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$SU(2)$ Yang–Mills Theory: Waves, Particles, and Quantum Thermodynamics

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Abstract: We elucidate how Quantum Thermodynamics at temperature T emerges from pure and classical $SU(2)$ Yang–Mills theory on a four-dimensional Euclidean spacetime slice $S_1 \times \mathbf{R}^3$. The concept of a (deconfining) thermal ground state, composed of certain solutions to the fundamental, classical Yang–Mills equation, allows for a unified addressation of both (classical) wave- and (quantum) particle-like excitations thereof. More definitely, the thermal ground state represents the interplay between nonpropagating, periodic configurations which are electric-magnetically (anti)selfdual in a non-trivial way and possess topological charge modulus unity. Their trivial-holonomy versions—Harrington–Shepard (HS) (anti)calorons—yield an accurate a priori estimate of the thermal ground state in terms of spatially coarse-grained centers, each containing one quantum of action $\hbar$ localized at its inmost spacetime point, which induce an inert adjoint scalar field ϕ ($|\phi|$ spatio-temporally constant). The field ϕ, in turn, implies an effective pure-gauge configuration, $a_\mu^{\rm gs}$, accurately describing HS (anti)caloron overlap. Spatial homogeneity of the thermal ground-state estimate $\phi, a_\mu^{\rm gs}$ demands that (anti)caloron centers are densely packed, thus representing a collective departure from (anti)selfduality. Effectively, such a "nervous" microscopic situation gives rise to two static phenomena: finite ground-state energy density $\rho_{\rm gs}$ and pressure $P_{\rm gs}$ with $\rho_{\rm gs} = -P_{\rm gs}$ as well as the (adjoint) Higgs mechanism. The peripheries of HS (anti)calorons are static and resemble (anti)selfdual dipole fields whose apparent dipole moments are determined by $|\phi|$ and T, protecting them against deformation potentially caused by overlap. Such a protection extends to the spatial density of HS (anti)caloron centers. Thus the vacuum electric permittivity ϵ_0 and magnetic permeability μ_0, supporting the propagation of wave-like disturbances in the $U(1)$ Cartan subalgebra of $SU(2)$, can be reliably calculated for disturbances which do not probe HS (anti)caloron centers. Both ϵ_0 and μ_0 turn out to be temperature independent in thermal equilibrium but also for an isolated, monochromatic $U(1)$ wave. HS (anti)caloron centers, on the other hand, react onto wave-like disturbances, which would resolve their spatio-temporal structure, by indeterministic emissions of quanta of energy and momentum. Thermodynamically seen, such events are Boltzmann weighted and occur independently at distinct locations in space and instants in (Minkowskian) time, entailing the Bose–Einstein distribution. Small correlative ramifications associate with effective radiative corrections, e.g., in terms of polarization tensors. We comment on an $SU(2) \times SU(2)$ based gauge-theory model, describing wave- and particle-like aspects of electromagnetic disturbances within the so far experimentally/observationally investigated spectrum.

Keywords: Harrington–Shepard caloron; (anti)selfduality; electric and magnetic dipole densities; vacuum permittivity and permeability; Poincaré group; quantum of action; Boltzmann weight; Bose–Einstein distribution function

1. Introduction

Boltzmann's statistical approach to kinetic gas theory can be considered an anticipation of Quantum Physics. Assuming for simplicity a single atomic species of mass m, his equation reads

$$\left(\partial_t + \dot{x}_i\partial_{x_i} + \frac{1}{m}F_i\partial_{\dot{x}_i}\right) f(\vec{x},\dot{\vec{x}},t) = \partial_t f|_{\text{coll}} \quad (i = 1,2,3)\,, \tag{1}$$

where $\vec{F}$ is an external force field, and the term on the right-hand side denotes the collision integral—a functional of the phase-space probability distribution f. Equation (1) would describe a time-reversal invariant evolution of f like in Classical Mechanics (that is, if $f(\vec{x},\dot{\vec{x}},t)$ satisfies Equation (1) then so does $f(\vec{x},-\dot{\vec{x}},-t)$) if the collision integral was identically zero or determined from Classical Mechanics itself. However, the collision integral's intrinsic indeterminism, expressed through probabilistic changes from initial to final scattering states (*molecular chaos*), selfconsistently underlies the concept of the probability distribution f and its time-reversal non-invariant evolution. Presently, we use Quantum Mechanical or Quantum Field Theoretical amplitudes to compute $\partial_t f|_{\text{coll}}$ from first principles for dilute gases (typical scattering lengths smaller than mean interparticle distance). The arrow of time, expressing the asymptotic attainment of an ergodic (thermal) equilibrium state of maximum entropy as a consequence of f's evolution via Boltzmann's Equation (1), thus is a direct consequence of the indeterminism inherent to the collision integral, and our modern understanding of *molecular chaos* is that this integral be expanded into positive powers of $\hbar$—Planck's (reduced) quantum of action (In the formal limit $\hbar \to 0$ the quantity $\partial_t f|_{\text{coll}}$ is given by Classical Mechanics or vanishes). The purpose of the present article is to discuss the emergence of Quantum Thermodynamics in pure $SU(2)$ Yang–Mills theories and to explore some of its consequences which appear to extend beyond thermodynamics.

In contrast to thermalization of a dilute gas of massive (bosonic) particles, such as atoms or molecules, by virtue of the collision integral in Boltzmann's equation the emergence of the Bose–Einstein quantum distribution is a much more fundamental and direct affair in pure Yang–Mills theory. This is because classical $SU(2)$ Yang–Mills theory on the Euclidean spacetime $\mathbf{R}^4$ provides for (anti)selfdual and temporally periodic gauge-field configurations—so-called Harrington–Shepard (HS) (anti)calorons (HS (anti)calorons represent the basic constituents of the thermal ground state in the deconfining phase of $SU(2)$ Yang–Mills Quantum Thermodynamics [1–3]) of topological charge modulus $|k| = 1$ and trivial holonomy [4–14]—which, in the singular gauge used to construct them, exhibit boundary behavior around a central spacetime point x_0 defining $\hbar$. Indeterminism of *Minkowskian* processes involving (anti)caloron centers is an immediate consequence which, together with spatial independence, implies the Bose–Einstein distribution for thermal photons.

