Action horizon: on the controllability of complex systems, moving towards management for energy systems

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Abstract. Building robust and resilient systems is a major challenge for engineering. To this end, analytical indicators may be useful for managers in making decisions. In this paper, a new analytical construct is presented: the action horizon. This construct is devoted to assisting in the control of a system when an event driving the system to its collapse is detected. This construct is useful for analysing the best moment to start executing a corrective action. In this paper, the action horizon is experimentally evaluated in the predator-prey system and we discuss how it could help with managing power systems.

Keywords: complex system, action horizon, controllability, power systems

1 Introduction

Instability in power grids can provoke a dramatic power drop, which may lead to service disruption. This situation can happen due to natural environmental changes, machinery failure and/or human actions.

The practical importance of stability analysis, for events that cannot be predicted, is that it helps in selecting countermeasures to avoid system collapse and enhances stability. There are short-term and long-term countermeasures that comprise system planning, operational planning and real time monitoring.

However, once the destabilising event has been identified and the countermeasure has been selected, the following questions arise: when should this action be executed and, more specifically, what is the best moment to execute it in order to minimise costs?

In general, the controllability of a system involves not only the selection of the countermeasure, but also the analysis of the optimal moment to start executing it. However, due to the dynamism of a complex system, it is interesting to delay the execution of the countermeasure since other events could occur and allow the system to recover stability without taking any action. In some cases, therefore, it could be better to do nothing.

This paper introduces the construct of action horizon which helps decide the best moment to execute a countermeasure improving the controllability of complex systems. The structure of the paper is defined as follows: firstly, the controllability concept is presented. After that, this concept is discussed with respect to energy systems and the challenges to improve it. Then, the action horizon is formally defined. This construct is evaluated experimentally in the predator-prey system. The final sections discuss the benefits of the action horizon for the predator-prey and how it could help to manage energy systems.

2 Complex systems and controllability

Complex Systems science has historically emerged from domains such as Sociology and Biology. Complexity theory aims to understand how these usually natural, large and very dynamic systems function, which are their rules and patterns and how global emergence leads to an equilibrium in which the system remains. The concept of Emergence, which describes a global behaviour that cannot fully be inferred by the behaviours of individual agents or, as Aristote long ago observed: when the whole is more than the sum of the parts, is characteristic to all Complex Systems. In an effort to understand Emergent phenomena, different methods of the domain have been used, for example modelling of individual parts of the system by reflecting their behaviour in a synthetic way. This has led to a better understanding of the system; exploring it through detailed inspection and analysis of the individual elements, and reconstructing their interactions and dynamics in order to understand the aggregated behaviour at different levels of this system.

For many years now, however, Complexity Theory has not only helped to improve our knowledge of existing natural systems; it has also has been successfully applied to understanding the behaviour of large man-made complex systems. Evolution theories have been applied to urban systems [3, 2], and the world-wide web, as one of the largest and most widely distributed systems, has been shown to comply to many of the laws that were previously discovered through the exploratory approach. The difference here is that man-made systems are usually designed to fulfill a concrete purpose. Complex systems science as a means for exploring and understanding systems has already lead to scientific breakthroughs (examples of understanding of firefly synchronisation, behaviour in bee hives, etc.).

Going one step further and understanding how we can modify the trajectory of these systems however is an important challenge. [6] raised this topic in their paper entitled "Controllability of complex networks", focusing on finding the entities of the system which have a greater impact on its *controllability*, using the statistical physics approach. Indeed, the methods of exploring complex systems to better know the means by which we can drive those systems into a desired trajectory could be seen as one of the main challenges in this field. Complexity theory could help us better understand engineered systems, such as power grids or other infrastructure networks. However, a transfer of these rather *theoretical* approaches to concrete application cases is needed. In this paper, we initially try to shed light on controllability from a conceptual point of view, then we will try to illustrate its possibilities for typical academic complex system examples, and further apply these methods to a real engineered system. As mentioned above, energy systems are complex man-made systems, nowadays with a tendency towards more decentralised and distributed management patterns. With a view to ensuring both the reliability and sustainability of those systems, a complex systems approach might help to improve their controllability and ensure stability.

