



Available online at www.sciencedirect.com



Nuclear Physics B 913 (2016) 318-326



www.elsevier.com/locate/nuclphysb

Low-energy electromagnetic radiation as an indirect probe of black-hole evaporation

Slava Emelyanov

Institute for Theoretical Physics, Karlsruhe Institute of Technology (KIT), 76131 Karlsruhe, Germany Received 19 February 2016; received in revised form 22 June 2016; accepted 18 September 2016 Available online 21 September 2016

Editor: Stephan Stieberger

Abstract

We study the influence of black-hole evaporation on light propagation. The framework employed is based on the non-linear QED effective action at one-loop level. We show that the light-cone condition is modified for low-energy radiation due to black-hole evaporation. We discuss conditions under which the phase velocity of this low-energy radiation is greater than c. We also compute the modified light-deflection angle, which turns out to be significantly different from the standard GR value for black-hole masses in the range $M_{\rm Pl} \ll M \lesssim 10^{19} M_{\rm Pl}$.

© 2016 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP³.

1. Introduction

A propagation of light in a non-trivial, i.e. non-Minkowskian, quantum state can be modified in quantum electrodynamics (QED). Moreover, the field operator $\hat{A}_{\mu}(x)$ of the electromagnetic field can have a non-standard structure due to non-trivial boundary conditions that leads to a modification of its action on the Minkowski vacuum. As a consequence, the photon propagator alters as well. Specifically, a low-energy electromagnetic wave propagating through a thermal gas turns out to be subluminal [1,2], while superluminal when propagating in-between conducting plates in the Casimir set-up [3]. These two effects can be described at the same time by considering the effective action of the electromagnetic field with integrated out fermion degrees of freedom [2].

http://dx.doi.org/10.1016/j.nuclphysb.2016.09.011

E-mail address: viacheslav.emelyanov@kit.edu.

^{0550-3213/© 2016} The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP³.

It was further realised that a sign of the renormalised energy density and pressure determines whether the phase velocity of low-energy electromagnetic radiation is greater or smaller than c as measured under the standard conditions [4,5].

In curved spacetime extra curvature-dependent terms appear in the effective action in quantum electrodynamics [6]. At the leading α -order these terms are quadratic with respect to the field strength of the electromagnetic field. This implies in particular that the Drummond–Hathrell term is oblivious to the quantum state at the α -order approximation, but not to the spacetime geometry.

In the current paper we study the Maxwell field equations modified by the Euler–Heisenberg term as well as the Drummond–Hathrell term in the Schwarzschild black-hole geometry [6–8]. Under the assumption the vector-field operator modifies when the black-hole horizon forms, rather than the Fock space representation of the field operators as argued in [9], one can *a priori* expect a non-trivial effect in spacetime with the black hole analogous to that in-between the conducting plates. Thus, our main concern in this paper is to investigate how quantum fluctuations of the electromagnetic field in the form of the Hawking radiation $(\langle \hat{T}_{\mu\nu} \rangle \neq 0)$ influence the propagation of the long-wavelength radiation in quantum electrodynamics.

Throughout this paper the fundamental constants are set to unity, $c = G = k_{\rm B} = \hbar = 1$.

2. Effective field equations

Integrating out fermion degrees of freedom in QED, we obtain a non-linear effective action for the electromagnetic field alone. This is exactly what we mean by the non-linear QED. In curved spacetime this leads to adding the Drummond–Hathrell term [6] and the Euler– Heisenberg term [7] to the standard Maxwell action (see also [8]). We denote this effective action as $\Gamma_{\text{eff}}[A, g]$ below. The vector-field equation is thus modified in quantum electrodynamics and reads

$$\nabla_{\mu}F^{\mu\nu} - 4\left(\frac{4\alpha^2}{90m_e^4}F^{\mu\nu}F_{\lambda\rho} + \frac{7\alpha^2}{90m_e^4}\tilde{F}^{\mu\nu}\tilde{F}_{\lambda\rho} - \frac{\alpha}{360\pi m_e^2}R^{\mu\nu}_{\ \lambda\rho}\right)\nabla_{\mu}F^{\lambda\rho} = 0, \qquad (1)$$

