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## Introducing health gains in location-allocation models: A stochastic model for planning the delivery of long-term care

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**Abstract.** Although the maximization of health is a key objective in health care systems, location-allocation literature has not yet considered this dimension. This study proposes a multi-objective stochastic mathematical programming approach to support the planning of a multi-service network of long-term care (LTC), both in terms of services location and capacity planning. This approach is based on a mixed integer linear programming model with two objectives – the maximization of expected health gains and the minimization of expected costs – with satisficing levels in several dimensions of equity – namely, equity of access, equity of utilization, socioeconomic equity and geographical equity – being imposed as constraints. The augmented  $\epsilon$ -constraint method is used to explore the trade-off between these conflicting objectives, with uncertainty in the demand and delivery of care being accounted for. The model is applied to analyze the (re)organization of the LTC network currently operating in the Great Lisbon region in Portugal for the 2014-2016 period. Results show that extending the network of LTC is a cost-effective investment.

### 1. Introduction

Long-term care (LTC) aims at improving the quality of life of individuals who are dependent on help with basic activities of daily living due to chronic illness and/or disability [1]. LTC includes the organization of both nonmedical and medical services, with these varying across countries [1]. In particular, different services can be provided (with services ranging from short-term to longer institutionalizations, home-based and ambulatory services) and different entities can be responsible for providing care (e.g., family, public, private and not-for-profit entities).

The European LTC sector has been facing several challenges, as there has been an increasing demand for LTC, mainly due to the ageing phenomenon and to the increasing prevalence of chronic diseases [2]. Furthermore, the current supply of LTC is clearly not enough to meet this growing demand [2]. Within this context, planning the delivery of LTC is a top policy priority in many European countries. Particularly, when facing a strong pressure to decrease and control public spending in health care, countries based on a National Health Service (NHS) structure need to plan networks of LTC that minimize costs and account for other top level system objectives, such as the attainment of health gains and of equity. To help such planning, decision support tools are required.

Mathematical programming models have been broadly used in the health care planning literature [3], in particular for planning the location-allocation of services. Nevertheless, there is a lack of studies comprehensively modeling key features of health systems that are critical for proper planning



in real settings. In particular, although a vast literature in the area of health care planning relies on the use of single objective mathematical programming models [4], only recently models with multiple objectives have been proposed in this area (examples can be found in [5-7]). Within the existing models, equity [8], costs [7] and health benefits [9] related objectives have been accounted for, but to the authors' knowledge, no study has addressed the joint attainment of these different dimensions. Also, a vast health care planning literature relies on equity or cost related objectives (such as [7] [8,10]), whereas health benefits has seldom been used (examples can be found in [9,11]). Also, few research has addressed uncertainty in an integrated way, for instance using stochastic models [12]. To the authors' knowledge, the only stochastic models proposed for strategic and tactical planning of health care services can be found in [6,7,13]. Furthermore, few studies have modelled multi-service systems, and decomposed the planning horizon into periods [7,14]. Additionally, very few studies were dedicated to the LTC sector [15], with no LTC study comprehensively addressing all the features that are relevant for health systems.

This paper develops a multi-objective two-stage stochastic mixed integer linear programming (MILP) model to support location-allocation decisions in the LTC sector in the context of a NHS-based country. The proposed model assists LTC planners on how to (re)organize a multi-service network of LTC (including a wide range of institutional, home-based and ambulatory care services), providing information related to capacity planning and location selection. A key feature of the study is related to the modeling of health gains and its inclusion in the objective function, and to be analyzed against the cost dimension. These two objectives are dealt with through the use of the augmented  $\epsilon$ -constraint method. Equity considerations in several dimensions (equity of access, equity of utilization, socioeconomic equity and geographical equity) are accounted for in the model through the imposition of satisficing equity levels as model constraints. LTC demand and supply uncertainty are also considered in the model. A case study in Portugal for the 2014-2016 period is used to show the applicability of the model. For the purpose of this case study, health gains are proxied by the Quality Adjusted Life Years (QALYs) gained with the delivery of each type of LTC service. QALYs are a standard measure used to inform resource allocation decisions in the health care sector [16].

The remaining of this paper presents the methodology in Section 2, and the case study and key results in Section 3. Key conclusions and lines for further research are presented in Section 4.

## 2. Methodology

This study proposes a multi-objective stochastic approach based on a MILP model to support capacity planning and location decisions in the LTC sector. This section starts by presenting background information related to this problem, and then describes the mathematical structure of the model.

