated policy shocks to give full weight to the macro growth and trend inflation in tracing out their effects on asset prices.

2.2 Nominal Bonds

Expectations about future Fed Funds rates as well as risk premiums determine the prices of nominal bonds. We assume that risk premiums for nominal bonds depend on the macro variables X_t^o. Let $\lambda_t^\$$ denote the vector of risk premiums at date t, which we specify as

$$\lambda_t^\$ = \lambda_0^\$ + \lambda_1^\$ X_t^o \tag{3}$$

where $\lambda_0^\$$ is a three dimensional column vector and $\lambda_1^\$$ is a 3×3 matrix, following Constantinides (1992), Duffee (2002), and others. A consequence of equation (3) is that nominal bond prices reflect the predictable component of inflation dynamics, to which the Fed adjusts short

rates in equation (1), and not the unpredictable deviations from trend inflation. Inflation surprises are reflected in real discount rates, as we explain below.

The nominal pricing kernel, $M_{t+1}^{\$}$, takes the standard exponential form

$$M_{t+1}^{\$} = \exp\left(-r_t^{\$} - \frac{1}{2}\lambda_t^{\$'}\lambda_t^{\$} - \lambda_t^{\$'}\varepsilon_{t+1}\right), \tag{4}$$

where the shocks to the nominal pricing kernel, ε_{t+1}, are the same unpredictable shocks to the macro variables X_{t+1}^o in equation (2).

The price of a nominal zero-coupon bond of maturity n, $P_t^{\$}(n)$, is given by

$$P_t^{\$}(n) = \mathrm{E}_t\left[M_{t+1}^{\$}P_{t+1}^{\$}(n-1)\right].$$

We can equivalently express this under the risk-neutral pricing measure, Q:

$$P_t^{\$}(n) = \mathrm{E}_t^Q\left[\$1 \cdot e^{-\sum_{i=0}^{n-1} r_{t+i}^{\$}}\right]. \tag{5}$$

Note that the discounting of the nominal unit payoff in n periods is done using the future path of nominal short rates, $\{r_u^{\$}\}_{u=t}^{n-1}$. Under Q, the observable state vector X_t^o follows

$$X_{t+1}^o = \mu^Q + \Phi^Q X_t^o + \Sigma\varepsilon_{t+1}^Q, \tag{6}$$

where

$$\mu^Q = \mu - \Sigma\lambda_0^{\$} \qquad \text{and} \qquad \Phi^Q = \Phi - \Sigma\lambda_1^{\$}.$$

Following standard recursion arguments using equation (5) (see, for example, Ang and Piazzesi (2003)), the price of the nominal zero-coupon bond is given by

$$P_t^{\$}(n) = \exp(A_n^{\$} + B_n^{\$'}X_t^o), \tag{7}$$

where the loadings solve the difference equations

$$\begin{aligned}
A_{n+1}^{\$} &= A_n^{\$} + B_n^{\$'}\mu^Q + \frac{1}{2}B_n^{\$'}\Sigma\Sigma'B_n^{\$'} + A_1^{\$}, \\
B_{n+1}^{\$'} &= B_n^{\$'}\Phi^Q + B_1^{\$'},
\end{aligned}$$

with $A_1^{\$} = -c$ and $B_1^{\$} = -\delta_1^{\$}$. Nominal bond yields, $y_t^{\$}(n)$, are then given by

$$y_t^{\$}(n) = a_n^{\$} + b_n^{\$'}X_t^o, \tag{8}$$

where $a_n^{\$} = -A_n^{\$}/n$ and $b_n^{\$} = -B_n^{\$}/n$.

When the macro variables are not priced, that is $\lambda_0^\$ = \lambda_1^\$ = 0$, then the yield on a nominal bond of maturity n is simply the average of future Fed Funds rates (ignoring the Jensen's inequality term) as given by the Expectations Hypothesis. Priced macro factors enter into μ^Q and Φ^Q causing the risk-neutral dynamics to differ from the process of X_t^o in the physical measure. The resulting effects on the loadings $A_n^\$$ and $B_n^\$$ are able to capture a constant risk premium and time-varying risk premium, respectively, in the dynamics of nominal yields, as shown by Dai and Singleton (2002), and others.

2.3 Inflation

We assume that observed inflation rates, π_t, are a noisy realization of trend inflation at the beginning of the period, π_{t-1}^e:[6]

$$\pi_t = \pi_c + \pi_{t-1}^e + \Sigma^{\pi'}\varepsilon_t + \sigma_\pi \varepsilon_t^\pi. \tag{9}$$

Ignoring the constant π_c, which matches the mean of inflation as the state variables are demeaned, realized inflation, π_t, is equal to trend inflation at the beginning of the period, π_{t-1}^e, plus an inflation shock, $\Sigma^{\pi'}\varepsilon_t + \sigma_\pi \varepsilon_t^\pi$, where $\varepsilon_t^\pi \sim$ i.i.d. $N(0,1)$ is orthogonal to the factor shocks, ε_t. The inflation surprise correlated with shocks to the state variables X_t^o are spanned by nominal bonds while the inflation-specific shock, ε_t^π, is completely hedged only by real bonds, as we now explain.

2.4 Real Short Rates

We denote the real pricing kernel, which prices real claims, as M_{t+1}^r. The real and nominal kernels are linked through realized inflation. Denoting the logs of the real and nominal pricing kernels as m_t^r and $m_t^\$$, respectively, the log of the real stochastic discount factor is equal to the log of the nominal stochastic discount factor plus inflation:

$$m_{t+1}^r = m_{t+1}^\$ + \pi_{t+1}, \tag{10}$$

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[6] Equation (9) assumes that trend inflation is an unbiased estimate of actual inflation. This is true in data. A regression of future realized inflation over the next quarter on trend inflation, which is the median one-quarter ahead inflation forecast from the Survey of Professional Forecasters, produces a coefficient on trend inflation of 0.86 with a standard error of 0.05. If the year-on-year quarterly change in realized inflation is used as the regressand, the coefficient on trend inflation is 1.11 with a standard error of 0.04. In both cases, we fail to reject that trend inflation is an unbiased predictor of future realized inflation at the 95% level.

where $m^r_{t+1} \equiv \ln M^r_{t+1}$ and $m^\$_{t+1} \equiv \ln M^\$_{t+1}$. The conditional expected value and conditional volatility of both sides of equation (10) must coincide in order to prevent arbitrage. Thus, M^r_{t+1} also takes a standard exponential form:[7]

$$M^r_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda^{r'}_t \lambda^r_t - \lambda^{r'}_t \varepsilon_{t+1} + \sigma_\pi \varepsilon^\pi_{t+1}\right), \tag{11}$$

where r_t is the real short rate and $\lambda^r_t = \lambda^r_0 + \lambda^r_1 X^o_t$ is the 3×1 column vector of real market price of risk with λ^r_0 a 3×1 vector and λ^r_1 a 3×3 vector.

The real short rate, r_t, is generated endogenously in the model after nominal bonds and inflation are specified and is obtained by equating the conditional expected values in equation (10). The real short rate is given by

$$r_t = \delta^r_0 + \delta^{r'}_1 X^o_t, \tag{12}$$

where

$$\delta^r_0 = c - \pi_c - \frac{1}{2}\Sigma^{\pi'}\Sigma^\pi + \Sigma^{\pi'}\lambda^\$_0 - \frac{1}{2}\sigma^2_\pi$$

$$\delta^r_1 = \delta^\$_1 - e_2 + \left(\lambda^{\$'}_1\Sigma^\pi\right),$$

where e_2 is a vector of zeros with a one in the second position, which extracts π^e_t from X^o_t. The real short rate depends only on the macro factors influencing nominal bonds, inflation, and inflation risk.[8]

It is instructive to analyze the spread between the nominal Fed Funds rate, $r^\$_t$, and the real short rate, r_t:

$$r^\$_t - r_t = \frac{1}{2}\Sigma^{\pi'}\Sigma^\pi + \frac{1}{2}\sigma^2_\pi + \pi_c + e'_2 X^o_t - \Sigma^{\pi'}\lambda^\$_0 - (\Sigma^{\pi'}\lambda^\$_1)' X_t. \tag{13}$$

This consists of a Jensen's inequality term, $\frac{1}{2}\Sigma^{\pi'}\Sigma^\pi + \frac{1}{2}\sigma^2_\pi$, expected inflation, $\pi_c + e'_2 X^o_t = \pi_c + \pi^e_t$, and an inflation risk premium, $-\Sigma^{\pi'}\lambda^\$_0 - (\Sigma^{\pi'}\lambda^\$_1)' X_t$. Only if the inflation risk premium is equal to zero does a version of the pure Fisher Hypothesis hold, where the nominal short rate

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[7] Our model follows David and Veronesi (2009), Lemke and Werner (2009), Koijen, Lustig and Van Nieuwerburgh (2010), Lettau and Wachter (2010), Campbell, Sunderam and Viceira (2010), and others in assuming that trend inflation appears in the real stochastic discount factor. While these authors exogenously assume trend inflation directly enters, in our model trend inflation enters the real pricing kernel through the Taylor rule, which is a function of trend inflation, operating on nominal short rates, combined with realized inflation being trend inflation plus inflation surprises. Piazzesi and Schneider (2006, 2010) and Ulrich (2010) develop equilibrium models showing why the real stochastic discount factor can depend on trend inflation.

[8] In Lettau and Wachter (2010), the real short rate depends on real dividend growth. We separate equity cashflow risk and real interest rate risk.

equals the real short rate plus expected inflation. If $\lambda_1^\$ \neq 0$ and/or $\lambda_0^\$ \neq 0$, then there are risk premiums on output, trend inflation, and monetary policy, which are reflected in the spread between the overnight real short rate and the nominal Fed Funds rate. Equation (13) shows that these risk premium adjustments are potentially important; empirically the real short rate is not observed, but using the model we can infer real short rates from the nominal Fed Funds rate and inflation given the prices of risk.

In equation (11), the real kernel, M_{t+1}^r, depends explicitly on inflation shocks, ε_{t+1}^π, but inflation shocks do not enter the nominal kernel in equation (4). In the Taylor rule (1), the Fed responds to trend inflation, not realized inflation. This corresponds to Fed practice in concentrating on forward-looking inflation measures and preferring to use core inflation, which excludes relatively volatile food and energy prices, as its main inflation measure. A temporary inflation shock leaves the Fed Funds rate unchanged. From equation (13), the real short rate falls. Thus, temporary inflation shocks increase the value of a real bond. The investor is willing to pay a positive premium to hedge exposure to temporary inflation shocks, which are not reflected in the nominal Fed Funds rate. In equation (11), the premium for this inflation hedge is $-\sigma_\pi$ per unit of inflation.

2.5 Real Bonds

We define a real zero-coupon bond as a security where the face value is indexed to the price index, or the payoff is constant in real terms. The nominal payoffs of real bonds depend explicitly on the path of realized inflation. The yields of these bonds constitute the term structure of real rates.

From equation (10), the real and nominal market prices of risk are linked by

$$\lambda_0^r = \lambda_0^\$ - \Sigma^\pi \qquad \text{and} \qquad \lambda_1^r = \lambda_1^\$, \tag{14}$$

where $\lambda_t^r = \lambda_0^r + \lambda_1^r X_t^o$. The real and nominal prices of risk differ by the covariation of inflation with the state variables, $\lambda_t^r = \lambda_t^\$ - \Sigma^\pi$ because our VAR in equation (2) is homoskedastic.

The real bond price of maturity n, $P_t^r(n)$, satisfies the Euler equation

$$P_t^r(n) = \mathrm{E}_t[M_{t+1}^r P_{t+1}^r(n-1)],$$

or we can price the real zero-coupon bond under Q:

$$P_t^r(n) = \mathrm{E}_t^Q \left[e^{-\sum_{i=0}^{n-1} r_{t+i}} \right], \tag{15}$$

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where the unit payoff in n periods is discounted using real short rates, $\{r_u\}_{u=t}^{n-1}$. The corresponding risk-adjusted dynamic of the observable state vector X_t^o is given by

$$X_{t+1}^o = \hat{\mu}^Q + \hat{\Phi}^Q X_t^o + \Sigma \varepsilon_{t+1}^Q, \tag{16}$$

and the conditional mean parameters are given by

$$\hat{\mu}^Q = \mu - \Sigma \lambda_0^r \qquad \text{and} \qquad \hat{\Phi}^Q = \Phi - \Sigma \lambda_1^r.$$

Real bond prices are exponential affine in X_t^o:

$$P_t^r(n) = \exp(A_n^r + B_n^{r'} X_t^o), \tag{17}$$

where the loadings satisfy

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\begin{aligned}
A_{n+1}^r &= A_n^r + B_n^{r'} \hat{\mu}^Q + \frac{1}{2} B_n^{r'} \Sigma \Sigma' B_n^{r'} + A_1^r \\
B_{n+1}^{r'} &= B_n^{r'} \hat{\Phi}^Q + B_1^{r'},
\end{aligned}
$$

subject to the initial conditions $A_1^r = -\delta_0^r$ and $B_1^r = -\delta_1^r$. The yield of the real bond of maturity n, $y_t^r(n)$, is affine in the state variables, X_t^o:

$$y_t^r(n) = a_n^r + b_n^{r'} X_t^o, \tag{18}$$

where $a_n^r = -A_n^r/n$, and $b_n^r = -B_n^r/n$.

Average real and nominal yields of a given maturity differ not only because the real and nominal short rates are different, but also because the real and nominal constant prices of risk are dissimilar, $\lambda_0^r \neq \lambda_0^\$$ from equation (14). Intuitively, the nominal short rate is driven by macro factors, X_t^o, and these factors together with how they are correlated with inflation affect the implied real short rate (see equation (13)). In addition to the mean effect, real and nominal yields also exhibit different conditional behavior. Although the time-varying components of the real and nominal price of risk are identical, that is $\lambda_1^r = \lambda_1^\$$ and so $\hat{\Phi}^Q = \Phi^Q$, the starting conditions of the B_n^r and $B_n^\$$ recursions are different. The nominal bond recursions for $B_n^\$$ start with the Taylor rule coefficient $-\delta_1^\$$. In contrast, the real bond recursions for B_n^r lower the nominal rate by expected inflation and a risk premium adjustment involving the covariance of inflation with macro factors, Σ^π, and the time-varying price of macro risk, $\lambda_1^\$$. This allows the model to capture a rich array of both real and nominal yield curve dynamics.