where α is the fine structure constant, m_e the electron mass. We have taken into account that $R_{\mu\nu} = 0$ in the Schwarzschild geometry

$$ds^{2} = f(r)dt^{2} - \frac{dr^{2}}{f(r)} - r^{2}d\Omega^{2}, \quad f(r) = 1 - 2M/r, \quad (2)$$

and neglected higher-order derivative terms of the electromagnetic field strength $F_{\mu\nu}$ in order to have the same order of the approximation in the Euler–Heisenberg and the Drummond– Hathrell action. We have also omitted the derivative of the Riemann tensor focusing only on light wavelengths λ_{γ} being much smaller than a characteristic curvature scale λ_c . Furthermore, the Euler–Heisenberg action is valid for the light wavelengths being much larger than the Compton length of the electron λ_e . Thus, the equation (1) must be reliable in the regime $\lambda_e \ll \lambda_{\gamma} \ll \lambda_c$.

The vector-field equation (1) follows from variation of $\Gamma_{\text{eff}}[A, g]$ with respect to $A_{\mu}(x)$ and, thus, is classical in the sense that $A_{\mu}(x)$ is not quantised. However, the full effective action $\Gamma_{\text{1PI}}[A, g]$ differs from $\Gamma_{\text{eff}}[A, g]$. We now want to take into account the influence of quantum fluctuations of the field $A_{\mu}(x)$ on the light propagation. Since we do not know an exact expression of $\Gamma_{\text{1PI}}[A, g]$, we follow [10] to compute one-loop correction to the classical non-linear equation. Specifically, we consider $\hat{A}'_{\mu}(x) = A_{\mu}(x) + \hat{a}_{\mu}(x)$, such that $\hat{a}_{\mu}(x)$ has an ordinary photon propagator in the Schwarzschild geometry. Substituting $\hat{A}'_{\mu}(x)$ in (1) and taking then its vacuum expectation value we get at the linear order in $A_{\mu}(x)$ in the one-loop approximation

$$\nabla_{\mu}F^{\mu\nu} - 4\left(\frac{4\alpha^2}{90m_e^4}\langle\hat{f}^{\mu\nu}\hat{f}_{\lambda\rho}\rangle_{\rm ren} + \frac{7\alpha^2}{90m_e^4}\langle\hat{f}^{\mu\nu}\hat{f}_{\lambda\rho}\rangle_{\rm ren} - \frac{\alpha}{360\pi m_e^2}R^{\mu\nu}_{\ \lambda\rho}\right)\nabla_{\mu}F^{\lambda\rho} = 0\,,\tag{3}$$

where we have neglected terms being of the order of $\lambda_e/\lambda_c \ll 1$ and $\lambda_{\gamma}/\lambda_c \ll 1$. It is worth mentioning that our procedure of deriving (3) is equivalent to the background-field method of taking into account quantum field fluctuations at one-loop level (e.g., see [11]).

A few remarks are in order. First, the role of the fermion field is to provide self-interacting terms for the field $A_{\mu}(x)$ in the effective action $\Gamma_{\text{eff}}[A, g]$. Second, the effect of our interest crucially depends on having quantum fluctuations of the electromagnetic field. Third, the equation (3) follows from the 1PI effective action computed at the lowest-order approximation by promoting the electromagnetic field to a quantum operator in $\Gamma_{\text{eff}}[A, g]$. Fourth, all non-linear terms with respect to $A_{\mu}(x)$ in (3) have been omitted, because we want to study how low-energy electromagnetic waves behave in the vacuum when propagating through spacetime. In other words, these waves are our probe or test of the electromagnetic properties of the vacuum (e.g., see [12]).

