Supersymmetric Explanation of *CP* Violation in $K \to \pi\pi$ Decays

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Recent progress in the determination of hadronic matrix elements has revealed a tension between the measured value of ϵ'_K/ϵ_K, which quantifies direct *CP* violation in $K \to \pi\pi$ decays, and the standard-model prediction. The well-understood indirect *CP* violation encoded in the quantity ϵ_K typically precludes large new-physics contributions to ϵ'_K/ϵ_K and challenges such an explanation of the discrepancy. We show that it is possible to cure the ϵ'_K/ϵ_K anomaly in the minimal supersymmetric standard model with squark masses above 3 TeV without overshooting ϵ_K. This solution exploits two features of supersymmetry: the possibility of large isospin-breaking contributions (enhancing ϵ'_K) and the Majorana nature of gluinos (permitting a suppression of ϵ_K). Our solution involves no fine-tuning of *CP* phases or other parameters.

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Measurements of charge-parity (*CP*) violation are sensitive probes of physics beyond the standard model (SM). *CP* violation in $K \to \pi\pi$ decays is characterized by the two quantities, ϵ_K and ϵ'_K, which describe indirect and direct *CP* violation, respectively. $|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$ measures *CP* violation in the K^0-$\bar{K}^0$ mixing amplitude, in which the strangeness quantum number S changes by two units [1]. ϵ'_K quantifies *CP* violation in the $|\Delta S| = 1$ amplitude triggering the decay $K \to \pi\pi$. To predict ϵ'_K in the SM, one must calculate hadronic matrix elements of four-quark operators with nonperturbative methods. A determination of all operators by lattice QCD has been obtained only recently [2], and the predicted ϵ'_K lies substantially below the experimental value [3]:

$$\frac{\epsilon'_K}{\epsilon_K} = \begin{cases} (16.6 \pm 2.3) \times 10^{-4} & \text{(PDG [1])} \\ (1.0 \pm 4.7 \pm 1.5 \pm 0.6) \times 10^{-4} & \text{(SM-NLO).} \end{cases} \quad (1)$$

Our SM prediction [4,5] is based on the next-to-leading order (NLO) calculation of Wilson coefficients and anomalous dimensions [8,9] and the hadronic matrix elements of Refs. [2,10]. As in Ref. [7], we exploit *CP*-conserving data to reduce hadronic uncertainties. The two numbers in Eq. (1) disagree by 2.9σ [4,7]. This tension is underpinned by results found with the $1/N_c$ expansion (dual QCD approach) [11–13], which is a completely different calculational method [7]. In the near future, the increasing precision of lattice calculations will sharpen the SM prediction in Eq. (1) further and answer the question about new physics (NP) in ϵ'_K.

An explanation of the puzzle in Eq. (1) by physics beyond the SM calls for a NP contribution which is seemingly even larger than the SM value. On general grounds, however, one expects that NP effects in a $|\Delta F| = 1$ four-quark process are highly suppressed once constraints from the corresponding $|\Delta F| = 2$ transition are taken into account. Here F denotes the flavor quantum number, and $F = S$ in our case of $K \to \pi\pi$ decays. To explain the NP hierarchy in $|\Delta F| = 1$ vs $|\Delta F| = 2$ transitions, we specify to ϵ'_K and ϵ_K: The SM contributions to both quantities are governed by the combination

$$\tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - i0.6) \times 10^{-3} \quad (2)$$

of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix with $\epsilon_K'^{\mathrm{SM}} \propto \mathrm{Im}\tau/M_W^2$ and $\epsilon_K^{\mathrm{SM}} \propto \mathrm{Im}\tau^2/M_W^2$. If the NP contribution comes with the $\Delta S = 1$ parameter δ and is mediated by heavy particles of mass M, one finds $\epsilon_K'^{\mathrm{NP}} \propto \mathrm{Im}\delta/M^2$, $\epsilon_K^{\mathrm{NP}} \propto \mathrm{Im}\delta^2/M^2$, and therefore the experimental constraint $|\epsilon_K^{\mathrm{NP}}| \leq |\epsilon_K^{\mathrm{SM}}|$ leads to

$$\left|\frac{\epsilon_K'^{\mathrm{NP}}}{\epsilon_K'^{\mathrm{SM}}}\right| \leq \frac{|\epsilon_K'^{\mathrm{NP}}/\epsilon_K'^{\mathrm{SM}}|}{|\epsilon_K^{\mathrm{NP}}/\epsilon_K^{\mathrm{SM}}|} = \mathcal{O}\left(\frac{\mathrm{Re}\tau}{\mathrm{Re}\delta}\right). \quad (3)$$

