Performance evaluation of closed-loop logistics systems with generally distributed service times

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Abstract

The performance evaluation of hospital logistics is becoming more and more important to guarantee efficient services in health establishments. Therefore, we propose a new discrete-time approach for the steady-state analysis of closed-loop queueing systems with arbitrary topology and generally distributed service times. Based on a finite Markov chain, it is possible to compute the complete cycle time distribution of service systems with a population constraint. In addition, the distribution of the number of customers at each service station can be obtained. The method is applied to the analysis of a sterilization process of medical devices. To verify the method, we compare the results of our discrete-time approach to the results that are obtained by a simulation model.

Keywords: Performance evaluation, Queueing networks, Closed-loop, General service process, Cycle time distribution, Hospital logistics

1. Introduction

As health establishments are faced with growing health expenses, more and more research is done in order to make service processes more efficient. Therefore, some authors started with the performance analysis of support services

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in health establishments, e.g., the sterilization of medical devices. Starting with a simulation model of a generic sterilization process (see Di Mascolo et al. (2009)), analytical methods which were previously used to analyze the material flow in production systems, were applied to analyze the sterilization service in health establishments (see Matzka (2011), Stoll and Di Mascolo (2013)). Discrete-time queueing models have been used as a method to achieve a fast and quite accurate way of determining performance figures of service systems. While with classical general queueing models in continuous-time domain, characteristic values are calculated only on the basis of means and variances (for a detailed overview of methods for closed queueing networks see Lagershausen (2012)), in discrete-time modeling all input and output variables are described with discrete probability distributions. This enables the derivation of quantiles of performance measures, which are often needed for the design of logistics systems.

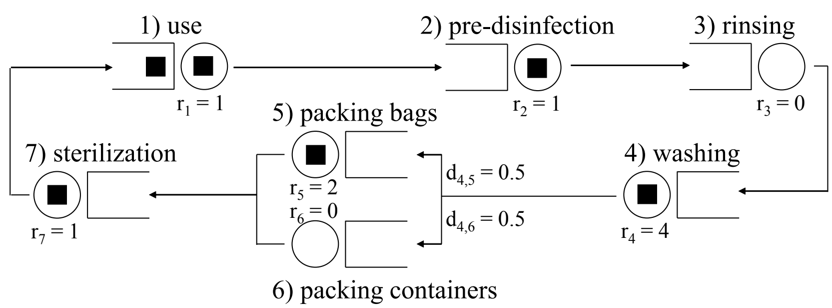

Figure 1: Representation of a sterilization process (see Matzka (2011)) as a closed queueing system

In the previous works, the sterilization process was modeled as an open discrete-time queueing network, as the according discrete-time queueing models were only applicable to open networks (see Matzka (2011), Stoll and Di Mascolo (2013)). As the sterilization process becomes a loop when the use-step is integrated (see figure 1), a more realistic model is achieved, when the sterilization of medical devices is modeled as a closed queueing network. In this paper, we present a method for the performance evaluation of closed-loop queueing systems in discrete-time domain with generally distributed service times that can be used to analyze the sterilization process of health establishments.

2. System description

The system under investigation is a closed queueing system in discrete-time domain with arbitrary topology. It consists of V stations with one server and one waiting room each, as well as $\hat{K}$ customers that circulate in the system. The routing of the customers to the subsequent stations depends on the routing matrix D. Its generic element $d_{h,i}$ defines the probability to be routed from station h to station i. The system is observed at equally spaced time periods with a length of t_{inc}. It is assumed that the beginning and the end of service as well as the routing to the subsequent stations take place immediately prior to the periods. The customers that cannot be processed immediately stay in the waiting room, which has infinite queueing capacities, and are served based on a first come first serve discipline. The service time at station i is assumed to be independent and identically distributed (i.i.d.) according to the random variable B_i, where $b_{i,j}$ denotes the probability that B_i takes value j. Furthermore, the service time distributions have finite upper supports $b_{i,max}$. The performance measures of interest are the distribution of the number of customers K_i in the stations at random periods and the total cycle time distribution S through the system starting from station $\hat{i}$, i.e., that for each customer the entry in that station determines the end of the current cycle time and the beginning of a new one. A closed queueing system, which will be used in the following section to clarify the algorithm to determine the performance measures, is depicted in figure 2.