Now the simplest way to obtain the light-cone condition is to employ the geometric optics approximation. This yields

$$k^{2}\epsilon^{\nu} - 8\left(\frac{4\alpha^{2}}{90m_{e}^{4}}\langle\hat{f}^{\mu\nu}\hat{f}_{\lambda\rho}\rangle_{\rm ren} + \frac{7\alpha^{2}}{90m_{e}^{4}}\langle\hat{f}^{\mu\nu}\hat{f}_{\lambda\rho}\rangle_{\rm ren} - \frac{\alpha}{360\pi m_{e}^{2}}R^{\mu\nu}_{\ \lambda\rho}\right)k_{\mu}k^{\lambda}\epsilon^{\rho} = 0\,,\quad(4)$$

where we have used the Bianchi identity $k_{(\mu}F_{\lambda\rho)} = 0$ with k_{μ} being a wave vector, i.e. $\nabla_{\mu}F^{\lambda\rho} = ik_{\mu}F^{\lambda\rho}$, and the vector ϵ^{μ} specifies light polarisation in the Lorentz gauge.

2.1. Modified radial propagation

A computation of the light-cone condition for the radial propagation of the electromagnetic wave is considerably simplified in the Newman–Penrose formalism. Accordingly, one introduces the null tetrad $e_a^{\mu} = \{l^{\mu}, n^{\mu}, m^{\mu}, \bar{m}^{\mu}\}$, such that $l^{\mu}n_{\mu} = -m^{\mu}\bar{m}_{\mu} = 1$ and the rest possible products vanish. Thus, we choose

$$k^{\mu} = l^{\mu} + \delta l^{\mu} \quad \text{and} \quad \epsilon^{\mu} = \alpha_1 m^{\mu} + \alpha_2 \bar{m}^{\mu} \tag{5}$$

for the radial propagation. Substituting these in (4) and looking for a non-trivial solution for coefficients α_1 and α_2 , we find that there exist two non-trivial polarisations ϵ_{\pm}^{μ} , such those

$$k^{2} + \frac{4\alpha^{2}}{45m_{e}^{4}}\epsilon_{\pm}\langle\hat{T}_{\mu\nu}\rangle k^{\mu}k^{\nu} = 0, \qquad (6)$$

where $\langle \hat{T}_{\mu\nu} \rangle$ is a renormalised stress tensor for $\hat{a}_{\mu}(x)$ and

$$\epsilon_{+} = 4 \quad \text{and} \quad \epsilon_{-} = 7. \tag{7}$$

It is worth noting that the Drummond–Hathrell action does not influence the radial light propagation [6]. The higher-order curvature-dependent terms have also no influence on the radial light propagation for the Schwarzschild black holes [13,14].



Fig. 1. Left: $\Theta_H(r) = (\langle \hat{T}_t^t \rangle_H - \langle \hat{T}_r^r \rangle_H)/\gamma$ as a function of r, where $\gamma = \frac{\pi^2}{45}T_H^4$, T_H is the Hawking temperature [15]. The distance from the black-hole centre when $\Theta_H(r)$ vanishes is approximately $r_n \approx 3.3 \times M$. Right: $\Theta_U(r) = (\langle \hat{T}_t^t \rangle_U + 2\varepsilon f(r) \langle \hat{T}_r^t \rangle_U - \langle \hat{T}_r^r \rangle_U)/\gamma$ as a function of r [17,18]. The solid and dashed line correspond to the outgoing and ingoing light wave for which $r_n = +\infty$ and $r_n = 0$, respectively.

The formula (6) can be employed to get a change of the phase velocity of the low-energy electromagnetic radiation due to non-trivial renormalised stress tensor $\langle \hat{T}_{\mu\nu} \rangle$ of the quantum field in the thermal state as well as in-between the conducting plates [1–4].