### 2.1. LTC planning background

The location-allocation model proposed in this study supports the (re)organization of a LTC network in the context of a NHS-based country, such as the one that currently operates in Portugal [17]. The model departs from a multi-service network of LTC that ensures the provision of institutional (IC), home-based (HBC) and ambulatory (AC) care services [18]. IC comprise convalescence care (CC), medium-term and rehabilitation care (MTRC), long-term and maintenance care (LTMC) and palliative care (PC), with these different services being characterized by different lengths of stay (LOS). We consider that the state is responsible for establishing contracts with public and private providers so as to ensure the provision of IC, whereas HBC and AC are mainly provided by the state [18].

The planning model provides guidance on i) where and when multiple LTC services should be delivered, ii) how much capacity should be available in each service (both in terms of beds and human resources), iii) how to distribute this capacity across services and patient groups, and iv) which changes are needed in this network over time, including capacity increasing or reduction and the closure and opening of services. These decisions are evaluated in a context of severe budget constraints or limited increase in health care spending, and so the planning of LTC networks needs to consider the minimization of costs. Furthermore, it is a cornerstone of any health system the

maximization of health gains for a given level of available resources. The augmented  $\epsilon$ -constraint method is employed to explore the trade-off between these conflicting objectives. In this article, other important policy objectives regarding equity – in particular, equity of access (EA), equity of utilization (EU), socioeconomic equity (SE) and geographical equity (GE) – are dealt with as model constraints, assuming that the policy maker accepts achieving equity satisficing levels (following the satisficing concept from Simon [19]).

Since future LTC demand cannot be predicted with total confidence and there is inherent uncertainty in LTC delivery, data uncertainty is modeled either in terms of the number of individuals requiring LTC and in terms of the amount of services required by each of those individuals, as captured by the LOS. A scenario tree approach [20] is selected for handling the uncertainty associated with this uncertain data. Within this two-stage stochastic model, first-stage decisions include the opening and closure of services and investment in new beds, whereas second-stage decisions comprise decisions related to the allocation and reallocation of patients and resources. The objective of the proposed model is thus to minimize expected costs and maximize expected health gains over the uncertain demand scenarios, while simultaneously ensuring the achievement of pre-defined satisficing levels of equity.

## 2.2. Mathematical formulation

2.2.1. *Notation.* The following indices, sets, parameters and variables will be used.

### Indices

$d$	Demand points	$s, p$	LTC services	$r$	Human resources
$g$	Socioeconomic groups	$t, w$	Time periods		
$l, j$	Locations for services	$n, k$	Scenario tree nodes		

### Sets

$D$	Time periods
$G = G_P \cup G_{NP}$	Socioeconomic groups, divided into subsets $G_P$ (individuals with priority as a result of having lower levels of income) and $G_{NP}$ (other individuals).
$L = L_I \cup L_O$	Locations for services, divided into subsets $L_I$ (locations for IC services) and $L_O$ (locations for HBC and AC services).
$R$	Human resources
$S = S_I \cup S_O$	LTC services, divided into subsets $S_I$ (IC services) and $S_O$ (HBC and AC services)
$T$	Time periods
$N$	Scenario tree nodes
$A = \{(s, r) : s \in S, r \in R\}$	Services $s$ provided by human resources $r$
$F = \{(d, l) : d \in D, l \in L\}$	Demand points $d$ that can receive LTC in locations $l$
$P = \{(k, n) : k, n \in N\}$	Predecessors $k$ of the scenario tree node $n$
$Q = \{(t, n) : t \in T, n \in N\}$	Time periods $t$ associated to scenario tree nodes $n$
$U = \{(s, l) : s \in S, l \in L\}$	Services $s$ that can be provided in locations $l$
$V = \{(s, l) : s \in S, l \in L\}$	Services $s$ provided in locations $l$ at $t=0$
$Z = \{(s, l) : s \in S, l \in L\}$	Services $s$ not provided in locations $l$ at $t=0$

### Parameters

$D_{dgstn}$	Number of individuals from demand point $d$ and socioeconomic group $g$ requiring service $s$ at $t$ in scenario tree node $n$
$gD_{tn}$	Number of individuals from lower income groups ( $g \in G_P \subseteq G$ ) requiring LTC at $t$ in scenario tree node $n$
$rD_{dm}$	Number of individuals from demand point $d$ requiring LTC at $t$ in scenario tree node $n$
$uD_{stn}$	Number of individuals requiring service $s$ at $t$ in scenario tree node $n$
$eB_{sl}^0$	Number of beds available in IC service $s$ ( $s \in S_I \subseteq S$ ) located in $l$ ( $l \in L_I \subseteq L$ ) at $t=0$
$mB_s / MB_s$	Minimum/maximum bed capacity allowed per IC service $s$ ( $s \in S_I \subseteq S$ )
$mI_s / MI_s$	Minimum/maximum number of individuals allowed per HBC/AC service $s$ ( $s \in S_O \subseteq S$ )