3. Computation of the performance measures

3.1. Steady-state probability computation

To obtain the performance measures, a finite Markov chain which governs the behavior of the system by $2 \cdot V$ parameters is created. V parameters k_i are needed to define the number of customers at each station and V more r_i to define the residual time of their currently served customer ($r_i = 0$ if there are no customers in the i^{th} station). As a result, a system state can be represented as follows:

$$\begin{gathered} z = (\ \ r_1\ r_2\ ...\ r_V \quad k_1\ k_2\ ...\ k_V\ \ ) \\ \text{with } r_i \in \{0, 1, ..., b_{i,max}\}, k_i \in \{0, 1, ..., \hat{K}\}, i \in \{1, ..., V\} \end{gathered} \tag{1}$$

For example, the system state of the queueing network depicted in figure 2, is defined by $z = (\ 1\ 1\ 1\ 1\ )$.

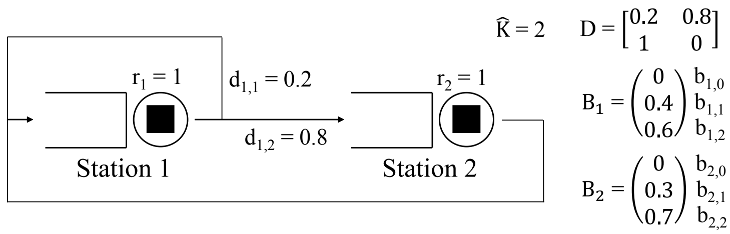

Figure 2: Example of a closed queueing system

The possible system states are determined iteratively by starting from one initial possible state and by computing all the possible states for the next time period. In particular, the beginning of a new time period reduces the residual time of each customer in service by 1 and the end of the service (r_i from 1 to 0) leads to a customer transition to a subsequent station. If a new service starts, the residual time can assume any value of the service time distribution B_i. The routing of the leaving customers is dependent on the routing matrix D, whereas the residual time at the beginning of a service is dependent on the service time distribution B_i. The combination of the possible customer transitions and new residual times determines the possible number of new states, while the routing probabilities and probability distribution of the service times are used to determine the transition probabilities to the correspondent subsequent states. If new states are found, the same reasoning is applied to them to find new states, otherwise the iteration stops. At the end of the iterations, all N possible states are found and denoted as z_n and their set $\mathcal{Z}$ is created. The state transition matrix U can be then computed, i.e., a matrix which contains the probabilities $u_{m,n}$ to go from the state z_m to z_n in the next time period. Once the matrix U is defined, the steady-state probabilities p_n can be computed using standard techniques like the Power Method (Bolch et al., 1998) or the ones explained in Stewart (2009).

3.2. Distribution of the number of customers K_i

The distribution of the number of customers K_i in station i at random periods can directly be obtained from the steady-state probabilities. Hereby, $k_{i,j}$

denotes the probability that K_i takes value j.

$$k_{i,j} = \sum_{\forall z_n \mid k_i = j} p_n \tag{2}$$

3.3. Calculation of the total cycle time distribution S through a chosen station $\hat{i}$

To obtain the total cycle time distribution S, the computation of the contribution S_n to the cycle time for each state z_n is carried out. The method uses a similar strategy as Colledani et al. (2015), where the computation of time distributions starts when a new customer enters the considered subsystem and ends when it leaves the subsystem. In our case, the computation starts when a new customer enters station $\hat{i}$ and ends when he comes back to station $\hat{i}$. In particular, for each z_n the set of the following states $\mathcal{Z}_n^0$ (referring to the period $t = 0$ of the cycle time computation) is computed along with their probability $p_{n,m}^0$ (with $m \in \{1, ..., M_n^0\}$ and M_n^0 the state number contained in $\mathcal{Z}_n^0$).