We now apply this formula for the radially propagating electromagnetic wave in the background of the Hawking radiation. We find

$$\frac{\delta c}{c} = -\frac{2\alpha^2}{45m_e^4} \epsilon_{\pm} \left(\langle \hat{T}_t^t \rangle + 2\varepsilon f(r) \langle \hat{T}_r^t \rangle - \langle \hat{T}_r^r \rangle \right), \tag{8}$$

where f(r) is a lapse function given in the equation (2), and ε is either +1 or -1 for an outgoing or ingoing light wave, respectively.

In the case of the eternal Schwarzschild black hole physical vacuum corresponds to the Hartle–Hawking state which is regular on both past and future horizon. Employing results of [15] for the renormalised stress tensor of the electromagnetic field in the Hartle–Hawking state, we find that the radially outgoing or ingoing radiation is superluminal for $r \in (2M, r_n)$, but subluminal for $r > r_n$, where $r_n \approx 3.3 \times M$ (see Fig. 1). Thus, the superluminal radial propagation between 2M and r_n resembles that in the Scharnhorst effect [3], although the analogy is not complete (see below). It is worth noting that violation of the null energy condition in our case is qualitatively similar for the case of the scalar field model conformally coupled to gravity [16].

In the case of a physical black hole, i.e. the black hole formed through the collapse of matter, physical vacuum corresponds to the Unruh state. We employ an approximate analytic expression of the renormalised stress tensor in the Unruh state [17,18] to analyse the influence of quantum fluctuations on the light propagation. The outgoing radiation turns out to be superluminal at any distance from the black hole, while the ingoing one is subluminal right up to the horizon. Qualitatively the same picture of violation of the null energy condition holds for the conformal scalar field model [19].

However, the Euler–Heisenberg action in the case of the Unruh state starts to dominate over the Maxwell action for distances roughly less than

$$2M\left(10^3 \, \frac{M_{\rm Pl}^2}{Mm_e}\right)^4\tag{9}$$

from the black-hole horizon, where M_{Pl} is the Planck mass. Therefore, (9) is negligibly small if the black-hole mass M is sufficiently large, i.e.

$$M \gg 10^3 \frac{M_{\rm Pl}^2}{m_e} \approx 10^{25} M_{\rm Pl} \approx 10^{-13} M_{\odot}.$$
 (10)

Thus, the approximation is reliable even close to the horizon for the black-hole masses being much larger than $10^{-15} M_{\odot}$. Note that violation of the weak gravity approximation may occur in the vicinity of the horizon. However, this is not the case for the radially propagating light whenever its wavelength λ_{γ} is much smaller than λ_c .

2.2. Modified light deflection

We now consider an electromagnetic wave propagating in the $\theta = \frac{\pi}{2}$ plane. Working in the notations of [22], we obtain the same formula (6), but now with k^{μ} given by the solution of the geodesic equation depending on an impact parameter *d* plus a correction of the α^2 -order. This is only possible in our context if the Drummond–Hathrell term is omitted. We are interested in the regime when the correction to the light deflection induced by this term is negligibly small, we thus study in the following a value of the light deflection being only due to the Euler–Heisenberg term.

Computing the deflection of light in the weak gravity limit, i.e. $r \gg 2M$, we find in the leading-order approximation

$$\Delta\phi_H \approx \left(1 - \frac{\epsilon_{\pm} \alpha^2}{(720\pi)^2} \left(\frac{M_{\rm Pl}^2}{Mm_{\rm e}}\right)^4\right) \Delta\phi_{\rm GR} \tag{11}$$

for the Hartle–Hawking state, where $\Delta \phi_{\text{GR}} = \frac{4M}{r_0}$ with r_0 being the closest distance to the black hole. It is worth noting that the term in the parentheses of (11) is of the order of the deviation of the phase velocity of the light wave from *c*.

