Karlsruhe Institute of Technology

Untersuchungen zum 3HDM
Investigations within the 3HDM

Bachelor's Thesis of

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Declaration

I hereby declare that I have developed and written the enclosed thesis independently, and have not used sources or means without declaration in the text. Any work of others or literal quotations are clearly cited.

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1 Motivation and Goals

During the Large Hadron Collider (LHC) Run 1 at CERN, with a c.m. energy of $\sqrt{s} = 7$ and 8 TeV, a Higgs boson with a mass of about 125 GeV was discovered by the LHC experiments ATLAS [1] and CMS [2]. Its measured properties are consistent with those of a Standard Model (SM) Higgs boson [3–6]. Now, we are sure that Electroweak Symmetry Breaking (EWSB) is triggered by the vacuum expectation value of a scalar field, which in its turn triggers the Higgs mechanism that generates masses for the $W^{\pm}$ and Z gauge bosons. Despite the discovered Higgs boson behaving very SM-like, there is still room for beyond the SM (BSM) interpretations. The question then is: Have we discovered the SM Higgs boson, or just the SM-like Higgs boson of a BSM extension?

BSM theories often incorporate multiple Higgs fields or other scalars [7–19]. The global symmetries in these theories determine the phenomenological behavior of the models. There exist numerous phenomenological investigations in the framework of the Two-Higgs Doublet Model (2HDM) [7, 11, 12]. The Three-Higgs Doublet Models (3HDMs) [15–19] with a richer charged, scalar and pseudo-scalar sectors present greater possibilities and may entail interesting phenomenological consequences. Until now, no extensive study has been carried out that would compare 2HDMs to 3HDMs, since the latter class of models has not been studied to as great detail as the former. This thesis provides an investigation of the 3HDM to gain first insights into this model.

This brings us to the goals of the current thesis. These are as follows: The analytical computation of the physical Higgs' spectrum, i.e. of their masses and their couplings to the SM particles; the investigation of the effects resulting from the respective chosen potential on the symmetry of the Higgs masses and their couplings, and on the possible existence of Dark Matter (DM) candidates; more generally, the investigation of the symmetry-induced relations between the Higgs bosons.

2 The 3HDM

One of the interesting properties of New Physics models that employ multiple Higgs fields is that they induce discrete symmetry groups in the Higgs and flavour sectors. At the global minimum of the Higgs potential, these discrete symmetry groups are often broken completely, or partially down to a proper subgroup. This has a strong impact on the phenomenology in the Higgs and flavour sectors, as well as interesting astroparticle consequences.

In 3HDM, if we consider only unitary transformations, then there are only ten realizable finite symmetry groups [15]:

$$\begin{array}{cccc} \mathbb{Z}_2, & \mathbb{Z}_3, & \mathbb{Z}_4, & \mathbb{Z}_2 \times \mathbb{Z}_2, \\ D_3 \cong S_3, & D_4, & T \cong A_4, & O \cong S_4, \\ \multicolumn{2}{c}{(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_2 \cong \Delta(54)/\mathbb{Z}_3,} & \multicolumn{2}{c}{(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4 \cong \Sigma(36),} \end{array} \tag{2.1}$$

where $N \rtimes M$ is the semidirect product of a normal subgroup N and a subgroup M, and $G \cong H$ denotes that the group G is isomorphic (structurally identical) to the group H.

If we try to impose any additional finite symmetry group of the Higgs-family transformations different from the ten listed in (2.1), this results in a potential with a continuous symmetry [15].

The current thesis deals with a 3HDM that is symmetric under the $\Delta(54)$ symmetry group [15]. More details, specifically on the generators of this group, are given in section 7.3 of [19]. Figure 2.1 visualizes the relations between the different symmetry groups possible in a 3HDM.

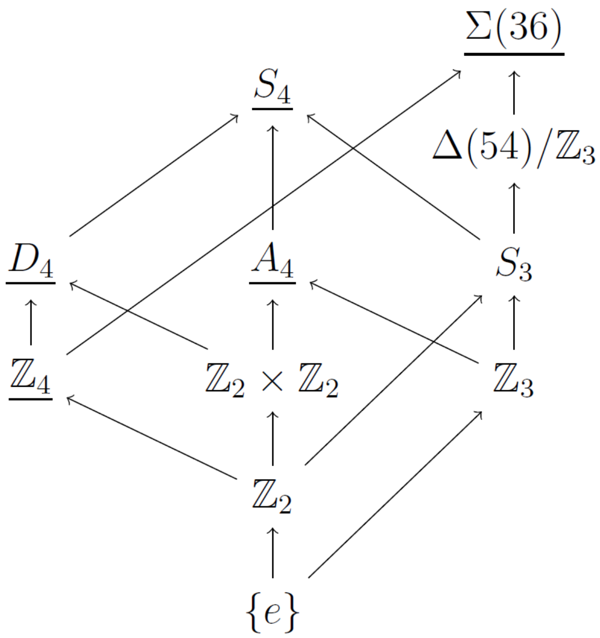

Figure 2.1: Relations between finite realizable groups of the 3HDM Higgs-family transformations. Groups that are underlined lead to automatic explicit CP-conservation. $A \to B$ stands for $A \subset B$ [15].

2.1 3HDM $\Delta(54)$ Potential

The scalar potential for the $\Delta(54)$ family of discrete symmetry groups is given by [15]

$$\begin{aligned} V = &- m^2 \left[\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3\right] + \\ &+ \lambda_0 \left[\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3\right]^2 + \\ &+ \lambda_1 \Big[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2 + (\Phi_3^\dagger\Phi_3)^2 - \\ &\quad -(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\Phi_2^\dagger\Phi_2)(\Phi_3^\dagger\Phi_3) - (\Phi_3^\dagger\Phi_3)(\Phi_1^\dagger\Phi_1)\Big] + \\ &+ \lambda_2 \left[|\Phi_1^\dagger\Phi_2|^2 + |\Phi_2^\dagger\Phi_3|^2 + |\Phi_3^\dagger\Phi_1|^2\right] + \\ &+ \left\{\lambda_3 \left[(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_3) + (\Phi_2^\dagger\Phi_3)(\Phi_2^\dagger\Phi_1) + (\Phi_3^\dagger\Phi_1)(\Phi_3^\dagger\Phi_2)\right] + h.c.\right\} \end{aligned} \tag{2.2}$$

with the parameters

$$m^2, \lambda_0, \lambda_1, \lambda_2 \in \mathbb{R}, \text{ and } \lambda_3 \in \mathbb{C} \; . \tag{2.3}$$

The Φ_i for $i \in \{1, 2, 3\}$ are the three Higgs doublets. These are parametrized differently in each of the four cases of the global minimum (see section 2.2).

In the following, we will consider only the CP-conserving model, i.e. $\lambda_3 \in \mathbb{R}$.

2.2 Four Cases of the Global Minimum

There can only be four different types of the global minimum in this model. All other minima of the potential are equivalent to these by a symmetry of the model. The corresponding vacuum expectation value (vev) alignments (v_1, v_2, v_3), with the overall vev scale factored out in each case, are [15]:

$$A : (1, 0, 0) \qquad B : (1, 1, 1) \qquad C : (\omega, 1, 1) \qquad D : (\omega^2, 1, 1) \tag{2.4}$$

where

$$\omega = \exp(2\pi i/3)\ . \tag{2.5}$$

In the following, these four cases will be studied. For each of them the scalar potential will be diagonalized, and the Higgs masses and the Higgs couplings to the gauge bosons will be presented. Possible DM candidates will be listed as well. Additionally, for the scenario B: $(1, 1, 1)$, the $\Delta(54)/\mathbb{Z}_3$ finite symmetry group will be reduced to a $\mathbb{Z}_2$ symmetry group, the Higgs couplings to fermions will be calculated and the decays of the Higgs bosons will then be analyzed in the form of the branching ratios.

3 Case A: (1,0,0)

The ground state is realized when the doublets adopt their vevs:

$$\langle \Phi_j \rangle = \langle 0|\Phi_j|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v_j \end{pmatrix} \, , \qquad j \in \{1,2,3\} \, . \tag{3.1}$$

Expanding around the minima, the 3 doublets can thus be parametrized as

$$\Phi_j = \begin{pmatrix} w_j^+ \\ \frac{1}{\sqrt{2}}(v_j + h_j + i a_j) \end{pmatrix} \, , \qquad j \in \{1,2,3\} \, , \tag{3.2}$$

with a complex field w_j, and real fields h_j and a_j.

For this specific case that we will call $(1,0,0)$, the vevs of the corresponding doublets are

$$v_1 = v \, , \qquad v_2 = v_3 = 0 \, . \tag{3.3}$$

Here, the vev v is

$$v = \frac{1}{\sqrt{\sqrt{2} G_F}} \approx 246\,\text{GeV} \, , \tag{3.4}$$

where G_F denotes the Fermi constant [20]

$$G_F = 1.166\,378\,7 \cdot 10^{-5}\ \text{GeV}^{-2} \, . \tag{3.5}$$

A more detailed description of this case, focusing on the implication of two inert doublets on DM candidates, has been carried out in [21].

3.1 Minimum Conditions

The ground state is given by the minimum of the potential in eqn. (2.2). The doublets in eqn. (3.2) are expressed through their corresponding vevs in the ground state, and the following conditions hold:

$$\left. \frac{\partial V}{\partial \Phi_i^\dagger} \right|_{\Phi_j = \langle \Phi_j \rangle} = 0 \, , \qquad i,j \in \{1,2,3\} \, . \tag{3.6}$$

When differentiating with respect to $\Phi_1^\dagger$ this yields

$$m^2 = v^2\ (\lambda_0 + \lambda_1). \tag{3.7}$$

When differentiating with respect to $\Phi_2^\dagger$ and $\Phi_3^\dagger$, the minimum conditions are trivially satisfied because of $v_2 = v_3 = 0$.