In order to track the customers through the system, a new state vector a_n^t, which contains also the station number i_s and the position i_p in the station, must be used. The position i_p takes value 1 if the customer is in service and value $2, ..., \hat{K}$ when he waits for service. The vector represents a generic state after t periods which has been originated from the n^{th} state of the system. As a result, a system state can be represented as follows:

$$\begin{gathered} a_n^t = (\ r_1\ r_2\ ...\ r_V \quad k_1\ k_2\ ...\ k_V \quad i_s\ i_p\ ) \\ \text{with } r_i \in \{0, 1, ..., b_{i,max}\}, k_i \in \{0, 1, ..., \hat{K}\}, \\ i_s \in \{1, ..., V\}, i_p \in \{1, ..., \hat{K}\}, i \in \{1, ..., V\} \end{gathered} \tag{3}$$

A new set $\mathcal{A}_n^0$ is created. It contains all the states at t=0 originated from z_n, which are identified with the index $q \in \{1, ..., Q_n^0\}$, where Q_n^0 is the number of states in $\mathcal{A}_n^0$. They are filled according to the following procedure:

- The transitions from the state z_n to all states $z_{n,m}^0$, i.e., the ones included in $\mathcal{Z}_n^0$, are considered.

- For each transition, if no customer enters the station $\hat{i}$, no states are added to $\mathcal{A}_n^0$.
- If c customers have just entered the station $\hat{i}$, c state vectors a_n^0 are added to $\mathcal{A}_n^0$. Those vectors are created in the following way: the first $2 \cdot V$ parameters coincide with the correspondent vector in $\mathcal{Z}_n^0$, the station number i_s equals $\hat{i}$ and the corresponding probability $p_{n,q}^0$ coincides with the probability $p_{n,m}^0$ of the correspondent state in $\mathcal{Z}_n^0$. They only differ for i_p, since the customer of interest may enter the station as the $1^{st}, ..., c^{th}$ customer.

For the previously depicted queueing system (see figure 2), a numerical example of the computation of the set $\mathcal{A}_1^0$ is provided in figure 3, i.e, for set $\mathcal{A}_{n=1}^0$ that comes out from the first system state. For the sake of simplicity, the probabilities at each step are computed as if $p_1 = 1$, but in general $p_n < 1$.

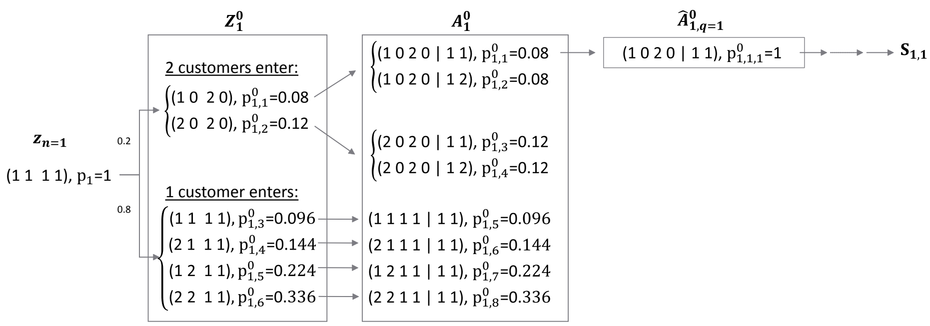

Figure 3: Numerical example of the computation of the set A_n^0

At this point, the cycle time distribution $S_{n,q}$ for each state of $\mathcal{A}_n^0$, namely the probability that the tracked customer returns to the station $\hat{i}$ at each time period, must be computed as follows:

1. Initialize t=0 and create the set $\hat{\mathcal{A}}_{n,q}^0$.
2. Add a copy of $a_{n,q}^0$ to the new set and assign a correspondent unitary probability to it. That new state is denoted as $\hat{a}_{n,q,1}^0$
3. Increment to t = t + t_{inc}.
4. Considering all the states of $\hat{\mathcal{A}}_{n,q}^{t-t_{inc}}$, determine the states for the current time step. It must be considered that if the tracked customer leaves a station, the i_s and i_p indexes must be recomputed accordingly. Add the

new states to the set $\hat{\mathcal{A}}_{n,q}^{t}$. The index $w \in \{1, ..., W_{n,q}^{t}\}$ denotes each state $\hat{a}_{n,q,w}^{t}$, whereas $W_{n,q}^{t}$ corresponds to the total number of states included in $\hat{\mathcal{A}}_{n,q}^{t}$.