Interactions of photons with massive vector modes (adjoint Higgs mechanism) are mediated by effective vertices, which occur through (anti)caloron centers, are feeble, and, for sufficiently high temperatures, amount to a slight rescaling of $T \to T'$ in thermodynamical quantities [15]. The transition from $T \to T'$ can be interpreted as a re-thermalization due to a collision integral in the sense of Equation (1), associated with loop integration in the photon's polarization tensor [15,16].

The very notion of a Minkowskian spacetime and the Poincaré group, however, relates to the static structure of HS (anti)calorons spatially far from point x_0 [17] if at least two gauge-group factors $SU(2)$ of disparate Yang–Mills scales [18] are invoked. More specifically, we will argue that the spatial peripheries of HS (anti)calorons enable the propagation of coherent, wave-like disturbances, as introduced, e.g., by classically oscillating electric charges, through undulating polarizations of electric and magnetic dipole densities. The wave-particle duality of electromagnetic disturbances would thus be understood in terms of the spatial peripheries and centers of (anti)selfdual, Euclidean field configurations in $SU(2)$ Yang–Mills theory.

This paper is organized as follows. In Section 2 the construction of the thermal ground state for deconfining $SU(2)$ Yang–Mills thermodynamics, the properties of its effective thermal excitations, and the physics of effective radiative corrections are sketched for the reader's convenience. Section 3 reviews observational and theoretical reasons for the postulate that an $SU(2)$ rather than a $U(1)$ gauge principle underlies the fundamental description of thermal photon gases [2]. A discussion of how the spatial periphery of HS (anti)calorons provides electric and magnetic dipole densities, which (i) are protected against (anti)caloron overlap; are (ii) associated with the alternating electric and magnetic polarization of the ground state as induced by external disturbances $\vec{E}$ and $\vec{B}$; and (iii) propagate these disturbances at a finite speed, being independent of frequency and intensity within certain bounds governed by the Yang–Mills scale Λ, is performed in Section 4. Both, propagation of an isolated monochromatic electromagnetic wave and of waves in a thermal ensemble are addressed and confronted with experiment. Section 5 elucidates how short wave lengths, which probe (anti)caloron centers, provoke indeterministic responses. Quantities, which associate with (anti)caloron centers, are interpretable in a Minkowskian spacetime—a concept induced by overlapping (anti)caloron peripheries—if they do not depend on analytic continuation from imaginary to real time. Only integral, gauge-invariant quantities qualify. In particular, the integral of the Chern–Simons current K_μ over the 3-sphere of vanishing 4D radius (topological charge), centered at the inmost spacetime point of an (anti)caloron, and therefore the (anti)caloron action $S_{\mathrm{C,A}} = 8\pi^2/e^2$ is physical in this sense. Here e denotes the effective gauge coupling. It turns out that $S_{\mathrm{C,A}} = \hbar$ [19,20]. As it seems, $\hbar$ is the only physical quantity which can be associated with the center of an (anti)caloron. What can be measured in response of probing such a center is $\hbar$ in combination with classical physical quantities such as frequency or wave number of a disturbance which, temporarily, is propagated by spatial (anti)caloron peripheries. We show that the statistical independence of the emission of (monochromatic) quanta of energy and momenta implies the Bose–Einstein distribution. Section 6 summarizes the present work.

If not stated otherwise we work in (super-)natural units $c = \hbar = k_B = 1$ from now on, c denoting the speed of light in vacuum and k_B Boltzmann's constant.

2. Mini-Review on Deconfining Thermal Ground State, Excitations, and Radiative Corrections

Let us briefly review how the thermal ground state in the deconfining phase of thermal $SU(2)$ Quantum Yang–Mills theory emerges from HS (anti)calorons of topological charge modulus $|k| = 1$ [4]. A crucial observation is that the energy momentum tensor $\theta_{\mu\nu}$ vanishes identically on these (anti)selfdual, periodic Euclidean field configurations when considered in isolation. This implies that HS (anti)calorons do not propagate. Moreover, their spatial peripheries are static, meaning that an adiabatically slow approach of centers, inducing a finite density thereof, does not generate any propagating disturbance on distances larger than the spatial radius R (to be specified below) that is associated with an (anti)caloron center. On the other hand, given that the Euclidean time dependence of field-strength correlations within the central region set by R spatially can be coarse grained into a mere choice of gauge for the inert, adjoint scalar field ϕ of space-time independent modulus $|\phi|$ [1,2] and taking into account that there is a preferred value $R \sim |\phi|^{-1}$ at a given temperature T [19] it is clear that no (potentially to be continued) gauge-invariant Euclidean time dependencies leak out from an (anti)caloron center to the periphery. (Static peripheries cannot resolve and therefore deform centers. However, their spatial overlap, as facilitated by dense packing of centers, introduces a departure from (anti)selfduality and thus finite energy density and pressure [2].) Moreover, topologically trivial, effectively propagating disturbances are governed by an action which is of the same form as the fundamental Yang–Mills action since their (Minkowskian) time dependence can be introduced adiabatically into the physics of overlapping (anti)caloron peripheries and since off-shellness, introduced by just-not-resolved and thus integrated edges of (anti)caloron centers, does not change the form of the action thanks to perturbative renormalizability, see below.

Let us be more specific. For gauge group $SU(2)$ the Harrington–Shepard (HS) caloron (C)—a gauge-field configuration whose components A_μ ($\mu = 4, 1, 2, 3$) assume values in the $SU(2)$ Lie algebra $SU(2)$—is given as follows (antihermitian group generators t_a ($\mathrm{tr}\, t_a t_b = -\frac{1}{2}\delta_{ab}$ with $a, b = 1, 2, 3$)):

$$A_\mu = \bar{\eta}^a_{\mu\nu} t_a \partial_\nu \log \Pi(\tau, r)\,, \tag{2}$$

where $r \equiv |\vec{x}|$, $\bar{\eta}^a_{\mu\nu}$ denotes the antiselfdual 't Hooft symbol [13], $\bar{\eta}^a_{\mu\nu} = \epsilon^a_{\mu\nu} - \delta_{a\mu}\delta_{\nu 4} + \delta_{a\nu}\delta_{\mu 4}$ ($\epsilon^a_{\mu\nu}$ the totally antisymmetric symbol in three dimension with $\epsilon^1_{23} = 1$ and $\epsilon^a_{\mu\nu} = 0$ for $\mu = 4$ or $\nu = 4$). The prepotential $\Pi(\tau, r)$ with