Repeating these computations for the physical black hole, i.e. in the Unruh state, we find

$$\Delta\phi_U \approx \Delta\phi_{\rm GR} - \frac{\epsilon_{\pm} \alpha^2}{960} LM^2 \left(\frac{M_{\rm Pl}^2}{Mm_{\rm e}}\right)^4 \left(\Delta\phi_{\rm GR}\right)^2 \tag{12}$$

where L is a luminosity equalling $L \approx 2.68 \times 10^{-6} \frac{4\pi}{M^2}$ for the electromagnetic field [23].¹ Expressing this correction to the angle of the light deflection through the change of the phase velocity of the electromagnetic wave at $r = r_0 \gg 2M$, we obtain

$$\Delta\phi_U \approx \Delta\phi_{\rm GR} - \frac{3\pi}{8} \left| \frac{\delta c_U}{c} \right| \approx \left(1 - \epsilon_{\pm} \frac{2M}{r_0} \left(3.32 \times 10^{19} \, \frac{M_{\rm Pl}}{M} \right)^4 \right) \Delta\phi_{\rm GR} \,. \tag{13}$$

The Drummond–Hathrell contribution to the light deflection is negligibly small with respect to the Euler–Heisenberg one if

$$2M\left(1.28 \times 10^{19} \,\frac{M_{\rm Pl}}{M}\right)^{-2} \ll r_0\,. \tag{14}$$

Note that the higher-order curvature/derivative terms are also suppressed in comparison with the Euler–Heisenberg term. The angle of the light deflection could be of the order one or even larger with respect to the standard result of general relativity (GR) if

¹ Note that there is a correction of the order $(\Delta \phi_{GR})^2$ due to general relativity only which we have omitted as being small in comparison with the term due to the Euler–Heisenberg action.



Fig. 2. Vacuum polarisation diagrams contributing to the photon self-energy up to the α^2 -order.

$$2M\epsilon_{\pm}^{\frac{1}{2}} \left(6.75 \times 10^{19} \, \frac{M_{\rm Pl}}{M} \right)^2 \lesssim r_0 \, \lesssim \, 2M\epsilon_{\pm} \left(3.32 \times 10^{19} \, \frac{M_{\rm Pl}}{M} \right)^4, \tag{15}$$

where the lower bound is due to our assumption $|\delta c_U/c| \leq 1/10$. Therefore, we come to a conclusion that black-hole evaporation considerably influences the light propagation if the black-hole mass is sufficiently small, i.e.

$$M \lesssim 10^{19} M_{\rm Pl} \approx 10^{-19} M_{\odot}.$$
 (16)

Note that the condition (14) as well as the weak gravity condition is then automatically satisfied if the black-hole mass lies in this range. However, the semi-classical approximation is reliable if the black hole is not too small, namely $M \gg M_{\rm Pl}$ should be fulfilled. Thus, the above effects of black-hole evaporation on the low-energy electromagnetic wave propagation are still trustable if the black-hole mass M is much bigger than the Planck mass $M_{\rm Pl}$, so that $M_{\rm Pl} \ll M \lesssim 10^{19} M_{\rm Pl}$.

2.3. Two-loop dominance

In terms of Feynman diagrams, the effect occurs due to the non-trivial modification of the photon propagator. The photon self-energy gets a correction at the level of two-loop diagrams depicted in Fig. 2. This has been taken into account in the effective action. However, the electron/positron propagator also changes and, hence, makes a contribution to the modification of the photon propagator. This is the main difference in comparison with the Scharnhorst effect. The Hartle–Hawking state is the Kubo–Martin–Schwinger state [24] with respect to the Killing vector ∂_t for the field operators having a vanishing support in the causal complement to the "right" Schwarzschild space. The renormalised stress tensor is mathematically indistinguishable, but not physically [9], from that for the thermal radiation at T_H sufficiently far from the black hole ($r \gg 2M$). Thus, the modification of the fermion propagator can be neglected as long as $M \gg 10^{21} M_{\rm Pl}$, i.e. when T_H is much smaller than the electron mass m_e . However, the photon acquires a thermal mass of the order of $\alpha^{\frac{1}{2}}T_H$ at $T_H \gg m_e$ due to the one-loop vacuum polarisation diagram depicted in Fig. 2. Therefore, our results for the Hartle–Hawking state are definitely reliable and do not lead to $m_\gamma \gg m_e$ if $T_H \ll m_e$ which is a regime of the two-loop dominance [25].²