2.6 Equity

2.6.1 Real Cashflows

The term structure of real yields gives us real discount rates which apply to securities with constant real payoffs. Equity has stochastic real payoffs. We now complete the model by specifying the stochastic stream of real dividends $\{D_t^r\}_{t=1}^{\infty}$. We denote the continuously compounded growth rate of real dividends as d_t,

$$d_t = \ln(D_t^{\$}/D_{t-1}^{\$}) - \pi_t, \tag{19}$$

where $D_t^{\$}$ is the nominal dividend at time t.

Real cashflow growth, d_t, follows the process

$$d_t = d_c + \Phi_{dg}g_{t-1} + \Phi_{d\pi^e}\pi_{t-1}^e + \Phi_{df}f_{t-1} + \Phi_{dd}d_{t-1} + \Phi_{dL}L_{t-1} + \sigma_d\varepsilon_t^d, \tag{20}$$

where the expected dividend growth rate depends on lagged observable aggregate macro state variables, X_{t-1}^o, past real dividend growth, d_{t-1}, and on a latent equity premium factor, L_{t-1}. Thus, the predictable components of real dividend growth depend on multiple factors which exhibit large autocorrelations. This allows the model to potentially capture highly persistent components of cashflows which Bansal and Yaron (2004), among others, argue is a stylized feature of the data. We specify the real dividend shock, $\varepsilon_t^d \sim$ i.i.d. $N(0,1)$ to be orthogonal to the other shocks for simplicity.[9]

2.6.2 Real Cashflow Risk

The price of risk of real cashflows, λ_t^d, depends on both macro variables and the latent equity factor, L_t:

$$\lambda_t^d = \lambda_0^d + \lambda_{X^o}^{d\prime}X_t^o + \lambda_L^d L_t, \tag{21}$$

where λ_0^d is a scalar, $\lambda_{X^o}^d$ is a 3×1 vector, and λ_L^d is a scalar. The latent factor L_t follows the AR(1) process:

$$L_t = \phi_L L_{t-1} + \sigma_L \varepsilon_t^L, \tag{22}$$

where ε_t^L is a standard i.i.d. Gaussian shock which is independent of the other error terms.

The factor L_t can be interpreted in several ways. First, it represents a time-varying predictable component in real cashflow growth which the Fed does not explicitly take into account

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[9] In empirical estimations allowing for cross-correlations, these estimates are very close to zero.

in its policy rule. This could be technological change, for example, of the sort modeled by Pástor and Veronesi (2009) that affects equity cashflows, but not bond yields. More broadly, L_t captures any effect on equity cashflows and risk not captured by output, trend inflation, and monetary policy.

From equation (21), L_t can be interpreted as a time-varying price of risk factor for real dividends. In equation (21), the equity risk premium has two components: the first is due to observable macro factors, X_t^o, and the second is driven by the equity factor, L_t. The factor L_t enters the time-varying price of dividend growth risk and represents a priced risk premium factor through the pricing of cashflow risk.[10] Note we do not price L_t itself: this makes our model similar to structural models like Campbell and Cochrane (1999) where factor risks in equity are priced through their covariation with cashflows. An alternative interpretation is that L_t captures a preference or sentiment shock affecting equities, and we investigate this in our empirical work.

Similar to Brennan, Wang and Xia (2004) and Lettau and Wachter (2007), the latent price of risk follows an AR(1) process in equation (22), but we allow it to also affect future cashflows. This potentially captures the predictability of dividend growth by components that also drive expected returns, like the L_t process, consistent with Lettau and Ludvigson (2005) and Ang and Bekaert (2007). We identify L_t through its impact on the cashflow process and the dividend yield. In contrast, other studies like Brandt and Kang (2004) and Rytchkov (2010) estimate persistent unobservable components of expected returns, except the latent components are identified with realized return variation. Identifying persistent risk premium factors with cashflows and dividend yields is more precise because returns are substantially more conditionally volatile than dividend yields. Binsbergen and Koijen (2010) also estimate latent expected return factors from price-dividend ratios, but they do not allow the risk premium factor to also influence cashflows, through Φ_{dL}, or account for the effect of macro risk through X_t^o.

Finally, L_t can be econometrically viewed as a "goodness-of-fit" test of the ability of X_t^o to explain equity prices by comparing an estimation without L_t to an estimation with only observable macro factors and cashflows. A large discrepancy between the two models may indicate model mis-specification or that the required variation in equity risk premiums cannot be captured by variations in our observed aggregate macro factors X_t^o. We examine estimations

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[10] We do not allow the market price of real cashflow risk to depend on real dividend growth d_t. An unexpected change in real dividend growth, as reflected by a change in d_t, changes the cashflow but not the premium for cashflow risk. Thus, d_t has a pure interpretation of being a cashflow factor and its time-varying price of risk depends on L_t.

with and without L_t in our empirical work and show that the observed aggregate macro factors X_t^o alone account for most of the variation in dividend yields.

2.6.3 Equity Prices

Under no arbitrage, the price of a stock equals the risk-neutral expected value of future real dividends, discounted by real short rates:[11]

$$\frac{P_t^r}{D_t^r} = \frac{P_t^\$}{D_t^\$} = \mathrm{E}_t^Q \left[\sum_{s=1}^{\infty} \exp \left(\sum_{k=1}^{s} d_{t+k} - r_{t+k-1} \right) \right], \tag{23}$$

where $P_t^r/D_t^r = P_t^\$/D_t^\$$ is the price-dividend ratio in both real and nominal terms. In equation (23), the timing of the real dividend growth and real short rates are offset because the real short rates are known at the beginning of the period.

To price equity, we collect the whole set of factors in $\tilde{X}_t = (g_t \, \pi_t^e \, f_t \, d_t \, L_t)'$. We write the dynamics of $\tilde{X}_t$ compactly as a VAR(1):

$$\tilde{X}_t = \tilde{\mu} + \tilde{\Phi}\tilde{X}_t + \tilde{\Sigma}\tilde{\varepsilon}_{t+1}, \tag{24}$$

where $\tilde{\varepsilon}_{t+1} = (\varepsilon'_{t+1} \, \varepsilon_{t+1}^d \, \varepsilon_{t+1}^L)'$ and the parameters $\tilde{\mu}$, $\tilde{\Phi}$, and $\tilde{\Sigma}$ are determined by stacking equations (2), (20), and (22).

We define the prices of risk corresponding to $\tilde{X}_t$ as

$$\tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{X}_t, \tag{25}$$

where

$$\tilde{\lambda}_0 = \begin{pmatrix} \lambda_0^r [3 \times 1] \\ \lambda_0^d \\ 0 \end{pmatrix} \quad \text{and} \quad \tilde{\lambda}_1 = \begin{pmatrix} \lambda_1^r \ [3 \times 3] & 0 \ [3 \times 2] \\ \lambda_{X^o}^{d'} [1 \times 3] & 0 & \lambda_L^d \\ 0 \ [1 \times 5] \end{pmatrix},$$

where the dimensions of the matrices are given in square brackets and all other parameters are scalars. Under the risk-neutral measure Q, the extended state vector that determines equity prices, $\tilde{X}_t$, follows

$$\tilde{X}_{t+1} = \tilde{\mu}^Q + \tilde{\Phi}^Q X_t + \tilde{\Sigma}\tilde{\varepsilon}_{t+1}^Q,$$

where

$$\tilde{\mu}^Q = \tilde{\mu} - \tilde{\Sigma}\tilde{\lambda}_0 \quad \text{and} \quad \tilde{\Phi}^Q = \tilde{\Phi} - \tilde{\Sigma}\tilde{\lambda}_1.$$

---

[11] Alternatively, one can also discount the value of future nominal dividends by the nominal short rate under the risk-neutral measure. Both approaches yield identical equity values.

Using standard techniques (see, for example, Ang and Liu (2001)), the price-dividend ratio takes the form

$$\frac{P_t^r}{D_t^r} = \frac{P_t^\$}{D_t^\$} = \sum_{n=1}^{\infty} \exp(a_n + b_n' \tilde{X}_t), \tag{26}$$

where a_n and b_n follow the recursions

$$a_{n+1} = a_n - \delta_0^r + (e_4 + b_n)' \tilde{\mu}^Q + \frac{1}{2}(e_4 + b_n)' \tilde{\Sigma}\tilde{\Sigma}'(e_4 + b_n)$$
$$b_{n+1} = -\tilde{\delta}_1^r + \tilde{\Phi}^{Q'}(e_4 + b_n),$$

where $a_1 = -\delta_0^r + e_4' \tilde{\mu}^Q + \frac{1}{2}e_4' \tilde{\Sigma}\tilde{\Sigma}' e_4$, $b_1 = -\tilde{\delta}_1^r + \tilde{\Phi}^{Q'} e_4$, $\tilde{\delta}_1^r = (\delta_1^{r'} \, 0 \, 0)'$ and $e_4 = (0\,0\,0\,1\,0)'$.

The price-dividend ratio naturally depends on real cashflow growth, which enters the recursions explicitly through the terms involving e_4. It also depends on the state of the macro economy, X_t^o, and the equity premium latent factor L_t. These factors affect both the forecasts of future real cashflows, through $\tilde{\Phi}$, and the time-varying price of risk of dividends, through $\tilde{\lambda}_1$.

2.7 The Term Structures of Risk Premiums

The previous sections derived the prices of nominal bonds, real bonds, and equity. From these prices, we now define and compute expected equity returns and equity risk premiums (see Appendix A for analytical expressions).

We define the expected k-period total holding return on equity as

$$\mathrm{E}_t[R_t^{E,\$}(k)] = \mathrm{E}_t\left[\ln\left(\frac{P_{t+1}^\$ + D_{t+1}^\$}{P_t^\$}\right) + ... + \ln\left(\frac{P_{t+k}^\$ + D_{t+k}^\$}{P_{t+k-1}^\$}\right)\right], \tag{27}$$

where prices, $P^\$$, and dividends, $D^\$$, are in nominal terms. We also refer to this as the total expected equity return. Varying maturity k, we have a term structure of expected equity returns.

The expected k-period real holding return on equity is defined as

$$\mathrm{E}_t[R_t^{E,r}(k)] = \mathrm{E}_t\left[\ln\left(\frac{P_{t+1}^r + D_{t+1}^r}{P_t^r}\right) + ... + \ln\left(\frac{P_{t+k}^r + D_{t+k}^r}{P_{t+k-1}^r}\right)\right], \tag{28}$$

where all quantities are now real. The difference between the total and real expected holding period returns is expected inflation, $\mathrm{E}_t[\pi_t(k)]$:

$$\mathrm{E}_t[R_t^{E,\$}(k)] = \mathrm{E}_t[R_t^{E,r}(k)] + \mathrm{E}_t[\pi_t(k)], \tag{29}$$

where

$$\pi_t(k) = \pi_{t+1} + ... + \pi_{t+k}$$

14

is the cumulative (log) inflation rate from time t to $t + k$.

For a given horizon k, we can decompose the expected total equity return into several components. We start with the real short rate, r_t. Now we consider a real zero-coupon bond with maturity k. The real yield, $y_t^r(k)$, is the expected holding period return on this bond from now to k. This compensates the investor for real duration risk. Next, we could hold a nominal zero-coupon bond of maturity k with yield $y_t^\$(k)$. The inflation risk present in this bond must be compensated in equilibrium by a higher real return. We define the expected real return over k periods on a nominal zero-coupon bond of maturity k, $y_t^{\$,r}(k)$, as the nominal bond's holding period return (which is the same as the nominal bond yield) less expected inflation, $y_t^{\$,r}(k) = y_t^\$(k) - \mathrm{E}_t[\pi_t(k)]$. The difference between the real yield on the nominal bond and the real bond represents an inflation risk premium. Finally, we have the expected nominal equity return, $\mathrm{E}_t[R_t^{E,\$}(k)]$. Real equity cashflows are stochastic, and so the difference between the expected nominal equity return and the nominal bond yield represents a real cashflow premium.

We summarize this as:

$$
\begin{aligned}
\mathrm{E}_t[R_t^{E,\$}(k)] &= \mathrm{E}_t[R_t^{E,r}(k)] + \mathrm{E}_t[\pi_t(k)] \quad && \text{Total equity return} \\
&= r_t && \text{Real short rate, } r_t \\
&+ (y_t^r(k) - r_t) && \text{Real duration premium, } DP_t(k) \\
&+ (y_t^{\$,r}(k) - y_t^r(k)) && \text{Inflation risk premium, } IRP_t(k) \\
&+ (y_t^\$(k) - y_t^{\$,r}(k)) && \text{Expected inflation, } \mathrm{E}_t[\pi_t(k)] \\
&+ (\mathrm{E}_t[R_t^{E,\$}(k)] - y_t^\$(k)) && \text{Real cashflow risk premium, } CFP_t(k)
\end{aligned}
$$

Thus, we decompose the total expected equity return as:

$$
\mathrm{E}_t[R_t^{E,\$}(k)] = r_t + DP_t(k) + IRP_t(k) + \mathrm{E}_t[\pi_t(k)] + CFP_t(k). \tag{30}
$$

Each of these risk premiums themselves have a term structure across horizons k.

The cashflow risk premium, $CFP_t(k)$, can be interpreted as an "equity risk premium," as it is the difference between expected total equity returns and the expected return on a nominal bond. Practitioners often use this definition (see, for example, Asness (2000); Ibbotson and Chen (2003)), except they generally use yields on coupon bonds rather than zero-coupon bonds.[12] We prefer the more precise term "cashflow risk premium" to indicate that it is the incremental reward for bearing stochastic dividend risk in excess of nominal bonds. The cashflow

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[12] In contrast, many academics prefer to define the equity risk premium as the difference between total equity returns and short rates (or cash returns), following Mehra and Prescott (1985).

premium is equivalently given by the difference between expected real equity returns and real returns on nominal bonds:

$$CFP_t(k) = \mathrm{E}_t[R_t^{E,\$}(k)] - y_t^{\$}(k) = \mathrm{E}_t[R_t^{E,r}(k)] - y_t^{\$,r}(k). \tag{31}$$

We also refer to the cashflow risk premium as the "equity risk premium over nominal bonds."