$$\Pi(\tau, r) = 1 + \frac{\pi\rho^2}{\beta r} \frac{\sinh\left(\frac{2\pi r}{\beta}\right)}{\cosh\left(\frac{2\pi r}{\beta}\right) - \cos\left(\frac{2\pi\tau}{\beta}\right)} \tag{3}$$

is derived by an infinite superposition of the temporally shifted prepotential $\Pi_0(x)$ of a singular-gauge instanton [13,14] with topological charge $k = 1$ on $\mathbf{R}^4$ to render $\Pi(\tau, r)$ periodic in τ. One has

$$\Pi_0(x) = 1 + \frac{\rho^2}{x^2}\,, \tag{4}$$

where ρ is the instanton scale parameter. The associated antiselfdual field configuration (A) is obtained in replacing $\bar{\eta}^a_{\mu\nu}$ by $\eta^a_{\mu\nu}$ (selfdual 't Hooft symbol) in Equation (2). Configuration (2) is singular at $\tau = r = 0$ where the topological charge $k = 1$ on $S_1 \times \mathbf{R}^3$ is localized in the sense that the integral of the Chern–Simons current K_μ with

$$K_\mu = \frac{1}{16\pi^2} \epsilon_{\mu\alpha\beta\gamma} \left(A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3} \epsilon^{abc} A^a_\alpha A^b_\beta A^c_\gamma \right) \tag{5}$$

over a three-sphere S^3_δ of radius δ, which is centered at $\tau = r = 0$, is unity for $\delta \to 0$. Since the configuration C of Equation (2) is selfdual (and the associated configuration A is antiselfdual) the action of the HS (anti)caloron is given in terms of its topological charge $k = \pm 1$ and the gauge-coupling constant g as

$$S_C = S_A = \frac{8\pi^2 |k|}{g^2} = \frac{8\pi^2}{g^2} \int_{S^3_\delta} d\Sigma_\mu K_\mu = \frac{8\pi^2}{g^2}\,. \tag{6}$$

Moreover, since Equation (6) holds in the limit $\delta \to 0$ the action $S_C = S_A$ admits a Minkowskian interpretation. Based on [21–25] and on the fact that the thermal ground state emerges from $|k| = 1$ caloron/anticalorons, whose scale parameter ρ essentially coincides with the inverse of maximal resolution, $|\phi|^{-1}$, in deconfining $SU(2)$ Yang–Mills thermodynamics, it was argued in [19], see also [20], that S_C and S_A both equal $\hbar$ if the effective theory emergent from the spatial coarse-graining, see below, is to be interpreted as a local quantum field theory. With [26], see also [17], we now investigate how the field strengths of C and A look like away from their centers at $\tau = r = 0$.

For $|x| \ll \beta$ one has

$$\Pi(x) = (1 + \frac{\pi}{3}\frac{s}{\beta}) + \frac{\rho^2}{x^2} + O(x^2/\beta^2)\,, \tag{7}$$

where s is given as

$$s \equiv \pi \frac{\rho^2}{\beta}\,. \tag{8}$$

From Equations (2) and (7) one obtains the following expression for $F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\kappa\lambda} F_{\kappa\lambda} \equiv \tilde{F}_{\mu\nu}$ on the caloron (C)

$$F^a_{\mu\nu} = -4\rho'^2 \frac{\bar{\eta}^a_{\alpha\beta}}{(x^2 + \rho'^2)^2} I_{\alpha\mu} I_{\beta\nu} + O(x^2/\beta^4)\,, \tag{9}$$

where $I_{\alpha\mu} \equiv \delta_{\alpha\mu} - 2\frac{x_\alpha x_\mu}{x^2}$. At small four-dimensional distances $|x|$ from the caloron center the field strength thus behaves like the one of a singular-gauge instanton with a renormalized scale parameter $\rho'^2 = \frac{\rho^2}{1+\frac{\pi}{3}\frac{s}{\beta}}$. For $|x| \ll \beta$ the field strength tensor $F_{\mu\nu}$ thus exhibits a dependence on τ which would give rise to a nontrivial analytic continuation with no Minkowskian interpretation. For $r \gg \beta$ the selfdual electric and magnetic fields E_i^a and B_i^a are *static*:

$$E_i^a = B_i^a \sim -\frac{\frac{\hat{x}^a \hat{x}_i}{r^2} - \frac{1}{rs}(\delta_i^a - 3\hat{x}^a \hat{x}_i)}{(1+\frac{r}{s})^2}\,. \tag{10}$$

Here $\hat{x}_i \equiv \frac{x_i}{r}$ and $\hat{x}^a \equiv \frac{x^a}{r}$. A simplification of Equation (10) occurs for $\beta \ll r \ll s$ as

$$E_i^a = B_i^a \sim -\frac{\hat{x}^a \hat{x}_i}{r^2}\,. \tag{11}$$

This is the field of a static non-Abelian monopole of unit electric and magnetic charges (dyon). For $r \gg s \gg \beta$ Equation (10) reduces to

$$E_i^a = B_i^a \sim s\,\frac{\delta_i^a - 3\,\hat{x}^a \hat{x}_i}{r^3}\,, \tag{12}$$

representing the field strength of a static, selfdual non-Abelian dipole field. The dipole moment p_i^a of the latter is given as

$$p_i^a = s\,\delta_i^a\,. \tag{13}$$

For A one simply replaces $E_i^a = B_i^a$ by $E_i^a = -B_i^a$ in Equations (10)–(12).