In the case of the Unruh state, the stress tensor vanishes as $T_H^4 (2M/r)^2$ far from the hole. Taking this into account as well as the structure of the one-loop vacuum polarisation diagram, the effective photon mass squared is expected to be of the order of $\alpha T_H^2 (2M/r)^2$ far from the hole. At high temperatures, $T_H \gg m_e$, one thus has that our approximation is reliable if the light wavelength λ_{γ} is much less than $\alpha^{\frac{1}{2}}\lambda_e(T_H/m_e)$, which is much greater than λ_e for $M \lesssim 10^{19} M_{\rm Pl}$. There are still propagating waves for distances greater than the lower bound of r_0 given

² It is worth emphasising that eternal black holes are of no physical interests anyway, because these are not formed through the gravitational collapse. As one can see, if there is a small eternal black hole, then the effective photon mass is huge. Moreover, $\lambda_{\gamma} \lesssim \lambda_{e}$ should hold to have propagating light waves. This is beyond of our approximation.

in (15), because the effective plasma frequency is suppressed by the geometrical factor 2M/r. The α^3 -order term is expected to be of the order of $(T_H/m_e)^6 (2M/r)^4$ which is suppressed far from the black hole. Specifically, the closest distance r_0 should approximately be larger than the lower bound given in (15).

We analytically compute the effective photon mass m_{γ} at one-loop level in QED in the background of the small Schwarzschild black hole in [26].

3. Concluding remarks

We have analysed the influence of the vacuum polarisation induced by the black holes in quantum electrodynamics on the propagation of the low-energy electromagnetic radiation. This results in the violation of the null energy condition and the superluminal/subluminal phase velocity of the radial outgoing/ingoing electromagnetic radiation, respectively.

Black-hole evaporation should be observable through the angle of the light deflection. Specifically, for the black-hole masses in the range

$$M_{\rm Pl} \ll M \lesssim 10^{19} \, M_{\rm Pl} \tag{17}$$

and for the closest distance r_0 to the black hole lying in the annulus (15), one can expect a significant deviation of the light-deflection angle from the standard GR value. Note that this angle is different for different types of the light polarisations. Practically, this implies that a source of unpolarised light has an image stretched in the direction from the hole. It is also worth emphasising this might be a physical effect being appropriate for discovering *only* small black holes.³

The electromagnetic wave moving along a circular geodesic around the black hole propagates with the phase velocity less than c at $r \gtrsim 27.9 \times M$. The circular velocity approaches c as $(2M/r)^2$ at $r \gg 2M$. The radial outgoing light velocity approaches c as $(2M/r)^5$, while the ingoing one as $(2M/r)^2$. The ratio of the phase velocities of the radial ingoing wave and the circular wave is

$$\frac{c_{in,\pm}}{c_{cir,\mp}} \approx 1 - \left(2.68 \times 10^{19} \frac{M_{\rm Pl}}{M}\right)^4 \left(\frac{2M}{r}\right)^2 \left(4\epsilon_{\pm} - \epsilon_{\mp}\right),\tag{18}$$

where the indices + and - refer to the light polarisations. For the supermassive black hole in the centre of the Milky Way, we find an extremely small value 10^{-121} of the anisotropy of the phase velocities. This is much less than the anisotropy due to the Drummond–Hathrell term, i.e. 10^{-84} , because the constraint (14) is not fulfilled. Note that the anisotropy due to black-hole evaporation becomes larger whenever the black hole is lighter and closer to earth.

As pointed out above, the imprint of black-hole evaporation on the light propagation is due to the modification of the vector-field and fermion operator when the event horizon forms. This is analogous to the Casimir effect, wherein the modification is however due to the boundary conditions satisfied by the electromagnetic field on the conducting plates. This picture is fully consistent with unitary black-hole evaporation [9].