$HRp_{rs}$	Hours of care to be provided by human resource $r$ for each individual requiring service $s$
$EA_t / EU_t / GE_t / SE_t$	Target defined for EA/EU/GE/SE at $t$
$iC_{st}$	Investment cost per new bed installed in IC service $s$ ( $s \in S_l \subseteq S$ ) at $t$
$oC_{st}$	Operational cost per service $s$ per period $t$
$rC_{st}^1 / rC_{st}^2$	Cost of reallocating a bed to IC service $s$ ( $s \in S_l \subseteq S$ ) from a service delivered in a different location/from another service delivered in the same location at $t$
$r_t$	Inflation rate at $t$
$LOS_{sn}$	Average length of stay (LOS, in days) in IC service $s$ ( $s \in S_l \subseteq S$ ) in scenario tree node $n$
$TP$	Number of days per time period
$\alpha_{st}$	Minimum level of demand that need to be satisfied per service $s$ at $t$
$\beta$	Maximum travel time allowed for patients accessing IC services (in minutes)
$\theta_{dl}$	Travel time between demand point $d$ and service location $l$ (in minutes)
$\varepsilon_s$	Efficiency factor associated with the provision of service $s$
$\rho_n$	Probability of scenario tree node $n$

### Variables

$X_{slt}$	Equal to 1 if service $s$ is located in $l$ at $t$ ; 0 otherwise
$aB_{slt}$	Number of additional beds to invest in for IC service $s$ ( $s \in S_l \subseteq S$ ) located in $l$ ( $l \in L_l \subseteq L$ ) at $t$
$B_{dgsln}$	Number of beds required for individuals from demand point $d$ belonging to socioeconomic group $g$ and receiving IC service $s$ ( $s \in S_l \subseteq S$ ) located in $l$ ( $l \in L_l \subseteq L$ ) at $t$ in scenario tree node $n$
$rB_{slpjm}^{in} / rB_{slpjm}^{out}$	Number of beds reallocated to/from IC service $s$ ( $s \in S_l \subseteq S$ ) located in $l$ ( $l \in L_l \subseteq L$ ) from/to IC service $p$ ( $p \in S_l \subseteq S$ ) located in $j$ ( $j \in L_l \subseteq L$ ) at $t$ in scenario tree node $n$
$gR_m$	Number of individuals from lower income groups receiving LTC at $t$ in scenario tree node $n$
$R_{dgsln}$	Proportion of individuals from demand point $d$ and socioeconomic group $g$ receiving service $s$ in location $l$ at $t$ in scenario tree node $n$
$rR_{dm}$	Number of individuals from demand point $d$ receiving LTC at $t$ in scenario tree node $n$
$uR_{sm}$	Number of individuals receiving service $s$ at $t$ in scenario tree node $n$
$HR_{rdgsln}$	Number of hours of care provided by human resource $r$ for individuals from demand point $d$ belonging to socioeconomic group $g$ and receiving service $s$ located in $l$ at $t$ in scenario tree node $n$
$U_m^{tot}$	Total travel time (in minutes) at $t$ for scenario tree node $n$
$U_m^{pen}$	Penalty (in minutes) attributed to individuals not receiving IC at $t$ in scenario tree node $n$
$U_m^{max}$	Maximum total travel time (in minutes) at $t$ in scenario tree node $n$
$TIC_m / TOC_m$	Total investment/operational cost at $t$ for scenario tree node $n$

**2.2.2. Building the objective function.** The two objectives considered include the minimization of expected costs (equations (1-3)) and the maximization of expected health gains (equation (4)). Costs include both investment (equation (2)) and operational costs (equation (3)). Investment costs are related to the investment in new beds (first term in equation (2)) and in the reallocation of beds between services (second and third terms). Operational costs include costs associated with the operation of beds in IC services (first term in equation (3)) and to the provision of HBC and AC services (second term). Health gains vary with the type of service delivered ( $HG_s$ ) (equation (4)).