It is instructive to discuss a slight deformation of the HS caloron towards non-trivial holonomy, keeping s fixed and maintaining selfduality [11]. A holonomy $u \ll \pi/\beta$ then produces a nearly massless and de-localized magnetic monopole (charged w.r.t. $U(1) \subset SU(2)$ left unbroken under $A_4(|\vec{x}| \to \infty) = ut^3 \neq 0$ where t^a ($a = 1,2,3$) now denotes the hermitian generator of $SU(2)$, normalized to tr $t^a t^b = \frac{1}{2}\,\delta^{ab}$) and its localized massive antimonopole, the latter centered at $r \sim 0$—a position which nearly coincides with the spatial locus of topological charge of the (anti)caloron [11]. The centers of the mass densities of both particles are separated by s. For $r \ll s$ the massive antimonopole appears like a purely magnetic charge. However, as r increases beyond s this magnetic charge is increasingly screened by the presence of the delocalized magnetic monopole such that the non-Abelian, selfdual field strength of Equation (12) prevails (no reference to the scale u) (In contrast, the definition of an Abelian field strength, see [27], requires that $u \neq 0$ to be able to define the $SU(2)$ unit vector $\hat{A}_4$ everywhere except for the two central points of the magnetic charge distributions). Moreover, it was shown in [12] that $u \ll \pi/\beta$ leads to monopole-antimonopole attraction under the influence of small field fluctuations. This renders the interpretation of s as the scale of magnetic monopole-antimonopole separation irrelevant for the physics of (slightly deformed) HS (anti)caloron peripheries.

Let us now come back to the question how field ϕ emerges thanks to HS (anti)calorons. We have discussed in [3] why the following definition of a family of phases associated with the inert field ϕ is unique and, as a whole, transforms homogeneously under fundamental gauge transformations

$$\begin{aligned}\{\hat{\phi}^a\} \;\equiv\; & \textstyle\sum_{C,A} \mathrm{tr}\int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0})\,\{(\tau,\vec{0}),(\tau,\vec{x})\} \\ & \times\, F_{\mu\nu}(\tau,\vec{x})\{(\tau,\vec{x}),(\tau,\vec{0})\},\end{aligned} \tag{14}$$

where the Wilson line $\{(\tau,\vec{0}),(\tau,\vec{x})\}$ is defined as

$$\begin{aligned}\{(\tau,\vec{0}),(\tau,\vec{x})\} &\equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right],\\ \{(\tau,\vec{x}),(\tau,\vec{0})\} &\equiv \{(\tau,\vec{0}),(\tau,\vec{x})\}^\dagger,\end{aligned} \tag{15}$$

the integration in Equation (15) is along the straight spatial line connecting the points $(\tau,\vec{0})$ and $(\tau,\vec{x})$, and the sum is over configuration C of Equation (2) and its antiselfdual partner A. Moreover, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu]$ denotes the Yang–Mills field-strength tensor, and the symbol $\mathcal{P}$ demands path ordering. On C and A path ordering actually is obsolete since the spatial components of the gauge field represent a hedge-hog configuration which fixes the direction in $SU(2)$ in terms of the direction in 3-space. As a consequence, all factors, associated with infinitesimal line elements, contributing to the group element of Equation (15) commute, and therefore their order is irrelevant. One can show [1,3] that in performing the integrations over $\vec{x}$ and ρ in Equation (14) and by re-instating temporal shifts $\tau \to \tau + \tau_{C,A}$, the family $\{\hat{\phi}^a\}$ is parameterized, modulo global gauge rotations, by four real parameters, two for each "polarization state" for harmonic motion in a plane of $SU(2)$. This uniquely associates a linear differential operator $\mathcal{D}$ of order two with $\{\hat{\phi}^a\}$: $\mathcal{D} \equiv \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$. Moreover, one shows that the result of the ρ-integration, which depends cubically on its upper cutoff ρ_u, hence is sharply dominated by ρ_u whatever the value of this cutoff turns out to be.

Operator $\mathcal{D}$ exhibits an explicit temperature (β) dependence. However, due to the fact that the action in Equation (6), which determines the weight in the partition function that is introduced by HS (anti)calorons, is not temperature dependent such an explicit temperature dependence must not appear in the effective, thermal Yang–Mills action (ETYMA) obtained from a spatial coarse-graining in combination with integrating out these (anti)calorons. Therefore, in deriving the part of ETYMA, which is solely due to the field ϕ, by demanding it to be stationary w.r.t. variations in ϕ at a fixed value of β (Euler–Lagrange equation) the explicit β dependence in $\mathcal{D}$ is to be absorbed into the ϕ-derivative of a potential $V(\phi)$. Demanding consistency of a first-order Bogomol'nyi-Prasad-Sommerfield (BPS) equation, which needs to be satisfied by ϕ owing to the fact that it embodies spatial field-strength correlations on (anti)selfdual gauge-field configurations, one derives the following first-order equation for V

$$\frac{\partial V(|\phi|^2)}{\partial|\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2}, \tag{16}$$

whose solution reads

$$V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}. \tag{17}$$

Here Λ denotes an arbitrary mass scale (the Yang–Mills scale). This implies that

$$|\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}. \tag{18}$$

Scale $|\phi|^{-1}$ represents a minimal length scale in evaluating the consequences of ETYMA. Therefore, $\rho_u \sim |\phi|^{-1}$. The condition $s_u \gg \beta$, which is required for Equations (11) and (12) to actually represent static field strengths, is always satisfied provided that the dimensionless temperature $\lambda \equiv \frac{2\pi T}{\Lambda} \gg 1$. Namely, one then has

$$\frac{s_u}{\beta} \equiv \pi\left(\frac{\rho_u}{\beta}\right)^2 \sim \pi\left(\frac{\lambda^{3/2}}{2\pi}\right)^2 = \frac{\lambda^3}{4\pi} \gg 1. \tag{19}$$

Also, it is true that

$$\frac{\rho_u}{\beta} \sim \frac{\lambda^{3/2}}{2\pi} \gg 1. \tag{20}$$

That the condition $\lambda \gg 1$ is satisfied in the deconfining phase of $SU(2)$ Yang–Mills thermodynamics is a consequence of the evolution equation for the effective coupling e. This evolution follows from the demand of thermal consistency of the Yang–Mills gas of non-interacting thermal quasi-particle fluctuations and their thermal ground state [2], based on ETYMA density

$$\mathcal{L}_{\rm eff}[a_\mu] = {\rm tr}\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)\,. \tag{21}$$