We define the "equity risk premium over real bonds" or the "real risk premium" as:

$$RRP_t(k) = \mathrm{E}_t[R_t^{E,r}(k)] - y_t^r(k)), \tag{32}$$

which is the difference between the expected real equity return and the real bond yield. The difference between the nominal and real equity risk premiums is expected inflation plus the inflation risk premium, $\mathrm{E}_t[\pi_t(k)] + IRP_t(k)$. Put another way, the real risk premium is the sum of the cashflow premium and the inflation risk premium:

$$RRP_t(k) = CFP_t(k) + IRP_t(k). \tag{33}$$

In our empirical work, we focus on the 10-year horizon ($k = 40$ quarters) for our benchmark results. This is a benchmark maturity in fixed income and is the horizon often chosen to correspond to "long-term" forecasts in surveys (such as the Survey of Professional Forecasters and surveys of industry professionals like Graham and Harvey (2005), for example). But, we also consider the term structure of risk premiums over all k.

3   Data

We work at the quarterly frequency and take data from 1982:Q1 to 2008:Q4. Over 1979-1982 the Fed set explicit targets for monetary aggregates and so we start our estimation in 1982:Q1 to avoid this period. The sample on real bonds starts later in 2003:Q1 due to the non-availability and liquidity problems of real bonds in the earlier part of the sample.

We take the output gap for g_t, the median one-quarter ahead inflation forecast from the Survey of Professional Forecasters (SPF) for π_t^e, and construct f_t as the residual from the Taylor-rule regression (1) for f_t. The inflation rate, π_t, is the change in the consumer price index over the past year. Ang, Bekaert and Wei (2008) show that the median inflation forecast from the SPF has the best performance for forecasting inflation among a comprehensive collection of Phillips curve models, time-series models, and macro term structure models.

Figure 1 plots the output gap, trend inflation, and the monetary policy shock. All variables are demeaned. The output gap, g_t, has decreased in all recessions and reaches its lowest point

16

during the 2008 recession (the "Great Recession"). Trend inflation, π_t^e, has become less volatile since the early 1990s, as documented by Clark and Davig (2009), and others, but remains anchored to the end of the sample. Both g_t and π_t^e are highly persistent with autocorrelations of 0.97 and 0.92, respectively. The monetary policy shock, f_t, is the least persistent process, with an autocorrelation of 0.52, and reaches its maximum during the 1987 Savings and Loan crisis and its minimum during the 1991 recession.

The bond data comprise the Fed Funds rate, nominal bond yields, and real bond yields. All bond yields are expressed as continuously compounded rates. We take zero-coupon bonds for nominal and real bonds from the Board of Governors of the Federal Reserve System, which are constructed following the method of Gürkaynak, Sack and Wright (2007, 2010). We take nominal bonds of maturities 1, 3, 5, 7, 10, 12, and 15 years. We deliberately do not take the very long part of the term structure (the 30-year maturity) because of the repurchase of long-dated bonds and the temporary cessation of the issue of 30-year bonds during the early 2000s due to Federal government surpluses at that time.

The data on real zero-coupon bonds are constructed from TIPS and we take maturities of 5, 7, 10, 12, and 15 years starting in 2003:Q1. Although the first TIPS were issued in 1997, the TIPS market was very illiquid for the first few years. We take data starting 2003:Q1 to mitigate these effects (see, among others, D'Amico, Kim and Wei (2007); Pflueger and Viceira (2011)). We do not take short maturity TIPS as these are only available later in the sample and require adjustments for indexation lags and the deflation put.[13] For this reason, we take only horizons greater than five years when we discuss the real duration premium, $DP_t(k)$, and the real equity risk premium, $RRP_t(k)$.

Equity dividend yields are constructed using the CRSP value-weighted stock index. We construct dividend yields by summing dividends over the past four quarters to remove seasonality. Our year-on-year real dividend growth rates at the quarterly frequency are also constructed to remove seasonality. Appendix B contains further details on the data.

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[13] While most analysis with TIPS does not take into account indexation lags, Evans (1998) considers indexation lags for UK real bonds. His analysis does not find evidence that the indexation lag affects market prices significantly. The deflation put refers to the property of TIPS where the principal does not fall below par when inflation is negative (see, for example, Jacoby and Shiller (2008)).

17

4 Empirical Results

4.1 Parameter Estimates

We report the parameter estimates of the model in Table 1.[14] The top panel reports estimates of the Taylor rule (equation (1)) and shows that the Fed responses on the output gap and trend inflation are 0.31 and 2.44, respectively, with both coefficients being highly significant. These signs are consistent with those in the literature where the Fed lowers the Fed Funds rate in response to weakening economic growth and raises the Fed Funds rate when inflation expectations increase. The reaction of the Fed to trend inflation is very strong at 2.44. This is consistent with Clarida, Galí and Gertler (2000), Boivin (2006), Ang et al. (2011), and others, who find that the response of the Fed to inflation since the post-Volcker era has been, on average, much larger than one. These and other studies, however, generally find lower coefficients than 2.44 because they tend to use realized inflation, rather than trend inflation as we do. The R^2 of this regression is 70%, so the policy rule explains a large amount of the variation in the Fed Funds rate.

As expected, the VAR dynamics in Table 1 (equations (2), (20), and (22)) reflect the high persistence in the output gap and trend inflation (see Figure 1). The latent factor is less persistent with $\phi_L = 0.16$. There is evidence of Granger-causality in both directions between the output gap and trend inflation, where increases in either variable predict increases in the other variable next period. The real cashflow equation shows that all variables, including lagged cashflows, predict next-period real cashflows. Increases in trend inflation Granger-cause large decreases in future real dividend growth, with a coefficient of -2.45. This is consistent with a large literature in macroeconomics finding that inflation is negatively related with real production (see, for example, Fama (1981)). Empirically, nominal price rigidity is pervasive (see, for example, Nakamura and Steinsson (2008)) and cost increases can only be passed on in stages, rather than continuously. In models like Diamond (1993), menu costs reduce market power as consumers search for products which have not had price increases. Thus, rising inflation reduces profit margins and consequently reduces real payouts. In the equation for d_t, real dividend growth also decreases with positive Fed policy surprises, with a coefficient of -0.21. Thus, active monetary policy that is more aggressive than implied by the Taylor rule further stifles real equity cashflows.

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[14] Details of the estimation are in Appendix C.

4.2 Real and Nominal Short Rates

Real short rates are empirically unobserved, but endogenously determined in the model. Real short rate dynamics follow:

$$r_t = \begin{matrix} 0.0074 \\ (0.0015) \end{matrix} + \begin{matrix} 0.1993 \ g_t \\ (0.3334) \end{matrix} + \begin{matrix} 1.7112 \ \pi_t^e \\ (0.2652) \end{matrix} + \begin{matrix} 1.3547 \ f_t, \\ (0.0697) \end{matrix} \tag{34}$$

with standard errors in parentheses. In the nominal Taylor rule, the coefficient on trend inflation, π_t^e, is 2.44 (see Table 1). In equation (34), the coefficient on π_t^e is 1.71 and thus the real rate is not simply the nominal rate minus trend inflation as per the Fisher relation. Non-neutrality arises from three sources. First, the Fed is very aggressive in the nominal Taylor rule (equation (1)), with a response to π_t^e well above one. This causes the real rate loading on π_t^e to be greater than zero. In fact, the coefficient on π_t^e is a very large 1.71 indicating that increases in trend inflation coincide with more than one-for-one increases in real rates. This finding is consistent with Woodford (2003) and others who argue that the Fed tries to affect real interest rates through active nominal interest rate policies.

If the real rate were simply the nominal rate less trend inflation, the coefficient on π_t^e should simply be $2.44 - 1 = 1.44$. In our model, the real rate coefficient on π_t^e is 1.71, which is higher than 1.44 due to the positive inflation risk premium (see equation (13)). Monetary policy moves both the real rate, with a coefficient of 1.35, more than it moves the nominal rate, where f_t has a coefficient of one by definition, due to the price of real rate risk. Thus, monetary policy actions taken by the Fed have a larger effect in real versus nominal terms.

Figure 2 plots the real short rate implied by the model, r_t, and the Fed Funds rate, $r_t^\$$. In periods where trend inflation is stable and the output gap is low, the Fed can set real rates to be negative by setting the Fed Funds rate to be lower than predicted by the Taylor rule (so there are negative monetary policy shocks). The real short rate is negative during the 1991, 2001, and 2008 recessions, and bottoms at -1.81% during the 2008 financial crisis. This reflects the Fed's strategy of aggressively lowering the Fed Funds rate to low levels when the economy is in recession, even if trend inflation is positive, producing negative real short rates. There is a high correlation between r_t and $r_t^\$$ of 0.95. The real short rate has been, on average, 2.98% lower than the Fed funds rate. This difference reflects the sum of trend inflation and the time-varying inflation risk premium (see equation (13)). The smallest difference between the real rate and nominal rate is 0.34% in 1986:Q4, which corresponds to the largest deviation from the Taylor rule in our data sample.

4.3 The Term Structure of Nominal and Real Bonds

Figure 3 plots the model-implied average nominal yield curve and for comparison the average yields, with two standard error bounds, in data. Like most affine models fitted on nominal yield curve data, the fit is excellent. However, while most term structure models require latent factors to fit the yield curve well, our model closely matches nominal yield curves with only observable factors. In the level and slope principal component interpretation of Litterman and Scheinkman (1991), trend inflation, π_t^e, tracks the level of the yield curve well and the monetary policy factor, f_t, closely matches the slope factor.

In Figure 4, we plot the term structure of real yields implied by the model and the average real yields in data, with two standard error bounds. The model-implied curve lies between two standard deviation bounds for all yields except the 60-quarter maturity. The inferior, but still relatively good, fit to the real yield curve compared to the nominal yield curve in Figure 3 is due largely to the shorter estimation period for real bonds, 2003:Q1-2008:Q4, compared to the full sample which starts in 1982:Q1. Note that TIPS are well known to have significant liquidity effects even over the post-2003 sample, as documented by D'Amico, Kim and Wei (2008) and Pflueger and Viceira (2011), among others. Our model does not incorporate an extra liquidity factor to fit the TIPS curve, as D'Amico, Kim and Wei do, and uses the same factors to fit both the nominal and real curves. In this light, the fit to the real yield curve is excellent.

The bottom panel of Figure 4 shows the model-implied real yield term structure interpolated for the full sample. The real term spread between the 60-quarter and 20-quarter maturities is 0.74%.[15] Our model produces both an upward-sloping nominal and real yield curve through the positive prices of risk (the negative λ_0 parameters) for the output gap and inflation. Economically, the positive real term spread can reflect, among other things, consumption growth risk (see Wachter (2006) or uncertainty about the effectiveness of benevolent government interventions in the business cycle (see Ulrich, 2011b).

4.4 Equity

Figure 5 graphs dividend yields. We plot the model-implied dividend yield without the latent factor, L_t, in the dashed line, and the dividend yield in data in the solid line. By construction, we obtain an exact match with the dividend yield in data with L_t. Impressively, the model is

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[15] Ang, Bekaert and Wei (2008) estimate real yield curves over earlier periods when TIPS are not traded and find that the real yield curve is fairly flat. They do not use traded TIPS and their ending maturity is 20 quarters, which is the first maturity of our real bonds in data.

able to match closely the dividend yield in data without L_t. To gain some intuition on how the model matches dividend yields, we can interpret the loadings on the various factors in the log-linearized dividend yield:

$$\ln\left(1 + \frac{D_t}{P_t}\right) \simeq 0.0063 - 0.0149g_t + 0.7976\pi_t^e - 0.0185f_t - 0.0022d_t + 0.0094L_t. \quad (35)$$

The high correlation between dividend yields and long-term bond yields in data is often called the "Fed model". In our sample, the correlation between the dividend yield and the 10-year nominal bond yield is 0.87, which our model matches exactly. There are two main reasons why our model nails this correlation. First, in equation (35) there is a large coefficient of 0.80 on trend inflation, π_t^e. Trend inflation has the largest effect on dividend yields of all the variables. An increase in trend inflation decreases real cashflow growth, increases the real interest rate, the inflation risk premium and the cashflow premium. This simultaneously increases yields on bonds and the dividend yield.

Second, the dividend yield, like nominal interest rates, is countercyclical. In equation (35), the negative loading on the output gap, g_t, and real dividend growth, d_t, indicates that in expansions g_t and d_t are high, expected returns are low and prices are high, and thus dividend yields are low. The Fed also moves nominal Fed Funds rates countercylically in the Taylor rule (equation (1), and thus macro variables cause both the dividend yield and nominal bond yield to comove strongly.

The negative coefficient on f_t implies that a surprise move by the Fed to inject liquidity beyond that suggested by the Taylor rule (a negative f_t shock) increases dividend yields. Thus, a surprise loosening of monetary policy tends to decrease stock prices and increase dividend yields. This result is due to the effect of monetary policy surprises on real dividend risk. While surprise decreases in the Fed Funds rate spur, on average, increases in real dividends next period, which we see by the coefficient $\Phi_{df} = -0.21$ in equation (20) and reported in Table 1, the price of risk of cashflow growth increases, as λ_f^d is negative. Surprise decreases, therefore, in the Fed Funds rate do increase firm cashflows, but these cashflows are discounted at higher rates. The discount rate effect dominates and this causes the dividend yield to increase. On the other hand, during recessions g_t is low and π_t^e is low, so the Fed Funds rate, $r_t^\$$, falls as predicted by the Taylor rule. A negative surprise in f_t sends $r_t^\$$ lower than implied by the Taylor rule. In our model this is risky, and the discount rate increases to reflect that risk.

4.4.1 Latent Equity Factor

When L_t is included, we match the dividend yield exactly so L_t is a non-linear function of the difference between the dividend yield implied by the model and the actual dividend yield in data in Figure 5. Note that from equation (35), high L_t causes low prices, or high dividend yields. The correlation between dividend yields and L_t is 0.46.