³ Our estimate of the α^3 -order term made in Sec. 2.3 may be too optimistic. If it turns out that the three-loop contribution to the light-cone condition is of the order of $(T_H/m_e)^6 (2M/r)^2$, then the higher loop contributions are in general not negligible at $T_H \gg m_e$ as in the ordinary hot plasma. This would imply that the perturbation theory gets out of control. Thus, the perturbation series needs to be resumed. Nevertheless, the two-loop dominance still occurs for black-hole masses $10^{18} M_{\rm Pl} \lesssim M \lesssim 10^{19} M_{\rm Pl}$.

The Drummond–Hathrell term leads to violation of the strong principle of equivalence, whereas the Euler–Heisenberg term is completely consistent with this principle. These terms allow in particular to have superluminal propagation of the low-energy electromagnetic radiation. However, the superluminality here does not necessary imply a violation of causality [13, 20,21,27-29]. To establish whether causality is not broken, one needs to know how high-energy photons propagate in the background of the evaporating black hole. Indeed, the wave-front velocity corresponding to the signal velocity is given by the phase velocity of the high-energy radiation [30]. This is certainly beyond of our approximation. Nevertheless, one might expect that causality is *not* violated as we have started with a perfectly causal theory, namely QED, in the classical geometrical background.

It is not obvious whether the so-called "horizon" theorem [13] still holds for the radially outgoing low-energy radiation, i.e. whether the light cone remains $k^2 = 0$ for the outgoing light wave on the event horizon of a black hole formed through the gravitational collapse. Although the perturbation theory breaks down near r = 2M for the outward light, one can still have $k^2 < 0$ for the outgoing radiation very close to the horizon for sufficiently large black holes, $M \gg 10^{25} M_{\text{Pl}}$. However, we expect that this theorem still holds on the horizon, because $k^2 > 0$ close to it inside the hole as it can be shown by performing an analytic continuation of the Schwarzschild coordinates inside the black hole.

Acknowledgements

It is a pleasure to thank Frans Klinkhamer for valuable discussions and many helpful comments on an early version of this paper. We are grateful to the anonymous referee for many fruitful questions/comments which helped to improve the manuscript.

References

- [1] R. Tarrach, Thermal effects on the speed of light, Phys. Lett. B 133 (1983) 259.
- [2] G. Barton, Faster-than-c light between parallel mirrors. The Scharnhorst effect rederived, Phys. Lett. B 237 (1990) 559.
- [3] K. Scharnhorst, On propagation of light in the vacuum between plates, Phys. Lett. B 236 (1990) 354.
- [4] J.I. Latorre, P. Pascual, R. Tarrach, Speed of light in non-trivial vacua, Nucl. Phys. B 437 (1995) 60, arXiv:hep-th/ 9408016.
- [5] W. Dittrich, H. Gies, Light propagation in non-trivial QED vacua, Phys. Rev. D 58 (1998) 025004, arXiv:hep-ph/ 9804375.
- [6] I.T. Drummond, S.J Hathrell, QED vacuum polarization in a background gravitational field and its effect on the velocity of photons, Phys. Rev. D 22 (1980) 343.
- [7] W. Heisenberg, H. Euler, Folgerungen aus der Diracsche Theorie des Positrons, Z. Phys. 98 (1936) 714.
- [8] F. Bastianelli, J.M. Dávila, Ch. Schubert, Gravitational corrections to the Euler–Heisenberg lagrangian, J. High Energy Phys. 03 (2009) 086, arXiv:0812.4849 [hep-th].
- [9] S. Emelyanov, Can gravitational collapse and black-hole evaporation be a unitary process after all?, arXiv:1507. 03025 [hep-th].
- [10] V.F. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, 2005.
- [11] M.D. Schwartz, Quantum Field Theory and the Standard Model, Cambridge University Press, 2014.
- [12] W. Dittrich, H. Gies, Probing the Quantum Vacuum. Perturbative Effective Action Approach in Quantum Electrodynamics and Its Application, Springer-Verlag, 2000.
- [13] G.M. Shore, Faster than light photons in gravitational fields II. Dispersion and vacuum polarisation, Nucl. Phys. B 633 (2002) 271, arXiv:gr-qc/0203034.
- [14] G.M. Shore, A local effective action for photon-gravity interactions, Nucl. Phys. B 646 (2002) 281, arXiv:gr-qc/ 0205042.

