

An efficient numerical approach for transient simulation of multiphase flow behavior in centrifuges

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Abstract

The separation process of particles and liquids in centrifuge is a widely used operation in industry. In spite of the long term usage of centrifuges, the knowledge for a detailed characterization of the interaction of the macroscopic processes due the separation is lacking. Reason for this is the complex multiphase flow and the sediment built-up within the machines. The sediment shape influences the flow behavior. The shape is dependent of the rheological behavior of the sediment. Computational fluid dynamics (CFD) is suitable for the numerical investigation of the complex flow behavior. For consideration of impact of the sediment built-up and rheological behavior on the flow behavior a new low time consumption and in time and place resolved simulation method was invented. The different rheological behavior of suspension and sediment are modelled with separate approaches. Based on an empiric numerical parameter study for the rheological behavior of the sediment the suitable use of the method is approved.

Keywords

Separation process, rheological behavior, simulation, sediment built-up, centrifuge

Introduction

The mechanical solid-liquid separation is a common used process operation in industry, e.g. chemical, pharmaceutical, waste water treatment and food industry (Anlauf, n.d.; Leung, 1998). Centrifuges can be utilized for a wide range of applications e.g. clarification, dewatering, classification (Leung, 1998). Tubular bowl centrifuges are designed for efficient separation or classification of fine solids due to realization of very high centrifugal acceleration within the machines (Fernández and Nirschl, 2009; Konrath et al., 2015). The separation process is determined by the flow conditions. During the separation process a sediment emerges. The rheological behavior of the sediment influences the sediment shape. However the sediment shape has a significant impact on the flow conditions within the centrifuge. This results in a time-dependent separation efficiency (Stahl et al., 2008). Centrifuges are dimensioned by a simplified analytical approach, so-called sigma theory (Ambler, 1959). The complex transient and turbulent flow conditions and the emerging sediment are neglected. Due

to this derivations occur between the theoretical and separation efficiency in operation (Romani Fernandez, 2012).

With computational fluid dynamics (CFD) fully resolved characterizations of flow conditions within centrifuges are achievable. Different kind of approaches are known for the simulation of turbulent multiphase flows in literature (Paschedag, 2004). A few researchers investigated the separation process in centrifuges. The used approaches differ from self-developed methods to standard methods in the CFD (Breitling et al., 2003; Bürger and Concha, 2001; Pang et al., 2012; Romani Fernandez, 2012). It was observed, that fully resolved simulations of flow conditions are needed for describing the separation process in semi- or continuous centrifuges (Breitling et al., 2003). Simplified models considering only the difference flow of particles and liquid and the sediment built-up provide a good accordance for batch centrifuges (Bürger and Concha, 2001).

To investigate the influence of the sediment built-up according to the rheological behavior a time-efficient and fully in time and place resolved simulation method was invented. For consideration of the varying rheological behavior of suspension and sediment different characterization approaches are used. An empiric numerical study of the sediment built-up according to the rheological behavior is presented.

Theory

Sedimentation in a centrifugal field

One crucial parameter for the separation efficiency in centrifuges is the sediment velocity of the particles. A first approach for a theoretical characterization of the sedimentation process was given by Stokes (Stokes and others, 1901). The approach neglects particle interference and is limited only for small Reynolds numbers. Nevertheless, it is still the basis for process and machine designs in the mechanical solid-liquid separation (Stahl et al., 2008). On the basis of Stokes approach a general equation for the characterization of the sediment behavior was evolved (Beiser, 2006) and is given in Eq. (1)

$$v_{sed} = \left(\frac{4}{3} \cdot \frac{\rho_s}{\rho_l} \cdot \frac{d_p}{c_D(Re_p)} \cdot \left(1 - \frac{\rho_s}{\rho_l} \right) \cdot a \right)^{\frac{1}{2}} \quad (1)$$

v_{sed} is the sedimentation velocity for a single particle, ρ_s the density of the particle, ρ_l the density of the fluid, $c_D(Re_p)$ is the drag force coefficient as a function of the Reynolds number of the particle Re_p and a the acceleration. The acceleration in a centrifugal field is dependent of the radius r and the angular velocity ω and is shown in Eq. (2)

$$a = r \cdot \omega^2 \quad (2)$$

For this work the correlation from Kürten, Raasch and Rumpf (Kürten et al., 1966) is used for the characterization of the drag force coefficient. It is valid up to $Re_p < 2 \cdot 10^5$ and is given in Eq.(3)