3.2 Mass Matrices

The mass matrix of the charged Higgs bosons $\mathcal{M}_C$ is given by

$$(\mathcal{M}_C)_{i,j} = \frac{\partial^2 V}{\partial w_i^- \partial w_j^+} \, , \qquad i,j \in \{1,2,3\} \, . \tag{3.8}$$

Choosing the (w_1^+, w_2^+, w_3^+) ordering in the rows and (w_1^-, w_2^-, w_3^-) in the columns of $\mathcal{M}_C$, respectively, i.e.

$$\mathcal{M}_C = \begin{array}{c} \\ w_1^- \\ w_2^- \\ w_3^- \end{array} \begin{array}{c} \begin{array}{ccc} w_1^+ & w_2^+ & w_3^+ \end{array} \\ \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \end{array} \, , \tag{3.9}$$

and using the minimum condition given in eqn. (3.7) one obtains:

$$\mathcal{M}_C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{3}{2}v^2\lambda_1 & 0 \\ 0 & 0 & -\frac{3}{2}v^2\lambda_1 \end{pmatrix} \, . \tag{3.10}$$

The mass matrices of the scalar $\mathcal{M}_S$ and of the pseudo-scalar $\mathcal{M}_{PS}$ Higgs bosons can be obtained by similar calculations:

$$(\mathcal{M}_S)_{i,j} = \frac{\partial^2 V}{\partial h_i \partial h_j} \, , \qquad i,j \in \{1,2,3\} \tag{3.11}$$

and

$$(\mathcal{M}_{PS})_{i,j} = \frac{\partial^2 V}{\partial a_i \partial a_j} \, , \qquad i,j \in \{1,2,3\} \, . \tag{3.12}$$

Here, no mixing takes place between the scalar and the pseudo-scalar Higgs bosons. This, however, will not be the case for the scenarios C and D, investigated in chapters 5 and 6, respectively.

Choosing the ordering of the matrix elements in rows and columns as (h_1, h_2, h_3) and (a_1, a_2, a_3) for the scalar and for the pseudo-scalar Higgs mass matrices, respectively, one obtains:

$$\mathcal{M}_S = \begin{pmatrix} 2v^2(\lambda_0 + \lambda_1) & 0 & 0 \\ 0 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2) & \frac{1}{2}v^2\lambda_3 \\ 0 & \frac{1}{2}v^2\lambda_3 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2) \end{pmatrix} \tag{3.13}$$

and

$$\mathcal{M}_{PS} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2) & -\frac{1}{2}v^2\lambda_3 \\ 0 & -\frac{1}{2}v^2\lambda_3 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2) \end{pmatrix} \, . \tag{3.14}$$

The zeros in the $(0,0)$ entries in eqns. (3.10) and (3.14) correspond to the massless charged and neutral Goldstone bosons.

3.3 Diagonalization of the Mass Matrices

The charged Higgs mass matrix is already in a diagonal form, hence we can easily read off the mass squares:

$$M^2_{H_2^\pm} = M^2_{H_3^\pm} = -\frac{2}{3}v^2\lambda_1 \ . \tag{3.15}$$

Here, $H_i^\pm$ for $i \in \{1,2,3\}$ denote the physical mass eigenstates. Note, that $H_1^\pm$ is identified with the massless charged Goldstone boson, which gives mass to the charged $W^\pm$ boson through the Higgs mechanism.

The general expression for the rotation matrix that diagonalizes the scalar mass matrix is

$$\mathcal{R}_H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_H) & \sin(\alpha_H) \\ 0 & -\sin(\alpha_H) & \cos(\alpha_H) \end{pmatrix} \ . \tag{3.16}$$

With this rotation matrix we change from the non-physical fields into the mass basis given by the physical fields H_i, $i \in \{1,2,3\}$:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \mathcal{R}_H \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \ . \tag{3.17}$$

Hence it holds:

$$\mathcal{M}_H = \mathcal{R}_H \mathcal{M}_S \mathcal{R}_H^T = \text{diag}\left(M^2_{H_1}, M^2_{H_2}, M^2_{H_3}\right) \ . \tag{3.18}$$

The large symmetry of the $\Delta(54)$ family of symmetry groups, however, allows for a parameter independent diagonalization of the mass matrices.

Thus the following form of the rotation matrix,

$$\mathcal{R}_H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \tag{3.19}$$

diagonalizes the scalar mass matrix, and we obtain the diagonal mass matrix

$$\mathcal{M}_H = \begin{pmatrix} 2v^2(\lambda_0 + \lambda_1) & 0 & 0 \\ 0 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2 + \lambda_3) & 0 \\ 0 & 0 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2 - \lambda_3) \end{pmatrix} \ . \tag{3.20}$$

Similarly for the pseudo-scalar Higgs bosons, the rotation matrix is

$$\mathcal{R}_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} , \tag{3.21}$$

defined by

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \mathcal{R}_A \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} , \tag{3.22}$$

where A_i, $i \in \{1, 2, 3\}$, represent the pseudo-scalar Higgs bosons in the mass basis. $\mathcal{R}_A$ diagonalizes the pseudo-scalar mass matrix, and we obtain

$$\mathcal{M}_A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2 + \lambda_3) & 0 \\ 0 & 0 & \frac{1}{2}v^2(-3\lambda_1 + \lambda_2 - \lambda_3) \end{pmatrix} . \tag{3.23}$$

We identify the $(0, 0)$ entry of the $\mathcal{M}_A$ mass matrix with the one of the massless neutral Goldstone boson. Hence, A_1 corresponds to the neutral Goldstone boson G^0 that is absorbed to give mass to the massive Z boson through the Higgs mechanism.

It is interesting to note, that the masses of the scalar Higgs boson H_2 and the pseudo-scalar A_2 are the same, as well as the masses of H_3 and A_3 are the same. The reason for this is again the high symmetry of the $\Delta(54)$ family of symmetry groups.

3.4 Couplings to Gauge Bosons

In order to calculate the Higgs couplings to the gauge bosons, we first calculate the kinetic part of the Higgs Lagrange density,

$$\mathcal{L}_{\text{Higgs}} = \sum_{i=1}^{3} \left(\mathrm{D}_\mu \Phi_i\right)^\dagger \left(\mathrm{D}^\mu \Phi_i\right) . \tag{3.24}$$

Here, D_μ is the covariant derivative:

$$\mathrm{D}_\mu = \partial_\mu + \frac{ig}{2}\vec{\sigma}\vec{W}_\mu + \frac{ig'}{2}B_\mu , \tag{3.25}$$

where $\vec{\sigma}$ is the Pauli vector defined by

$$\vec{\sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} \tag{3.26}$$

with the Pauli matrices σ_i ($i \in \{1, 2, 3\}$), and $\vec{W}_\mu$ is defined as

$$\vec{W}_\mu = \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \end{pmatrix} . \tag{3.27}$$

Introducing the charged $W^\pm$ boson defined as

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \ , \tag{3.28}$$

the covariant derivative can be simplified further to

$$\mathrm{D}_\mu = \begin{pmatrix} \partial_\mu + \frac{ig}{2}W_\mu^3 + \frac{ig'}{2}B_\mu & \frac{ig}{\sqrt{2}}W^+ \\ \frac{ig}{\sqrt{2}}W^- & \partial_\mu - \frac{ig}{2}W_\mu^3 + \frac{ig'}{2}B_\mu \end{pmatrix} \ , \tag{3.29}$$

with the rotations into the photon A_μ and the neutral Z boson

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3 \ , \tag{3.30}$$
$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3 \ , \tag{3.31}$$

or, rearranging,

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \ , \tag{3.32}$$
$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \ , \tag{3.33}$$

where θ_W is the Weinberg angle.

Considering the terms of $\mathcal{L}_{\mathrm{Higgs}}$ trilinear in the fields, assigning an additional factor i to each term, carrying out the substitution $\partial^\mu X \to -ik_X^\mu X$ for the partial derivative acting on the field X, where k_X^μ is the momentum of the corresponding particle X, and multiplying with an extra symmetry factor of 2 for two identical particles, we obtain the Feynman rules for the Higgs couplings to the gauge bosons [22].

These are listed in the appendix 8.1. The couplings are given with all momenta taken as in-going.

H_1 is the SM-like Higgs here, so it possesses the entire coupling to the gauge bosons, while the couplings of H_2 and H_3 to the gauge bosons vanish in order to satisfy the sum rule for the gauge boson couplings, which arises from the requirement that unitarity has to be fulfilled [7],

$$\sum_{i=1}^{3} g_{H_iVV}^2 = g_{HVV}^2 \ , \tag{3.34}$$

where g_{H_iVV} denote the Higgs couplings to the massive gauge bosons $V = Z, W^\pm$ and g_{HVV} the SM coupling. This then forbids the Higgs bosons other than H_1 to couple to the gauge bosons. Furthermore, the SM-like Higgs H_1 does not have any non-SM-like couplings like, for example, $ZA_{2,3}H_1$ or $W^\pm H_{2,3}^\mp H_1$.

Note, in particular, that there are also couplings between one gauge boson and two different Higgs bosons (other than H_1). This leads to interesting Higgs decays into a pair consisting of a Higgs boson and a gauge boson, which is not possible in the SM. Such decays can be exploited as alternative Higgs discovery channels, provided the corresponding branching ratios are large enough.

Concerning the Higgs couplings to fermions, the following can be said: As only the H_1 has a non-zero vev, in order to give masses to the fermions through the Higgs mechanism, all fermions have to couple to H_1. Additionally, we require them to couple only to H_1 in order to avoid Flavor Changing Neutral Currents (FCNCs) at tree level [12]. Therefore, the couplings of H_1 to fermions are exactly the couplings of the SM-like Higgs to fermions, and there are no further Higgs to fermion couplings in this specific case.

Here and in the following three chapters the trilinear and quartic Higgs self-couplings were not calculated, as this goes beyond the scope of the current thesis. Such decays are interesting, however, as they allow for the determination of the Higgs self-couplings which can then be used to reconstruct the Higgs potential [23–25]. Furthermore, Higgs-to-Higgs decays can be exploited as alternative discovery channels for the heavier Higgs bosons. Such investigations are left for future work.

3.5 Dark Matter Candidates

The lightest neutral field from the second or third doublet, stabilized by the remaining $\mathbb{Z}_3$ symmetry, is a possible DM candidate [18]. This could be a scalar or a pseudo-scalar Higgs, since the corresponding masses are the same, i.e.

$$M_{H_2}^2 = M_{A_2}^2 = \frac{1}{2}v^2(-3\lambda_1 + \lambda_2 + \lambda_3) \,, \tag{3.35}$$

$$M_{H_3}^2 = M_{A_3}^2 = \frac{1}{2}v^2(-3\lambda_1 + \lambda_2 - \lambda_3) \,. \tag{3.36}$$

Whether the DM candidate is from the second or the third doublet depends on the actual mass value. It has to be the lighter of the two, as it then cannot decay into a pair of lighter Higgs bosons.

4 Case B: (1,1,1)

The parametrization of the doublets is the same as in case A and is given by eqn. (3.2). However, for the vevs it now holds

$$v_1 = v_2 = v_3 =: v' = \frac{1}{\sqrt{3}} v \ . \tag{4.1}$$

They satisfy the sum rule

$$\sum_{i=1}^{3} v_i^2 = v^2 \ , \tag{4.2}$$

which is required by phenomenology.

4.1 Minimum Conditions

Similarly to case A, described in chapter 3, using eqn. (3.6), we obtain

$$m^2 = \frac{1}{3} v^2 (3\lambda_0 + \lambda_2 + 2\lambda_3) \tag{4.3}$$

as the minimum condition, since differentiating with respect to $\Phi_1^\dagger$, $\Phi_2^\dagger$ or $\Phi_3^\dagger$ yields the same equation, because of $v_1 = v_2 = v_3$.