We investigate if movements in L_t reflect factors not captured in the model, especially credit risk, volatility risk, sentiment, and liquidity. We measure credit risk by the Baa-Aaa credit spread, volatility risk as the VIX volatility index, sentiment by the Baker and Wurgler (2007) sentiment factor, and take the liquidity factor from Pástor and Stambaugh (2003). In Table 2, we run contemporaneous regressions of L_t onto these various factors. In regressions I-IV, we consider univariate relations. The latent equity factor L_t has the strongest relation with the Baker-Wurgler sentiment factor, with a t-statistic of -5.42 and an adjusted R^2 of 28%. Thus, L_t, which represents the portion of equity price movements not captured by macro variables and cashflows, can be interpreted partly as a sentiment factor in line with Baker and Wurgler. In fact, this is the only factor that exhibits a significant correlation with L_t. It remains the only significant relation in the joint regression V. In regression VI where we add controls for macro variables, both sentiment and trend inflation carry significant loadings.

Figure 6 plots L_t together with the Baker-Wurgler sentiment factor. Periods of low sentiment tend to coincide with periods of high L_t and vice versa. The sentiment factor reaches a sharp peak in 2001:Q1 where equity prices are very high. While the equity factor L_t is low during this period, the low values of L_t are not unusual. During this time, the output gap is high and inflation expectations are low, as shown in Figure 1. This causes both interest rates and discount rates to be low, leading to macro-based equity prices to be very high. Thus, the model accounts for low dividend yields in the late 1990s and early 2000s by low macro risk similar to Lettau, Ludvigson and Wachter (2008), rather than an unusual increase in sentiment. In contrast, Shiller (2000) and Baker and Wurgler (2007), among others, attribute low dividend yields during this time almost entirely to high sentiment.

4.5   Equity Risk Premiums

4.5.1   The Term Structure of Equity Risk Premiums

We now decompose the total equity return into various risk premium components following Section 2.7. We start by examining the total expected return, $E_t[R_t^{E,\$}(k)]$, the cashflow risk

premium, $CFP_t(k)$ (the equity risk premium over nominal bonds), and the real risk premium, $RRP_t(k)$ (the equity risk premium over real bonds), in Figure 7. As we vary the horizon, k, we trace out the term structure of total equity returns and equity risk premiums. As our starting real bond in data has a maturity of 20 quarters, we start the real risk premium curve at this maturity. We end at 60 quarters, which is the longest maturity of both nominal and real bonds in data.

Figure 7 shows that the term structure of total equity returns is downward sloping. The downward-sloping curve implies that equity becomes less risky, or that total expected holding period returns decrease, as the horizon increases. Similar downward-sloping term structures have been estimated by Lettau and Wachter (2010), Binsbergen, Brandt and Koijen (2011), and Binsbergen et al. (2011), among others. At a one-year horizon, the total expected equity return is 12.6% and this reduces to 11.0% for the 15-year horizon. Using the joint term structures of nominal and real bonds, we plot the cashflow risk premiums and real risk premiums in the dotted and dotted-dashed lines, respectively. Figure 7 shows that the cashflow risk premium has a similar downward-sloping pattern, decreasing from 6.7% at the one-year horizon to 3.9% at the 15-year horizon. The real risk premium is also sloped downwards, with real risk premiums of 4.5% and 4.4% at the five and 15-year horizons, respectively.

The term structure of equity returns is downward sloping due to the risk premium associated with expected inflation decreasing with horizon. Intuitively, equity in the long run has some inflation-hedging ability as its risk premium to trend inflation falls. The risk due to the output gap and monetary policy shocks increases with maturity. The monetary policy shock is quickly mean-reverting, so it lowers the cashflow premium at small horizons and has effectively no effect on horizons greater than five quarters. The offsetting effects of the falling inflation premium and the increasing output gap premium, combined with the very short-acting monetary policy shock premium, contribute to the small hook in the term structure of expected returns at short horizons. In the long run, the decreasing trend inflation risk premium dominates, and equity becomes less risky at longer horizons.

4.5.2 Decomposing Equity Risk Premiums

Using our model, we can further decompose the total expected return into the real return, real duration premiums, expected inflation and inflation risk premiums, and the cashflow premium. This is done in Figure 8 and Table 3. The real short rate implied by the model is 2.98%, on average. The reward for bearing real duration risk is very small at 0.1% at the five-year horizon increasing to 0.81% at the 15-year horizon. Expected inflation is approximately 3%.

The inflation risk premium, which is the difference between nominal and the Fisher Hypothesis-implied real bond yields, is slightly upward sloping past the five-year horizon and increases from 0.41% to 0.62% from five to 15 years. While these real duration and inflation risk premiums are increasing, the cashflow premium is decreasing across horizon. This makes the overall expected total return curve downward sloping.

Table 3 reports a variance decomposition of the total expected equity return. At the 10-year horizon, there is substantial variation due to the real rate, with a variance decomposition of 37%. In our model, the real rate is a function of macro factors and these same macro factors also drive expected equity returns. Real duration risk accounts for 18% of the variance. The contribution from expected inflation and the inflation risk premium is negligible. Finally, the variation in the cashflow premium accounts for 43% of variance in total expected equity returns. Hence, most of the contribution of the total expected return comes from real rates and cashflow risk premiums. Whereas the variance attributions of the real rate increase from 19% to 44% from five to 15 years, the variance attributions of the cash flow risk premium decrease from 72% to 32% over those horizons. Thus, cashflow risk premiums drive most variation of total expected equity returns at short horizons while real rate variation dominates at long horizons; equity, in the long run, is a real security.

The difference between the equity risk premium over nominal bonds, CFP, and the equity risk premium over real bonds, RRP, is the inflation risk premium, IRP (see equation (33)). Table 3 reports that the cashflow premium and the real risk premium are very similar because the inflation risk premium is modest. At the 10-year horizon, the cashflow premium is 3.84% and the real risk premium is 4.45%, implying a 10-year inflation risk premium of 0.60%. The inflation risk premium does not exhibit much variation as a proportion of the variation of the expected total equity return, with a variance decomposition of approximately 1% at the 10-year horizon. This is why the variance decomposition of the expected equity return into the cashflow premium and the real risk premium components are very similar at around 43-44%.

How much variation in the risk premiums can we attribute to the macro and other factors? We answer this in Table 4 by computing factor variance decompositions at the 10-year horizon. Macro factors matter a lot for all premium components: the real short rate, real duration premium, expected inflation, inflation risk premium, and the cashflow premium. For the real short rate, trend inflation and monetary policy shocks account for 97% of variance. Monetary policy actions are responsible for driving almost all variation in real duration premiums. Not surprisingly, variation in expected inflation accounts for most variation in the inflation risk premium.

24

For the 10-year equity risk premiums (the cashflow and real risk premiums) reported in Table 4, economic growth dominates with variance decompositions above 60%. At longer horizons, the effect of economic growth is even more important with the variance decomposition at the 15-year horizon being over 70% for both the cashflow and real risk premiums. At long horizons, trend inflation, monetary policy actions, and real dividend growth account for little of the variation in equity risk premiums; we attribute the remainder of the variance in equity risk premiums to the equity latent factor, L_t, but this effect is approximately half the effect of economic growth. The output gap is a persistent process, with an autocorrelation of 0.9715: long-run economic growth has the same effect as persistent long-run consumption growth in the Bansal and Yaron (2004) setting and constitutes a long-run equity risk premium factor.

4.5.3 Dynamic Responses of Equity Risk Premiums

We further examine dynamic responses of the cashflow risk premium in Figure 9, which plots impulse responses for the 10-year horizon. We consider 1% moves in the shocks to each factor.[16] Figure 9 reveals that monetary policy shocks and shocks to the latent factor L_t are only short lived and die out within one year. In contrast, there are very persistent dynamics for the cashflow premium from shocks to trend inflation and shocks to the output gap. A 1% increase in the output gap (signalling "good times") lowers the cashflow risk premium by approximately 93 basis points. This is a very persistent shock: in five years, the cashflow risk premium is still lowered by 87 basis points.

Figure 9 shows a highly non-monotonic response of the cashflow premium to inflation shocks. A 1% shock to trend inflation (signalling "bad times" for future cashflows) increases the cashflow risk premium by 40 basis points. In five years, the cashflow risk premium has fallen to -60 basis points. It bottoms at -70 basis points after 10 years before mean-reverting back to zero. This implies the effect of an increase in trend inflation leads to highly non-linear, long-lasting effects on equity prices and risk premiums. The initial response of the cashflow risk premium is to spike when inflation rises, causing equity prices to fall. After one year, the cashflow premium turns negative, leading to increasing stock prices. The non-monotonicity in the impulse response of the cashflow risk premium to inflation is caused by two opposing effects from the output gap and trend inflation. Initially an increase in trend inflation increases the cashflow risk premium, but it also Granger-causes the output gap to increase during the following periods. Increases in the output gap cause the cashflow risk premium to gradually fall.

---

[16] In our model, real dividend growth is a pure cashflow factor and does not affect discount rates.

The output gap effect dominates in the medium to long run because of the greater persistence of the output gap, which causes the cashflow risk premium to gradually turn negative after an initial shock to trend inflation.

4.6 The Time Series of Risk Premiums

Figure 10 shows the time series of 10-year risk premiums. In Panel A, we graph the expected total equity return (left-hand axis) along with the dividend yield (right-hand axis). As dividend yields fall, total expected equity returns also decrease. Expected returns on equity are above 20% during the early 1980s and decrease through the 1980s and 2000s, reaching a low of 2.8% during 2000:Q3. This is consistent with low equity returns as macroeconomic volatility declines during the "Great Moderation" as noted by Lettau, Ludvigson and Wachter (2008), and others. After this low, the expected equity return increases during the 2001 recession and remains flat around 10% during the mid-2000s. During the financial crisis expected returns on equity increase to 19% in 2008:Q4. The correlation between the expected 10-year total equity return and dividend yields is 0.90, consistent with Campbell and Shiller (1988), Cochrane (1992), and others, who find that discount rate variation accounts for a large fraction of dividend yield variation.

In Panel B, we break down the total expected equity return into its various components. The real short rate is the same as Figure 2 and is included for completeness. The much lower volatility, and lower average real short rate post the mid-1980s, reflects the Great Moderation. The real duration premium exhibits pronounced swings and increases during every recession. The real duration premium is negative in the early 1980s when real rates were high and volatile. It reaches a low of -8% during the 1987 Savings and Loan crisis, turns positive in the early 1990s, and peaks at 5% during the 1991 recession. During the late 1990s, the real duration premium is around zero, sharply increases during the 2000s recession, slowly falls below zero before the on-set of the financial crisis, and during the financial crisis reaches 3% at 2008:Q4. Economically, during recessions the market prices long-term real bonds with a large duration premium to reflect the increased risk that economic growth is slow during these periods. The duration premium correlates highly with monetary policy shocks (See Table 4), with a correlation of -0.98. The duration premium is highest in recessions because the Fed tends to lower the Fed Funds rates more aggressively than suggested by the Taylor rule during those times.

The next panel graphs the inflation risk premium and expected inflation. The 10-year expected inflation is fairly stable during the sample, and gradually falls from 3.6% at 1982:Q1 to

2.5% at 2008:Q4. The inflation risk premium reflects this gradual decrease in expected inflation. The highest inflation risk premium occurs at the beginning of the sample at 2.4% in 1982:Q1. It is relatively volatile during the 1980s, which reflects the higher volatility of inflation and the real short rate at this time (see also Figure 1). Since 1992, the inflation risk premium has been close to zero. It is even slightly negative during the early 2000s. In our model, we account for the decreasing inflation risk premium by the strong anchoring of trend inflation since the mid-1990s.

The final panel plots the cashflow and real risk premiums. There is a small difference between the two risk premiums prior to the early 1990s, which reflects the larger inflation risk premium during that time. Post-1992, the low and stable inflation risk premiums cause the cashflow and real risk premiums to be almost identical. The equity risk premiums generally trend downwards over the 1980s and 1990s. There is a strong increase in risk premiums during the early 1990s which is not fully reflected in higher dividend yields or higher total expected equity returns (see Panel A). This is due to a drop in the output gap during this period, which lowers the real short rate (see Panel B), and lowers the equity risk premium. The cashflow and real risk premiums are negative during the late 1990s and early 2000s and reach a low of -4% in 2000:Q2. Through the early 2000s equity risk premiums rise to approximately 5%. There is a substantial increase in the equity risk premiums during the financial crisis, which end the sample at 16%. During the financial crisis, the output gap falls substantially (see Figure 1) and the Fed was extremely aggressive in supplying liquidity, so the monetary policy shock is very negative. The increasing macro risk captured by these two factors alone causes an increase in the equity risk premiums. There is also a shock to the equity latent factor, perhaps as a result of additional negative sentiment, which contributes to high risk premiums.

5 Conclusion

We show that a standard, no-arbitrage macro-finance model with the output gap, trend inflation, and a monetary policy shock can fit the nominal and real yield curves well and, augmented with real cashflow risk and real cashflow risk premiums, can match dividend yields. We show that trend inflation endogenously affects real discount rates through the actions of the Federal Reserve operating on nominal short rates. The nominal and real yield curves are upward sloping due to increasing real duration and inflation risk premiums, while equity risk premiums decrease as horizon increases, which is due to the decreasing effect that trend inflation has on the cashflow

risk premium.

Our model reveals that the average 10-year expected return on equity is 10.9%. Decomposing this into its various components, we can break this down into a 3.0% real risk-free rate and a 0.5% real duration premium, which is the premium for holding 10-year real bond. We next move to holding 10-year nominal bonds, which reflect expected inflation of 3.0% and an inflation risk premium of 0.6%. The cashflow risk premium, which is the excess return of equity over nominal bonds, is 3.8%. The corresponding 10-year real equity risk premium, which is the difference between expected equity returns and real bond yields, is 4.4%. At the 10-year horizon, over half the variation of the expected equity return is attributed to variations in real rates and real duration premiums and the remainder is due to the cashflow risk premium. Approximately 70% of the variance of the expected equity return over this horizon is due to fluctuations in the output gap and trend inflation. For the 10-year cashflow and real risk premiums, over 60% of variation is due to the output gap. Thus, over long horizons, equity prices reflect economic growth risk and trend inflation – which are also important components of nominal and real bond prices. We find that the remaining part has some link to the Baker and Wurgler (2007) sentiment factor.