With $M \gtrsim 1$ TeV, NP effects can be relevant only for $|\delta| \gg |\tau|$, and Eq. (3) seemingly forbids detectable NP contributions to ϵ'_K. In this Letter, we show that Eq. (3) can be overcome in the minimal supersymmetric standard model (MSSM) and one can reproduce the central value of the measured ϵ'_K in Eq. (1) with squark and gluino masses in the multi-TeV range. Our solution involves no fine-tuning of *CP* phases or other parameters.

ϵ'_K In the MSSM.—The MSSM is a good candidate for physics beyond the SM, because it alleviates the hierarchy problem, improves gauge coupling unification, and provides dark-matter candidates. Present collider bounds [14] (and the largish Higgs mass of 125 GeV [15,16]) push the

masses of colored superpartners into the TeV range, which makes supersymmetry an imperfect solution to the hierarchy problem but actually improves gauge coupling unification.

The master equation for ϵ'_K reads [7]

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\rm exp}|{\rm Re}A_0^{\rm exp}}\left\{\frac{{\rm Im}A_2}{\omega_+} - (1-\hat{\Omega}_{\rm eff}){\rm Im}A_0\right\}, \quad (4)$$

with $\omega_+ = (4.53 \pm 0.02)\times 10^{-2}$, the measured $|\epsilon_K^{\rm exp}|$, $\hat{\Omega}_{\rm eff} = (14.8 \pm 8.0)\times 10^{-2}$, and the amplitudes $A_I = \langle(\pi\pi)_I|\mathcal{H}^{|\Delta S|=1}|K^0\rangle$ involving the effective $|\Delta S| = 1$ Hamiltonian $\mathcal{H}^{|\Delta S|}$. $I = 0$, 2 labels the strong isospin of the final two-pion state. ${\rm Im}A_2$ is under good control for some time [10]; the recent theory progress of Refs. [2,12,13] concerns the QCD penguin contribution to ${\rm Im}A_0$. Prior to the first reliable lattice result for ${\rm Im}A_0$ [2], SM predictions for ϵ'_K were based on analytic methods, the dual QCD method of Refs. [11–13], or chiral perturbation theory [17]. The second method gives a larger value for ϵ'_K because of an enhancement of ${\rm Im}A_0$ from final-state interaction. In the calculation of Ref. [17], this effect is strictly correlated with a (phenomenologically welcome) enhancement of ${\rm Re}A_0$. In the dual QCD method, this correlation is absent [13]. With shrinking errors, lattice gauge theory will settle the issue of ${\rm Im}A_0$ soon. It is important to state that the lattice calculation of Ref. [2] *does* include final-state interaction along the line of Ref. [18].

The MSSM contribution to ϵ'_K simply adds to the SM piece. Supersymmetric contributions to ϵ'_K/ϵ_K have been widely studied [19–22,24,25] in the past, but for a supersymmetry-breaking scale M_S in the ballpark of the electroweak scale, so that the suppression mechanism inferred from Eq. (3) is avoided.

In the absence of sizable left-right squark mixing, the low-energy Hamiltonian reads

$$\mathcal{H}^{|\Delta S|=1}_{\rm eff,SUSY} = \frac{G_F}{\sqrt{2}}\sum_q\left[\sum_{i=1}^{2} c_i^q(\mu)Q_i^q(\mu) + \sum_{i=1}^{4}[c_i'^q(\mu)Q_i'^q(\mu) + \tilde{c}_i'^q(\mu)\tilde{Q}_i'^q(\mu)]\right] + {\rm H.c.}, \quad (5)$$