3 Energy system controllability

Energy system complexity comes from the great variety of technical installations (e.g. plants, grids), energy sources and carriers (e.g. gas, electricity), actors (e.g. producer, network operator, industrial and private consumer), and is strongly influenced by various economic (e.g. energy prices, market systems), political (e.g. taxes, subsidies) environmental (e.g. weather) and social (e.g. acceptance of technologies) factors. Energy systems nowadays have changed tremendously, in comparison to earlier energy systems. The share of renewable energy generation (REG) in electricity systems is increasing, new actors, for example, traders, have entered the systems, other actors have changed their role (for example, consumers becoming producers), markets systems have changed from a highly regulated to a more liberalized operation, acceptance of certain technologies (e.g. nuclear power) has decreased in certain countries, and information technology has found its way with an extensive application into the systems (see [4]). A wide range of ideas to cope with these new system elements and influences have been developed: inter alia: system integration of REG via smart grids, demand side management (DSM), interconnection of the different energy systems with the electricity system for transport (e.g. via electric vehicles), heat (e.g. via heat pumps), gas (e.g. via power-to-gas), increased use of storage, grid expansion (see [9] for further concepts).

With a growing complexity of energy systems, the corresponding models which aim to analyse and optimize their performance have increased in complexity, too. To keep these models manageable (and, in turn, the real systems they model) they have been simplified, concentrating on the improvement of a subset of objectives ([7]). In most of the cases this simplification merely consists of decreasing the number of system elements and target variables by e.g. limiting the observation period, its differentiation and the geographical extension. Depending on the chosen methodology the complex interactions and procedures normally are mapped as precisely as possible, within the defined subset of targets to study. Therefore, these models represent adequate states for the modelled time horizons and impacts of simulated events and actions. Numerous energy models study a large number of the influences of decisions (or actions, or countermeasures) on the system. Non-deterministic simulations have become an important means of modelling energy systems, as they represented these extremely complex systems very well. Multi agent models (using the same methodology as the model we will investigate later) have become more and more common ([10]). Controllability of energy systems has always been an issue, but with deterministic model actions to control the mode (and thus the represented system), it is relatively easily determined (type and point in time). With multi agent models, controllability becomes a bigger issue.

To illustrate this problem we will present two (of course there are many more!) problems in energy systems: The first problem comes from the increasing penetration of photovoltaics and other small-scale electricity generations in many distribution grids around the world. As such investments have been subsidised for a long time, a lot of electricity consumers have chosen to invest and become producers as well. In fact, during some hours of the day, this generation outstrips consumption, which leads to a feed-in of energy at distribution grid level. The problem here is that most existing distribution grids were not designed for such usage. Because of high security standards, grids in countries like Germany are heavily oversized in regard to effective power transmission, a fact which has allowed the grids to cope with the new application until now. Other aspects of the electrical grids, such as maintenance of voltages and stability frequency may pose problems, as there are strict regulations of the level for all of them (power quality: $400V \pm 10\%$, frequency: $50Hz \pm 0.2\%$).

New types of big consumers, such as electric vehicles, may also contribute to this problem. Owners of the electric vehicles may decide to recharge them at the same moment at home, which will result in a huge peak load. To deal with these issues, strategies such as DSM in smart grids have been developed. In this approach, flexible demand is shifted using different kinds of incentives, such as electricity prices varying during the day, or a direct control by an operator, for example an electric vehicle fleet operator that schedules vehicle charging. As effective power supply cannot be predicted very well (using weather forecasts, etc.) maintenance of voltage and stability of frequency is more complicated and depends on grid conditions and other short-term factors. Scheduling actions to prevent problems and using these actions optimally (concerning grid conditions and costs) is an issue studied in a lot of energy model simulations.

The second problem arises on a greater scale. As regenerative decentralized electricity generation is increasingly being adopted to confront climate change in many countries, its share in the generation mix increases. This generation is mainly fluctuating and not evenly distributed geographically, due to weather and climate conditions. Therefore, the role of balancing energy, supply and demand, becomes more and more important. To enhance this balancing, energy storage and increasing flexibility of demand may become important. As mentioned before, power grid regulations and safety management are strict. Therefore, in a possible future energy system with a high percentage of uncertain and unequally distributed generation, the question of how much balancing energy has to be stored, and when and where this will be done, is becoming more and more important.