$$\text{Min } f_1 = \sum_{n \in N} \sum_{t:(t,n) \in Q} \rho_n (TIC_m + TOC_m) \quad (1)$$

$$TIC_m = \sum_{s \in S_I} \sum_{\substack{l \in L_I \\ l:(s,l) \in U}} \left( aB_{slt} \frac{iC_{st}}{(1+r_t)^t} + \sum_{p \in S_I} \sum_{\substack{j \in L_I \\ j:(p,j) \in U \\ j \neq l}} rB_{slpjm}^{in} \frac{rC_{st}^1}{(1+r_t)^t} + \sum_{\substack{p \in S_I \\ p:(p,l) \in U}} rB_{slplm}^{in} \frac{rC_{st}^2}{(1+r_t)^t} \right) \forall (t,n) \in Q \quad (2)$$

$$TOC_m = \sum_{d \in D} \sum_{g \in G} \left( \sum_{s \in S_I} \sum_{\substack{l \in L_I \\ l:(s,l) \in U \\ l:(d,l) \in F}} B_{dgsstm} \frac{oC_{st}}{(1+r_t)^t} + \sum_{s \in S_O} \sum_{\substack{l \in L_O \\ l:(s,l) \in U \\ l:(d,l) \in F}} R_{dgsstm} D_{dgsstm} \frac{oC_{st}}{(1+r_t)^t} \right) \forall (t,n) \in Q \quad (3)$$

$$Max f_2 = \sum_{n \in N} \rho_n \left[ \sum_{s \in S: (t,n) \in Q} HG_s \left( \sum_{d \in D} \sum_{g \in G} \sum_{\substack{l \in L \\ l:(s,l) \in U \\ l:(d,l) \in F}} R_{dgsstm} D_{dgsstm} \right) \right] \quad (4)$$

2.2.3. *Defining the constraints of the model.* The proposed model makes use of a set of constraints:

- Opening and closure of services constraints – opening/closing an IC service is not allowed after deciding upon closing/opening it in a previous time period (equations (5-6)). No openings or closures are considered for HBC and AC (equation (7)), given that these services are provided within the scope of a primary care network, which is already established in any NHS-based country;

$$X_{slw} \leq X_{slt} \quad \forall s \in S_I, l \in L_I : (s,l) \in (U \cap V), (t,w) \in T, w > t \quad (5)$$

$$X_{slw} \geq X_{slt} \quad \forall s \in S_I, l \in L_I : (s,l) \in (U \cap Z), (t,w) \in T, w > t \quad (6)$$

$$X_{slt} = 1 \quad \forall s \in S_O, l \in L_O : (s,l) \in U, t \in T \quad (7)$$

- Minimum level of demand satisfaction – equation (8) ensures that a minimum level of satisfied demand per service  $s$  (captured by the parameter  $\alpha_{st}$ ) should be guaranteed per time period  $t$ .

$$\sum_{n:(t,n) \in Q} \rho_n \left( \frac{uR_{stm}}{uD_{stm}} \right) \geq \alpha_{st} \quad \forall s \in S, t \in T \quad \text{with } uR_{stm} = \sum_{d \in D} \sum_{g \in G} \sum_{\substack{l:(s,l) \in U \\ l:(d,l) \in F}} R_{dgsstm} D_{dgsstm} \quad \forall s \in S, (t,n) \in Q \quad (8)$$

- Single and closest assignment constraints – individuals should receive the care they need in the closest available service (equation (9)), and cannot access services that are not within a maximum travel time (equation (10)). According to equation (9), if location  $l$  provides service  $s$  and is closer to demand point  $d$  than another location  $j$ , it means that patients from  $d$  should not receive the required care at  $j$ ;

$$X_{slt} + R_{dgsstm} \leq 1 \quad \forall g \in G, s \in S, (l,j) \in L : (s,l) \in U, (s,j) \in U, (d,l) \in F, (t,n) \in Q, \theta_{dl} < \theta_{dj}, l \neq j \quad (9)$$

$$R_{dgsstm} = 0 \quad \forall g \in G, s \in S, l \in L : (s,l) \in U, (d,l) \in F, (t,n) \in Q, \theta_{dl} > \beta \quad (10)$$

- Resources requirement constraints – number of beds (equation (11)) and human resources (equation (13)) that should be made available per LTC service; and for the particular case of beds, it is also imposed that the bed capacity that needs to be in place in each service and location cannot be less than the bed capacity required in each scenario (equation (12); similarly to [20]). Since in health it is not expected a full occupancy of services, an efficiency factor ( $\epsilon_s$ ), which takes the value of 1 when occupancy rates are equal to 100%, and lower values for lower occupancy rates, is included in equations (11) and (13);

$$B_{dgsstm} = \frac{LOS_{sn}}{TP} R_{dgsstm} D_{dgsstm} \frac{1}{\epsilon_s} \quad \forall g \in G, s \in S_I, l \in L_I : (s,l) \in U, (d,l) \in F, (t,n) \in Q \quad (11)$$