where G_F is the Fermi constant and

$$\begin{aligned} Q_1^q &= (\bar{s}_\alpha q_\beta)_{V-A}(\bar{q}_\beta d_\alpha)_{V-A}, & Q_2^q &= (\bar{s}q)_{V-A}(\bar{q}d)_{V-A}, \\ Q_1'^q &= (\bar{s}d)_{V-A}(\bar{q}q)_{V+A}, & Q_2'^q &= (\bar{s}_\alpha d_\beta)_{V-A}(\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_3'^q &= (\bar{s}d)_{V-A}(\bar{q}q)_{V-A}, & Q_4'^q &= (\bar{s}_\alpha d_\beta)_{V-A}(\bar{q}_\beta q_\alpha)_{V-A}. \end{aligned} \quad (6)$$

Here $(\bar{s}d)_{V-A}(\bar{q}q)_{V\pm A} = [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{q}\gamma^\mu(1\pm\gamma_5)q]$, α and β are color indices, and opposite-chirality operators $\tilde{Q}_i'^q$ are found by interchanging $V-A \leftrightarrow V+A$. In the presence of moderate left-right mixing, also the chromomagnetic penguin operator $Q_{8g} = m_s g_s/(16\pi^2)\bar{s}T^a\sigma_{\mu\nu}(1-\gamma_5)dG^{\mu\nu\,a}$ can be relevant and is included in our discussion below. Our solution exploits two special features of supersymmetric theories. First, there are loops governed by the strong interaction which contribute to ${\rm Im}A_2$ entering Eq. (4) with the enhancement factor $1/\omega_+ = 22.1$ [24,25]. These are gluino-box diagrams which feed the $(\pi\pi)_{I=2}$ final state if the right-handed up and down squarks ($\tilde{\bar{U}}$ and $\tilde{\bar{D}}$) have different masses (see Fig. 1). The flavor-changing neutral-current parameter is the (1,2) element of the left-handed down squark mass matrix M_Q^2 inducing $\tilde{s}_L$-$\tilde{d}_L$ mixing. Second, the Majorana nature of the gluino leads to a suppression of the gluino-squark contribution to ϵ_K, because there are *two* such diagrams (crossed and uncrossed boxes) with opposite signs. If the gluino mass $m_{\tilde{g}}$ equals roughly 1.5 times the average down squark mass M_S and if either left-handed or right-handed squark mixing is suppressed, both contributions to $\epsilon_K^{\rm SUSY}$ cancel [26]. For $m_{\tilde{g}} > 1.5M_S$, the gluino-box contribution approximately behaves as $[m_{\tilde{g}}^2 - (1.5M_S)^2]/m_{\tilde{g}}^4$, with a shallow maximum at $m_{\tilde{g}} \simeq 2.5M_S$, after which the $1/m_{\tilde{g}}^2$ decoupling sets in. In this parameter region also chargino, neutralino, and gluino-neutralino box diagrams are important [26] and are included in our numerics. The up-type squark mass matrix is $(VM_Q^2V^\dagger)_{ij}$ [up to negligible $\mathcal{O}(v^2)$ terms, where v is the electroweak vacuum expectation value], so that also chargino diagrams are affected by squark flavor mixing. The measured ϵ_K agrees well with the SM expectation, if the global CKM fit uses the $|V_{cb}|$ measured in inclusive semileptonic B decays [27], but exceeds $\epsilon_K^{\rm SM}$ for the smaller $|V_{cb}|$ inferred from exclusive decays [28,29]. Figure 2 shows that for both cases $\epsilon_K^{\rm SM} + \epsilon_K^{\rm SUSY}$ complies with $\epsilon_K^{\rm exp}$ over a wide parameter range without fine-tuning.