4 Defining the action horizon

An operator in the energy system should be able to analyse the optimal moment to start executing countermeasures. To this end, for controllability purposes, it is interesting to know: (1) the maximum time after an event occurs that the system is still guaranteed to recover its stability by executing the countermeasure; (2) the time after the event in which the action will be effective, with a probability rate; or (3) the time after the event in which the action will no longer be effective.

Understanding these questions evolves around the event horizon concept, which is defined in the general theory of relativity to explain how space time bends around a massive object. This theory predicts that there is a region near black holes from which light is unable to escape. The boundary of this region is the event horizon, or "a point of no return".

In an analogous manner, if we delay the execution of a countermeasure, there also could be a "point of no return". That is, a moment in time where this corrective action is not effective for recovering the stability of the system.

This is the proposed **action horizon** construct, defined as the moment up to which it is possible to recover the stability of the system by executing the selected corrective action for a probability of success.

Given a corrective action from a set of possible countermeasures, it is defined a function F_ω that assigns a success probability to the delay of starting the execution of the action.

$$F_\omega : \tau \to \mu$$

where $\omega \in \Omega$ is a corrective action from a set of countermeasures, $\tau \in [0, \infty)$ is the time from the moment when the event that destabilises the system occurs, and $\mu \in [0, 1]$ is the measure that represents the success probability of a corrective action.

Thus, given a probability μ, it could be calculated τ such that $F_w(\tau) = \mu$. The following properties can be defined for this function:

Non negativity:

$$F_w(\tau) \geq 0, \forall \omega \in \Omega \; and \; \forall \tau \in [0, \infty)$$

Monotonicity:

$$F_\omega(\tau_x) \geq F_\omega(\tau_y), \forall \tau_x, \tau_y \in [0, \infty) \; and \; \tau_x \leq \tau_y$$

Axiom 1. When the system is stable, $F_\omega(\infty) = 1, \forall \omega \in \Omega$, since the action is not required to be executed.

Axiom 2. When the system has collapsed, $F_\omega(0) = 0, \forall \omega \in \Omega$, since the corrective actions will not be able to recover the system at any moment

Then, the **action horizon** for a given action and a defined success probability, θ_ω^μ, is the time after the event occurs such that:

$$\theta_\omega^\mu \in [0, \infty)$$
$$F_\omega(\theta_\omega^\mu) = \mu$$
$$F_\omega(\tau_i) \leq F_\omega(\theta_\omega^\mu), \forall \tau_i \geq \theta_\omega^\mu$$

Since the most common desired success probability is 100%, a simplification of the action horizon notation is:

$$\theta_\omega \equiv \theta_\omega^1$$

5 Experimental evaluation of the action horizon: the predator prey system case

In this section, previously mentioned ideas are applied to a system that has been modelled as an agent-based model: the predator prey system. We have chosen this system as it is simple enough to illustrate the ideas explained in this paper, in addition to being a well-known system.

The model we have used for running all the experiments can be downloaded from here [8]. This model may be freely reused. Our only requirement is to cite us.

To find out the action horizon of the system, we have studied how the system behaves when parameters are stable. Then, an exogenous event causes the system to lose stability, provoking its collapse after some time. An action has been found that, when applied, prevents the collapse of the system. This action, that counteracts the exogenous event, avoids the collapse of the system if applied during the very first step in which the event starts. At this point, an analysis is run in order to calculate the probability of avoiding the collapse of the system when performing this action, depending on the time at which it is applied, after the start of the event. The “action horizon” will be the final time at which the application of the action has a hundred percent chance of success.

5.1 Predator prey model

The model used for analysing the action horizon is inspired by the model developed in [11]. This model considers three types of species: “grass”, “hares” and “foxes”. The predators of the grass are the hares and the predators of the hares are the foxes. This model represents a 2-dimensional toroidal world where every kind of agent is spatially located and the possibility of eating is related to the agents that are in the same cell, as well as their kind.

In this model, there are as many grass units as cells in world. This grass has a specific regrowth time which defines the steps to have new edible grass after the old grass unit has been eaten. This means that every grass unit has an attribute that defines if it is ready to be eaten or not. The regrowth time is the same for all the grass units.