4.2 Mass Matrices

The mass matrices $\mathcal{M}_C$, $\mathcal{M}_S$ and $\mathcal{M}_{PS}$, can again be derived by using eqns. (3.8), (3.11) and (3.12). Thus one obtains for $i, j \in \{1, 2, 3\}$:

$$(\mathcal{M}_C)_{i,j} = -\frac{1}{3} v^2 (\lambda_2 + 2\lambda_3) \ , \qquad i = j \ , \tag{4.4}$$

$$(\mathcal{M}_C)_{i,j} = \frac{1}{6} v^2 (\lambda_2 + 2\lambda_3) \ , \qquad i \neq j \ ; \tag{4.5}$$

$$(\mathcal{M}_S)_{i,j} = \frac{1}{3} v^2 (2\lambda_0 + 2\lambda_1 - \lambda_3) \ , \qquad i = j \ , \tag{4.6}$$

$$(\mathcal{M}_S)_{i,j} = \frac{1}{6} v^2 (4\lambda_0 - 2\lambda_1 + 2\lambda_2 + 5\lambda_3) \ , \qquad i \neq j \ ; \tag{4.7}$$

$$(\mathcal{M}_{PS})_{i,j} = -v^2\lambda_3 \ , \qquad i = j \ , \tag{4.8}$$

$$(\mathcal{M}_{PS})_{i,j} = \frac{1}{2}v^2\lambda_3 \ , \qquad i \neq j \ . \tag{4.9}$$

4.3 Diagonalization of the Mass Matrices

The rotation matrix for the charged Higgs sector is chosen as

$$\mathcal{R}_{H^\pm} = \begin{pmatrix} c_1c_2 & s_1c_2 & s_2 \\ -c_1s_2s_3 - s_1c_3 & c_1c_3 - s_1s_2s_3 & c_2s_3 \\ -c_1s_2c_3 + s_1s_3 & -c_1s_3 - s_1s_2c_3 & c_2c_3 \end{pmatrix} \ , \tag{4.10}$$

with

$$s_i := \sin\alpha_i^{H^\pm} \ , \qquad c_i := \cos\alpha_i^{H^\pm} \ , \tag{4.11}$$

where

$$\alpha_i^{H^\pm} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \ , \qquad i \in \{1,2,3\} \ . \tag{4.12}$$

Again, due to the large symmetry of the $\Delta(54)$ family of symmetry groups, a parameter-free diagonalization is possible, and the rotation matrix for the charged Higgs sector becomes

$$\mathcal{R}_{H^\pm} = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{6} & \sqrt{2/3} & -1/\sqrt{6} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \ . \tag{4.13}$$

Hence we obtain

$$M^2_{H^\pm_{2,3}} = -\frac{1}{2}v^2(\lambda_2 + 2\lambda_3) \ . \tag{4.14}$$

The scalar Higgs sector is diagonalized by the same rotation matrix as in the case of the charged Higgs sector,

$$\mathcal{R}_{H} = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{6} & \sqrt{2/3} & -1/\sqrt{6} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \ . \tag{4.15}$$

We thus obtain

$$M^2_{H_1} = \frac{2}{3}v^2(3\lambda_0 + \lambda_2 + 2\lambda_3) \ , \tag{4.16}$$

$$M^2_{H_{2,3}} = \frac{1}{6}v^2(6\lambda_1 - 2\lambda_2 - 7\lambda_3) \ . \tag{4.17}$$

The pseudo-scalar Higgs sector is also diagonalized by the same rotation matrix,

$$\mathcal{R}_{A} = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{6} & \sqrt{2/3} & -1/\sqrt{6} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \tag{4.18}$$

to obtain

$$M^2_{A_{2,3}} = -\frac{3}{2}v^2\lambda_3 \ . \tag{4.19}$$

Note, that $H_1^\pm$ and A_1 are identified with the massless Goldstone bosons $G^\pm$ and G^0, which are absorbed to give masses to the massive $W^\pm$ and Z bosons, respectively, through the Higgs mechanism.

4.4 Couplings to Gauge Bosons

We have derived the couplings to the gauge bosons in the same way as described in section 3.4. They are given in the appendix 8.2. The couplings are given with all momenta taken as in-going.

Just as in case A, chapter 3.4, H_1 is again the SM-like Higgs here, having the entire coupling to the gauge bosons, while the couplings of H_2 and H_3 to the gauge bosons must vanish. Again, the SM-like Higgs H_1 does not have any non-SM-like couplings like, for example, $ZA_{2,3}H_1$ or $W^\pm H^\mp_{2,3}H_1$. Note, that, in contrast to the Higgs couplings to the gauge bosons in case A, there are no couplings of the form $W^\pm A_{2/3}H^\mp_{3/2}$ or $W^\pm H_{2/3}H^\mp_{3/2}$, which leads to a fewer number of possible decays for the corresponding Higgs bosons.

4.5 Couplings to Fermions

So far there have been no investigations on the Higgs couplings to fermions for the $\Delta(54)$ symmetry group. This investigation is also beyond the scope of the current thesis. Therefore, here we break the $\Delta(54)$ symmetry group down to a $\mathbb{Z}_2$ symmetry group (c.f. fig. 2.1), and investigate only the types I, II, X and Y of the model. These types are defined analogously to the 2HDM, to avoid FCNCs at tree level [12]. Here, types I and II are the corresponding types I and II in the 2HDM, respectively. Types X and Y correspond to the Lepton Specific and Flipped types in the 2HDM, respectively. The four independent types of Yukawa interactions are listed in table 4.1. Here, the doublets coupling to up-type quarks, down-type quarks and leptons are given by Φ_u, Φ_d and Φ_l, respectively. Dependent on the type of the model these become Φ_1, Φ_2 or Φ_3. Note that here we consider a $\mathbb{Z}_2$ symmetry and the types I, II, X and Y, whereas in [16], from where table 4.1 is quoted, a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry and types I, II, X, Y and Z are investigated. The coupling of the third doublet Φ_3 to fermions appears in type Z in [16].

Doublets	Φ_u	Φ_d	Φ_l
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Type X	Φ_2	Φ_2	Φ_1
Type Y	Φ_2	Φ_1	Φ_2

Table 4.1: Four independent types of Yukawa interactions [16].

We follow here [16], where the Higgs couplings to fermions in the 3HDM for this symmetry group are given. To this end, we define the Higgs basis (Φ, Ψ_1, Ψ_2) in the 3HDM, where one of the three doublets contains the vev v, and the charged and pseudo-scalar Goldstone bosons. This can be done with an orthogonal 3×3 matrix R:

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} = R \begin{pmatrix} \Phi \\ \Psi_1 \\ \Psi_2 \end{pmatrix} , \tag{4.20}$$

so that we have

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \tilde{h} + iG^0) \end{pmatrix} , \qquad \Psi_j = \begin{pmatrix} \tilde{H}_j^+ \\ \frac{1}{\sqrt{2}}(\tilde{H}_j + i\tilde{A}_j) \end{pmatrix} , \qquad j \in \{2, 3\} , \tag{4.21}$$

where $G^{\pm}$ and G^0 are the charged and neutral Goldstone bosons, respectively.

Here, the rotation matrix R is expressed in terms of the vevs v_1, v_2, v_3 in the following way:

$$R = \begin{pmatrix} \frac{v_1}{v} & -\frac{v_2 v_1}{v_{13} v} & -\frac{v_3}{v_{13}} \\ \frac{v_2}{v} & \frac{v_{13}}{v} & 0 \\ \frac{v_3}{v} & -\frac{v_2 v_3}{v_{13} v} & \frac{v_1}{v_{13}} \end{pmatrix} = \begin{pmatrix} \cos\beta\cos\gamma & -\sin\beta\cos\gamma & -\sin\gamma \\ \sin\beta & \cos\beta & 0 \\ \cos\beta\sin\gamma & -\sin\beta\sin\gamma & \cos\gamma \end{pmatrix} , \tag{4.22}$$

where

$$v_{13} = \sqrt{v_1^2 + v_3^2} , \tag{4.23}$$

and v_1,v_2,v_3 are the vevs in eqn. (4.1).

Expressing the vevs v_i $(i \in \{1, 2, 3\})$ through v and the mixing angles β and γ,

$$v_1 = v \cos\beta \cos\gamma , \tag{4.24}$$

$$v_2 = v \sin\beta , \tag{4.25}$$

$$v_3 = v \cos\beta \sin\gamma , \tag{4.26}$$

we can calculate the rotation angles β and γ using eqn. (4.1) and obtain

$$\beta = \text{arccot}\sqrt{2} , \tag{4.27}$$

$$\gamma = \pi/4 , \tag{4.28}$$

if we restrict ourselves to β, $\gamma \in [-\pi/2, \pi/2]$. Here, again, because of the high symmetry group, the Higgs basis can be defined independently of the parameters, and the rotation angles β and γ adopt discrete values.

Hence R becomes:

$$R = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & \sqrt{2/3} & 0 \\ 1/\sqrt{3} & -1/\sqrt{6} & 1/\sqrt{2} \end{pmatrix} . \tag{4.29}$$

As it turns out, the two charged, the three CP-even and the two CP-odd states, calculated in the Higgs basis, are already the mass eigenstates. Hence we have the following relations between our original mass basis and the Higgs basis:

$$\begin{pmatrix} G^\pm \\ \tilde{H}_2^\pm \\ \tilde{H}_3^\pm \end{pmatrix} = \begin{pmatrix} G^\pm \\ H_2^\pm \\ H_3^\pm \end{pmatrix} , \qquad \begin{pmatrix} \tilde{h} \\ \tilde{H}_2 \\ \tilde{H}_3 \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} , \qquad \begin{pmatrix} G^0 \\ \tilde{A}_2 \\ \tilde{A}_3 \end{pmatrix} = \begin{pmatrix} G^0 \\ A_2 \\ A_3 \end{pmatrix} . \tag{4.30}$$

The Feynman rule for the coupling of the scalar Higgs bosons H_i $(i \in \{1,2,3\})$ to fermions $(f = u, d, l)$ is given by

$$\lambda_{H_i ff} = -i\frac{m_f}{v} g_{H_i ff} \ , \tag{4.31}$$

where m_f denotes the mass of the fermion. The Feynman rule for the coupling of pseudo-scalar Higgs bosons A_i $(i \in \{2,3\})$ to fermions reads

$$\lambda_{A_i ff} = -\frac{m_f}{v} g_{A_i ff} \gamma_5 \tag{4.32}$$

with the Dirac matrix γ_5 given by

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \ . \tag{4.33}$$

The Feynman rule for the coupling of charged Higgs bosons $H_i^\pm$ $(i \in \{2,3\})$ to a lepton and a neutrino is given by

$$\lambda_{H_i^\pm l\nu} = i\frac{1}{\sqrt{2}v} m_l g_{H_i^\pm ll} \left(\mathbb{1}_4 \pm \gamma_5\right) \ , \tag{4.34}$$

and the Feynman rule for the coupling of the charged Higgs bosons to a pair consisting of an up- and a down-type fermion is given by

$$\lambda_{H_i^\pm ud} = i\frac{1}{\sqrt{2}v}\left[\left(m_d g_{H_i^\pm dd} + m_u g_{H_i^\pm uu}\right) \pm \left(m_d g_{H_i^\pm dd} - m_u g_{H_i^\pm uu}\right)\gamma_5\right] \ . \tag{4.35}$$

The coupling factors $g_{H_i ff}$, $g_{A_i ff}$ and $g_{H_i^\pm ff}$ $(i \in \{2,3\})$ are given in table 4.2. Note, that H_1, respectively, $\tilde{h}$, corresponds to the SM Higgs boson with the coupling factors $g_{H_1 ff} = 1$ for all fermions. The bosons $H_1^\pm$ and A_1 are identified with the charged and neutral Goldstone bosons.