To get the desired large effect in ϵ'_K, we need a contribution to the operators $Q'_{1,2}$ with $(V-A)\times(V+A)$ Dirac structure, whose matrix elements are chirally enhanced by a factor $(m_K/m_s)^2$. Therefore, the flavor mixing has to be in the left-handed squark mass matrix. The opposite situation with right-handed flavor mixing and $\tilde{u}_L - \tilde{d}_L$ mass splitting is not possible, because $SU(2)_L$ invariance enforces $M^2_{\tilde{u}_L} - M^2_{\tilde{d}_L} = \mathcal{O}(v^2)$. Therefore, our

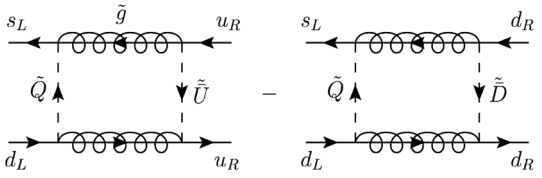

FIG. 1. Supersymmetric gluino box contribution to ϵ'_K/ϵ_K (called a *Trojan penguin* in Ref. [25]). It contributes to ${\rm Im}A_2$ for $m_{\bar{U}} \neq m_{\bar{D}}$ and is the largest contribution in our scenario. There are also crossed box diagrams.

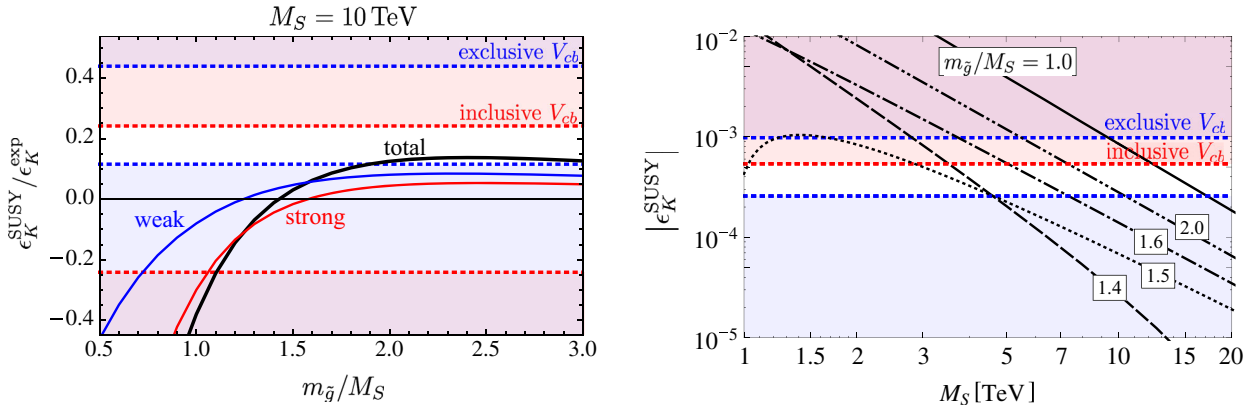

FIG. 2. The left plot shows $\epsilon_K^{\rm SUSY}/\epsilon_K^{\rm exp}$ as a function of the gluino-squark mass ratio $m_{\tilde g}/M_S$, where we take $M_S = m_Q = m_{\bar U} = m_{\bar D} = 10$ TeV. The red line shows the gluino-gluino box contribution (with the zero crossing near $m_{\tilde g}/M_S = 1.5$ [26]), while the blue line denotes the sum of the box contributions with one or two winos. The total contribution is shown in black. The red (blue) regions are excluded by the measurement of ϵ_K at the 95 % confidence level (C.L.), if the SM prediction uses the inclusive (exclusive) measurement of $|V_{cb}|$ [28]. On the right, the black lines show $|\epsilon_K^{\rm SUSY}|$ for several gluino-squark mass ratios as a function of the squark mass.

scenario involves flavor mixing between left-handed squarks only. We use the following notation for the squark mass matrices: $M^2_{X,ij} = m^2_X(\delta_{ij} + \Delta_{X,ij})$, with $X = Q$, $\bar U$, or $\bar D$. Throughout this Letter, we use $m^2_Q = m^2_{\bar D} = M^2_S$ and vary $m_{\bar U}$. We have calculated all one-loop contributions to the coefficients in Eq. (5) in the squark mass eigenbasis and will present the full results elsewhere [30]. For the dominant "Trojan penguin" contribution, we confirm the result of Ref. [24] and find a typo in the expression for c'_4 in Ref. [25]. The second-largest contribution to ϵ'_K stems from the chromomagnetic penguin operator, and our coefficient is in agreement with Refs. [31,32]. To our knowledge, the other coefficients have been obtained only in the mass insertion approximation [19], and our results agree upon expansion in $\Delta_{X,ij}$. Our results also comply with the loop diagram results collected in Ref. [33]. The individual contributions to ϵ'_K/ϵ_K are shown in Fig. 3.