$$aB_{slt} \geq \begin{cases} \sum_{d \in D} \sum_{g \in G} B_{dgslm} - eB_{sl}^0 - \left[ \sum_{p \in S_I} \sum_{\substack{j \in L_I \\ j:(p,j) \in U}} (rB_{slpjm}^{in} - rB_{slpjm}^{out}) \right] & \forall s \in S_I, l \in L_I; (s,l) \in U, (t,n) \in Q, t=1 \\ \sum_{d:(d,l) \in F} \sum_{g \in G} \left( B_{dgslm} - \sum_{k:(n,k) \in P} B_{dgsl(t-1)k} \right) - \sum_{p \in S_I} \sum_{\substack{j \in L_I \\ j:(p,j) \in U}} (rB_{slpjm}^{in} - rB_{slpjm}^{out}) & \forall s \in S_I, l \in L_I : (s,l) \in U, (t,n) \in Q, t > 1 \end{cases} \quad (12)$$

$$HR_{rdgslm} = R_{dgslm} D_{dgslm} HR_{prs} \frac{1}{\epsilon_s} \quad \forall g \in G, (d,l) \in F, (s,l) \in U, (s,r) \in A, (t,n) \in Q \quad (13)$$

- Minimum and maximum capacity constraints – a minimum and maximum number of beds (equation (14)) and individuals in need (equation (15)) are imposed per service;

$$mB_s X_{slt} \leq \sum_{d:(d,l) \in F} \sum_{g \in G} B_{dgslm} \leq MB_s X_{slt} \quad \forall s \in S_I, l \in L_I : (s,l) \in U, (t,n) \in Q \quad (14)$$

$$mI_s X_{slt} \leq \sum_{d:(d,l) \in F} \sum_{g \in G} R_{dgslm} D_{dgslm} \leq MI_s X_{slt} \quad \forall s \in S_O, l \in L_O : (s,l) \in U, (t,n) \in Q \quad (15)$$

- Reallocation constraints – a maximum number of beds is allowed to be reallocated from each IC service (equation (16)); and the number of beds reallocated to service  $s$  from service  $p$  should be equal to the number of beds removed from service  $p$  to service  $s$  (equation (17));

$$\sum_{p \in S_I} \sum_{\substack{j \in L_I \\ j:(p,j) \in U}} rB_{slpjm}^{out} \leq \begin{cases} eB_{sl}^0 & \forall s \in S_I, l \in L_I : (s,l) \in U, (t,n) \in Q, t=1 \\ \sum_{d:(d,l) \in F} \sum_{g \in G} \sum_{k:(n,k) \in P} B_{dgsl(t-1)k} & \forall s \in S_I, l \in L_I : (s,l) \in U, (t,n) \in Q, t > 1 \end{cases} \quad (16)$$

$$rB_{slpjm}^{in} = rB_{pjsltm}^{out} \quad \forall (s,p) \in S_I, (l,j) \in L_I : (s,l) \in U, (p,j) \in U, (t,n) \in Q \quad (17)$$

- Equity satisficing levels' constraints – pre-defined satisficing levels of equity are imposed for EA (equation (18)), EU (equation (19)), SE (equation (20)) and GE (equation (21)). The measures selected for defining EA, SE and GE are similar to the ones used in [10], whereas the EU measure is similar to the GE, although ensuring a minimum service level (in comparison to need) across different typologies of LTC services. These satisficing levels should be defined by decision-makers (DMs), corresponding to levels that should be attained (and not optimized) since one is in the context of scarce resources [19].

$$\sum_{n:(t,n) \in Q} \rho_n \left( \frac{U_m^{tot} + U_m^{pen}}{U_m^{max}} \right) \leq EA_t \quad \forall t \in T \quad \text{with} \quad \begin{cases} U_m^{tot} = \sum_{d \in D} \sum_{g \in G} \sum_{s \in S_I} \sum_{\substack{l \in L_I \\ l:(s,l) \in U \\ l:(d,l) \in F}} \theta_{dl} R_{dgslm} D_{dgslm} \quad \forall (t,n) \in Q \\ U_m^{pen} = \sum_{d \in D} \sum_{g \in G} \sum_{s \in S_I} \left( D_{dgslm} - \sum_{\substack{l \in L_I \\ l:(s,l) \in U \\ l:(d,l) \in F}} R_{dgslm} D_{dgslm} \right) \quad \forall (t,n) \in Q \\ U_m^{max} = \sum_{d \in D} \sum_{g \in G} \sum_{s \in S_I} \beta \times D_{dgslm} \quad \forall (t,n) \in Q \end{cases} \quad (18)$$

$$\sum_{n:(t,n) \in Q} \rho_n \left( 1 - \frac{uR_{stm}}{uD_{stm}} \right) \leq EU_t \quad \forall s \in S, t \in T \quad (19)$$