Analogous to [16], the table of factors for the Yukawa interactions is the following.

Factors for $X_{2,3}$	$H_2,\ A_2,\ H_2^\pm$			$H_3,\ A_3,\ H_3^\pm$		
Factors	g_{X_2uu}	g_{X_2dd}	g_{X_2ll}	g_{X_3uu}	g_{X_3dd}	g_{X_3ll}
Type I	$\pm\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	0	0
Type II	$\pm\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	$-\sqrt{3/2}$	$-\sqrt{3/2}$
Type X	$\pm\sqrt{2}$	$\sqrt{2}$	$-1/\sqrt{2}$	0	0	$-\sqrt{3/2}$
Type Y	$\pm\sqrt{2}$	$-1/\sqrt{2}$	$\sqrt{2}$	0	$-\sqrt{3/2}$	0

Table 4.2: The coupling factors for the different 3HDM types. The "+" in g_{X_2uu} applies to the neutral Higgs couplings, the "−" applies to the charged Higgs couplings.

4.6 Partial Decay Widths

4.6.1 Scalar and Pseudo-Scalar Higgs Decays into Gauge Bosons and Fermions

The following partial widths were derived from [7].

Since only the scalar Higgs boson H_1, which corresponds to the SM-like Higgs boson, couples to the $W^\pm$ and Z gauge bosons (see appendix 8.2), the partial widths for the decays of the scalar Higgs bosons into massive gauge bosons are given by

$$\Gamma(H_1 \to W^+W^-) = \frac{G_F m_{H_1}^3}{8\pi\sqrt{2}} \left(1 - \frac{4m_W^2}{m_{H_1}^2}\right)^{1/2} \left(1 - \frac{4m_W^2}{m_{H_1}^2} + \frac{12m_W^4}{m_{H_1}^4}\right) \,, \tag{4.36}$$

$$\Gamma(H_1 \to ZZ) = \frac{G_F m_{H_1}^3}{16\pi\sqrt{2}} \left(1 - \frac{4m_Z^2}{m_{H_1}^2}\right)^{1/2} \left(1 - \frac{4m_Z^2}{m_{H_1}^2} + \frac{12m_Z^4}{m_{H_1}^4}\right) \,. \tag{4.37}$$

Here $G_F = 1.166\,378\,7 \cdot 10^{-5}$ GeV^{-2} denotes the Fermi constant [20], and m_W and m_Z the gauge boson masses.

The partial widths for the decays of the charged Higgs boson into a pair consisting of a $W^\pm$ gauge boson and a scalar or pseudo-scalar Higgs boson are given by

$$\begin{aligned} \Gamma(H_j^\pm \to W^\pm H_j) = {} & \frac{G_F}{8\pi\sqrt{2}} \lambda(m_{H_j^\pm}, m_W, m_{H_j}) \frac{m_W^2}{m_{H_j^\pm}^3} \times \\ & \times \left(m_W^2 - 2(m_{H_j^\pm}^2 + m_{H_j}^2) + \frac{(m_{H_j^\pm}^2 - m_{H_j}^2)^2}{m_W^2} \right) \,, \qquad j \in \{2,3\} \,, \end{aligned} \tag{4.38}$$

$$\Gamma(H_j^\pm \to W^\pm A_j) = \frac{G_F}{8\pi\sqrt{2}} \lambda(m_{H_j^\pm}, m_W, m_{A_j}) \frac{m_W^2}{m_{H_j^\pm}^3} \times \\ \times \left(m_W^2 - 2(m_{H_j^\pm}^2 + m_{A_j}^2) + \frac{(m_{H_j^\pm}^2 - m_{A_j}^2)^2}{m_W^2} \right) , \qquad j \in \{2,3\} \, , \tag{4.39}$$

where

$$\lambda(x,y,z) = \left[(x^2+y^2-z^2)^2 - 4x^2y^2\right]^{1/2} \, . \tag{4.40}$$

The partial widths for the decays of the pseudo-scalar Higgs boson into a Higgs boson and the Z boson are given by

$$\Gamma(A_j \to Z H_j) = \frac{G_F}{8\pi\sqrt{2}} \lambda(m_{A_j}, m_Z, m_{H_j}) \frac{m_Z^2}{m_{A_j}^3} \times \\ \times \left(m_Z^2 - 2(m_{A_j}^2 + m_{H_j}^2) + \frac{(m_{A_j}^2 - m_{H_j}^2)^2}{m_Z^2} \right) , \qquad j \in \{2,3\} \, . \tag{4.41}$$

Note that the pseudo-scalar Higgs bosons cannot decay into a pair of massive gauge bosons.

For the partial decay widths of the scalar Higgs bosons H_i ($i \in \{1,2,3\}$) into a pair of fermions, we have [7]

$$\Gamma(H_i \to f\bar{f}) = \frac{N_C G_F}{4\pi\sqrt{2}} m_{H_i} \bar{m}_f^2 K_i^2 \left(1 - \frac{4(m_f^{pole})^2}{m_{H_i}^2}\right)^{3/2} , \tag{4.42}$$

where for $i \in \{2,3\}$ we have

$$K_i = \begin{cases} -g_{X_i uu} \, , & \text{up-type quarks} \\ g_{X_i dd} \, , & \text{down-type quarks} \\ g_{X_i ll} \, , & \text{leptons} \end{cases} , \tag{4.43}$$

with $g_{X_i uu}$, $g_{X_i dd}$ and $g_{X_i ll}$ given in table 4.2. For the SM-like Higgs H_1, as we have normalized with respect to the SM coupling, we have instead

$$K_1 = 1 \, . \tag{4.44}$$

N_C is the colour factor, given by

$$N_C = \begin{cases} 3 \, , & \text{quarks} \\ 1 \, , & \text{leptons} \end{cases} . \tag{4.45}$$

In the above and in the following decay formulae for the Higgs decays into fermions, we distinguish between the pole masses of the fermions m_f^{pole}, and the running masses of the fermions $\bar{m}_f$ to account for the QCD corrections, but only for the quarks, since there are no QCD corrections for the leptons. The use of the running $\overline{\text{MS}}$ fermion masses at the scale of 100 GeV in the Higgs couplings to the fermions allows to take into account the bulk of the higher order QCD corrections, see e.g. [26]. The running fermion masses at the scale of 100 GeV have been obtained with the help of [27–30].

Analogously, the decays of the pseudo-scalar Higgs bosons A_i into a pair of fermions can be cast into the form

$$\Gamma(A_i \to f\bar{f}) = \frac{N_C G_F}{4\pi\sqrt{2}} m_{A_i} \bar{m}_f^2 K_i^2 \left(1 - \frac{4(m_f^{pole})^2}{m_{A_i}^2}\right)^{1/2}, \qquad i \in \{2,3\}. \tag{4.46}$$

Note that, as stated above, we neglect Higgs decays into a pair of lighter Higgs bosons, which is beyond the scope of this thesis.

4.6.2 Charged Higgs Boson Decays into Leptons and Quarks

The partial decay widths of the charged Higgs bosons $H_j^\pm$ ($j \in \{2,3\}$) into a lepton and a neutrino or a pair of quarks are [7, 16]

$$\Gamma(H_j^\pm \to \ell^\pm \nu) = \frac{G_F}{4\pi\sqrt{2}} m_{H_j^\pm} m_{\ell^\pm}^2 g_{X_j ll}^2 \left(1 - \frac{m_{\ell^\pm}^2}{m_{H_j^\pm}^2}\right)^2, \tag{4.47}$$

$$\begin{aligned}\Gamma(H_j^\pm \to ud) = {} & \frac{3G_F}{4\pi\sqrt{2}} \left|V_{ud}\right|^2 \lambda(m_{H_j^\pm}, m_u^{pole}, m_d^{pole}) \frac{1}{m_{H_j^\pm}} \times \\ & \times \left[(\bar{m}_d^2 g_{X_j dd}^2 + \bar{m}_u^2 g_{X_j uu}^2)(1 - x_u^j - x_d^j) + 4\bar{m}_u \bar{m}_d \sqrt{x_u^j x_d^j} g_{X_j dd} g_{X_j uu}\right],\end{aligned} \tag{4.48}$$

where V_{ud} denotes the corresponding CKM matrix element [31], and

$$x_u^j = \left(\frac{m_u^{pole}}{m_{H_j^\pm}}\right)^2, \qquad x_d^j = \left(\frac{m_d^{pole}}{m_{H_j^\pm}}\right)^2, \tag{4.49}$$

and $g_{X_j uu}$, $g_{X_j dd}$, $g_{X_j ll}$ are given in table 4.2.

Again, we distinguish between pole masses in the kinematics and the running fermion masses in the Yukawa couplings to account for the bulk of the higher order QCD corrections. The running masses are obtained from [27–30].

4.7 Branching Ratios

The branching ratios for the decays of a Higgs Boson $\mathcal{H}$ into two particles X and Y are given by

$$BR(\mathcal{H} \to XY) = \frac{\Gamma(\mathcal{H} \to XY)}{\sum_{AB} \Gamma(\mathcal{H} \to AB)} \ . \tag{4.50}$$

The sum $\sum_{AB}$ extends over all possible Higgs decays into the final states A and B. As we do not take into account the Higgs decays into a pair of lighter Higgs bosons, we either have to choose in the following numerical analysis the scenarios such that these decays are kinematically forbidden, or we assume, for simplicity, that the involved trilinear Higgs self-couplings are very small, so that these decays are suppressed.