In Equation (21) $G_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - ie[a_\mu, a_\nu] \equiv G^a_{\mu\nu}\,t_a$ denotes the field strength of the *effective* trivial-topology gauge field $a_\mu = a^a_\mu\,t_a$, $D_\mu\phi = \partial_\mu\phi - ie[a_\mu,\phi]$, and e is the effective gauge coupling. The latter takes the value $e = \sqrt{8}\pi$ almost everywhere in the deconfining phase (in natural units $\hbar = k_B = c = 1$) [3,28,29]. One can show [3,19] that $\mathcal{L}_{\rm eff}$ is uniquely determined as in Equation (21), resting on the facts that the effective $k = 0$ field a_μ is governed by the first term due to perturbative renormalizability [30–32], gauge invariance fixes the second term, and no higher-dimensional mixed operators, involving fields a_μ and ϕ, may appear due to the impossibility of the former to resolve the physics leading to the latter (inertness). The action density of Equation (21) predicts the existence of one massless and two massive (adjoint Higgs mechanism, thermal quasi-particle excitations) directions in $SU(2)$ provided that their interactions are feeble and justifiedly expandable into a growing number of vertices. In unitary-Coulomb gauge (a completely fixed, physical gauge) constraints on admissible four-momentum transfers can be stated precisely. These constraints imply a rapid numerical convergence of radiative corrections [15,33], and, by counting the number of constraints versus the number of radial loop variables in dependence of loop number, it was conjectured in [3,34] that one-particle irreducible bubble diagrams vanish, starting from a finite loop number. Note that Equation (20) states the independence of ϕ's modulus on Euclidean time τ, and Equations (19) and (12) indicate that an (anti)selfdual static dipole field only emerges spatially far from the central region of an (anti)caloron, the latter being bounded by a spatial sphere of radius $|\phi|^{-1} \sim \rho_u$.

3. Mini-Review on the Postulate $SU(2)_{\rm CMB}$ (Thermal Photon Gases)

In [2] we have postulated that thermal photon gases, fundamentally seen, should be subject to an $SU(2)$ rather than a $U(1)$ gauge principle.

Theoretically, such a postulate rests on the facts that in the deconfining phase of Yang–Mills thermodynamics the gauge symmetry $SU(2)$ is broken to $U(1)$ by the field ϕ and that the interaction between massive and massless excitations is feeble with the exception of the low-frequency regime at temperatures not far above the critical temperature T_c for the deconfining-preconfining phase transition [15,16,35]. Observationally, however, the physics of the deconfining-preconfining phase boundary [3,36], the presence of a nontrivial thermal ground state, giving rise to massive quasi-particle fluctuations and therefore an equation of state $p = p(\rho) \neq \frac{1}{3}\rho$, and feeble radiative effects influencing the propagation properties of the massless mode [3,37,38] allow to confront the $SU(2)$ postulate with reality. As for the former, a highly significant cosmological radio excess at frequencies $\nu \leq 1\,\text{GHz}$ [39], when considered in the $SU(2)$ framework, links the evanescence of low-frequency electromagnetic waves belonging to the Cosmic Microwave Background (CMB) to an (incomplete) condensation phenomenon involving screened and ultralight electric charges (In units, where $\hbar$ is re-instated as a dimensionful quantity, one has $e = \sqrt{8}\pi/\sqrt{\hbar}$ almost everywhere in the deconfining phase. This and the fact that the thermal ground state is sharpy dominated by (anti)caloron radii $\rho_u \sim |\phi|^{-1}$, see Section 2, imply that the (anti)caloron action equals $\hbar$ [19]. The fact that the (unitless) Quantum Electrodynamics (QED) fine-structure constant α is given as $\alpha = \frac{Q^2}{4\pi\hbar}$, where Q denotes the charge of the electron, implies an electric-magnetically dual interpretation of the $U(1)$ charge content [27] of $SU(2)$ field configurations [19]: $Q \propto 1/e$). This gives rise to a (partial) Meissner effect, and hence frequencies smaller than the implied Meissner mass m_γ do not propagate but constitute an ensemble of evanescent waves (In Section 4 we show that these low frequencies, indeed, associate with classical waves). As a result, a re-shuffling of spectral power, creating a maximum at zero frequency, takes place at small CMB frequencies. Because m_γ is (critically and thus rapidly) increasing when T is decreased below T_c [2] the observation of a spectral-excess anomaly in the CMB at small frequencies implies that the present baseline temperature of the CMB, $T_0 = 2.725\,\mathrm{K}$, practically coincides with T_c. This fixes the Yang–Mills scale Λ of the theory by virtue of $\Lambda = \frac{2\pi}{\lambda_c} T_c \sim 10^{-4}\,\mathrm{eV}$ [36] ($\lambda_c \equiv 13.87$ [2]) which prompts the name $SU(2)_{\mathrm{CMB}}$. Based on the precise experimental match $T_c = T_0$ and on the availability of the (practically one-loop exact) equation of state $p = p(\rho)$ of deconfining $SU(2)$ Yang–Mills thermodynamics [2], a prediction of the CMB redshift (z)—temperature (T) relation is accomplished [40] which exhibits strong violations of conformal behavior at $T \sim 2\,T_0$ where $z \sim 2.1$ (conventionally: $z \sim 1$). As a consequence, the discrepancy between the redshift z_{re} for instantaneous re-ionization of the intergalactic medium, as extracted with $z_{\mathrm{re}} \sim 11$ from the depletion of peaks in the CMB TT angular power spectrum by appealing to the conventional, conformal z–T relation $T/T_0 = z + 1$ [41], and as observed with $z_{\mathrm{re}} \sim 6$ by detection of the Gunn–Peterson trough for $z \geq z_{\mathrm{re}}$ in high-redshift quasar spectra [42], is resolved [40]. Finally, with $T_c = T_0$ one predicts that the temperature dependence of *radiatively* induced effects at low frequencies such as anomalies in blackbody spectra [15,16,35] (spectral gap, extending from zero to about 17 GHz at $T \sim 5.4$ Kelvin) as well as the thermal excitation of longitudinally propagating magnetic-field modes [43] (several, partially superluminal, low-frequency branches whose combined energy densities match the order of magnitude of the field strength ($\sim 10^{-8}$ Gauss) squared of intergalactic magnetic fields extracted from small-angle CMB anisotropies [44]).

4. Dipole Densities: (Anti)caloron Peripheries and Thermal Wave Propagation

In this section, we discuss the vacuum parameters of Classical Electromagnetism—electric permittivity ϵ_0 and magnetic susceptibility μ_0—and their possible relation to the thermal-ground state properties caused by (anti)caloron peripheries, see also [17]. It will become clear that, in describing thermal photon gases, classical aspects of the thermal ground state of $SU(2)_{\mathrm{CMB}}$ are limited to very low frequencies.