$$\sum_{n:(t,n) \in Q} \rho_n \left( 1 - \frac{gR_m}{gD_m} \right) \leq SE_t \quad \forall t \in T \quad \text{with} \quad gR_m = \sum_{d \in D} \sum_{g \in G} \sum_{s \in S_I} \sum_{\substack{l:(s,l) \in U \\ l:(d,l) \in F}} R_{dgslm} D_{dgslm} \quad \forall (t,n) \in Q \quad (20)$$

$$\sum_{n:(t,n) \in Q} \rho_n \left( 1 - \frac{rR_{dm}}{rD_{dm}} \right) \leq GE_t \quad \forall d \in D, t \in T \quad \text{with} \quad rR_{dm} = \sum_{g \in G} \sum_{s \in S_I} \sum_{\substack{l:(s,l) \in U \\ l:(d,l) \in F}} R_{dgslm} D_{dgslm} \quad \forall d \in D, (t,n) \in Q \quad (21)$$

### 2.3. Solution approach



To deal with the two selected objectives, the augmented  $\epsilon$ -constraint method is used [21]. Instead of providing a single optimum solution, this method provides a subset of the Pareto-optimal set. According to Mavrotas [21] the augmented  $\epsilon$ -constraint method is a novel version of the conventional  $\epsilon$ -constraint method that solves its well-known pitfalls, namely, i) the calculation of the range of each objective function over the efficient set, and ii) the guarantee of efficiency for the obtained solution. For the purpose of this study, the augmented  $\epsilon$ -constraint method is applied by minimizing expected costs, whereas the objective related to expected health gains is imposed as constraint. Accordingly, the objective function is now given by equation (22), with  $\mu$  representing a small number, usually between  $10^{-3}$  and  $10^{-6}$ , and  $s_2$  representing a slack variable; and the health gains-related constraint is given by equation (23), with  $e_2$  depending on the minimum and maximum values for  $f_2$  and on the number of grids points selected for building the Pareto Frontier.

$$\text{Min} (f_1 - \mu \times s_2) \quad (22)$$

$$f_2 - s_2 = e_2 \quad (23)$$

### 3. Case-study

Section 3 analyzes the first results from applying the proposed multi-objective stochastic model to the county level in the Great Lisbon region in the 2014-2016 period. It starts by briefly describing how health gains were modeled, and then describes other data in use and presents some results.

#### 3.1. Modeling of health gains

Given that we did not find available data providing estimates of the health gains associated with different LTC interventions, we selected the QALYs gained with the delivery of a LTC service as a proxy for health gains. For estimating QALYs, we used the EQ-5D self-report questionnaire, as it ‘has become one of the valuation approaches recommended by several reimbursement authorities and academic bodies in European countries’ [22]. Accordingly, the following steps were used to estimate QALYs:

- i. The EQ-5D self-report questionnaire was first completed with information on disabilities for different LTC services [23];
- ii. Using the Portuguese EQ-5D value set built by Ferreira et al. [24] together with the information on the EQ-5D level for each type of LTC service, the value of the EQ-5D states before and after receiving LTC within the National Network of Long-Term Care (*Rede Nacional de Cuidados Continuados Integrados*, RNCCI) were calculated;
- iii. The QALYs gained per type of LTC service were then determined as the difference between the values of the EQ-5D states after and before receiving LTC.

#### 3.2. Remaining data-set

The model was implemented in the General Algebraic Modeling System (GAMS) 23.7 and was solved with CPLEX 12.0 on a Two Intel Xeon X5680, 3.33GHz computer with 12GB RAM.

The dataset used includes a wide range of information, namely:

- LTC supply at the end of 2013 in the Great Lisbon region [18];
- Operational and investment costs [25];
- Travel time (in minutes) between each county in the Great Lisbon region and each LTC service [26]; and maximum travel time allowed for LTC patients accessing IC services [27];
- Number of hours of care to be provided by physicians and nurses per individual in need [28];
- Efficiency factor associated to the provision of services with a score of 1, given that providers in Portugal are paid according to utilization [18];
- Satisficing equity levels defined by the Head of the Long-Term Care Coordination Team of the Lisbon and Tagus Valley Health Authority, which has given her opinion on these parameters, acting in the role of a real DM in the LTC sector;
- Information about physical, functional, mental and social disabilities of patients receiving care within the RNCCI during 2008, before and after receiving LTC [23].