Both animals, hares and foxes, have energy as an attribute and can perform these actions at each step: move, eat and reproduce. The energy is decremented at each step, meaning that an animal may die after having taken several steps without eating (energy = 0). Actions are described in the following list:

- **Move**: animals can only move to adjacent cells and their orientation is calculated randomly.
- **Eating**: as previously mentioned, eating is only possible when there is something to eat in the cell where the animal is. The restrictions are that a hare cannot eat another hare or a fox, nor can a fox eat another fox, or grass. Obviously, when a fox eats a hare, the hare dies and its energy becomes 0. When a hare eats grass, the grass cannot be eaten again until it is regrown.
- **Reproduce**: each kind of animal has a reproduction rate. In each step, a random number is calculated in order to see if the animal has a baby or not. Every animal is able to produce babies, and there is no gender consideration.

5.2 Stability of the system

The predator-prey system remains quite stable whenever all attributes, such as the reproduction rates, the regrowth time of the grass or the initial populations, stay the same along the simulation. In this section, the parameterisation of the system is illustrated in table 1.

Parameter	**Value**
World size	150 * 150 cells
Grass regrowth time	17 steps
Hares' reproduction rate	4%/step
Foxes' reproduction rate	3%/step
Initial edible grass	50% of all cells
Initial hares	2000 hares
Initial foxes	800 foxes

Table 1. Parameters of the system

As it can be seen in figure 1, the population of the species is stable along the time. Edible grass oscillates between 2,500 and 3,500 units. As it can be seen in figure 2, the fox population oscillates between 600 and 900 and hares between 1,600 and 2,000.

5.3 Destabilising the system

Keeping the system stable results in the action horizon being infinite. That is, if no action is made, the system can remain stable indefinitely. The calculus of

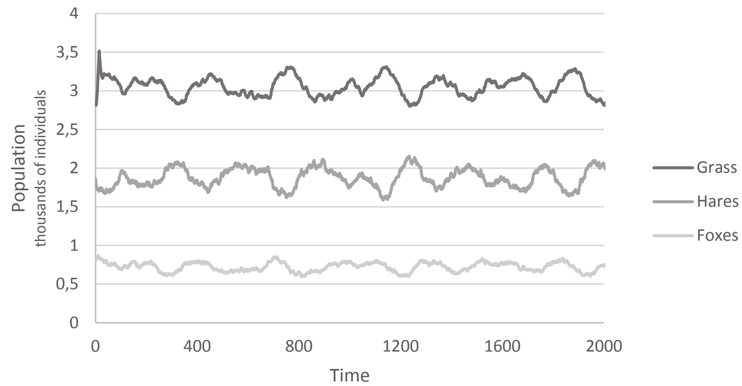

Fig. 1. Population evolution with stable parameters

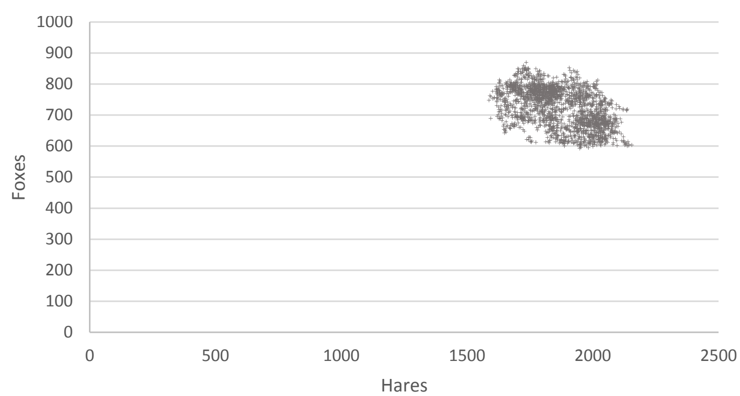

Fig. 2. Relation between hares and foxes populations

the action horizon must be considered whenever some indicators show that the system is moving out from its stable state.

There are many ways to destabilise this predator prey system we are analysing. We have chosen one of those most likely to happen: a drought. The way in which this drought has been implemented in the model is through increasing the regrowth time from 17 to 50. This means that there will not be as many edible grass units as before. Therefore, the hares will be in danger and if their population decreases, the foxes will be in danger too. The duration of this drought is 500 steps.

In figure 3, the evolution of the populations is presented in a simulation in which, after step 200, a drought starts which makes the regrowth time of the grass longer. In this figure, it can be seen how the quantity of grass is dramatically reduced, thus affecting the hare population. This reduction of the hare population provokes, on the one hand, the extinction of the foxes some steps later and, on the other hand, the stability of edible grass at around 2,500 units. When the drought finishes (step 700), a new stability situation is created for both the grass units and the hares. Unfortunately, the overall system has collapsed, as foxes have totally disappeared.