The methodology and findings can be extended in several ways. While we capture the effect of monetary policy on equity prices, a recent policy debate is on whether equity prices should enter policy rules (see, for example, De Grauwe (2008)). We have also abstracted from credit risk and the leverage in equity markets, which authors like Brunnermeier and Pedersen (2009) show are important in determining asset prices. Another interesting extension is to allow for time-varying policy rules following Ang et al. (2011) and how shifts in monetary stances affects the equity premium and stock prices. In addition, the model could be extended to explore the effects of government intervention other than through monetary policy following Pástor and Veronesi (2011a, 2011b) and Ulrich (2011b).

Appendix

A   Derivations

The recursions to price nominal bonds (equation (7)), real bonds (equation (17)), and equity (equation (26)) are derived elsewhere and are standard expressions given the affine framework.

Expected inflation over multiple horizons follows from the VAR set-up, which includes trend inflation as a state variable (equations (2) and (9)), and is an affine function of X_t^o.

In the next two sections, we detail expressions for a log-linear approximation of the dividend yield and the long-horizon expected holding period return of equity. The approximations differ from those previously employed by the literature, especially Campbell and Shiller (1988) and subsequent papers, because we linearize the exact, non-linear price-dividend ratio given by equation (26). In contrast, the Campbell-Shiller log-linear pricing models first linearize the definition of returns, where the approximation error is very small, and then iterate these first-order approximations forward and sum them to infinity, where the approximation error to the true non-linear price-dividend ratio can be much larger.

A.1   Approximate Dividend Yield

We derive $\ln\left(1 + \frac{D_t}{P_t}\right) \approx (h_0 - d_0) + (h_1 - d_1)'\tilde{X}_t$.

We have

$$\ln\left(1 + \frac{D_t}{P_t}\right) \equiv \ln\left(1 + \frac{P_t}{D_t}\right) - \ln\left(\frac{P_t}{D_t}\right), \tag{A.1}$$

where the price-dividend ratio is given by

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \exp(a_n + b_n'\tilde{X}_t) \tag{A.2}$$

restating equation (26).

We log-linearize both terms on the right-hand side of equation (A.1) around the constant $\tilde{\mu}$. In general, for the log function g we have

$$\ln(g(\tilde{X})) = \ln(g(\tilde{\mu})) + \frac{1}{g(\tilde{\mu})} \cdot \frac{\partial g(\tilde{X})}{\partial \tilde{X}}|_{\tilde{X}_t = \mu} \cdot (\tilde{X}_t - \tilde{\mu}) + O\left((\Delta\tilde{X}_t)^2\right)$$

where $O\left((\Delta\tilde{X}_t)^2\right)$ are error terms of second order.

We first linearize $\ln(P_t/D_t)$:

$$
\begin{aligned}
\ln\left(\frac{P_t}{D_t}\right) &= \ln\left(\sum_{n=1}^{\infty} \exp(a_n + b_n'\tilde{\mu})\right) \\
&\quad + \frac{1}{\sum_{n=1}^{\infty}\left(\exp(a_n + b_n'\tilde{\mu})\right)} \cdot \sum_{n=1}^{\infty}\left(\exp(a_n + b_n'\tilde{\mu})b_n'\right)\left(\tilde{X}_t - \tilde{\mu}\right) + O\left((\Delta\tilde{X}_t)^2\right)
\end{aligned}
$$

Grouping all constants into d_0, we can write

$$\ln\left(\frac{P_t}{D_t}\right) \approx d_0 + d_1'\tilde{X}_t. \tag{A.3}$$

We linearize $\ln(1 + P_t/D_t)$ as:

$$\ln\left(1 + \frac{P_t}{D_t}\right) = \ln\left(1 + \sum_{n=1}^{\infty} \exp(a_n + b_n'\tilde{\mu})\right)$$

$$+ \frac{1}{1 + \sum_{n=1}^{\infty}(\exp(a_n + b_n\tilde{\mu}))} \cdot \sum_{n=1}^{\infty}(\exp(a_n) + b_n)\tilde{\mu})b_n'\right)\left(\tilde{X}_t - \tilde{\mu}\right) + O\left((\Delta\tilde{X}_t)^2\right)$$

Collecting all constants into h_0, we can write

$$\ln\left(1 + \frac{P_t}{D_t}\right) \approx h_0 + h_1'\tilde{X}_t. \tag{A.4}$$

Equations (A.3) and (A.3) produce the log-linearized dividend yield.

We do not use the log-linearized dividend yield for estimation, and instead match the exact, non-linear dividend yield in equation (26). Further details on estimation are in Appendix C.

A.2   Expected Equity Holding Period Returns

We derive $\mathrm{E}_t[R_t^E(k)] \approx \hat{c}_0(k) + \hat{c}_1(k)\hat{X}_t$.

We restate the definition of long-horizon expected returns (equations (27) and (28) in nominal and real terms, respectively):

$$\mathrm{E}_t[R_t^E(k)] = \mathrm{E}_t\left(\ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) + ... + \ln\left(\frac{P_{t+k} + D_{t+k}}{P_{t+k-1}}\right)\right)$$

Using the equality

$$\ln\frac{P_{t+1} + D_{t+1}}{P_t} = \ln\left(\frac{P_{t+1}}{P_t}\right) + \ln\left(1 + \frac{D_{t+1}}{P_{t+1}}\right),$$

we have

$$\mathrm{E}_t(R_{t\to t+k}^E) = \sum_{m=1}^{k}\left(\mathrm{E}_t\left[\ln\frac{P_{t+m}}{P_{t+m-1}}\right] + \mathrm{E}_t\left[\ln\left(1 + \frac{D_{t+m}}{P_{t+m}}\right)\right]\right). \tag{A.5}$$

We consider each term in the brackets separately. First consider $\mathrm{E}_t[\ln P_{t+m}/P_{t+m-1}]$. From the log-linearized dividend yield from Appendix A.1 we have

$$\ln\frac{P_t}{D_t} \approx d_0 + d_1'(\tilde{X}_t - \tilde{\mu}).$$

This implies

$$\ln P_{t+m} = \ln D_{t+m} + d_0 + d_1'(\tilde{X}_{t+m} - \tilde{\mu}),$$

so we can write

$$\ln P_{t+m} - \ln P_{t+m-1} = \ln\frac{D_{t+m}}{D_{t+m-1}} + d_1'(\tilde{X}_{t+m} - \tilde{X}_{t+m-1}).$$

Now consider $\mathrm{E}_t[\ln(1 + D_{t+m}/P_{t+m})]$. From Appendix A.1 we have

$$\ln\left(1 + \frac{D_{t+m}}{P_{t+m}}\right) \approx (h_0 - d_0) + (h_1 - d_1)'(\tilde{X}_{t+m} - \tilde{\mu})$$

Combining both terms we have

$$\mathrm{E}_t[R_t^E(k)] = \sum_{m=1}^{k}\left(\mathrm{E}_t\left[\ln\frac{D_{t+m}}{D_{t+m-1}} + d_1'(\tilde{X}_{t+m} - \tilde{X}_{t+m-1})\right] + \mathrm{E}_t\left[(h_0 - d_0) + (h_1 - d_1)'(\tilde{X}_{t+m} - \tilde{\mu})\right]\right).$$

We substitute

$$\ln \frac{D_{t+m}}{D_{t+m-1}} = d_c + d'\mathrm{E}_t[\tilde{X}_{t+m-1}],$$

where the coefficients d_c and d are implied from the VAR in equation (24). This allows us to write the expected holding period return as a linear function of the state vector

$$
\begin{aligned}
E_t[R_t^E(k)] &= \sum_{m=1}^{k} (d_c + (h_0 - d_0) - (h_1 - d_1)'\tilde{\mu}) + \\
&\quad + \sum_{m=1}^{k} (d'\mathrm{E}_t[\tilde{X}_{t+m-1}] + d_1'(\mathrm{E}_t[\tilde{X}_{t+m}] - \mathrm{E}_t[\tilde{X}_{t+m-1}]) + (h_1 - d_1)'E_t[\tilde{X}_{t+m}]). \quad \text{(A.6)}
\end{aligned}
$$

Note that $\mathrm{E}_t[\tilde{X}_{t+m}]$ is affine from the VAR in equation (24):

$$\mathrm{E}_t[\tilde{X}_{t+m}] = (I - \tilde{\Phi})^{-1}(I - \tilde{\Phi}^m)\mu + \tilde{\Phi}^m \tilde{X}_t.$$

Plugging this result into equation (A.6) yields

$$E_t[R_t^E(k)] \approx \hat{c}_0(k) + \hat{c}_1(k)\tilde{X}_t, \quad \text{(A.7)}$$

where

$$
\begin{aligned}
\hat{c}_0(k) &= \sum_{m=1}^{k} c_0(m) + \bar{c}_0(m) \\
\hat{c}_1(k) &= \sum_{m=1}^{k} c_1(m) + \bar{c}_1(m) \\
\bar{c}_0(m) &= d_c + d'(I - \tilde{\Phi})^{-1}(I - \tilde{\Phi}^{m-1})\tilde{\mu} + d_1'\tilde{\Phi}^{m-1}\tilde{\mu} \\
\bar{c}_1(m) &= d'\tilde{\Phi}^{m-1} + d_1'(\tilde{\Phi} - I)\tilde{\Phi}^{m-1} \\
c_0(m) &= (h_0 - d_0) + (h_1 - d_1)'((I - \tilde{\Phi})^{-1}(I - \tilde{\Phi}^m) - I)\tilde{\mu} \\
c_1(m) &= (h_1 - d_1)'\tilde{\Phi}^m \tilde{X}_t
\end{aligned}
$$

B   Data

We follow Rudebusch and Svensson (2002) and others and define the output gap, g_t, as

$$g_t = \frac{1}{4}\frac{Q_t - Q_t^*}{Q_t^*}, \quad \text{(B.1)}$$

where Q_t is real GDP from the Bureau of Economic Analysis (BEA) constructed using chained 2000 dollars and Q_t^* is potential GDP from the Congressional Budget Office (CBO). The original CBO potential GDP series is constructed with chained 1996 dollars so we make both the BEA and CBO series comparable by translating real GDP into 1996 dollars. We divide by four to express the output gap in per quarter units.

We match quarterly realized inflation with the growth rate of the consumer price index (without food and energy). Data of the consumer price index is from the St. Louis Federal Reserve. Since the quarter-on-quarter growth rate, i.e. $\ln CPI(t+1) - \ln CPI(t)$, is seasonal, we take the change in inflation over the past four quarters expressed at the quarterly frequency:

$$\pi_t = \frac{1}{4}\frac{\ln CPI(t)}{\ln CPI(t-4)}, \quad \text{(B.2)}$$

where $CPI(t)$ is the level of the CPI index at time t. The time-series of inflation over our sample is not volatile (see the literature on the Great Moderation, e.g. Stock and Watson, 2002) and so our results are not sensitive to which inflation proxy is used and robust to other measures to treat seasonality (see also comments by Ang, Bekaert and Wei, 2008).

Our equity return is the CRSP value-weighted stock index including dividends. We construct non-seasonal year-on-year dividend yields and real dividend growth at the quarterly frequency. Quarterly dividend yields, which are given by

$$\frac{D_{t+1}}{P_{t+1}} = \left(\frac{P_{t+1}}{P_t}\right)^{-1} \left(\frac{P_{t+1} + D_{t+1}}{P_t} - \frac{P_{t+1}}{P_t}\right), \tag{B.3}$$

are highly seasonal. Instead, we sum dividends in each quarter to obtain the dividend yield:

$$dy_t = \frac{1}{4} \frac{D_t + D_{t-1} + D_{t-2} + D_{t-3}}{P_t}, \tag{B.4}$$

and the inverse is the empirical counterpart to equation (26). Summing dividends over the past year avoids the seasonality of using quarterly dividends.

We similarly construct a measure of year-on-year dividend growth at the quarterly frequency. Nominal dividend growth, $d_t^{\$}$ is given by

$$d_t^{\$} = \ln\left(\frac{dy_t}{dy_{t-4}} \cdot \frac{P_{t-4}}{P_t}\right),$$

which uses the sum of the last four quarters of real dividends to compute the year-on-year growth of real dividends at the quarterly frequency and and dy_t is defined in equation (B.4). We deflate by quarterly inflation to obtain real dividend growth, d_t:

$$d_t = d_t^{\$} - \pi_t. \tag{B.5}$$

C   Model Estimation

We conduct a three-step estimation method. First, we estimate all parameters associated with observable macro factors. This step includes the Taylor rule in equation (1), the VAR parameters for X_t^o in equation (2), and the inflation dynamics in equation (9). In the second step, we hold these parameters constant and set the latent factor L_t to zero and estimate the risk prices λ by minimizing the squared difference between model-implied nominal yields, real yields, and equity dividend yields with their data counterparts. By setting L_t to zero, this step assigns as much explanatory power as possible to the macro variables. In the third step, we estimate L_t by matching dividend yields exactly.

C.1   Estimation of Macro Observable X_t^o Processes

We first estimate the Taylor rule on the Fed Funds rate (equation (1)), from which we extract the monetary policy shocks, $\{f_t\}$. We then estimate the VAR for the macro variables X_t^o in equation (2) and the real cashflow growth equation (20). All these estimations are done by OLS. The latter is consistent because L_t is assumed to be orthogonal to all other factors.

The inflation dynamics in equation (9) are estimated the following way. We pin down π_c as the sample mean of realized inflation. The covariances Σ^π are identified through the covariance of $\Sigma\epsilon_{t+1}$ in equation (2) and $\Sigma^\pi\epsilon_{t+1}$ in equation (9). Then, the volatility of the inflation-specific shock is given by $\sigma_p = \sqrt{\text{var}(\pi_{t+1} - \pi_c - \pi_t^e) - \Sigma^\pi\Sigma^{\pi\prime}}$.