We have seen by virtue of Equations (19) and (20) that a probe being sensitive to spatial distances r from a given (anti)caloron center, which are much greater than the scale s_u (s_u itself being much greater than the coarse-graining scale $\rho_u \sim |\phi|^{-1}$), detects the static (anti)selfdual dipole field of Equation (12). The electromagnetic field, which propagates through the deconfining thermal ground state in absence of any explicit electric charges, is considered a monochromatic plane wave of wave length $l \sim r$. Such a field associates with a density of (anti)selfdual dipoles, see Equation (12). Because they are given by $p_i^a = s_u \delta_i^a$ their dipole moments align along the direction of the exciting electric or magnetic field both in space and in $SU(2)$. Note that at this stage the definition of what is to be viewed as an Abelian direction in $SU(2)$ is a global gauge convention such that *all* spatial directions of the dipole moment p_i^a are a priori thinkable. In a thermal situation and unitary gauge $\phi = 2\,t^3\,|\phi|$ we would thus set $a = 3$ which implies that $\vec{p} = s_u \hat{e}_3$.

Per spatial coarse-graining volume V_{cg} of radius $|\phi|^{-1} = \rho = \sqrt{\frac{\Lambda^3}{2\pi T}}$ with

$$V_{\mathrm{cg}} = \frac{4}{3}\pi|\phi|^{-3}\,, \tag{22}$$

the center of a selfdual HS caloron or the center of an antiselfdual HS anticaloron [3] resides. Note the large hierachy between s_u (the minimal spatial distance to the center of a (anti)caloron, which allows to identify the static, (anti)selfdual dipole) and the radius of the sphere $|\phi|^{-1}$ defining V_{cg},

$$\frac{s_u}{|\phi|^{-1}} = \frac{1}{2}\lambda^{3/2} = 25.83\left(\frac{\lambda}{\lambda_c}\right)^{3/2}. \tag{23}$$

If the exciting field is electric, $\vec{E}_e$, then it sees *twice* the electric dipole p_i^a (cancellation of magnetic dipole between caloron and anticaloron), if it is magnetic, $\vec{B}_e$, it sees *twice* the magnetic dipole p_i^a (cancellation of electric dipole between caloron and anticaloron, $\vec{E} = -\vec{B} \quad \Leftrightarrow \quad -\vec{E} = \vec{B}$). To be definite, let us discuss the electric case in detail, which is characterized by $\vec{E}_e$. The modulus of the according dipole density $\vec{D}_e || \vec{E}_e$ is given as

$$|\vec{D}_e| = \frac{2s_u}{V_{\text{cg}}} = \frac{3}{4\pi}\Lambda^2 \lambda_c^{1/2} \left(\frac{\lambda}{\lambda_c}\right)^{1/2} . \tag{24}$$

In Classical Electromagnetism the relation between the fields $\vec{E}_e$ and $\vec{D}_e$ is

$$\vec{D}_e = \epsilon_0 \vec{E}_e \,, \tag{25}$$

where

$$\epsilon_0 = 5.52703 \times 10^7 \, \frac{Q}{\text{V m}} \tag{26}$$

is the electric permittivity of the vacuum, and $Q = 1.602 \times 10^{-19}$ Ampere seconds (A s) denotes the elementary unit of electric charge (electron charge), both quoted in SI units.

According to electromagnetism the energy density ρ_{EM} carried by an external electromagnetic wave with $|\vec{E}_e| = |\vec{B}_e|$ is

$$\rho_{\text{EM}} = \frac{1}{2}(\epsilon_0 |\vec{E}_e|^2 + \frac{1}{\mu_0}|\vec{B}_e|^2) = \frac{1}{2}(\epsilon_0 + \frac{1}{\mu_0})|\vec{E}_e|^2 \,. \tag{27}$$

In natural units we have $\epsilon_0 \mu_0 = 1/c^2 = 1$, and therefore (To set $\epsilon_0 \mu_0 = 1$ is a short cut. This would have come out if we had treated the magnetic case explicitly.) one has $\mu_0 = 1/\epsilon_0$. Thus

$$\rho_{\text{EM}} = \epsilon_0 |\vec{E}_e|^2 \,. \tag{28}$$

The $\vec{E}_e$-field dependence of ρ_{EM} is converted into a fictitious temperature dependence by demanding that the temperature of the thermal ground state of $SU(2)_{\text{CMB}}$ adjusts itself such as to accommodate ρ_{EM} in terms of its ground-state energy density ρ_{gs} [2],

$$\rho_{\text{EM}} = \rho_{\text{gs}} = 4\pi\Lambda^3 T \quad \Leftrightarrow \quad |\vec{E}_e| = \Lambda^2 \sqrt{2\frac{\lambda_c}{\epsilon_0}} \left(\frac{\lambda}{\lambda_c}\right)^{1/2} . \tag{29}$$

Equation (29) generalizes the thermal situation of ground-state energy density (see below), where ground-state thermalization is induced by a thermal ensemble of excitations, to the case where the thermal ensemble is missing but the probe field induces a fictitious temperature and energy density to the ground state. Combining Equations (24), (25) and (29), and introducing the ratio ξ between the non-Abelian monopole charge Q' in the dipole and the (Abelian) electron charge (In natural units, the actual charge of the monopole constituents within the (anti)selfdual dipole is $1/g$ where g is the undetermined fundamental gauge coupling. This is absorbed into ξ.) Q, we obtain

$$\begin{aligned} \epsilon_0[Q(\text{V m})^{-1}] &= \tfrac{3}{\sqrt{32}\pi}\left(\tfrac{\Lambda[\text{m}^{-1}]}{\Lambda[\text{eV}]}\right)^{1/2} \xi Q \sqrt{\epsilon_0[Q(\text{V m})^{-1}]} \quad \Leftrightarrow \\ \epsilon_0[Q(\text{V m})^{-1}] &= \tfrac{9}{32\pi^2}\tfrac{\Lambda[\text{m}^{-1}]}{\Lambda[\text{eV}]}(\xi Q)^2 \,. \end{aligned} \tag{30}$$

Notice that ϵ_0 does not exhibit any temperature dependence and thus no dependence on the field strength $\vec{E}_e$. It is a universal constant. In particular, ϵ_0 does *not* relate to the state of fictitious ground-state thermalization which would associate to the rest frame of a local heat bath. To produce the measured value for ϵ_0 as in Equation (26) the ratio ξ in Equation (30) is required to be

$$\xi \equiv \frac{Q'}{Q} = 19.56\,. \tag{31}$$

Thus, compared to the electron charge, the charge unit associated with a (anti)selfdual non-Abelian dipole, residing in the thermal ground state, is gigantic. The discussion of μ_0 proceeds in close analogy to the case of ϵ_0. (It would be μ_0^{-1} defining the ratio between the modulus of the magnetic dipole density and the magnetic flux density $|\vec{B}|$.) Here, however, the comparison between non-Abelian magnetic charge and an elementary, magnetic, and Abelian charge is not facilitated since the latter does not exist in electrodynamics.