The scattered plot presented in 4, shows the evolution of both the hare and fox populations. It can be seen how, during the drought, the hare population decreases and, as a consequence, the fox population also decreases until they are totally extinct. After, the drought, the hare population increases, reaching around 4,000.

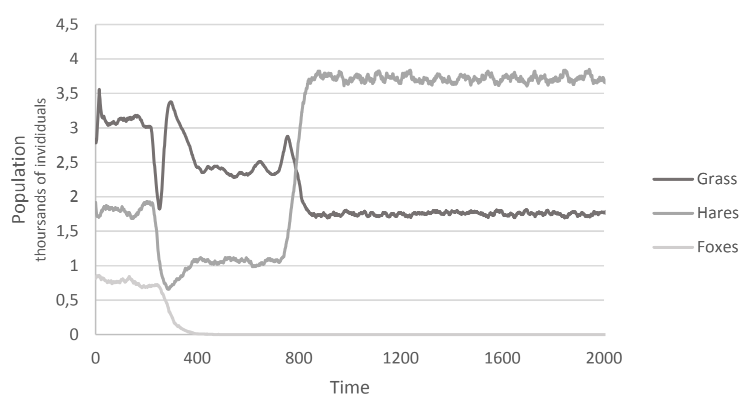

Fig. 3. Population evolution in case of drought

5.4 Stabilising the system

Before analysing the action horizon, an action must be found which avoids the collapse of the system. There are many actions that can be performed. However,

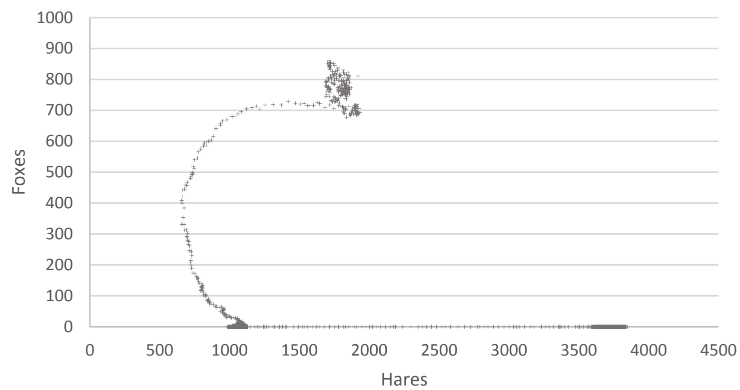

Fig. 4. Relation between hares and foxes populations in case of drought

the most ecologically-friendly we have seen is to artificially feed hares by placing some grass in the world, in order to avoid a high decrease in the hare population.

Specifically, the idea consists in leaving some grass (3% of the cells of the world) at every step after the drought event starts. However, this grass is only left whenever the population of hares is lower than 1,800. In figure 5, the evolution of the populations can be observed. There is a high oscillation of the grass population as consequence of the action. Furthermore, the hare population remains stable at around 1,800 whereas the fox population decreases slightly. After the drought, grass, hares and foxes are still alive, so that after some steps the situation before the drought event is recovered. The evolution of both hares and foxes can be observed in the scattered plot presented in figure 6.

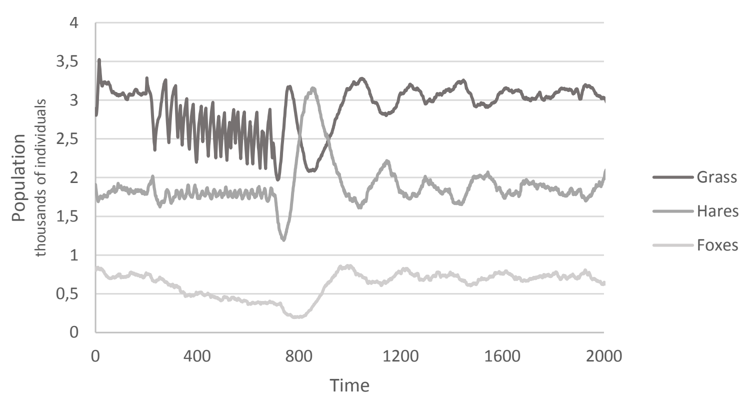

Fig. 5. Population evolution in case of drought and the corrective action is executed