C.2 Fitting Nominal Bond, Real Bond, and Equity Yields

The second step of the estimation fixes the parameters of the first step and matches jointly seven nominal yields, five real yields and the dividend yield. We minimize the squared differences between the model-implied yields and the yields implied in the data. The model-implied nominal and bond yields are affine in the state variables (equations ((18)) and ((8)), respectively). In our estimation, we use the exact, analytical expression for the dividend yield in equation ((26)).[17] In this second step we estimate 16 market price of risk parameters: $\lambda_0^\$$, $\lambda_1^\$$, and λ^d. As shown in equation (14), the real market prices of aggregate risk λ_0^r and λ_1^r are also identified because they are deterministic functions of $\lambda_0^\$$, $\lambda_1^\$$, and Σ^π.

C.3 Estimating the Equity Latent Factor, L_t

In the last step of the estimation, we estimate the equity latent factor, L_t, which affects the expected real cashflow growth rate and the cashflow risk premium. To estimate the parameters associated with L_t, we fix all other parameters from the previous two estimation steps and optimize only over $\theta_L := (\phi_{dL}\,\sigma_L\,\phi_L\,\sigma_d)$. For every parameter vector θ_L, we invert the log-linearized dividend yield to get an estimate for $\{L_t\}$, similar to Chen and Scott (1993). We optimize the parameter vector θ_L and the time-series estimate for $\{L_t\}$ by maximizing the joint likelihoods for L_t, which is an AR(1) following equation (22), and for d_t. The likelihood for d_t is given by the VAR in equation (24) and is also conditionally normally distributed. By matching L_t with all other parameters held fixed, we effectively assign the pricing error in the dividend yield not picked up by the observable macro factors and the prices of risk to L_t.

C.4 Standard Errors

We compute standard errors using the combined likelihood for all equations of motion and state variables. Due to the orthogonality assumptions, the three-step estimation procedure ensures the combined likelihood is maximized. We compute standard errors using the outer product of the score of the log-likelihood function and determine the score numerically with a second order approximation scheme.

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[17] Estimation with the log-linearized dividend yield produces very similar results.

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Table 1: Parameter Estimates

	g	π^e	f	d	L	σ_π

Taylor Rule Parameters

	c	a	b	
R	0.0149	0.3140	2.4355	1.0000
	(0.0004)	(0.1578)	(0.0659)	–

Inflation Parameters

	π_c	Σ_π				σ_π
π	0.0083	-0.0005	-0.0007	0.0003		0.0015
	(0.0010)	(0.0010)	(0.0008)	(0.0008)		(0.0006)

VAR Parameters

	d_c	$\tilde{\Phi}$				
g	-1.0e-4	0.9950	0.0885	0	0	0
	(0.0000)	(0.0039)	(0.0068)	–	–	–
π^e	-1.0e-4	0.0055	0.8897	0	0	0
	(0.0000)	(0.0004)	(0.0007)	–	–	–
f	0	0	0	0.5257	0	0
	–	–	–	(0.0080)	–	–
d	0.0050	-0.3920	-2.4525	-0.2097	0.2609	0.0554
	(0.0000)	(0.0050)	(0.0110)	(0.0081)	(0.0010)	(0.0010)
L		0	0	0	0	0.1564
		–	–	–	–	(0.0600)

Volatility

	$\tilde{\Sigma}$				
g	0.0015				
	(0.0001)				
π^e	0.0000	0.0010			
	(0.0000)	(0.0000)			
f	0	0	0.0036		
	–	–	(0.0000)		
d	0	0	0	0.0010	
	–	–	–	(0.0000)	
L	0	0	0	0	0.4014
	–	–	–	–	(0.0300)

Risk Premia Parameters

	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$				
g	-1.0493	223.85	-368.81	-482.30	0	0
	(0.0102)	(2.2182)	(3.8623)	(6.5380)	–	–
π^e	-0.2999	-5.5667	-95.169	-89.171	0	0
	(0.0019)	(0.4118)	(0.7163)	(1.2021)	–	–
f	0.5502	-53.460	144.61	247.01	0	0
	(0.0036)	(0.7757)	(1.3513)	(2.2988)	–	–
d	-3.8595	-1756.9	-1466.0	-11507	0	998.80
	(9.5000)	(633.49)	(1173.0)	(43320)	–	(67.710)

Note to Table 1

The table reports parameter estimates of the model in Section 2. The variable g denotes the output gap, π^e is trend inflation, f is a monetary policy shock, d is real dividend growth, and L is the latent equity premium factor. The Taylor rule is given by equation (1). The VAR is given by equation (24) and consists of the dynamics of $X^o = (g \, \pi^e \, f)'$, the real dividend growth equation (20), and the latent factor equation (22). The prices of risk are given in equation (25), which consists of the prices of risk of X^o in equation (3) and the price of risk of d in equation (21). The sample is from 1982:Q1 to 2008:Q4 and the frequency is quarterly.

Table 2: Latent Equity Factor Regressions

	I	II	III	IV	V	VI
Credit Spread	1.4616				-18.579	-27.068
	[0.08]				[0.15]	[-1.32]
VIX		-0.8198			0.1298	0.2018
		[-1.09]			[0.15]	[0.28]
Sentiment			-24.654		-26.496	-27.565
			[-5.42]		[-5.55]	[-6.03]
Liquidity				26.838	19.231	27.553
				[0.43]	[0.36]	[0.53]
g						-0.5245
						[-0.32]
π^e						-8.0747
						[-2.45]
f						-2.2255
						[-0.72]
Adjusted R^2	-0.01	0.02	0.28	-0.01	0.27	0.26

We regress the latent equity factor L_t onto various factors. The credit spread is the Baa-Aaa credit spread, VIX is the VIX volatility index, sentiment is the Baker and Wurgler (2007) sentiment factor, and liquidity is the Pástor and Stambaugh (2003) liquidity factor. The remaining factors are our macro factors: the output gap, g, trend inflation, π^e, and the monetary policy shock, f. The regressions use data from 1990:Q1 to 2007:Q4 as this is the only sample for which all data are available. Robust t-statistics are reported in squre brackets.

Table 3: Decomposing Total Expected Equity Returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Real rate	Real Duration Premium	Expected Inflation	Inflation Risk Premium	Cashflow Risk Premium	Real Risk Premium	Expected Total Return
k (yrs)	r	DP	$\mathrm{E}(\pi)$	IRP	CFP	RRP	$\mathrm{E}(R^{E,\$})$
Mean Decomposition							
5	2.9756	0.0604	3.0950	0.4142	4.7139	5.1281	11.259
10	2.9756	0.5020	3.0028	0.6048	3.8409	4.4457	10.926
15	2.9756	0.8012	2.9460	0.6241	3.7018	4.3259	11.049
Variance Decomposition							
5	0.1888	0.0810	0.0043	0.0036	0.7223	0.7259	1.0000
10	0.3745	0.1781	0.0030	0.0112	0.4332	0.4444	1.0000
15	0.4376	0.2229	0.0020	0.0133	0.3242	0.3375	1.0000

We report decompositions of the expected total equity return, $\mathrm{E}_t(R_t^{E,\$}(k))$, and its variance into the various risk premiums; real short rate, r_t; real duration premium, $DP_t(k)$; inflation risk premium, $IRP_t(k)$; expected inflation, $\mathrm{E}_t(\pi_t(k))$; real risk premium, $RRP_t(k)$, and the real cashflow risk premium, $CFP_t(k)$, as defined in Section 2.7. The total expected return in (7) is the sum of columns (1) to (5). The real risk premium is the sum of columns (4) and (5). Horizons k are given in years. The numbers in the mean decomposition are in annualized percentage terms. The premiums are computed at the sample mean of the factors.

Table 4: Factor Variance Decompositions of Risk Premiums

	g	π^e	f	d	L
Real Short Rate	0.0294	0.3774	0.5932	0.0000	0.0000
Real Duration Premium	0.0062	0.0473	0.9465	0.0000	0.0000
Expected Inflation	0.1160	0.8840	0.0000	0.0000	0.0000
Inflation Risk Premium	0.0125	0.9211	0.0664	0.0000	0.0000
Cashflow Risk Premium	0.6186	0.0174	0.0415	0.0000	0.3225
Real Risk Premium	0.6015	0.0429	0.0422	0.0000	0.3134
Expected Total Return	0.2703	0.5007	0.0018	0.0000	0.2272

We report decompositions of the variance of the real short rate and various risk premiums, as defined in Section 2.7. The horizon is 10 years for all risk premiums and expected inflation, except for the real short rate.

Figure 1: Output Gap, Trend Inflation, and Monetary Policy Shocks
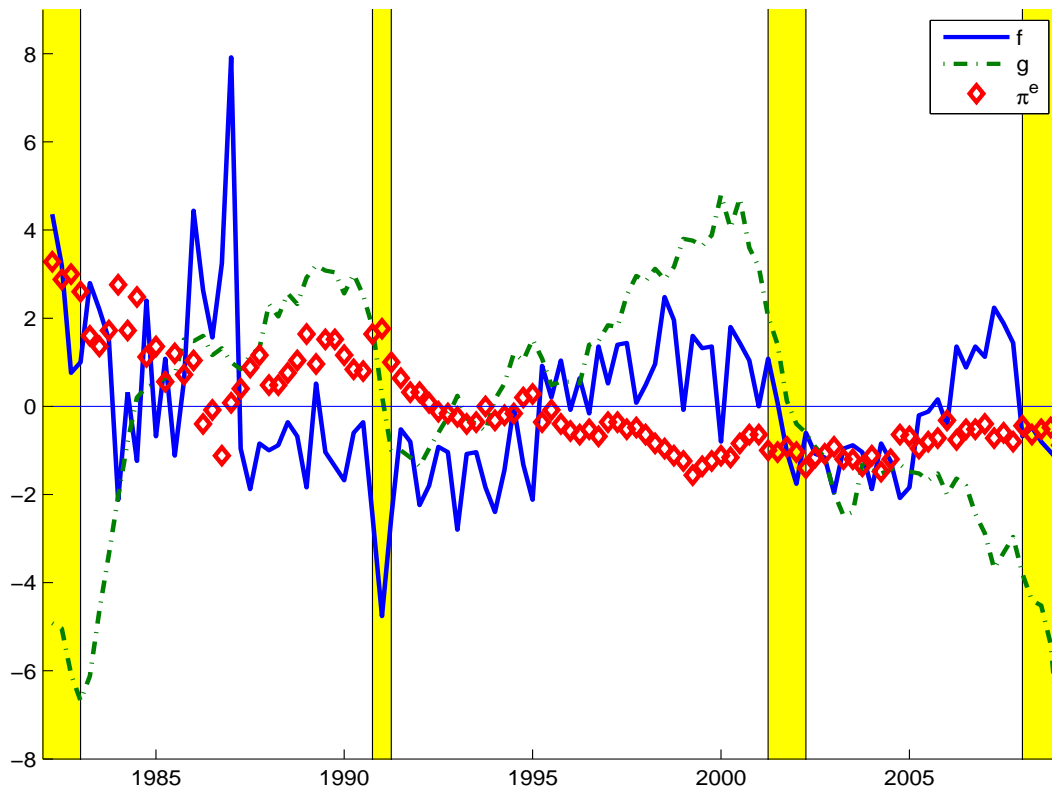
We plot the output gap, g_t, trend inflation, π_t^e, and the monetary policy shock, f_t. All variables have been demeaned and are annualized. Shaded areas represent NBER recessions. The sample is from 1982:Q1 to 2008:Q4 and the frequency is quarterly.

Figure 2: Real and Nominal Short Rates
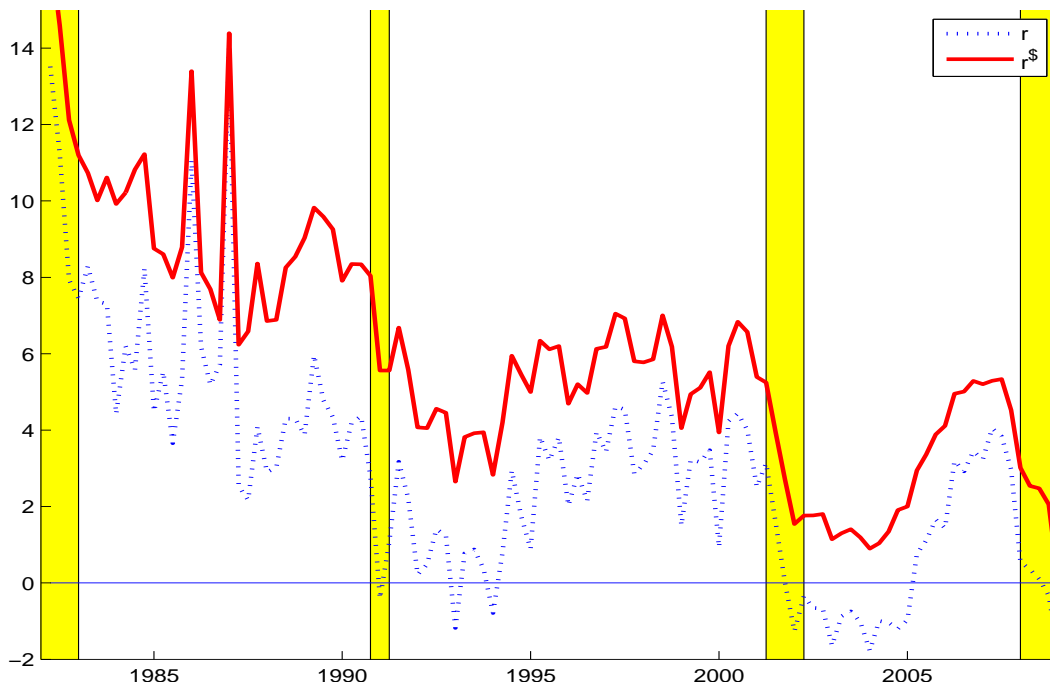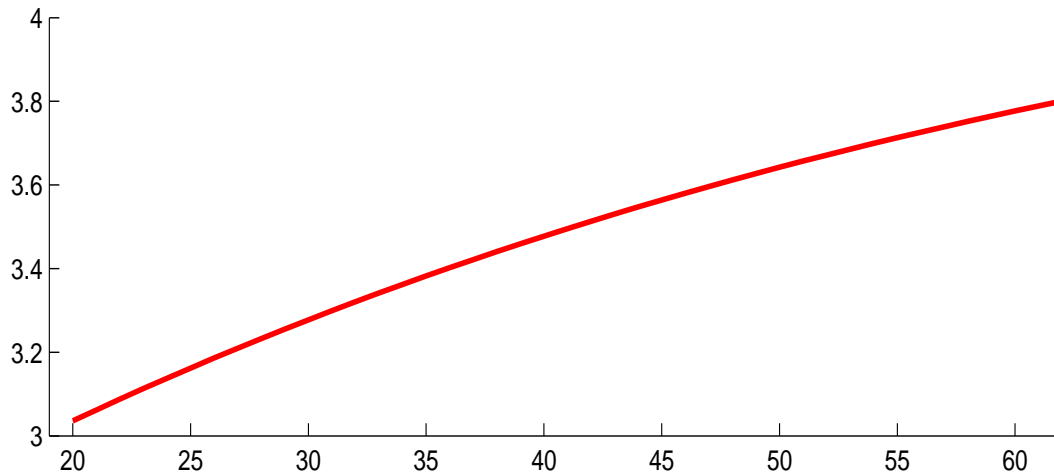
We plot the real short rate, r, which is implied by the model in equation (12) and unobserved in data, and the nominal short rate, $r^\$$, which is the Fed Funds rate. Shaded areas represent NBER recessions. The sample is from 1982:Q1 to 2008:Q4 and the frequency is quarterly.