The consideration above, linking the density of (anti)selfdual static dipoles in the thermal ground state to an exciting field-strength modulus $|\vec{E}_e|$ via a *fictitious* temperature T, which represents the energy density of the thermal ground state in terms of the classical field-energy density introduced by $|\vec{E}_e|$, has assumed isolated propagation of a monochromatic plane wave. How would the argument that the thermal ground state associates with the classical vacuum parameters ϵ_0 and μ_0 have to be modified if a thermodynamical equilibrium subject to a genuine *thermodynamical* temperature T prevails? The condition that wavelength l must be substantially larger than s_u amounts to

$$l = \frac{2\pi}{xT} \gg s_u = \frac{2\pi^2 T^2}{\Lambda^3} \quad \Leftrightarrow \quad x \ll \frac{1}{\pi}\left(\frac{\Lambda}{T}\right)^3 , \tag{32}$$

where $x \equiv \frac{2\pi\nu}{T}$, and ν denotes the frequency of the wave. In particular, for $T = T_c$ (32) states that

$$x \ll \frac{1}{\pi}\left(\frac{2\pi}{\lambda_c}\right)^3 \sim 0.0296 \qquad (\lambda_c = 13.87)\,. \tag{33}$$

Considering that the maximum of Planck's spectral energy density $u_{\mathrm{Planck}} = \frac{2}{\pi}T^3\frac{x^3}{e^x-1}$ occurs at $x = 2.82$ we conclude that wave-like propagation in a thermodynamical situation is restricted to the deep Rayleigh–Jeans regime where spectral energy density is (classically) given (Radiative effects in $SU(2)$ Yang–Mills thermodynamics alter the low-frequency behavior of the Rayleigh–Jeans spectral intensity [16]: there is a spectral gap ν_g such that no radiance is predicted to occur for $\nu \leq \nu_g$ (screening), and there is, compared to the conventional Rayleigh–Jeans spectrum, an exponentially decaying overshoot (anti-screening). However, $\nu_g \propto T^{-1/2}$ and therefore this radiative modification of the Rayleigh–Jeans spectrum can be neglected at temperatures much higher than $T_0 = 2.725\,\mathrm{K}$.) as

$$u_{\mathrm{RJ}} = \frac{2}{\pi}T^3 x^2 = 8\pi T\nu^2\,. \tag{34}$$

To convert u_{RJ} into an energy density it needs to be multiplied by a (constant) band width $\Delta\nu$. Notice that both, $\rho_{\mathrm{RJ}} \equiv u_{\mathrm{RJ}}\Delta\nu = 8\pi T\nu^2\Delta\nu$ and the energy density of the thermal ground state $\rho_{\mathrm{gs}} \equiv 4\pi T\Lambda^3$, compare with Equation (29), depend linearly on T. Therefore, an average electric field-strength modulus $|\vec{E}_e|$ in the Rayleigh–Jeans regime, defined as

$$\rho_{\mathrm{RJ}} = 8\pi T\nu^2\Delta\nu = \rho_{\mathrm{EM}} = \epsilon_0|\vec{E}_e|^2 \quad \Leftrightarrow \quad |\vec{E}_e| = 2\nu\sqrt{\frac{\lambda_c\Lambda\Delta\nu}{\epsilon_0}}\left(\frac{\lambda}{\lambda_c}\right)^{1/2} , \tag{35}$$

also yields temperature independence of ϵ_0,

$$\epsilon_0 \equiv \frac{|\vec{D}_e|}{|\vec{E}_e|} = \frac{9}{64\pi^2}\frac{\Lambda[\mathrm{m}^{-1}]}{\Lambda[\mathrm{eV}]}(\tilde{\xi}Q)^2 \times \frac{\Lambda^3}{\nu^2\Delta\nu}\left[1\right] , \tag{36}$$

where $\left[1\right]$ indicates that the preceding fraction is to be evaluated in natural units ($\hbar = k_B = c = 1$) so that it is dimensionless. The charge of a monopole in the dipole is represented by $\tilde{\xi}Q$. This charge now is perceived by the *ensemble* of waves with frequencies contained in the band $\Delta\nu$. Since ϵ_0 should be a frequency independent quantity we need to demand that

$$\tilde{\xi}^2 = C\frac{\nu^2\Delta\nu}{\Lambda^3}\,, \tag{37}$$

where $C = 2\xi^2$, compare with Equation (30). We conclude that the charge of a monopole making up the dipole as perceived by the ensemble of waves with frequencies contained in the band $\Delta\nu$ is increasingly screened with decreasing frequency ν.

Finally, from the condition $l \gg s_u$ and Equation (29) one obtains (natural units)