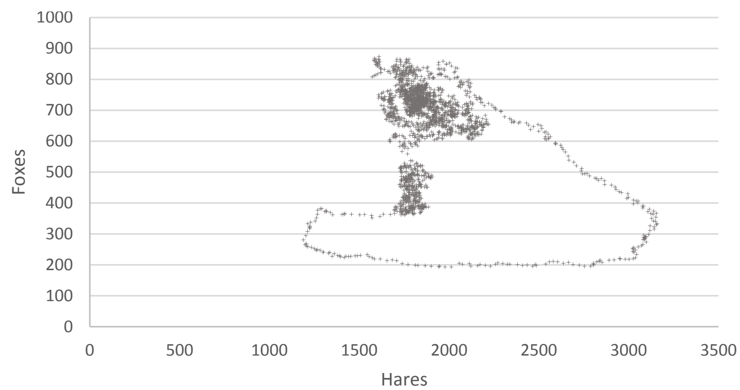

Fig. 6. Relation between hares and foxes populations in case of drought and corrective action executed

5.5 Evaluating the action horizon

After analysing the stability of the system, how to destabilise it and how to find a corrective action, a, to avoid its collapse, the action horizon can be evaluated. The idea of this last part of the experiment is to find the final time at which the previously mentioned action can be performed, so that the probability of the system recovering is 100%.

To run this experiment, a set of simulation has been defined in which the corrective action is applied at different times after the drought event starts. The set of times goes from 0 to 600, with a step of 10. That is, there will be simulations in which the action will start at the step 0, others at 10, 20, ..., 600 after the drought event starts. Every configuration will be simulated 2,000 times so that the probability of success in applying the action will be calculated for each of those temporal configurations.

In figure 7, the probability of success in applying the action after the drought starts are presented. As can be seen, the action can be applied with a 100% probability of success if applied within 110 steps after the drought event. Then, probability reduces until reaching 500 steps, where the probability of success is close to 0%. Thus, we can determine that the action horizon, θ_a, is 110 steps after the drought event considering action, a.

6 Discussion

In conducting these experiments, we have relied on three main factors that seem to be the most critical for the action horizon:

– **Event**: the event that produces the systems collapse determines how far away the action horizon is. If we had set the regrowth time to infinite, instead of 50, the action horizon would have definitely been quite shorter. Moreover,

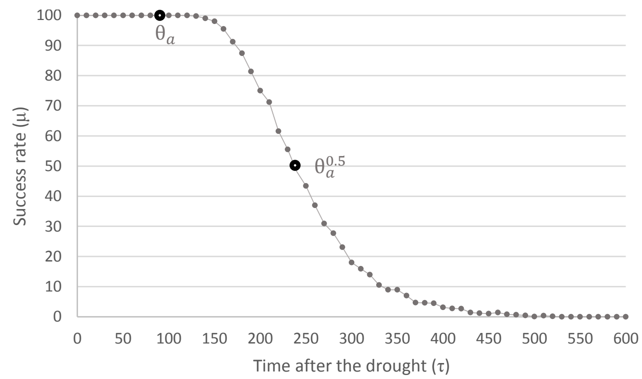

Fig. 7. Probabilities of succeeding in applying the action after the drought starts $F_a(\tau_i)$

the action that we applied to correct the situation (putting grass in the 3% of the cells) may not have been sufficient to recover the system.

- **Action**: the action to avoid the collapse of the system is also critical to determine how far away the action horizon is. In the case we have presented, the action horizon would have been further away if the grass had been set in more places, or the condition to leave grass had been when hares population was lower than 2,000. Obviously, if the action is too aggressive, it is possible to bring the action horizon closer, as a collapse may be caused by the action itself.
- **State before the event**: this last factor is not as critical as the other ones, but can determine how far away the action horizon is. If the system is running with low populations at the time in which the event happens, the action horizon will be closer, as an action to recover the system will be required before.

Another important factor to consider when evaluating the action horizon is the systems stochasticity. Under the same conditions in the three factors previously mentioned, the simulations can reach different action horizons as the evolution of the populations is determined by stochastic events.