For the calculation of ϵ'_K/ϵ_K, we must use the renormalization group (RG) equations to evolve the Wilson coefficients calculated at the high scale $\mu = M_S$ down to the hadronic scale $\mu_h = \mathcal{O}(1\ \text{GeV})$ at which the operator matrix elements are calculated. In order to use the well-known NLO 10×10 anomalous dimensions for the SM four-fermion operator basis [8], we switch from Eq. (5) to

$$\mathcal{H}^{|\Delta S|=1}_{\rm eff,SUSY} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} [C_i(\mu) Q_i(\mu) + \tilde C_i(\mu) \tilde Q_i(\mu)] + \text{H.c.}, \quad (7)$$

where $Q_{1,\ldots,10}$ are given in Refs. [8,9] and

$$\begin{aligned} C_{1,2}(\mu) &= c^u_{1,2}(\mu), \qquad \tilde C_{1,2}(\mu) = 0, \\ C_{3,4,5,6}(\mu) &= \frac{1}{3}[c'^u_{3,4,1,2}(\mu) + 2c'^d_{3,4,1,2}(\mu)], \\ C_{7,8,9,10}(\mu) &= \frac{2}{3}[c'^u_{1,2,3,4}(\mu) - c'^d_{1,2,3,4}(\mu)], \end{aligned} \quad (8)$$

and the coefficients $\tilde C_{3,\ldots,10}$ of the opposite-chirality operators are found from $C_{3,\ldots,10}$ by replacing $c'^q_i \to \tilde c'^q_i$. Note that $C_{7,8}$ receive the contribution of Fig. 1.

For the evolution of the coefficients from $\mu = M_S$ to $\mu = \mu_h$, we use a new analytical solution of the RG equations which avoids the problem of a singularity in the NLO terms discussed in Refs. [4,34]. For $\mathcal{H}^{|\Delta S|=1}_{\rm eff,SUSY}$, we employ proper threshold matching at the scales $\mu_{t,b,c}$ set by the top, bottom, and charm quark masses with the usual threshold matching matrices [9]. In our analysis we take $\mu_h = 1.3$ GeV. For the SM prediction in Eq. (1) and the calculation of the MSSM prediction, we have evolved the matrix elements of Refs. [2,10] (which are given at

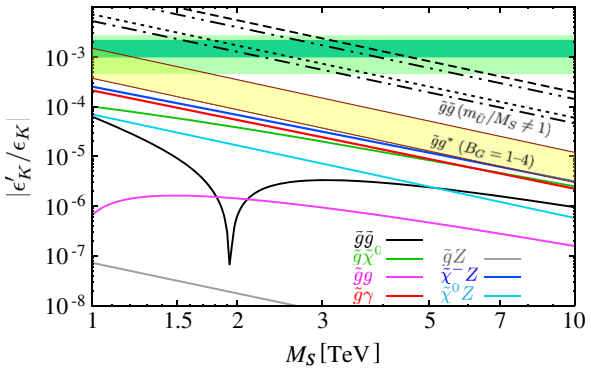

FIG. 3. Individual supersymmetric contributions to $|\epsilon'_K/\epsilon_K|$ as a function of $M_S = m_Q = m_{\bar D}$. $\tilde g\,\tilde g$, $\tilde g\tilde\chi^0$, $\tilde g g$, $\tilde g\gamma$, $\tilde g Z$, $\tilde\chi^- Z$, $\tilde\chi^0 Z$, and $\tilde g g^*$ represent the gluino-gluino and gluino-neutralino boxes, gluino gluon, photon, and Z penguins, chargino and neutralino Z penguins, and chromomagnetic contributions, respectively. The thick lines show the case of universal squark masses $m_{\bar U} = M_S$. The broken black lines are the gluino-gluino box contributions for $m_{\bar U}/M_S = 0.5$, 2.0, 0.8, 1.2 from top to bottom. The ϵ'_K/ϵ_K discrepancy is resolved at 1σ (2σ) in the dark (light) green band.