4.8 Numerical Analysis

In the following we will present the branching ratios for our investigated 3HDM scenario. To this end a Python code has been written where all formulae for the decay widths of sub-sections 4.6.1 and 4.6.2 and of section 4.7 have been implemented. The code takes the masses of the charged, scalar and pseudo-scalar Higgs bosons, and the type (I, II, X, or Y) of the model as input. From the implemented decay widths it then computes the various branching ratios. These are represented as plots for the corresponding decays of the Higgs bosons, which we will discuss in the following.

The SM-like Higgs boson mass was set according to [32] as

$$m_{H_1} = 125.09\,\text{GeV} \ . \tag{4.51}$$

For the Higgs masses in the plots below, the chosen input values are

$$m_{H_2} = m_{H_3} = 300.0\,\text{GeV} \ , \tag{4.52}$$

$$m_{H_2^\pm} = m_{H_3^\pm} = 500.0\,\text{GeV} \ , \tag{4.53}$$

$$m_{A_2} = m_{A_3} = 350.0\,\text{GeV} \ . \tag{4.54}$$

Following [20, 33], the SM parameters that we use, are

$$m_W = 80.385\,\text{GeV} \ , \tag{4.55}$$

$$m_Z = 91.1876\,\text{GeV} \ , \tag{4.56}$$

$$m_e = 0.510\,998\,928 \cdot 10^{-3}\ \text{GeV} \ , \tag{4.57}$$

$$m_\mu = 0.105\,658\,371\,5\,\text{GeV} \ , \tag{4.58}$$

$$m_\tau = 1.776\,82\,\text{GeV} \ , \tag{4.59}$$

$$m_c = 1.51\,\text{GeV} \ , \tag{4.60}$$

$$m_b = 4.92\,\text{GeV} \ , \tag{4.61}$$

$$m_t = 172.5\,\text{GeV} \ . \tag{4.62}$$

The following running masses $\bar{m}$ were obtained with HDECAY [27–30] at the scale which, for convenience, we choose for all Higgs decays to be 100 GeV [16]:

$$\bar{m}_c = 0.6277\,\text{GeV} \;, \tag{4.63}$$
$$\bar{m}_b = 2.846\,\text{GeV} \;, \tag{4.64}$$
$$\bar{m}_t = 169.611\,\text{GeV} \;. \tag{4.65}$$

The CKM matrix elements that we need are [20]

$$|V_{cb}| = 41.1 \cdot 10^{-3} \;, \tag{4.66}$$
$$|V_{tb}| = 1.021 \;. \tag{4.67}$$

The Fermi constant is [20]

$$G_F = 1.166\,378\,7 \cdot 10^{-5}\ \text{GeV}^{-2} \tag{4.68}$$

with the relation to the SM vev given by

$$v = \frac{1}{\sqrt{\sqrt{2}G_F}} \;. \tag{4.69}$$

We will now investigate the dependence of the branching ratios on the chosen Higgs boson and type of model. For this we will look at the branching ratios of the decays as a function of the mass of the decaying Higgs boson, which is varied in the range from 50 GeV to 1000 GeV, while all other masses are fixed.

4.8.1 Comparison Between Charged Higgs Bosons

Here we selected some representative plots for the comparison between the behavior of the decays of the two charged Higgs bosons $H_2^\pm$ and $H_3^\pm$ in the same type of the model (type II). In figs. 4.1 and 4.2 we show the branching ratios of $H_2^\pm$ and $H_3^\pm$, respectively, in the model type II as a function of the respective charged Higgs boson mass. All other parameters are kept fixed at the values given at the beginning of section 4.8.

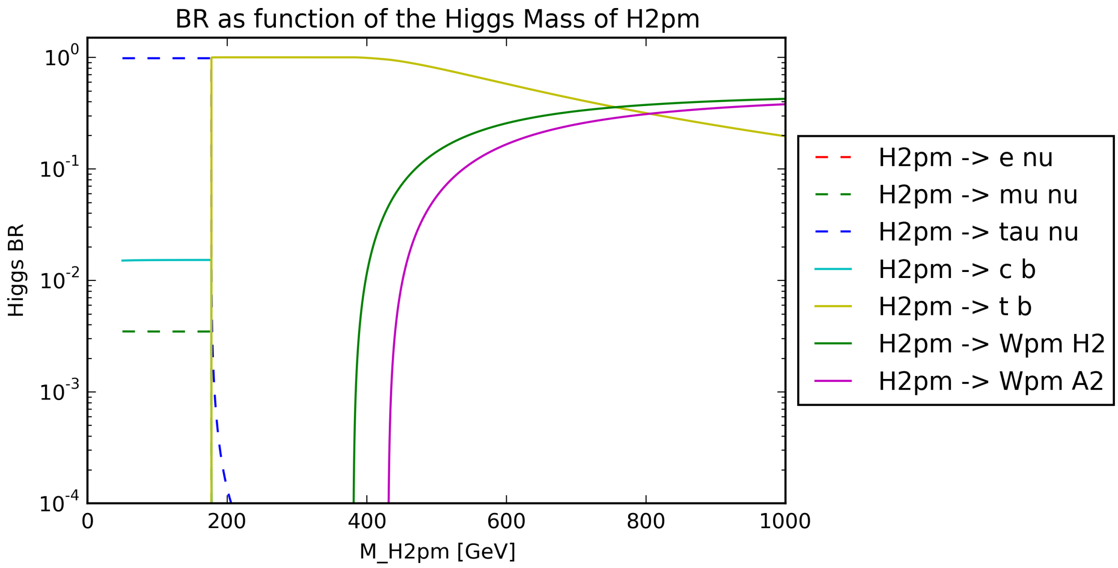

Figure 4.1: $H_2^{\pm}$ Type II plot.

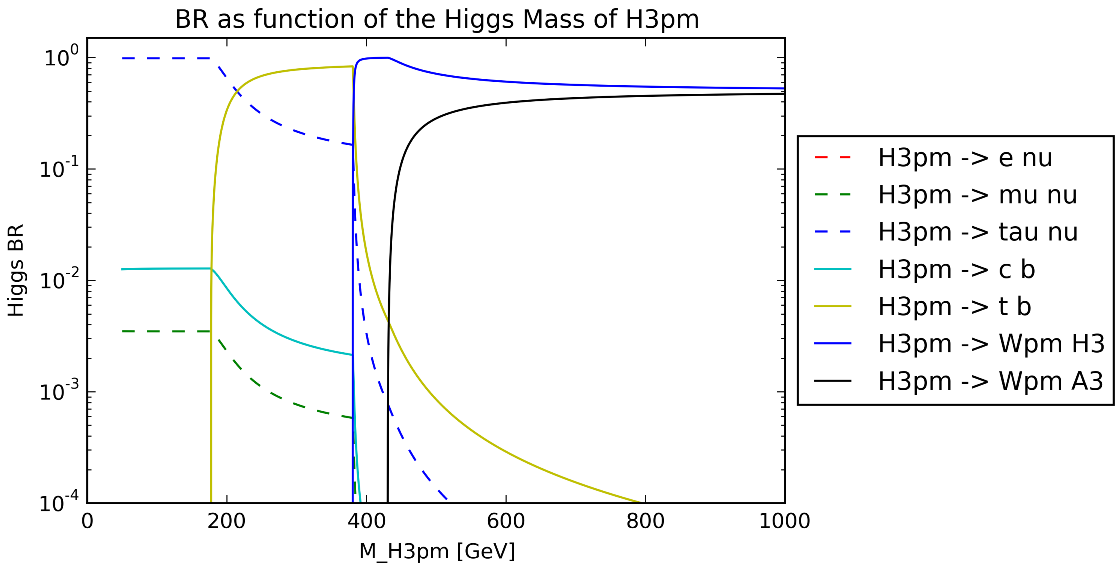

Figure 4.2: $H_3^{\pm}$ Type II plot.

A clear difference between the decays of $H_2^{\pm}$ and $H_3^{\pm}$ can be observed: Firstly, $H_2^{\pm}$ decays into $W^{\pm}H_2/A_2$ while $H_3^{\pm}$ decays into $W^{\pm}H_3/A_3$. This is because there are no couplings of the form $W^{\pm}A_{2/3}H_{3/2}^{\mp}$ or $W^{\pm}H_{2/3}H_{3/2}^{\mp}$. Secondly, the decay into the pair consisting of a top quark and a bottom quark grows much more rapidly and diminishes much more slowly in the case of $H_2^{\pm}$, which is due to the particular combination of the coupling factors g_{X_iuu} and g_{X_idd}, $i \in \{2, 3\}$, for the chosen type of the model (type II) i.e.

$$g_{X_2uu} = -\sqrt{2}\ , \qquad g_{X_2dd} = -1/\sqrt{2}\ , \tag{4.70}$$

$$g_{X_3uu} = 0\ , \qquad g_{X_3dd} = -\sqrt{3/2}\ . \tag{4.71}$$

These coupling factors appear in the partial decay width of the charged Higgs boson in eqn. (4.48). Thirdly, once the charged Higgs decay into a $W^\pm$ and a scalar or a pseudo-scalar Higgs becomes kinematically allowed, it strongly suppresses the branching ratio of the decay into the top and bottom quarks in the case of the decay of $H_3^\pm$ as it can achieve values close to one.

For the low mass range, i.e. below the threshold for the top-bottom channel, the dominant decays are the decays into a tau lepton and a tau-neutrino. Once the decay into a top and a bottom quark becomes kinematically allowed, this becomes the prevalent decay. The $H_2^\pm \to e\nu$ decay is not visible in the figure as its branching ratio lies far below the other branching ratios due to the very small masses of the involved fermions and hence the very small Yukawa couplings. For the decays into fermions, the Higgs mechanism is responsible for the fact that the couplings to heavier particles are larger and thus the corresponding decay is more important.

Once the decays $H_i^\pm \to W^\pm H_i$ and $H_i^\pm \to W^\pm A_i$, $i \in \{2, 3\}$, become kinematically allowed, the branching ratios of these decays become larger, diminishing the branching ratio of the decay into the top and a bottom quark. The decays with a gauge boson and a Higgs boson in the final state can become large and so represent interesting discovery channels.

Note, that the Higgs-to-Higgs decays are not possible here, since the decays of the form $H_i^\pm \to H_j H_k^\pm$ with $i, k \in \{2, 3\}$, $i \neq k$ and $j \in \{1, 2, 3\}$ are kinematically forbidden in this case of the model (case B), because the masses of $H_2^\pm$ and $H_3^\pm$ are equal (see eqn. (4.14)).

4.8.2 Comparison Between Different 3HDM Types

In figs. 4.3 and 4.4 the differences of the 3HDM for type I and type II are investigated for the example of the $H_3^\pm$ decays. Shown are the branching ratios as a function of the mass $m_{H_3^\pm}$ for type I (fig. 4.3) and type II (fig. 4.4). All other parameter values are kept fixed at the values given at the beginning of section 4.8.