Knowing the action horizon allows the managers to decide when is the best moment to act in the system so that effort can be reduced. In this case, one of the low-cost solutions might be to act within the action horizon time, as this means that effort was saved in all the previous steps in which nothing was done. Nevertheless, it is important to know the probability of succeeding after the action horizon, since this may be helpful in deciding what kind of action to apply, once this temporal point is reached. That is, if it is known that an action has a certain probability of success, perhaps it will be possible to come up with a better action that may have a higher probability of success.

7 Conclusion

In this paper we focused on a concrete example based on the predator-prey model in order to show the action horizon concept and how it supports the controllability of a complex system. By simulating a destabilising event, the system was disrupted, making it unstable. At this point, corrective actions were performed and tested in order to assess the so called action horizon. This is defined as the maximum time after the event occurs in which a corrective action has a desired success rate in achieving stability.

It is interesting to know the action horizon in order to delay the execution of the countermeasure. Due to the dynamism of a complex system, other events could occur and allow the system to recover stability without executing any action.

Another factor that should be considered by managers is the cost of executing the actions. The goal of minimising operational costs requires the analysis of the best moment to act in order to recover the system.

For example, in the case presented, it could be wise to wait for the action horizon with a 100% rate of success. In this way, new events can be seen that may restore the system stability, so that no intervention is needed (e.g. drought finishes), or other events that may push the system to other kinds of risks, in which case this corrective action could be harmful.

Future research on action horizon could analyse operational costs. In this way, it could be calculated what is the action horizon of the combination of two different actions or how several events occurring at the same time influence the action horizon. Further studies should also analyse the costs of recovering the system from its collapse, since this could be less costly than avoiding its collapse. For instance, feeding hares may be more costly than bringing foxes in from other territories after the drought has ended.

Furthermore, additional events could be considered in order to see how it could affect to the Action Horizon. Considering the combinatorial that this may have, in order to reduce the computational costs, it would be interesting to apply techniques for calculating the number of runs required to have a certain level of certainty when defining the Action Horizon [1].

8 Outlook

In the balancing of energy systems, the risk of blackouts has been thoroughly analysed. For example, frequency drops due to power unit failures may lead to blackouts and even complete desynchronisation of the system. In [5] the impact of distributed load management on system frequency was analysed. Furthermore, controllability of the energy system in terms of voltage control could be analysed. By optimising the use of flexible loads at the right moments and right places, voltage drops can be reduced. The analysis of the action horizon for these cases would lead to a measure that could serve as a stability indicator of the grid, as well as system frequency. Taking the flexibility potential into account, this indicator could show the added value of using local load management, and serve as risk assessment in real time for grid operation.

The concepts and methodology will be modelled within an energy system model that allows evaluating scalable, distributed agent-based load management strategies. The energy system will be a smart grid at distribution grid level: the modelled distribution network has certain flexibility potential. Therefore an improved organization and utilization of its elements will be performed, using a combined methodical approach based on agent-based simulation (ABS) and energy system model (ESM) optimization. As the optimization is deterministic, it will not take into account action horizons, but as the agents performing the optimization are placed in an ABS, the environment and its agents will be able to perform actions that have an action horizon. Flexible load (and generation) can be supplied by different technologies, such as some already existing appliances, demand side management (DSM), new system elements, for example, electric vehicles (EV), stationary batteries, or decentralized generation (for example with photovoltaic panels or microCHP). In order to ensure an efficient operation of the electricity system, these examples of flexibility can support the central generation system in dispatching and grid balancing which represents the previously mentioned countermeasures (or actions).

Conventional optimization of the integral system becomes very complex as the degrees of freedom in a system with large amounts of distributed resources normally is very high. Therefore, modular and scalable management techniques are required to tackle large amounts of distributed resources. For this reason, in our forthcoming work heuristics based on agent-based models simulation, including the concept of this paper and optimization of system parts will be analysed and tested to explore a scalable and multi-level solution in the given context.

The generic approach will be applied to a use case for which we will model a differentiated load modelling at household level (called micromodelling, for detailed representation of load shedding and demand-side management). Further on, the aggregation of loads to low-voltage transformer and the development of distributed, local demand side management mechanisms, based on local optimisation including technical and economical perspectives will be performed. Local and global constraints will be taken into account and the iteration with a grid model will allow identifying local grid constraints on the distribution network.

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