- [15] B.P. Jensen, A. Ottewill, Renormalized electromagnetic stress tensor in Schwarzschild spacetime, Phys. Rev. D 39 (1989) 1130.
- [16] M. Visser, Gravitational vacuum polarization. I. Energy conditions in the Hartle–Hawking vacuum, Phys. Rev. D 54 (1996) 5103, arXiv:gr-qc/9604007.
- [17] B.P. Jensen, J.G. McLaughlin, A.C. Ottewill, Renormalized electromagnetic stress tensor for an evaporating black hole, Phys. Rev. D 43 (1991) 4142.
- [18] J. Matyjasek, Stress-energy tensor of an electromagnetic field in Schwarzschild spacetime, Phys. Rev. D 55 (1997) 809;

J. Matyjasek, $(T_{\nu}^{\mu})_{ren}$ of the quantized conformal fields in the Unruh state in Schwarzschild spacetime, Phys. Rev. D 59 (1999) 044002, arXiv:gr-qc/9808019.

- [19] M. Visser, Gravitational vacuum polarization. IV. Energy conditions in the Unruh vacuum, Phys. Rev. D 56 (1997) 936, arXiv:gr-qc/9703001.
- [20] S. Liberati, S. Sonego, M. Visser, Faster than c signals, special relativity, and causality, Ann. Phys. 298 (2002) 167, arXiv:gr-qc/0107091.
- [21] G.M. Shore, Causality and superluminal light, in: I.I. Bigi, M. Faessler (Eds.), Time and Matter, World Scientific Publishing, 2006.
- [22] G.M. Shore, Superluminality and UV completion, Nucl. Phys. B 778 (2007) 219, arXiv:hep-th/0701185.
- [23] T. Elster, Vacuum polarization near a black hole creating particles, Phys. Lett. A 94 (1982) 205.
- [24] R. Haag, Local Quantum Physics. Fields, Particles, Algebras, Springer-Verlag, 1996.
- [25] H. Gies, QED effective action at finite temperature: two-loop dominance, Phys. Rev. D 61 (2000) 085021, arXiv: hep-ph/9909500.
- [26] S. Emelyanov, Effective photon mass from black-hole formation, arXiv:1603.01148 [hep-th].
- [27] G.M. Shore, Faster than light photons in gravitational fields causality, anomalies and horizons, Nucl. Phys. B 460 (1996) 379, arXiv:gr-qc/9504041.
- [28] T.J. Hollowood, G.M. Shore, Causality and micro-causality in curved spacetime, Phys. Lett. B 655 (2007) 67, arXiv:0707.2302 [hep-th];

T.J. Hollowood, G.M. Shore, The refractive index of curved spacetime: the fate of causality in QED, Nucl. Phys. B 795 (2008) 138, arXiv:0707.2303 [hep-th];

T.J. Hollowood, G.M. Shore, The causal structure of QED in curved spacetime: analyticity and the refractive index, J. High Energy Phys. 12 (2008) 091, arXiv:0806.1019 [hep-th].

- [29] T.J. Hollowood, G.M. Shore, R.J. Stanley, The refractive index of curved spacetime II: QED, Penrose limits and black holes, J. High Energy Phys. 08 (2009) 089, arXiv:0905.0771 [hep-th].
- [30] M.A. Leontovich, Wave-front propagation, in: L.I. Mandelshtam (Ed.), Lectures on Optics, Relativity, and Quantum Mechanics, Nauka, Moscow, 1972 (in Russian).