$$c_D(Re_p) = \frac{21}{Re_p} + \frac{6}{\sqrt{Re_p}} + 0.28 \quad (3)$$

The presented approach in Eq (1) is only valid for the sedimentation of a single particle in a viscous fluid. Richardson and Zaki adjusted the approach of Stokes to take particle interference into account (Richardson, 1954). However the predicted reduction of the sedimentation velocity of the particle interference is too low. Michaels and Burger enhanced the correlation of Richardson and Zaki. They relate the volume fraction of the particles to the critical volume fraction α_{gel} at which a framework of solids is build (Meeten, 1994). The modified Equation is shown in Eq. (4).

$$v_{Bulk} = \left(1 - \frac{\alpha}{\alpha_{gel}}\right)^n \cdot v_{Sed} \quad (4)$$

v_{Bulk} is the hindered sedimentation velocity cause of particle interference, α is the volume fraction of the solids, α_{gel} the volume fraction at which a framework of particles is build the first time and n an empirical parameter. An increase of the sedimentation velocity due to complex particle interference at lower particle concentrations is neglected.

Rheological behavior

The rheological behavior of the phases in simulations is considered by the viscosity in the Navier-Stokes equations. The dynamic viscosity η is the ratio of shear stress τ and shear rate $\dot{\gamma}$ and is given in Eq. (5)

$$\eta = \frac{\tau}{\dot{\gamma}} \quad (5)$$

Suspension

Material properties and flow conditions influence the viscosity of suspensions. In this work only the impact of the particle concentration is considered. In Literature a lot of approaches for characterization of the impact of the particle concentration are known (Hochstein, 1997). The approach of Quemada (Quemada, 1977) was chosen und is given in Eq. (6). η_{Susp} is the dynamic viscosity of the suspension and η_l of the pure liquid phase. A possible emerging yield point is neglected.

$$\eta_{Susp} = \eta_l \cdot \frac{1}{\left(1 - \frac{\alpha}{\alpha_{gel}}\right)^2} \quad (6)$$

Saturated sediment

With fluid saturated sediments show a complex rheological behavior. Two physical mechanism affect the behavior: Coulomb friction due to particle-particle contact and viscous friction due to the pore fluid (Erk, 2006; Mladenchev, 2007). Mladenchev (Mladenchev, 2007) and Erk (Erk, 2006) revealed an existing pressure dependent yield stress, so-called yield locus (Jenike, 1964), for a saturated sediment. Both characterized the rheological behavior with a Herschel-Bulkley approach, which is given in Eq (7).

$$\tau = \tau_0 + K \cdot \dot{\gamma}^n \quad (7)$$

τ_0 is the yield stress, K the consistency and n the rheological parameter. The yield stress for saturated sediment is dependent of the pressure on the framework. Therefore the Herschel-Bulkley approach has to be extended for each variable to be a function of the operating pressure in the framework. The value of the rheological parameter n affects the flow behavior extremely. Shear thinning behavior with a yield stress is characterized for the range $0 < n < 1$. Fluids with that kind of rheological behavior are called viscoplastic fluids. For $n = 1$ shows the shear stress a linear dependence of the shear rate, so-called Bingham plastic fluids (Nguyen and Boger, 1992). In Figure1 presents schematically the different rheological behaviors. For the parametric study the sediment was treated as a Bingham plastic fluid.

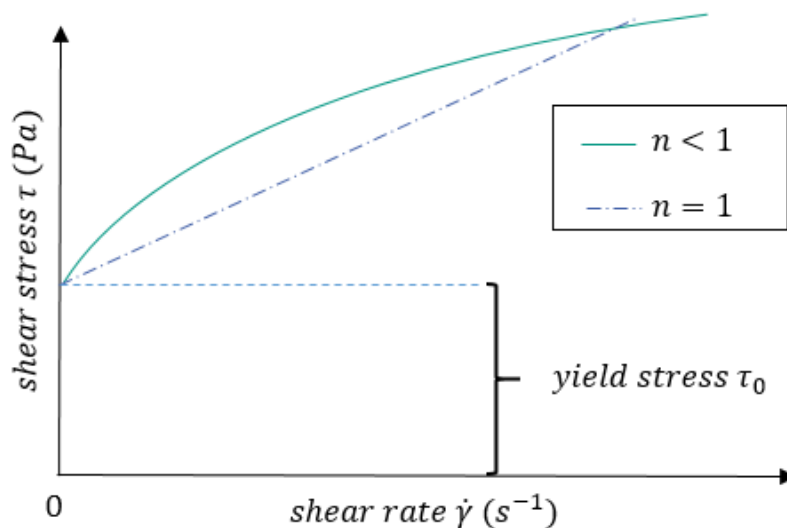


Figure 1: Schematic illustration of the rheological behavior of a viscoplastic ($0 < n < 1$) and a Bingham plastic ($n = 1$) fluid