Figure 3: Nominal Yield Curve
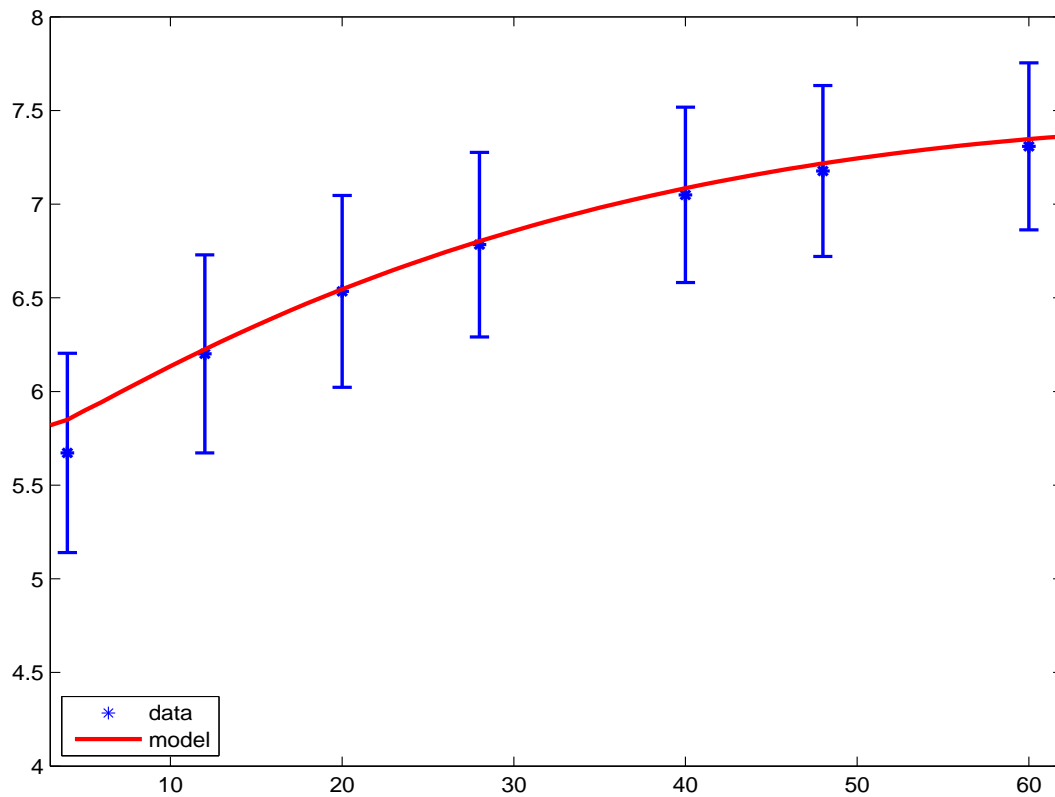
We plot the model-implied average term structure of nominal zero-coupon bonds (solid line) together with average yields, with two standard error bounds, in data. Maturities on the x-axis are in quarters and the yields are annualized on the y-axis. The sample is from 1982:Q1 to 2008:Q4.

Figure 4: Real Yield Curve
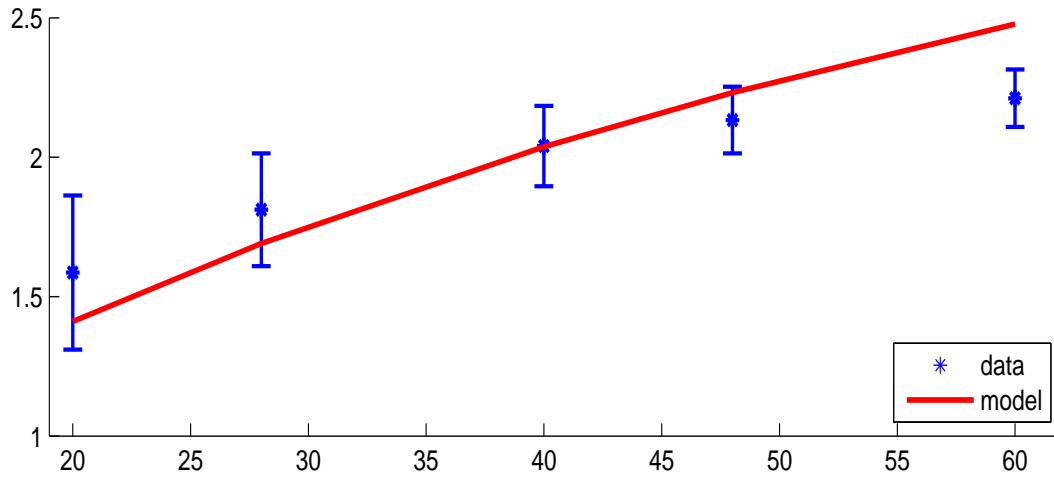
In the top panel, we plot the model-implied average term structure of yield zero-coupon bonds (solid line) together with average real yields, with two standard error bounds, in data. The sample period is from 2003:Q1 to 2008:Q4. The bottom panel plots the model-implied average term structure of real interest rates over the full sample 1982:Q1 to 2008:Q4. In both panels, maturities on the x-axis are in quarters and the yields are annualized on the y-axis.

47

Figure 5: Dividend Yields
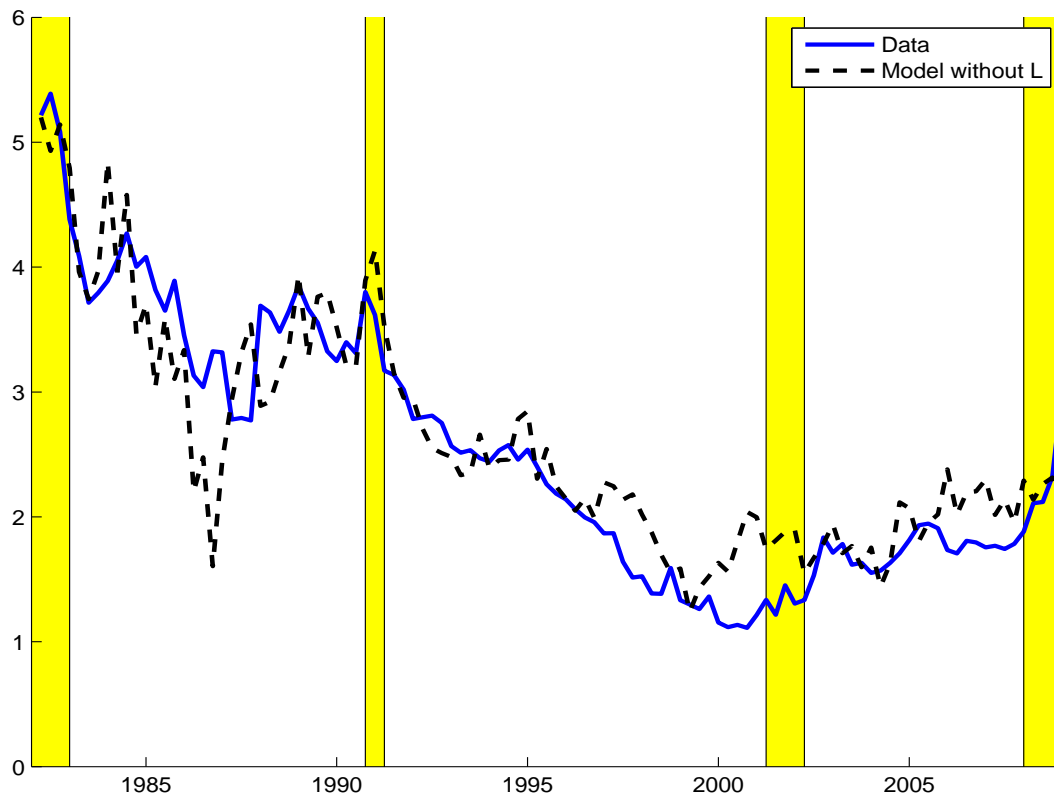
We plot the model-implied dividend yield where the model does not include the latent factor, L, (dashed line) and the dividend yield in data (solid line). The dividend yield is annualized and in percent. The sample is from 1982:Q1 to 2008:Q4.

Figure 6: Latent Equity Factor
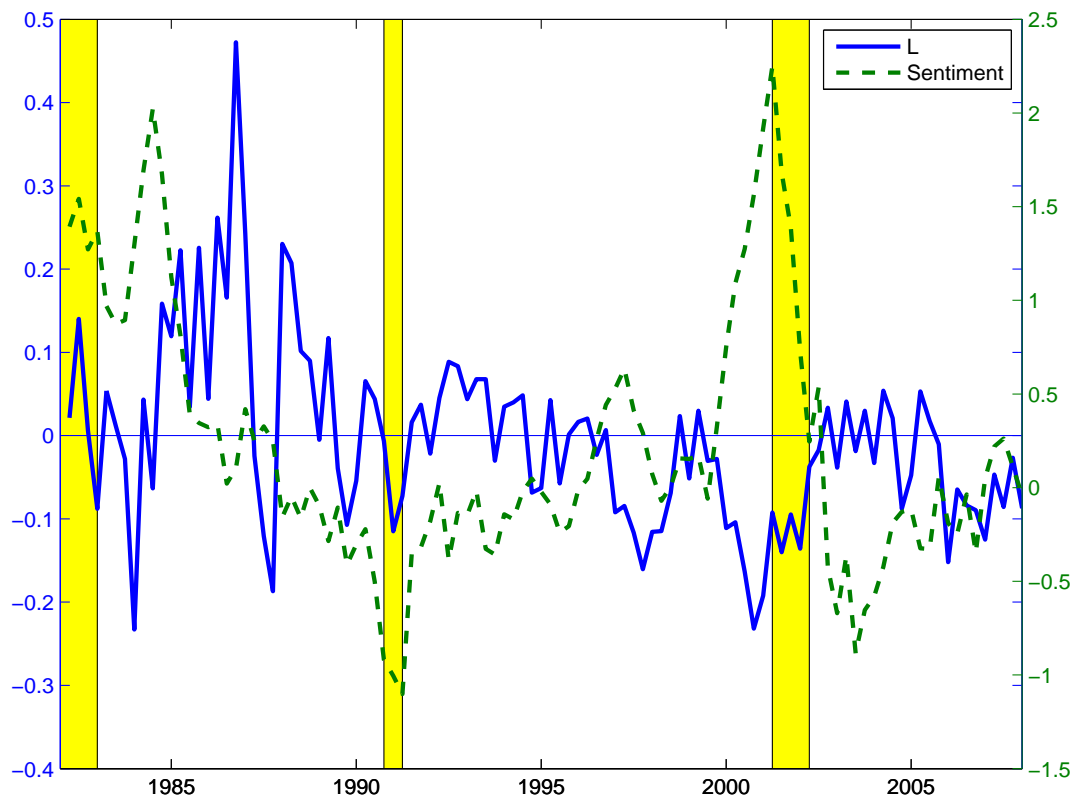
We plot the latent equity factor L (left-hand axis) in the solid line together with the Baker and Wurgler (2007) sentiment factor (right-hand axis) in the dashed line.

Figure 7: Term Structure of Equity Returns
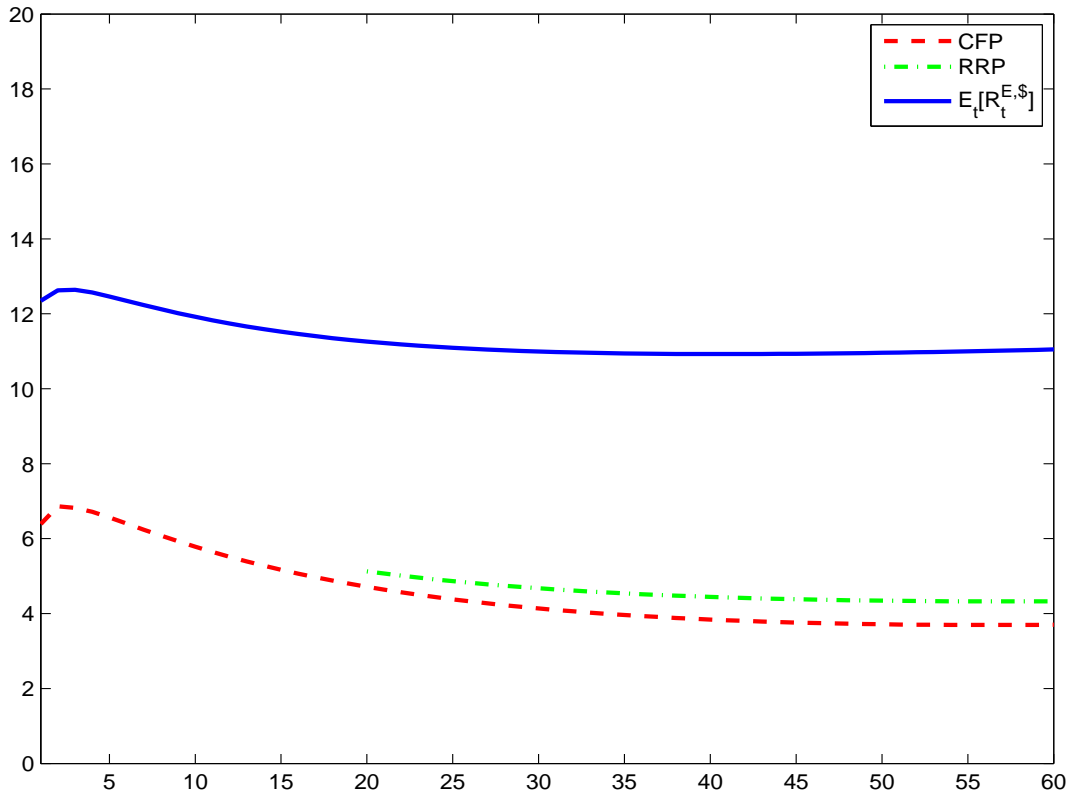
We plot the term structure of the total equity return, $\mathrm{E}_t[R_t^{E,\$}(k)]$, in the solid line, the cashflow premium, $CFP_t(k)$, defined in equation (31) in the dashed line, and the real equity risk premium, $RRP_t(k)$ defined in equation (32) in the dashed-dotted line for various k starting at the sample mean of the factors. Maturities on the x-axis are in quarters.