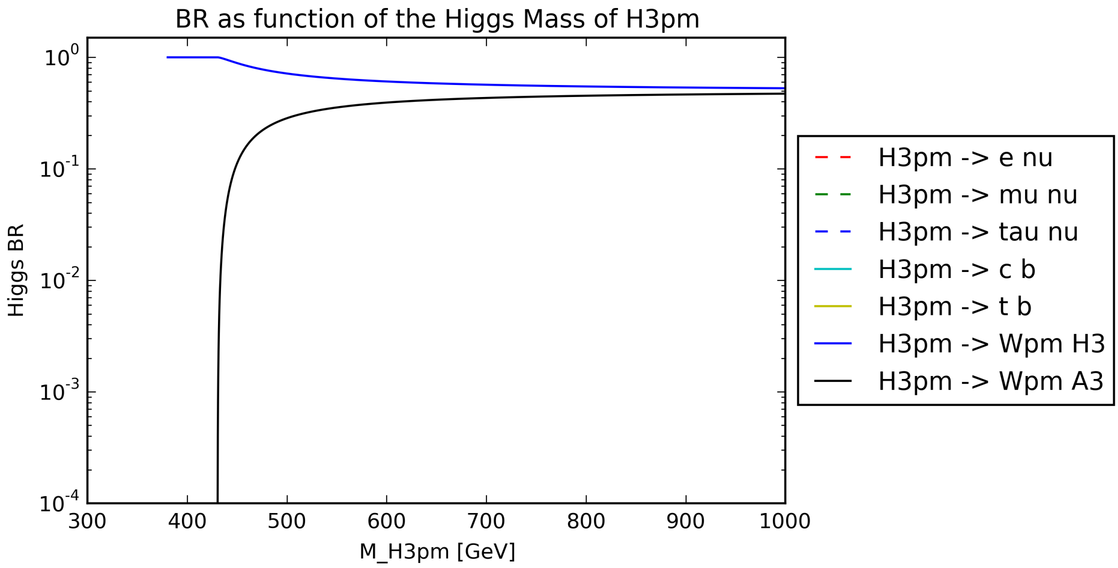

Figure 4.3: $H_3^{\pm}$ Type I plot.

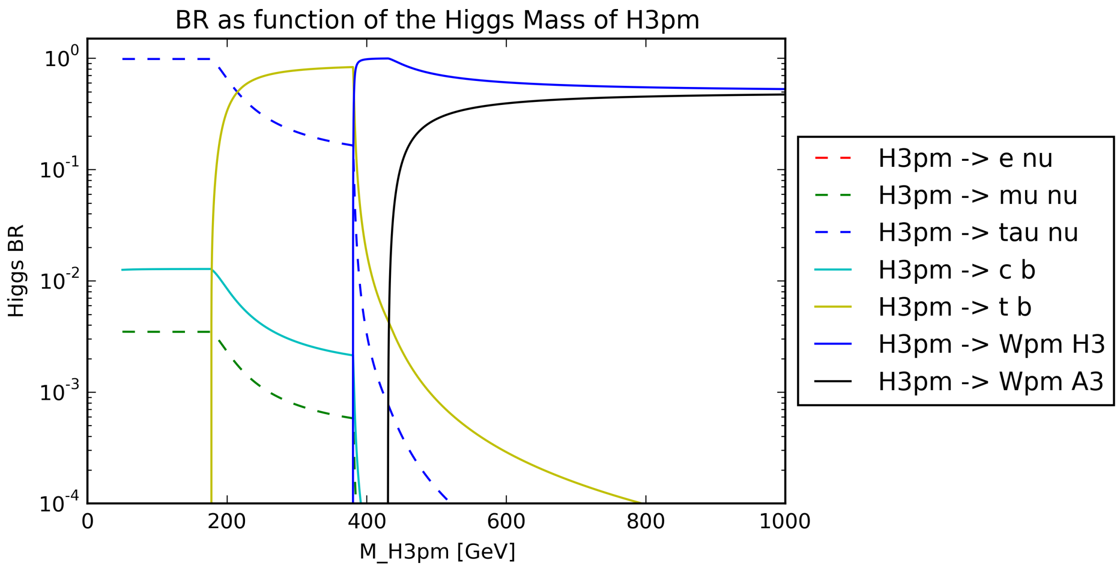

Figure 4.4: $H_3^{\pm}$ Type II plot.

The charged Higgs boson does not decay into quarks or leptons in the case of a type I model, which is clear from the corresponding coupling factors represented in table 4.2. Here again, the $H_3^{\pm} \to e\nu$ branching ratio in type II is strongly suppressed as compared to the other branching ratios.

Again, the decays with a gauge boson and a Higgs boson in the final state are interesting possible discovery channels. Furthermore, it is worthwhile noting that, just as before, for the high mass range the decay of $H_3^{\pm}$ into a top quark and a bottom quark is not the dominant channel, but precisely the decays $H_3^{\pm} \to W^{\pm}H_3$ and $H_3^{\pm} \to W^{\pm}A_3$ prevail. For the decays of $H_3^{\pm}$, the difference in the branching ratios of the $H_3^{\pm} \to W^{\pm}H_3$ and $H_3^{\pm} \to W^{\pm}A_3$ decays arises only from the difference in the masses of H_3 and A_3, i.e., as set at the beginning of section 4.8,

$$m_{H_3} = 300\,\mathrm{GeV}\ , \tag{4.72}$$
$$m_{A_3} = 350\,\mathrm{GeV}\ , \tag{4.73}$$

since the respective couplings are of the same absolute value (see appendix 8.2).

4.8.3 Comparison Between Scalar Higgs Bosons

Here we compare the branching ratios of the scalar Higgs bosons for the 3HDM type II. We show the branching ratios of H_1 (fig. 4.5), H_2 (fig. 4.6) and H_3 (fig. 4.7) as a function of their respective mass, while all other parameters are kept fixed at the values given above.

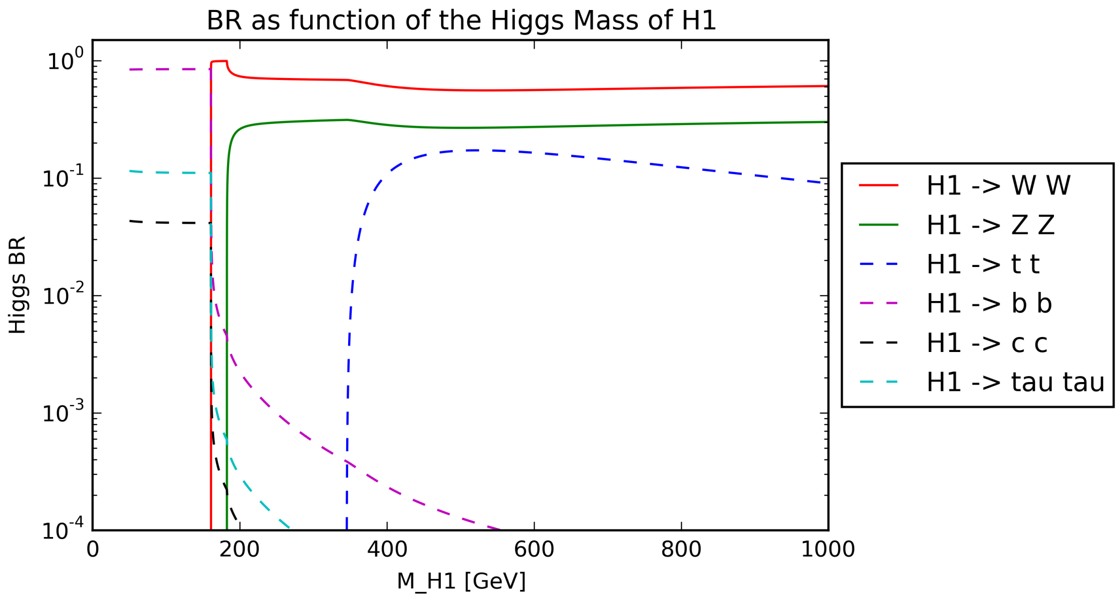

Figure 4.5: H_1 Type II plot.

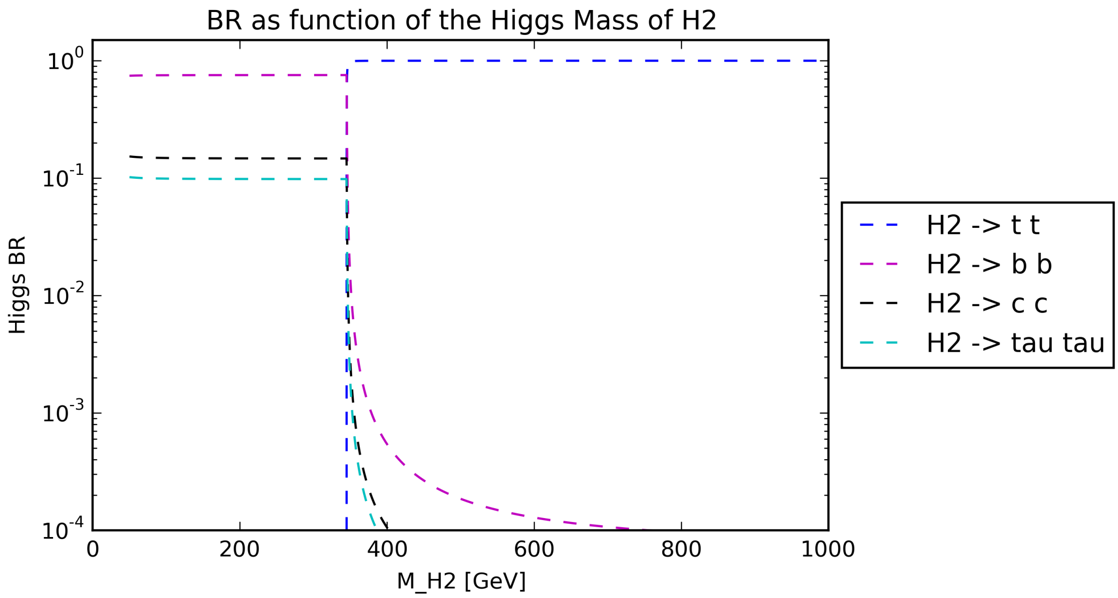

Figure 4.6: H_2 Type II plot.