$\mu = 1.531$ GeV for A_0 and at $\mu = 3.0$ GeV for A_2) to μ_h with three-flavor full NLO operator mixing. The use of NLO RG formulas for $\mathcal{H}^{|\Delta S|=1}_{\text{eff,SUSY}}$ involves a relative error of order $\alpha_s(M_S)$, because the two-loop corrections to the initial conditions of the Wilson coefficients are not included. However, the NLO corrections proportional to the much larger $\alpha_s(\mu_h)$ are all correctly included and independent of the renormalization scheme.

Phenomenology of ϵ_K and ϵ'_K.—In this section, we study ϵ_K and ϵ'_K/ϵ_K in the MSSM parameter region in which the discrepancy in Eq. (1) is removed. As input, we take $\alpha_s(M_Z) = 0.1185$, the grand-unified theory relation for gaugino masses, $m_{\tilde{g}}/M_S = 1.5$, and $m_Q = m_{\bar{D}} = \mu_{\text{SUSY}} = M_S$, where μ_{SUSY} is the Higgsino mass parameter. Furthermore, the trilinear supersymmetry-breaking matrices A_q are set to zero, $\tan\beta = 10$, and the only nonzero off-diagonal elements of the squark mass matrices are $\Delta_{Q,12,13,23} = 0.1\exp(-i\pi/4)$ and $(V\Delta_Q V^\dagger)_{ij}$ for the left-handed down and up sectors, respectively. For the CKM elements, we use CKMFITTER results [29].

Starting with ϵ_K, we first note that the phase of the SUSY contribution to the K^0-$\bar{K}^0$ mixing amplitude is essentially twice the phase of $\Delta_{Q,12}$. That is, our choice of $\pi/4$ for this phase maximizes the CP phase and is far away from a fine-tuned solution to suppress ϵ_K. We evaluate the MSSM Wilson coefficients for ϵ_K with the $\mathcal{O}(g_s^4, g_s^2 g^2, g^4)$ strong and weak contributions [26,35]. For the RG evolution of the MSSM contribution, the LO formula is sufficient [36]; lattice results for $|\Delta S| = 2$ hadronic matrix elements are available from several groups [37]. For an accurate SM prediction of ϵ_K one must include all NLO corrections [38] and the NNLO contributions involving the low charm scale [39]. At this level, ϵ_K^{SM} agrees with ϵ_K^{exp}, if the value of $|V_{cb}|$ measured in inclusive $b \to c\ell\nu$ decays is used for the calculation of the CKM elements. Figure 2 shows that the MSSM can accommodate this situation as well as the scenario with $|V_{cb}|$ taken from exclusive $B \to D^{(*)}\ell\nu$ decays [40], which calls for a new-physics contribution to ϵ_K. The left plot in Fig. 2 clearly reveals that the MSSM solution is not fine-tuned but merely requires $m_{\tilde{g}}/M_S \gtrsim 1.5$. For our chosen parameters, we roughly find $M_S \gtrsim 3$ TeV, with the possibility of slightly lighter squarks if the exclusive $|V_{cb}|$ is true.

We note that our results are stable if we switch on right-handed squark mixing as long as $\Delta_{\bar{D},12} \lesssim 10^{-5}$. Simultaneous sizable left-left and right-right sfermion mixing spoils the suppression of gluino box diagrams in ϵ_K^{SUSY} [26]. Although in our scenario $\Delta_{\bar{D},12}$ is generated by radiative corrections, the value is smaller than 10^{-5} thanks to the small down Yukawa coupling. A hierarchy $\Delta_{Q,12} \gg \Delta_{\bar{D},12}$ appears naturally in UV completions with a flavor symmetry; cf., e.g., Refs. [41,42] for models based on the discrete group S_3 and a gauged horizontal U(1), respectively.