Sediment build-up in a tubular centrifuge

Sediment build-up has a crucial effect on the separation process. Due to the separation process the particles are transported in radial direction within the centrifuge. This leads

to a concentration of the particles at the outer wall. The particles build up a porous framework, which is called sediment. The pores are completely filled with liquid. The sediment build-up is dependent of the flow conditions within the machine, the material properties of the solid and the liquid phase and process time. In turn the sediment effects the flow conditions, because of an increased flow resistance in the pores of the sediment (Ergun and Orning, 1949). Due to this, the flow conditions with time. This results in a time-dependent separation efficiency (Romani Fernandez, 2012).



Figure 2: Scheme of sediment distribution within a tubular centrifuge for products with different rheological behavior: 1) with a high yield stress, 2) with a low yield stress (Stahl et al., 2008)

The suspension flows in axial direction in a tubular centrifuge. Especially the in- and outflow geometries influences the flow conditions within the centrifuge. This can lead to an inhomogeneous sediment distribution. The other significant factor for the shape of the emerging sediment is the rheological behavior of it (Stahl et al., 2008). In Figure 2 two extreme cases are presented for the influence of the rheological behavior. With a high yield stress (1) of the sediment an in axial direction inhomogeneous sediment distribution with a certain response angle occurs in the separations process. A sediment with a lower yield stress evenly spreads in the centrifuge by reason of movement of the sedimented particles.

Simulation

Complex multiphase flows occurs in centrifuges. Layout of the centrifuge, process parameter and material characteristics influence the flow patterns. To design a centrifuge or a mechanical separation process the flow conditions have to be known. However the experimental investigation of the flow within centrifuges is highly sophisticated. Hence the CFD offers an appropriate alternative in opposite to experiments. Some researchers investigated the flow in centrifuges. Common or self-

developed numerical approaches for multiphase flows in centrifuges can be found in literature. Bürger et al (Bürger and Concha, 2001) models the sedimentation in centrifuges with a simplified approach. The sedimentation behavior is only described with a flux density function. The numerical results are in good agreement with experiments with a batch centrifuge. Romani et al (Fernández and Nirschl, 2009) investigated the sedimentation in a solid bowl centrifuge with an Euler-Lagrange Approach. The flow of all three in centrifuges existing phases was spatially and temporally resolved separately leading to a high time consumption. Therefore only a short time could be simulated. In general all approaches in literature have disadvantage a side of physical resolution or at the needed time. For realizing spatial and temporal resolved numerical simulation of the separation process in consideration of the sediment build-up and flow behavior a new method was evolved.

Method

For spatial and temporal numerical simulations an approach with a compromise between physical resolution and time consumptions equal to the fast Eulerian approach of Ferry (Ferry and Balachandar, 2001) was invented. The solid and liquid phase are approximated by a mixed phase. The gas phase is neglected and it is limited to an average particle size. The velocity field $v(\vec{x}, t)$ for the mixed phase is calculated by solving the Navier-Stokes equations. \vec{x} is the position in space and t in time. The particle velocity field $v_p(\vec{x}, t)$ can be evaluated explicitly from the velocity field of the mixed phase $v(\vec{x}, t)$ and the spatial sedimentation velocity v_{Bulk} . As a result, only one additional PDE has to be solved for the transport of the volume fraction of the particles $\alpha(\vec{x}, t)$. Due to this the approach has a lower time consumption compared to the classical Eulerian approach.