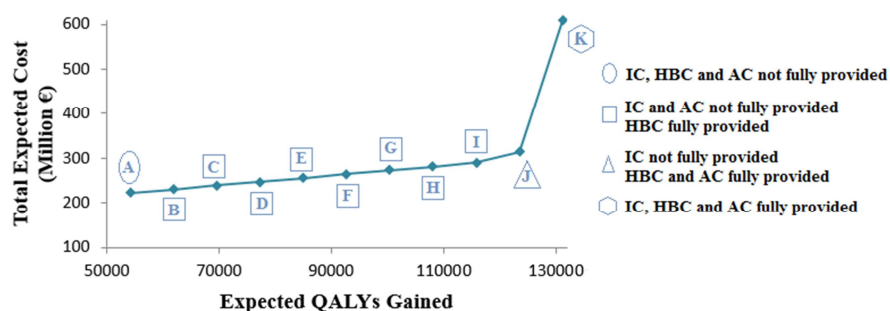
Regarding uncertainty, the following two parameters were estimated:

- i. The number of individuals requiring all the types of IC, HBC and AC, disaggregated by socioeconomic groups (very low income [VLI] and not very low income [NVLI]), as predicted by the detailed simulation model developed in [29];
- ii. The LOS associated to each type of IC service estimated with data from [30].

A scenario tree with 81 scenarios was used to describe combinations of these uncertain parameters. To build this scenario tree, the probability distributions associated to both parameters were taken from the outputs of the simulation model developed in [29] and [30], respectively, and were then converted into three annual scenarios using the extended Pearson-Tuckey method [31]. The whole dataset is available from authors, upon request.

### 3.3. Illustrative results

Figure 1 depicts the Pareto Frontier obtained when running the multi-objective stochastic model to the county level in the Great Lisbon region in the 2014-2016 period. Each solution was obtained by imposing a 120 minutes limit for the computation time, and optimality gaps were below 0.5% for all the solutions. Solution A represents the solution with the minimum expected cost and health gains as measured by QALYs gained, being characterized by the lowest levels of LTC provision. These levels of provision are the minimum levels required to meet the equity satisficing levels indicated by our DM. On the other hand, as we move from solution B to K, the costs of reorganizing and operating the corresponding networks increase, as well as the health gains achieved with LTC provision. The maximum expected health gains and costs are achieved under solution K, with the LTC network found under this solution allowing for full provision of LTC (both IC, HBC and AC).



**Figure 1** Pareto Frontier obtained when running the proposed multi-objective stochastic model. Legend: IC – Institutional Care; HBC – Home-Based Care; AC – Ambulatory Care.

In order to aid LTC policy makers deciding on how to reorganize the current network of LTC, the cost-effectiveness of each solution compared to the current provision of care is evaluated by determining the additional cost per QALY gained with LTC provision under each solution (also known as Incremental Cost Effectiveness Ratio [ICER] in health technology assessment literature; Table 1).

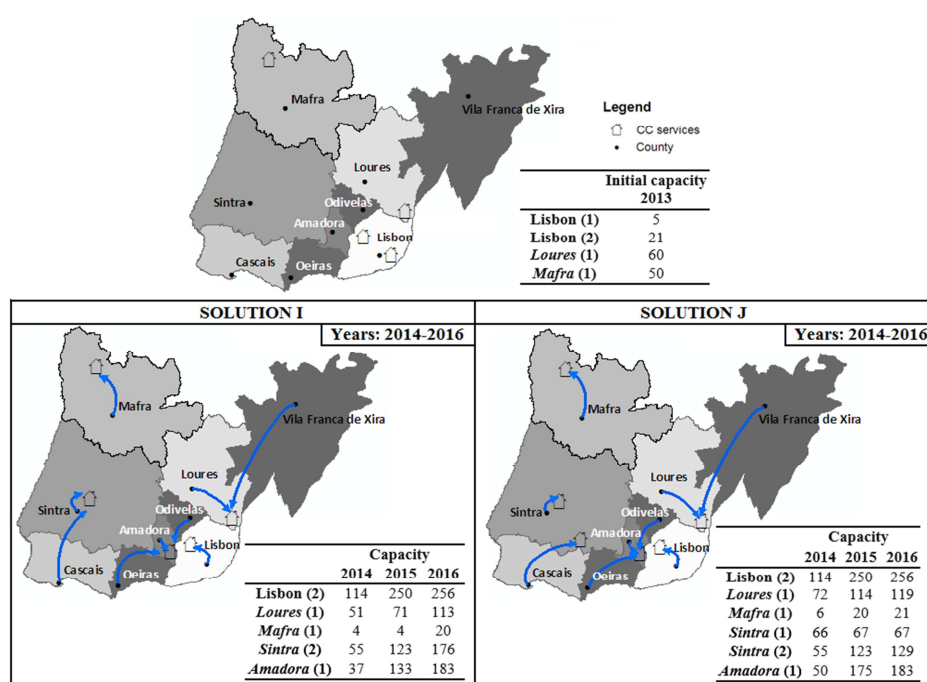
**Table 1.** Additional cost (in thousands of euros) per QALY gained, i.e., Incremental Cost Effectiveness Ratios (ICERs), in comparison to current practice for solutions A to K.