5. Determine the correspondent probabilities $p_{n,q,w}^{t}$ of the new states by considering the routing probabilities and service time probability distribution of all stations which serve a new customer.
6. If the tracked customer arrives at a station simultaneously with n other customers, all $n+1$ possible positions i_p must be considered and, as a result, $n+1$ different states must be added to $\hat{\mathcal{A}}_{n,q}^{t}$. Furthermore, the correspondent probability must be equally split and shared among those $n+1$ states.
7. If the tracked customer has just returned to station $\hat{i}$, add its probability $p_{n,q,w}^{t}$ to $S_{n,q}$ in the t^{th} position and delete the state from $\hat{\mathcal{A}}_{n,q}^{t}$.
8. Repeat the algorithm from step (3) until the set $\hat{\mathcal{A}}_{n,q}^{t}$ is empty or the total probability of its states is smaller than a given ϵ (in case of a possible infinite cycle time).

For the previously depicted queueing system (see figure 2), a numerical example of the computation of $S_{1,1}$ is provided in figure 4.

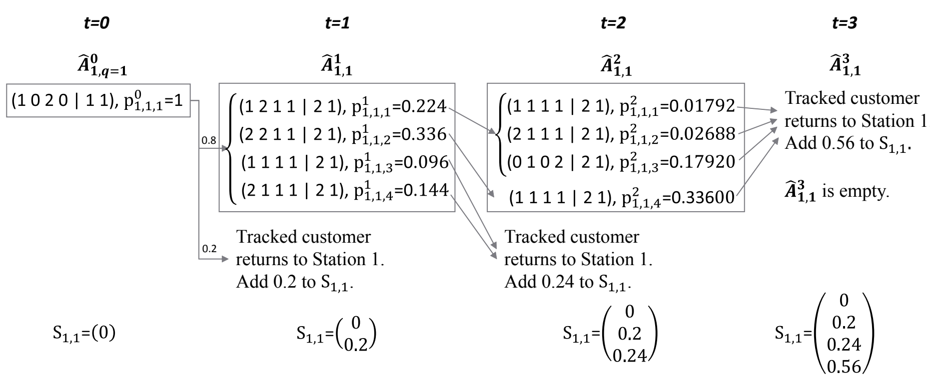

Figure 4: Numerical example of the computation of the cycle time distribution $S_{n,q}$

Once all the cycle time distributions of the tracked customers $S_{n,q}$ are computed, the cycle time distribution S_n of the n^{th} state is computed as follows:

$$S_n = \frac{\sum_{q=1}^{Q_n^0} p_{n,q}^0 \; S_{n,q}}{\sum_{q=1}^{Q_n^0} p_{n,q}^0} \tag{4}$$

The numerator results in a weighted sum of the cycle time distributions $S_{n,q}$, while the denominator normalizes the sum in such a way that the sum of the vector elements is equal to 1. In order to exactly compute the overall cycle time distribution S through the station $\hat{i}$, the cycle time contributions of all states are combined with the following formula:

$$S = \frac{\sum_{n=1}^{N} p_n \, S_n}{\sum_{n=1}^{N} p_n} \tag{5}$$

4. Numerical evaluation

In the following section, we apply the presented calculation method to the analysis of a sterilization process of medical devices. In a sterilization process, reusable medical devices are re-injected in the process after their use in the operation room. When we integrate the use step, the sterilization process becomes a sterilization loop as seen in figure 1.

Table 1: Discrete probability distributions of the single process steps

	Process Steps						
t_{inc}	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	0	0.5	1.0	0	0.9	0.1	0
2	0.1	0.5	0	0	0.1	0.9	0
3	0.3	0	0	0	0	0	1.0
4	0.1	0	0	1.0	0	0	0
5	0.2	0	0	0	0	0	0
6	0.3	0	0	0	0	0	0

The pre-disinfection step is done directly after the use in the operation room. The medical devices are placed in a disinfectant liquid to decrease the population of micro-organisms present on the soiled equipment, in order to protect the staff during the manipulation and to facilitate the later washing. During the pre-disinfection step, the used medical devices (MDs) are transferred from the operation rooms down to the sterilization area. At the sterilization area, the MDs are rinsed. Furthermore, the MDs are washed in machines to eliminate stains to obtain a clean medical device. After washing, the MDs are packed into containers or bags to constitute a barrier against micro-organisms. In the sterilization step, the MDs are placed in an autoclave where they are treated with saturated steam. After the sterilization, the MDs are stored close to the operating rooms.