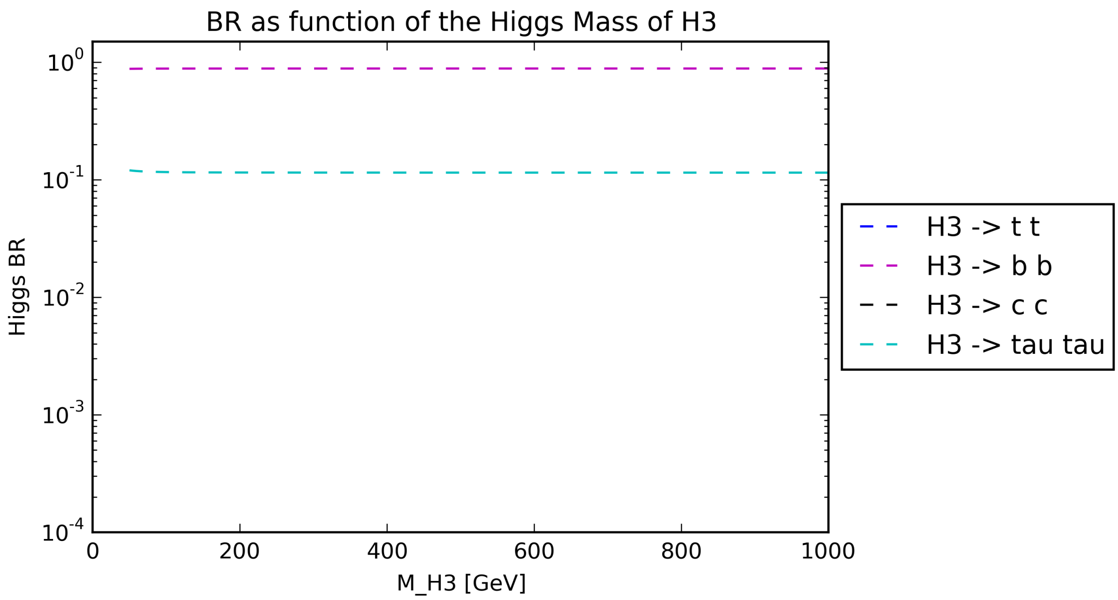

Figure 4.7: H_3 Type II plot.

H_2 and H_3 do not decay into $W^\pm$ or Z bosons, since they do not couple to the massive gauge bosons. This is obvious, as H_1 is the SM-like Higgs boson which couples with the SM coupling to the massive gauge bosons. The sum rule [7], which arises from the requirement that unitarity has to be fulfilled, then forbids the other Higgs bosons to couple to the gauge bosons. The sum rule has been given in eqn. (3.34) and we repeat it here for convenience,

$$\sum_{i=1}^{3} g^2_{H_iVV} = g^2_{HVV} \ . \tag{4.74}$$

Once the H_1 decay into the $W^\pm$ or the Z boson pair becomes kinematically allowed, these also become the dominant branching ratios. Below the respective thresholds the dominant branching ratio of the decay is the one into a pair of bottom quarks, since due to the Higgs mechanism the couplings to heavier particles are larger. For H_1 masses above the top quark threshold the $H_1 \to tt$ also becomes an important channel and can reach up to approximately 20% of the total decay width. The decay into a top quark pair is also the dominant one for the H_2 decays above the corresponding kinematic threshold. The H_3 does not decay into up-type quarks for the type II model (and, in fact, for all types), which can be read off from table 4.2.

For large H_1 masses, i.e. above approximately 350 GeV, the branching ratios of H_1 are exactly those of the SM-like Higgs boson and should accord well with [34], not taking into account higher order QCD and EW corrections, which are not included here. Note, that we do not include off-shell decays.

For small H_1 masses, however, we find differences because, again, we do not include off-shell decays into WW and ZZ, which can be important. Also for small H_1 masses the higher order QCD corrections have not been included. Note also that no $H_1 \to gg$, $H_1 \to \gamma\gamma$, $H_1 \to Z\gamma$ or $H_1 \to \mu\mu$ decays were considered.

Note, that there is no decay into a lighter Higgs boson and a gauge boson, since this is kinematically forbidden for the chosen mass values of the Higgs bosons. The Higgs-to-Higgs decays would in principle be possible in this CP-conserving case of the model (case B), i.e. $H_i \to H_j H_k$ (with $i, j, k \in \{1, 2, 3\}$ and $j, k \neq i$ in this particular decay), and $H_i \to A_j A_k$ and $H_i \to H_j^\pm H_k^\mp$ with $i \in \{1, 2, 3\}$ and $j, k \in \{2, 3\}$, if kinematically allowed, and provided the couplings between the involved Higgs bosons do not vanish because of the high symmetry of the potential. These decays are, however, not taken into account here. Depending on the relative strength of the decay, the branching ratios would change accordingly, as soon as the mass of the decaying Higgs boson exceeds the respective kinematic threshold.

For low mass ranges, i.e. below the mass of $W^\pm$, using eqns. (4.42) and (4.50), the ratio of the two BRs for the decay of an H_i into two pairs of fermions $f_1\bar{f}_1$ and $f_2\bar{f}_2$ follows approximately the relation

$$R_{f_1,f_2} = \frac{K_{i,f_1}^2 \bar{m}_{f_1}^2 N_{C,f_1}}{K_{i,f_2}^2 \bar{m}_{f_2}^2 N_{C,f_2}} , \tag{4.75}$$

where $\bar{m}_f$ is the running mass for quarks, and simply the pole mass for the leptons, since there are no QCD corrections for the leptons. Calculating, for example, $R_{b,\tau}$ for H_1 yields approximately 8. Comparing this value with the ratio of these BRs estimated from fig. 4.5 supports the validity of the above relation. Similar calculations can be done for the branching ratios involving other fermions.

4.8.4 Comparison Between Pseudo-Scalar Higgs Bosons

We now compare the decays of the pseudoscalar Higgs bosons exemplary for type II. The branching ratios of A_2 (fig. 4.8) and A_3 (fig. 4.9) are shown as a function of the respective pseudo-scalar mass, while all parameters are kept fixed at the previously defined values.

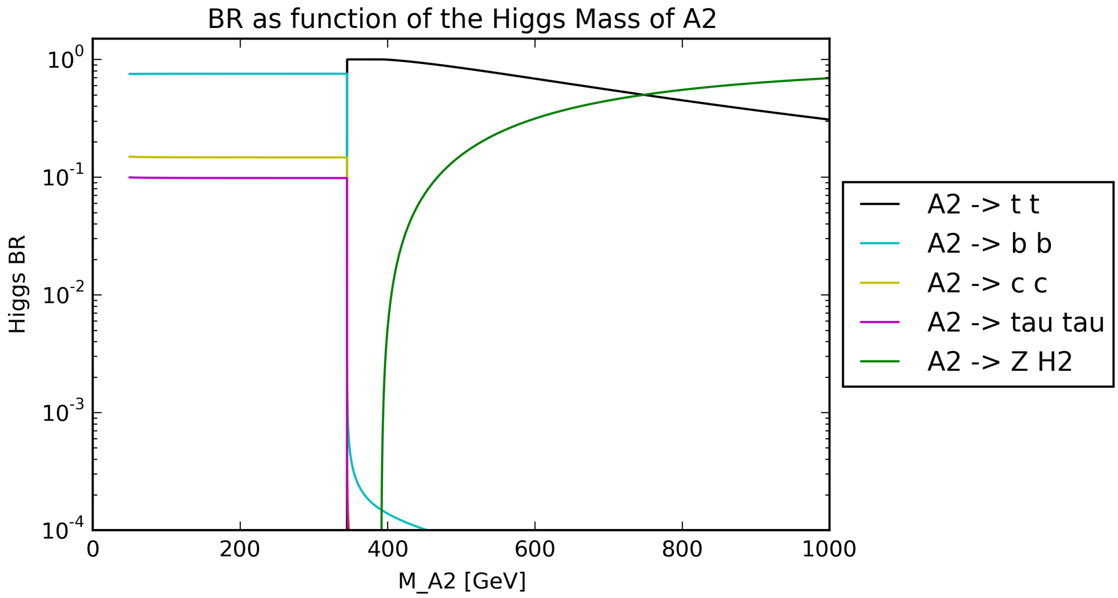

Figure 4.8: A_2 Type II plot.

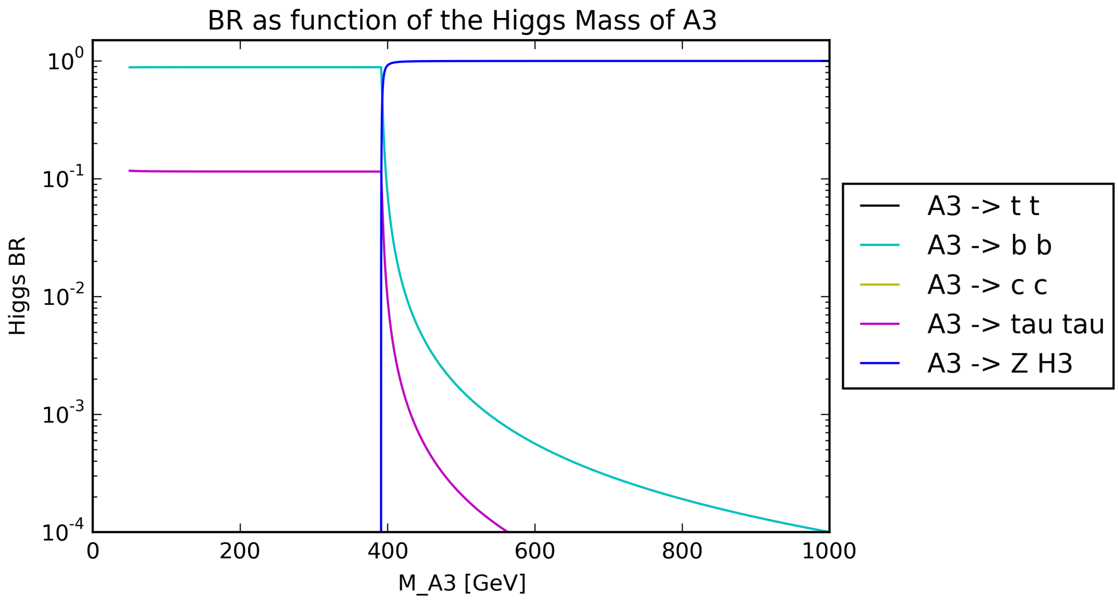

Figure 4.9: A_3 Type II plot.

As is again evident from table 4.2, the pseudo-scalar Higgs Boson A_3 does not decay into up-type quarks. It is interesting to note, that in the decay of A_2 the top pair final state and the ZH_2 channel play the dominant roles for the higher mass region, whereas in the case of A_3 the ZH_3 channel makes up almost the entire branching ratio, once the A_3 mass is large enough for this decay. This is clear, as the A_3 decay into top quarks does not take place due to the vanishing couplings of A_3 to up-type quarks.