Solution	A	B	C	D	E	F	G	H	I	J	K
ICER	2.83	2.58	2.40	2.25	2.14	2.05	1.97	1.90	1.81	1.86	4.19

According to Table 1, it can be seen that solutions I and J represent the two most cost-effective options (with a lower ICER). These two solutions are characterized by full provision of HBC (see Figure 1) but differ in terms of AC and IC provision. In particular, the LTC network found under solution J ensures full provision of AC, whereas under solution I there is only partial provision of AC. Moreover, comparing these two solutions in terms of IC provision allows for identifying several

differences. To illustrate these differences, Figure 2 shows the results obtained for CC services, with CC representing the type of IC service with the highest capacity requirements for both solutions.

Figure 2 provides information on the opening and closure of services, the allocation of individuals to services and the evolution of bed capacity over time for solutions I and J, for the scenario characterized by average demand. Figure 2 shows that, unlike solution I, solution J suggests operating a CC in *Sintra* (1). Also it can be seen that the total CC bed capacity characterizing solution J is higher than in solution I (according to Figure 2, by 2016, there should be 748 and 775 CC beds under solution I and J, respectively). The lower number of services and beds justifies the lower costs and health gains found under solution I when compared to solution J. Note that the higher provision found for CC is related to the high amount of QALYs gained with the provision of these services (the QALYs gained per individual receiving CC, MTRC, LTMC and PC amounts to 0.606, 0.707, 0.315 and 0.214, respectively) and to the higher number of individuals in need for CC, when compared to the remaining IC services.



**Figure 2** Changes in the provision of convalescence care (CC) services within the LTC network over the 2014-2016 period under solutions I and J (for the scenario characterized by average demand). For simplification purposes, institutions in each county are numbered – e.g. the two institutions in Lisbon are named Lisbon (1) and Lisbon (2).

Table 1 also shows that additional costs per QALY gained decrease as one moves from solution A to I, and increases from solution J onwards. More important than this, comparing the ICERs from A to K with thresholds discussed by some health technology assessment agencies (such as the National Institute for Health and Care Excellence, in the United Kingdom [32]) of around £20000-£30000, it seems that extending the LTC network in the Great Lisbon region in Portugal will be cost-effective.

A final result that can be observed in Figure 1 is that achieving the satisfying equity levels defined by our DM amounts to a minimum of 228 million euros for the three-year period (2014-2016), corresponding to the minimum cost solution (solution A). When compared to the budget available for reorganizing and operating the LTC network in the Great Lisbon region, 30 million euros per year, this value is 250% higher. This result makes it clear that a higher budget is needed for improving the delivery of LTC in the region.

#### 4. Conclusion

One of the top priorities common to the policy agenda of many European countries is related to the improvement in the supply of LTC, and this due to the current ageing phenomenon, as well as to the increasing prevalence of chronic diseases. Furthermore, to deal with the strong pressures that are currently in place to decrease public health care spending, the planning of networks of care requires accounting for cost, equity and health gains related objectives.

Within this context, a multi-objective two-stage stochastic MILP model is proposed to support location-allocation decisions in the LTC sector under the context of NHS-based countries. The model aims at informing the (re)organization of multi-service networks of LTC while maximizing expected health gains and minimizing expected costs (with satisficing levels of equity in several dimensions being achieved), with health gains representing a dimension usually ignored in location-allocation studies. The trade-off between these conflicting objectives is addressed through the use of the augmented  $\epsilon$ -constraint method. A case study in the Great Lisbon region in Portugal for the 2014-2016 period is explored. The estimation of health gains was done through the computation of QALYs.

Key results from the model application confirm that the budget currently available for improving LTC delivery in the Great Lisbon region is far from being able to achieve the satisficing equity levels defined by the DM; but investments in the LTC network are cost-effective. Results allowed for exploring the trade-off between cost and health gains, showing the relevance of considering both objectives when planning a network of LTC; and information on the incremental cost effectiveness ratio is particularly useful for policy makers in the LTC sector deciding on how to plan networks of care.

Several research topics should be pursued as further work. First, if a single solution is sought, rather than multiple solutions, after generating the Pareto optimal solutions, one may resort to Multiple Criteria Decision Making methods and use the generated solutions as discrete alternatives so as to assist decision makers selecting the most preferred solution. It is also relevant to explore the impact of adopting different health policy options in planning decisions. For instance, allowing for substituting institutional care by home-based care may result in higher health gains for the same level of cost, due to the lower QALYs gained with the provision of certain types of institutional services. Furthermore, alternative ways to capture benefits in the delivery of care can be explored. For instance, measures of social welfare – which are more holistic than health gains – can be explored.

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