Figure 8: Term Structure of Risk Premiums
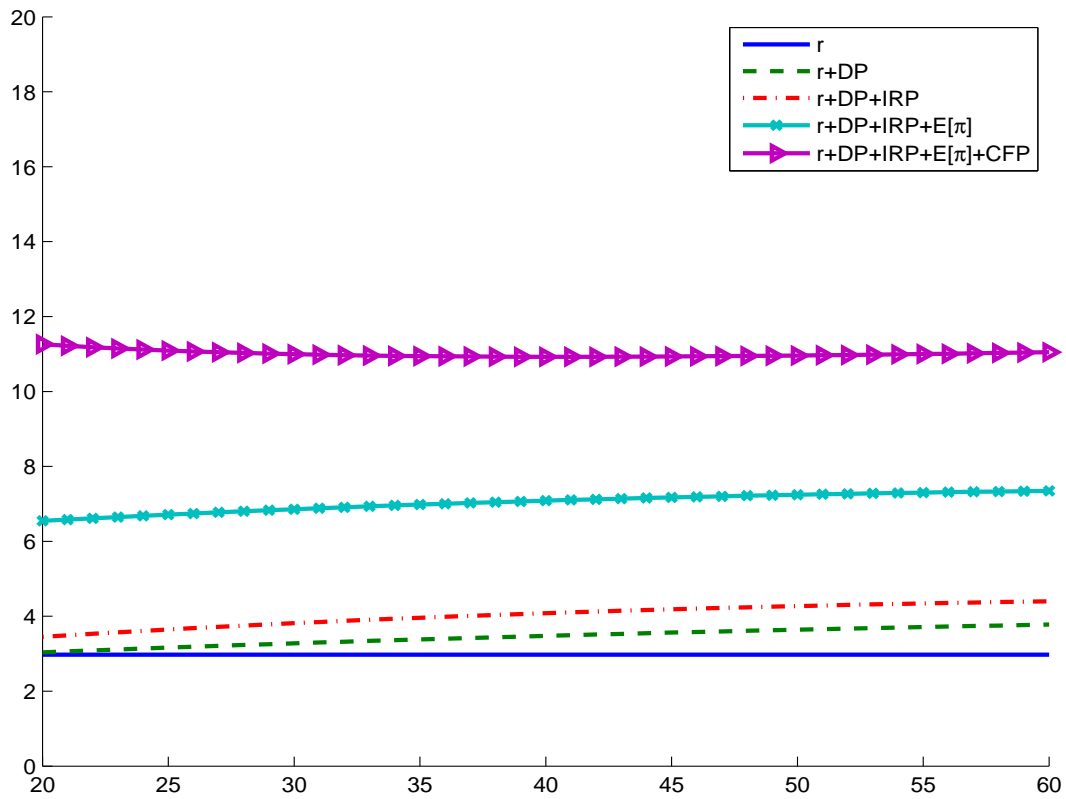
We plot the term structure of cumulative returns that consists of the real short rate, r, the duration premium, DP, the inflation risk premium, IRP, expected inflation, $E[\pi]$, and the cashflow premium, CFP. Maturities on the x-axis are in quarters.

Figure 9: Cashflow Risk Premium Impulse Responses
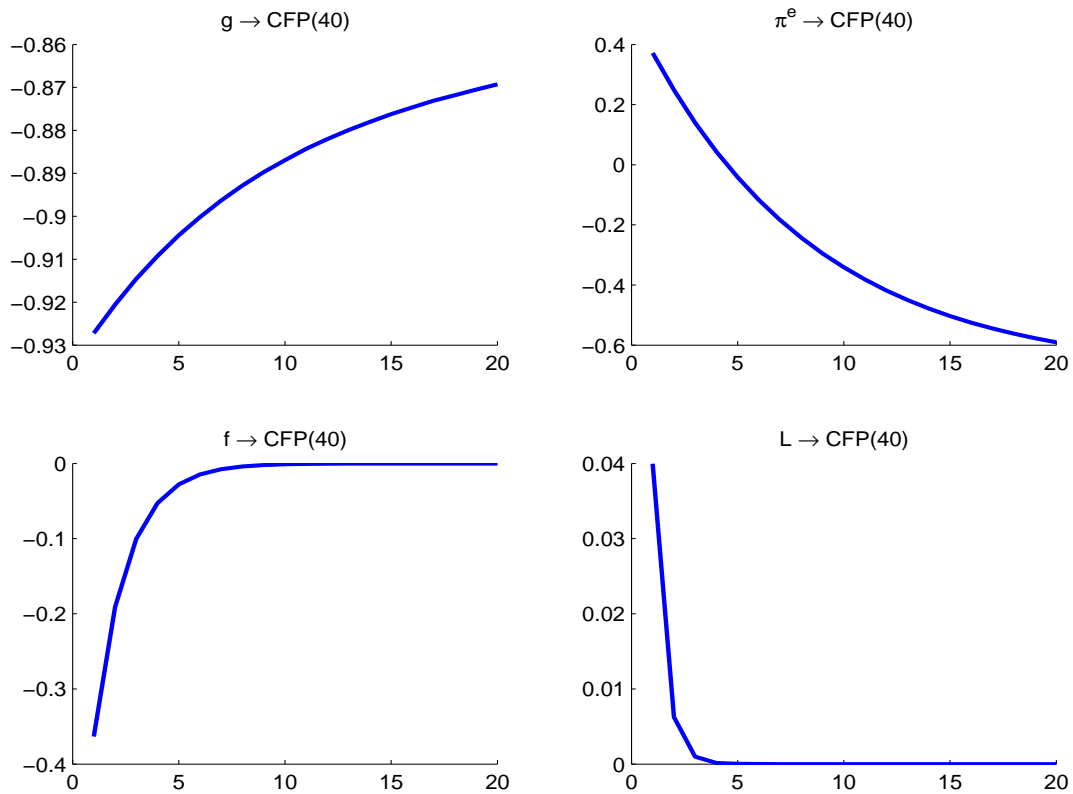
We plot impulse responses for the cashflow risk premium, CFP_t, or the equity risk premium over nominal bonds defined in equation (31) for a horizon of $k = 40$ quarters. We consider 1% moves in the shocks to each factor. The units of the x-axis are in quarters and the units on the y-axis are annualized and are in percent.

Figure 10: Risk Premiums

Panel A: Expected Total Equity Return
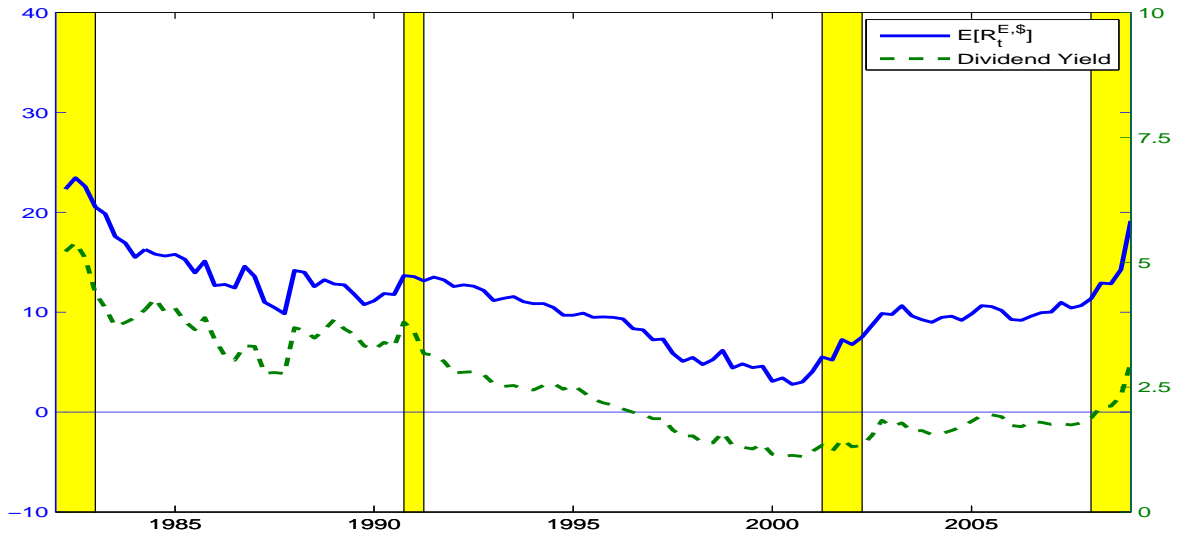
Panel B: Risk Premiums
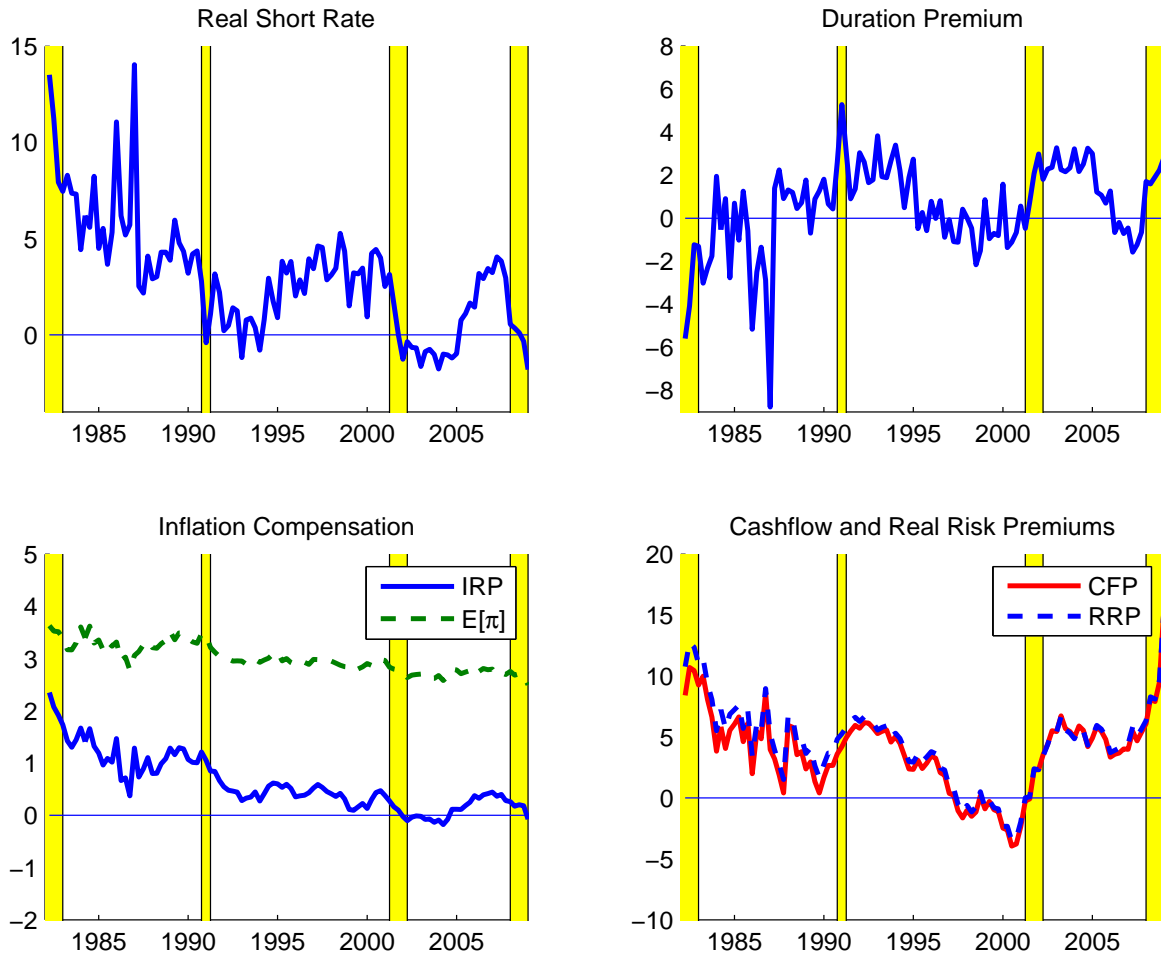
Note to Figure 10

In Panel A, we plot the total expected nominal equity return for a 10-year horizon, $E_t[R_t^{E,\$}(k)]$ for $k = 40$ quarters, implied by the model in the solid line. Units on the left-hand y-axis are in percent and correspond to the expected equity return. We overlay the dividend yield, which is plotted in the dotted line and has units on the right-hand y-axis in percent. In Panel B, we decompose the total expected equity return into the real short rate, real duration premium, expected inflation ($E(\pi)$) and the inflation risk premium (IRP), and the cashflow risk premium (CFP) and the real risk premium (RRP). All of these components correspond to a 10-year horizon and are defined in Section 2.7. Shaded areas represent NBER recessions. The sample is from 1982:Q1 to 2008:Q4 and the frequency is quarterly.