We next turn to the discussion of ϵ'_K: The thick lines in Fig. 3 show the individual contributions to $|\epsilon'_K/\epsilon_K|$ for the case of universal squark masses. The broken lines show that already a moderate $\bar{U}$-$\bar{D}$ mass splitting suffices to explain the measured value (indicated by the green bands). The second-largest contribution from the chromomagnetic penguin diagram comes with a poorly known hadronic matrix element [43]. The B parameter parametrizing this matrix element is estimated as $B_G = 1 \pm 3$ [22]. The yellow band in Fig. 3 is for $1 \le B_G \le 4$. Next, we remark that in our parameter region the gluino-photon (red line) and chargino-Z (blue line) penguins have opposite sign and almost cancel each other. This picture changes with nonzero trilinear terms; e.g., $|A_{d,21}| = 0.1 M_S$ ($|A_{u,31}A_{u,32}| = 0.1 M_S^2$) can lift the chromomagnetic (chargino-Z) contribution by about 40% (140%). We have neglected the gluino-W penguin and the gluino-chargino box contributions, which matches onto $c^u_{1,2}$ at $\mu = M_S$ and gives at most an $\mathcal{O}(10^{-5})$ contribution to ϵ'_K/ϵ_K.

Figure 4 shows our main result, the portion of the squark mass plane which simultaneous explains ϵ'_K/ϵ_K and ϵ_K. The figure uses the complete supersymmetric results except for the chromomagnetic contribution to ϵ'_K because of the uncertainty in B_G. The red region is excluded by the measurement of ϵ_K at 95 % C.L. in combination with the inclusive V_{cb}, while the region between the blue-dashed lines can explain the ϵ_K discrepancy at 95% C.L. for the

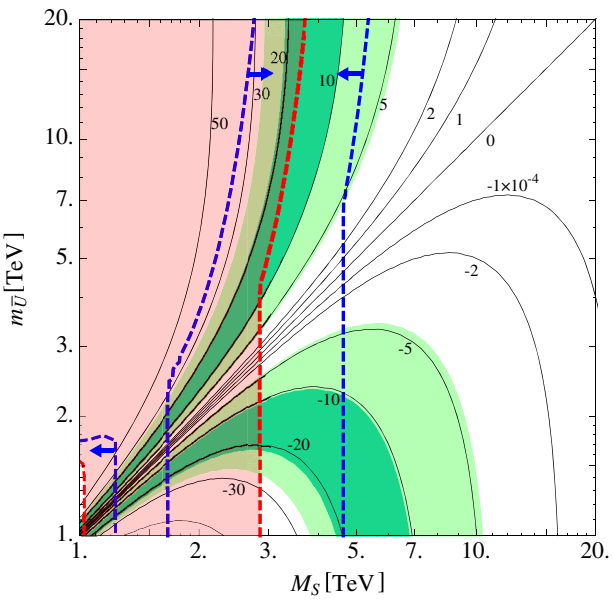

FIG. 4. Contours of the supersymmetric contributions to ϵ'_K/ϵ_K in units of 10^{-4}. The ϵ'_K/ϵ_K discrepancy is resolved at 1σ (2σ) in the dark (light) green region. The red shaded region is excluded by ϵ_K with inclusive $|V_{cb}|$ at 95% C.L., while the region between the blue-dashed lines can explain the ϵ_K discrepancy which is there for the exclusive $|V_{cb}|$. The green regions labeled with negative ϵ'_K/ϵ_K correspond to the change $\Delta_{Q,12,13,23} = 0.1\exp(-i\pi/4) \to \Delta_{Q,12,13,23} = 0.1\exp(i3\pi/4)$, which flips the sign of $\epsilon'^{\text{SUSY}}_K$ (making it positive) while leaving ϵ_K essentially unchanged.

exclusive value of $|V_{cb}|$. Note that we also found that there are no constraints from the mass difference of neutral kaon, D^0-$\bar{D}^0$ mixing [44], and the neutron electric dipole moment [45].

Conclusions.—In this Letter, we have calculated ϵ'_K in the MSSM and have shown that the large contributions needed to solve the discrepancy in Eq. (1) can be obtained for squark and gluino masses in the multi-TeV range. The constraint from ϵ_K, which in generic models of new physics precludes large effects in ϵ'_K, can be fulfilled without fine-tuning.

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