The rheological behavior of suspension and sediment in the centrifuges varies. To take the impact of the different behavior on the flow conditions into account, a spatial and temporal viscosity of the mixed phase $\eta_{MP}(\vec{x}, t)$ is defined and given in Eq.(8).

$$\eta_{MP}(\vec{x}, t) = A \cdot \eta_{Susp}(\vec{x}, t) + B \cdot \eta_{Sed}(\vec{x}, t) \quad (8)$$

$\eta_{MP}(\vec{x}, t)$ is calculated from the viscosity of the suspension $\eta_{Susp}(\vec{x}, t)$ and of the virtual viscosity of the sediment $\eta_{Sed}(\vec{x}, t)$. A and B are the spatial phase condition coefficients. A is the coefficient for suspension and B for sediment. The value of A and B varies between 0 and 1 dependent on the kind of phase condition, suspension or sediment, is temporarily and spatial present.

Geometry and Simulation parameters

The simulation method was implemented in the Open Source Software OpenFOAM. Figure 3 shows the used geometry for the simulations. The geometry approximates a simplified tubular centrifuge with a solid core. The remaining gas phase in the centrifuge can be neglected due to the solid core. In table 1 the dimensions of the geometry the process parameters and the material properties are shown. The

geometry is discretized with $2 \cdot 10^6$ cells. The grid has a grading towards all walls to increase the resolution near the wall. For modelling the turbulence of the flow within the centrifuge a kOmegaSST pattern was chosen.

Table 1: Dimensions of the simplified centrifuge and process parameters

Dimensions	
Length (mm)	167
Inner diameter (mm)	16
Outer diameter (mm)	38

Process parameters	
Angular velocity ω (s^{-1})	200
Volumetric flow rate \dot{V} ($\text{l}\cdot\text{min}^{-1}$)	0.5
Volume fraction of particles at the inlet α (-)	0.02

Material properties	
Density fluid ρ_l ($\text{kg}\cdot\text{m}^{-3}$)	1000
Density solid ρ_s ($\text{kg}\cdot\text{m}^{-3}$)	2500
Dyn. viscosity fluid η_l ($\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$)	$1\cdot 10^{-3}$
Diameter particle d_p (μm)	30

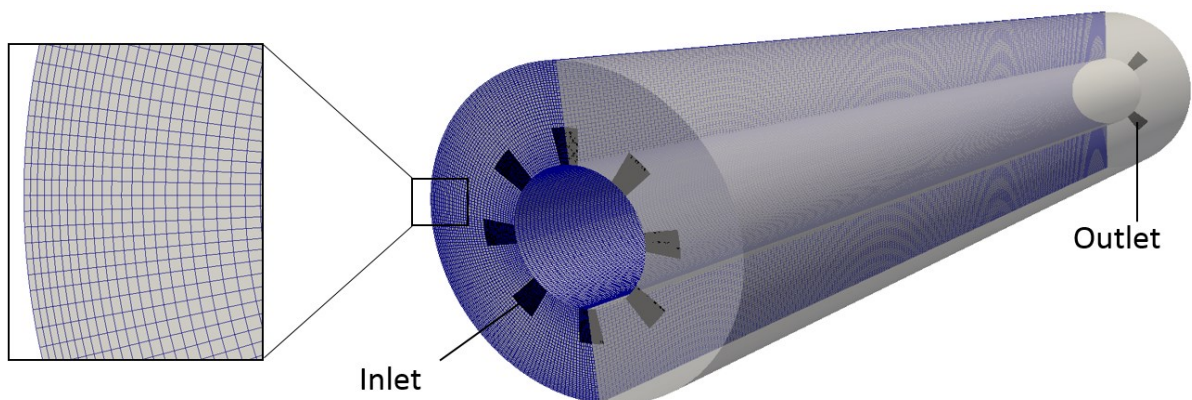


Figure 3: Geometry of a simplified tubular centrifuge presenting the used grid for the simulations

Results and Discussion

An empiric study of the influence of the rheological behavior onto the sediment built-up with CFD is presented at this work. As previous discussed, the rheological behavior of the suspension is modelled with the approach of Quemada and the sediment is