$$|\vec{E}_e|^4\nu \ll 8\Lambda^9\,. \tag{38}$$

Relation (38) needs to be obeyed by any classically propagating, monochromatic electromagnetic wave. Its violation indicates that the propagation of electromagnetic field energy no longer is mediated by an adiabatic time-harmonic modulation of the polarization state of electric and magnetic dipole densities of the vacuum, as provided by overlapping (anti)caloron peripheries, but by the quantum physics of (anti)caloron centers. Setting $\Lambda = \Lambda_{\rm CMB}$, (38) is a strong restriction on admissible frequencies at commonly occurring intensities in the propagation of electromagnetic waves. Such a restriction, however, is not supported by experience. In [18] it was therefore proposed to add flexibility to the value of Λ by postulating a product $SU(2)_{\rm CMB}\times SU(2)_e$ of gauge groups with $\Lambda_e \sim m_e \sim 0.5\,$MeV, see also [2,3], subject to a mixing angle of the unbroken (diagonal) subgroups which is adjusted depending on whether or not this gauge dynamics plays out in a thermal or nonthermal situation or any intermediate thereof. (In the present Standard Model of particle physics such a mixing between the $U(1)$ subalgebra of $SU(2)_W$ and $U(1)_Y$, the latter being regarded as a fundamental gauge symmetry, is subject to a *fixed* value of the associated Weinberg angle.) According to (38) the large value of Λ_e allows for the propagation of electromagnetic waves throughout the entire experimentally accessed frequency spectrum at commonly experienced intensities. However, by virtue of Equation (29) those intensities usually relate to (fictitious) temperatures that are much lower than $T_{c,e} \sim 2.21\,\Lambda_e$. As a consequence, the hierarchy between s_u and $|\phi|^{-1}$, taking place for $\lambda_c \geq \lambda$, actually is inverted in physical wave propagation subject to $SU(2)_e$. That is, the center of an (anti)caloron would extend well beyond a typical wavelength, thus in principle introducing hard-to-grasp nonthermal quantum behavior. Still, since (38) does not depend on the concept of a temperature anymore we may regard it as universally valid: it needs to be satisfied by any monochromatic, classically propagating electromagnetic wave.

5. Bose–Einstein Distribution: (Anti)caloron Centers and Indeterministic Emission of Quanta of Energy and Momentum

The derivation of the dipole density in Equation (24) has appealed to the independence and inertness of (anti)caloron centers in "sourcing" their respective peripheries, the latter supporting static dipole fields. This is consistent since fields propagating by virtue of peripheries never probe centers. The fact that the thermal ground state actually is a spatial arrangement of densely packed (anti)caloron centers, implying profound spatial overlaps of (anti)caloron peripheries, is implemented by Equation (29) which assigns a finite energy density to this ground state in terms of some temperature T which, in turn, is determined by the field-strength modulus $|\vec{E}_e|$ in the sense of an adiabatic deformation of the isotropic, thermal situation. (Anti)caloron centers are probed, however, if the wavelength l of a propagating disturbance approaches the value s_u—a situation when dipole moments induced by time-harmonic monopole accelerations, see Equation (11), yield inconsistencies [17]. This mirrors the fact that Maxwell's equations are void of magnetic sources (locality).

As the wave length l of a would-be propagating disturbance substantially falls below s_u we need to consider the physics inherent to the central region of an (anti)caloron which is anything but classical, see Section 3. Thus the classical quantities wave length l and frequency ν both cease to be applicable as physical concepts. On the other hand, the only trivially continuable and thus physical quantity associated with the central region of an (anti)caloron is the quantum of action $\hbar$. This can be used to transmute the no longer applicable classical concepts l and ν into valid concepts $|\vec{p}| = \hbar 2\pi l^{-1}$ (momentum modulus) and $E = \hbar 2\pi\nu$ (energy). As a consequence, it is the indeterministic emission of a quantum of momentum and energy (photon) that is expected as the response of an (anti)caloron center to disturbances whose classical propagation over distances larger than l is excluded. Since, apart from small correlative effects, which are induced by effective Yang–Mills vertices and computable in the theory (21), see [15,16,33,35,43], (anti)caloron centers act spatio-temporally independently, the derivation of the mean photon occupation number $\bar{n}$ proceeds as usual. Namely, the Boltzmann weight p_n of an n-fold photon event, each photon possessing energy E in the thermal ensemble, is the nth power of the Boltzmann weight p_1 of a single photon event

$$p_1(x) = \mathrm{e}^{-x} \quad \Rightarrow \quad p_n(x) = \mathrm{e}^{-nx} \quad \left(x \equiv \frac{E}{k_B T} \right) . \tag{39}$$

Therefore, the partition function $Z(x)$ reads

$$Z(x) = \sum_{n=0}^{\infty} p_n(x) = \frac{1}{1 - \mathrm{e}^{-x}} . \tag{40}$$

Finally, mean photon number $\bar{n}(x)$ is given as

$$\begin{aligned} \bar{n}(x) &= \tfrac{1}{Z(x)} \textstyle\sum_{n=0}^{\infty} n p_n(x) \\ &= -\tfrac{d \log Z(x)}{dx} = \tfrac{1}{\mathrm{e}^x - 1} \equiv n_B(x) , \end{aligned} \tag{41}$$

where $n_B(x)$ denotes the Bose–Einstein distribution function.

6. Conclusions

In this contribution we have given a sketchy overview on how the thermal ground state emerges in $SU(2)$ Yang–Mills theory in terms of a spatial coarse-graining over the field-strength correlation within the center of an electric-magnetically (anti)selfdual (anti)caloron gauge-field configuration of topological charge modulus unity and trivial holonomy [4], giving rise to an effective inert scalar field ϕ, and a pure-gauge solution a_μ^{gs} of the effective Yang–Mills field equations, sourced by ϕ. Details of this process can be studied in [1,3].

After motivating the postulate by observational facts that thermal photon gases should be subject to an $SU(2)$ rather than a $U(1)$ gauge principle we have subsequently addressed the question of how the $SU(2)$ thermal ground state remains a valid concept in supporting the propagation of electromagnetic waves. Namely, the electric permittivity ϵ_0 and the magnetic permeability μ_0 of the vacuum, which are parameters of Classical Electromagnetism, are related to their respective dipole densities emerging from the peripheries of (anti)calorons while their central regions, $r \leq |\phi|^{-1}$, are densely packed spatially. Both, ϵ_0 and μ_0 turn out to be temperature independent, and this derivation can be performed for an isolated, monochromatic wave and spectral bands within the deep Rayleigh–Jeans regime in a given (conventional) black-body spectrum.

The last part of this paper dealt with the physics implied by the central regions of (anti)calorons. We have argued here that the classical concepts frequency and wave length necessarily convert into quanta of energy and momentum (photon) by virtue of the localization of the quantum of action at the inmost spacetime point within the center of an (anti)caloron. Since, modulo small correlative effects—computable in terms of radiative corrections in the effective theory—central regions in (anti)calorons are, in a Minkowskian sense, spatiotemporally independent one concludes that, thermodynamically, the Boltzmann weight of an n-photon event in the gas factorizes into Boltzmann weights of a single-photon event. This implies the Bose–Einstein distribution function.

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