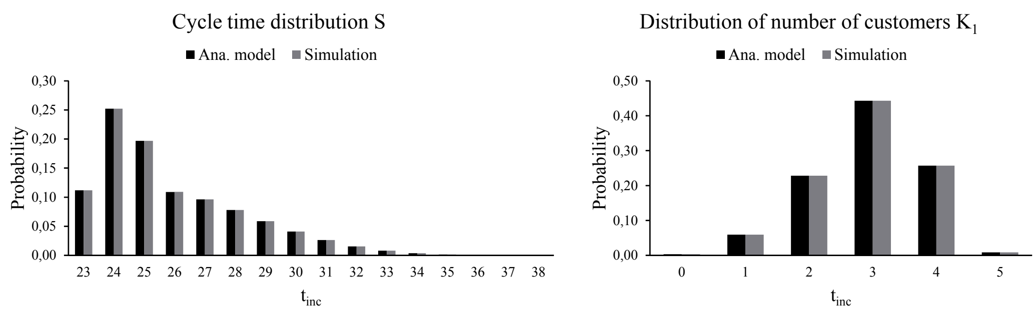

Figure 5: Comparison of analytical and simulation results of a closed network with 6 customers

In table 1, exemplary service time distributions of the single process steps are given ($t_{inc} = 10min$). We used this input data in order to compare the results of the discrete-time model with the ensemble averages of 10 independent replications of a discrete-event simulation that models the same assumptions as the analytical model. Each replication contains 10,000,000 customer cycles. We analyzed the closed queuing network with 2 to 6 MDs in the system. Figure 5 shows the probability distributions of the cycle time and the number of customers in queueing system 1 when the network contains 6 customers. Table 2 gives the results for the mean cycle time $E(S)$, the 95 % quantile of the cycle time $S_{0.95}$, the average number of MDs $E(K_1)$ in queueing system 1, and the probability $k_{1,0}$ that at station 1 zero MDs are present. For each parameter, we can see that the analytical results are equal to the simulation results, as our method is exact. We can see, that for 6 MDs, queueing system 1 is only empty in 0.34% of the cases and MDs are ready for use. Besides, we provide the total computation time for the analytical model and the simulation. We can see, that for 6 customers in the network, the analytical computation time for all parameters is already half of the simulation length. But note, that the computation of the system states only needs 1.4% of the total computing time. Therefore, our method provides performance figures in an acceptable time.

Table 2: Performance parameters for networks with 2 to 6 customers

	$E(S)[t_{inc}]$		$S_{0.95}[t_{inc}]$		$E(K_1)[cust.]$		$k_{1,0}[\%]$		Comp. time [s]	
$\hat{K}$	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.
2	15.59	15.59	18	18	0.57	0.57	44.83	44.83	0.43	434.92
3	16.24	16.24	19	19	0.90	0.90	20.58	20.58	4.44	408.58
4	18.21	18.21	22	22	1.39	1.39	5.56	5.56	9.00	450.75
5	21.78	21.77	26	26	2.08	2.08	1.26	1.26	54.32	396.04
6	25.89	25.89	31	31	2.92	2.92	0.34	0.34	224.36	401.20

5. Conclusion

In this paper, we propose a new approach for the steady-state analysis of closed-loop queueing systems with arbitrary topology and generally distributed service times. Based on a discrete-time Markov chain, the method provides an exact computation of the cycle time distribution and the distribution of the number of customers at each service station. The method is applied to the analysis of a sterilization process of medical devices. Moreover, we verify the results of our model by comparison to a discrete-event simulation. Future work will be dedicated to the extension of the approach to systems with finite queueing capacities and the application of this approach to case studies in health establishments.

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