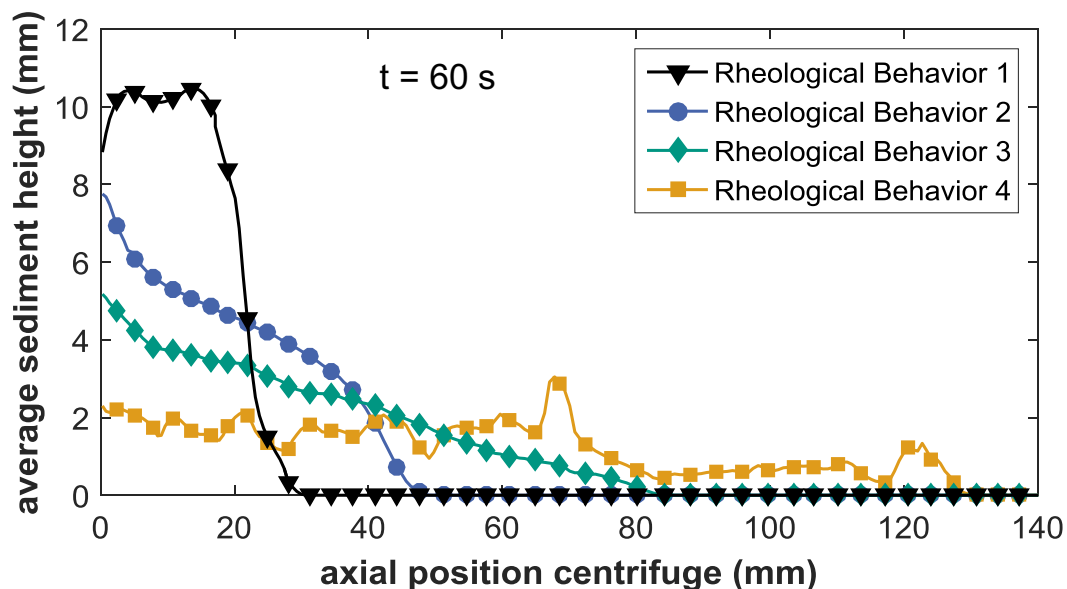
treated as a Bingham plastic fluid. The flow behavior of the sediment is given by Eq. (9).

$$\tau_{sed} = (k_1 \cdot p_s + k_2) + K \cdot \dot{\gamma} \quad (9)$$

τ_{sed} is the shear stress in the sediment, k_1 is a proportionality factor and k_2 is the yield stress at $p_s = 0$. In the empiric parametric study the influence of k_1 , k_2 and K is investigated. The settings of 4 exemplary chosen rheological behaviors (RB) are presented in table 2. The range for each parameter was chosen in consideration of the process parameters of the simulated separation process.

	k_1 (-)	k_2 (Pa)	K (Pa · s)
Rheological behavior 1	2.5	5000	$1 \cdot 10^{-4}$
Rheological behavior 2	0.5	500	$1 \cdot 10^{-6}$
Rheological behavior 3	0.001	30	$1 \cdot 10^{-3}$
Rheological behavior 4	0.001	30	$1 \cdot 10^{-4}$

The sediment after 60 seconds for the chosen flow behaviors are presented in Figure 4. Figure 4 shows the average sediment height over the axial position in the centrifuge. The sediment height is averaged in circumferential direction. The axial position 0 correspond to the inlet position of the suspension.



It can clearly be seen, the sediment built-up in a tubular centrifuge is inhomogeneous in axial direction. Further the influence of the rheological behavior of the sediment is indicated. The inhomogeneity in axial direction decreases with the decreasing yield stress and consistency. The sediment with rheological behavior 1 has a high yield stress. Due to this it is rigid and builds a steep front. The influence of the limitation to only one transported particle size can also be seen. A particle size distribution leads to a sedimentation velocity distribution. Due to this the particle would spread wider along in axial direction resulting in a less steep shaped sediment.

With decreasing yield stress and consistency (rheological behavior 2) the sediment front is less steep due to movement of the particle after the sedimentation process. The yield stress has a more significant impact compared to the consistency. But with decreasing yield stress increases the influence of the consistency. The difference between behavior 3 and 4 is the consistency. The consistency of RB 3 is ten times higher than of 4. The sediment with RB 3 builds up an inhomogeneous sediment with a flat angle. RB 4 leads to a wider spread wave-shaped sediment. With decreasing consistency disperse the sedimented particle more often. This leads to the wave-shaped form of the sediment.

Conclusion

The separation process in a tubular centrifuge is highly complex. Not only the material properties of the suspension and the process parameters, also the rheological behavior of the sediment influences the separation process. An approach to consider the complex rheological behavior of the sediment in CFD is presented. Different rheological behavior results in different shaped sediments. It was shown that the presented approach is appropriate for the simulation of the complex separation process in a tubular centrifuge considering the rheological behavior of the emerged sediment. The influence of the yield stress and the consistency is discussed. With decreasing yield stress the influence of the consistency onto the sediment shape increases.

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