Note that there are no decays into massive gauge bosons, as they are forbidden for pseudo-scalar Higgs bosons.

The Higgs-to-Higgs decays, i.e. $A_i \to A_j H_k$ with $i, j \in \{2, 3\}$, $i \neq j$ and $k \in \{1, 2, 3\}$, are kinematically not possible here since the masses of A_2 and A_3 are equal (see eqn. (4.19)).

4.9 Dark Matter Candidates

Since for the current numerical analysis H_3 was chosen lighter than A_3, since it cannot decay in massive gauge bosons because of the sum rule (see eqn. (4.74)), and because it does not decay into quarks or fermions in the type I model (c.f. table 4.2), it is a Dark Matter candidate under the condition that the decays into other Higgs bosons are kinematically forbidden or that its trilinear couplings with other Higgs bosons are zero.

Note, that both H_3 and A_3 do not couple to fermions in the type I model. Furthermore, note that A_3 cannot decay into $A_2 H_i$ ($i \in \{1, 2, 3\}$) because A_2 and A_3 have equal masses, as mentioned above. So, in principle, if one does not restrict the mass values as chosen for the current numerical analysis, the lighter Higgs boson among H_3 and A_3 is then a Dark Matter candidate in a type I model, but again only under the condition mentioned above.

5 Case C: (ω,1,1)

From here on, let $\Re(x)$ and $\Im(x)$ denote the real and complex parts of the variable x, respectively.

The doublets are now parametrized in the following way, since one of the vevs is now complex:

$$\Phi_j = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_j + i\psi_j \\ \Re(v_j) + h_j + i\left[a_j + \Im(v_j)\right] \end{pmatrix} , \qquad j \in \{1,2,3\} . \tag{5.1}$$

The vevs of the corresponding doublets are now

$$v_1 = \omega v' , \qquad v_2 = v_3 = v' , \qquad \text{with} \qquad \omega = \exp(2\pi i/3),\ v' = \frac{1}{\sqrt{3}} v . \tag{5.2}$$

5.1 Minimum Conditions

Similarly as before, we obtain from the minimum conditions

$$m^2 = (3\lambda_0 + \lambda_2 - \lambda_3)(v')^2 . \tag{5.3}$$

5.2 Mass Matrices

The charged and the neutral mass matrices $\mathcal{M}_C$ and $\mathcal{M}_N$ can be obtained by calculations analogous to previous chapters. Both the $\mathcal{M}_C$ and $\mathcal{M}_N$ mass matrices are symmetric in the $(\chi_1, ..., \chi_3, \psi_1, ..., \psi_3)$ and $(h_1, ..., h_3, a_1, ..., a_3)$ bases, respectively, and are given by

$$\begin{aligned}(\mathcal{M}_C)_{1,1} = (\mathcal{M}_C)_{2,2} = (\mathcal{M}_C)_{3,3} = (\mathcal{M}_C)_{4,4} = \\ = (\mathcal{M}_C)_{5,5} = (\mathcal{M}_C)_{6,6} = \frac{1}{3} v^2(-\lambda_2 + \lambda_3)\end{aligned} \tag{5.4}$$

$$(\mathcal{M}_C)_{1,2} = (\mathcal{M}_C)_{1,3} = (\mathcal{M}_C)_{4,5} = (\mathcal{M}_C)_{4,6} = \frac{1}{12} v^2(-\lambda_2 + \lambda_3) \tag{5.5}$$

$$(\mathcal{M}_C)_{1,4} = (\mathcal{M}_C)_{2,5} = (\mathcal{M}_C)_{2,6} = (\mathcal{M}_C)_{3,5} = (\mathcal{M}_C)_{3,6} = 0 \tag{5.6}$$

$$(\mathcal{M}_C)_{1,5} = (\mathcal{M}_C)_{1,6} = \frac{1}{4\sqrt{3}} v^2(-\lambda_2 + \lambda_3) \tag{5.7}$$

$$(\mathcal{M}_C)_{2,3} = (\mathcal{M}_C)_{5,6} = \frac{1}{6} v^2(\lambda_2 - \lambda_3) \tag{5.8}$$

$$(\mathcal{M}_C)_{2,4} = (\mathcal{M}_C)_{3,4} = \frac{1}{4\sqrt{3}} v^2(\lambda_2 - \lambda_3) \tag{5.9}$$

and

$$(\mathcal{M}_N)_{1,1} = \frac{1}{6}v^2(\lambda_0 + \lambda_1 + 4\lambda_3) \tag{5.10}$$

$$(\mathcal{M}_N)_{1,2} = (\mathcal{M}_N)_{1,3} = \frac{1}{6}v^2(-2\lambda_0 + \lambda_1 - \lambda_2 + 2\lambda_3) \tag{5.11}$$

$$(\mathcal{M}_N)_{1,4} = -\frac{1}{2\sqrt{3}}v^2(\lambda_0 + \lambda_1) \tag{5.12}$$

$$(\mathcal{M}_N)_{1,5} = (\mathcal{M}_N)_{1,6} = (\mathcal{M}_N)_{2,5} = (\mathcal{M}_N)_{3,6} = \frac{1}{2\sqrt{3}}v^2\lambda_3 \tag{5.13}$$

$$(\mathcal{M}_N)_{2,2} = (\mathcal{M}_N)_{3,3} = \frac{1}{6}v^2(4(\lambda_0 + \lambda_1) + \lambda_3) \tag{5.14}$$

$$(\mathcal{M}_N)_{2,3} = \frac{1}{12}v^2(8\lambda_0 - 4\lambda_1 + 4\lambda_2 - 5\lambda_3) \tag{5.15}$$

$$(\mathcal{M}_N)_{2,4} = (\mathcal{M}_N)_{3,4} = \frac{1}{2\sqrt{3}}v^2(2\lambda_0 - \lambda_1 + \lambda_2 - \lambda_3) \tag{5.16}$$

$$(\mathcal{M}_N)_{2,6} = (\mathcal{M}_N)_{3,5} = -\frac{1}{4\sqrt{3}}v^2\lambda_3 \tag{5.17}$$

$$(\mathcal{M}_N)_{4,4} = \frac{1}{2}v^2(\lambda_0 + \lambda_1) \tag{5.18}$$

$$(\mathcal{M}_N)_{4,5} = (\mathcal{M}_N)_{4,6} = 0 \tag{5.19}$$

$$(\mathcal{M}_N)_{5,5} = (\mathcal{M}_N)_{6,6} = \frac{1}{2}v^2\lambda_3 \tag{5.20}$$

$$(\mathcal{M}_N)_{5,6} = -\frac{1}{4}v^2\lambda_3 \ . \tag{5.21}$$

5.3 Diagonalization of the Mass Matrices

The neutral 6×6 rotation matrix $\mathcal{R}_N$ is parametrized as

$$\mathcal{R}_N = \mathcal{R}_1\mathcal{R}_2\mathcal{R}_3 \ , \tag{5.22}$$

where

$$\mathcal{R}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & -b & -c & -d \\ 0 & 0 & b & a & -d & c \\ 0 & 0 & c & d & a & -b \\ 0 & 0 & d & -c & b & a \end{pmatrix} \tag{5.23}$$

and

$$\mathcal{R}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & p & -q & -r & -s \\ 0 & 0 & q & p & s & -r \\ 0 & 0 & r & -s & p & q \\ 0 & 0 & s & r & -q & p \end{pmatrix} \tag{5.24}$$

with

$$a^2 + b^2 + c^2 + d^2 = 1 \tag{5.25}$$
$$p^2 + q^2 + r^2 + s^2 = 1 \tag{5.26}$$
$$a, b, c, d, p, q, r, s \in \mathbb{R} \ , \tag{5.27}$$

and the matrix $\mathcal{R}_3$, which diagonalizes the upper-left 2×2 block of the neutral mass matrix $\mathcal{M}_N$, given by

$$\mathcal{R}_3 = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ \frac{1}{2\sqrt{26}} & -\frac{3}{\sqrt{26}} & 0 & \frac{1}{2}\sqrt{\frac{13}{6}} & \sqrt{\frac{2}{39}} & \sqrt{\frac{2}{39}} \\ \sqrt{\frac{3}{26}} & \frac{1}{2}\sqrt{\frac{3}{26}} & 0 & 0 & 2\sqrt{\frac{2}{13}} & -\frac{5}{2\sqrt{26}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}\sqrt{\frac{3}{2}} \end{pmatrix} . \tag{5.28}$$

Here, the neutral mass matrix cannot be diagonalized in a parameter-free way, in contrast to cases A and B. Hence a, b, c, d, p, q, r, s are parameter-dependent.

The charged rotation matrix $\mathcal{R}_C$ is parametrized in the following way:

$$\mathcal{R}_C = \begin{pmatrix} \mathcal{U}_C & 0 \\ 0 & \mathcal{U}_C^* \end{pmatrix} \mathcal{U}_\mathcal{P} \ , \tag{5.29}$$

with

$$\mathcal{U}_\mathcal{P} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & i & 0 & 0 \\ 0 & 1 & 0 & 0 & i & 0 \\ 0 & 0 & 1 & 0 & 0 & i \\ 1 & 0 & 0 & -i & 0 & 0 \\ 0 & 1 & 0 & 0 & -i & 0 \\ 0 & 0 & 1 & 0 & 0 & -i \end{pmatrix} , \tag{5.30}$$

which rotates the fields from the interaction basis into the mass basis, and

$$\mathcal{U}_C = \begin{pmatrix} \frac{1}{6}(-3i - \sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1+i\sqrt{3}}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1+i\sqrt{3}}{2\sqrt{6}} & \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} \end{pmatrix} , \tag{5.31}$$

where $\mathcal{U}_C$ diagonalizes the upper left 3×3 block of the charged mass matrix $\mathcal{M}_C$, and $\mathcal{U}_C^*$ diagonalizes the lower right 3×3 block of $\mathcal{M}_C$.

The charged Higgs masses are given by

$$M^2_{H^\pm_{2,3}} = \frac{1}{2} v^2(-\lambda_2 + \lambda_3) \ . \tag{5.32}$$

Note, that the charged Higgs boson $H_1^\pm$ is identified with the massless charged Goldstone boson $G^\pm$, which gives mass to the charged $W^\pm$ boson through the Higgs mechanism.

For the neutral Higgs boson masses we obtain,

$$M_{\tilde{H}}^2 := M_{H_2}^2 = \frac{2}{3}v^2(3\lambda_0 + \lambda_2 - \lambda_3) \ , \tag{5.33}$$

$$M_{H_3}^2 = M_{H_4}^2 = \frac{1}{6}v^2 \left(3\lambda_1 - \lambda_2 + 4\lambda_3 - \right. \\ \left. - \sqrt{9\lambda_1^2 + \lambda_2^2 + \lambda_2\lambda_3 + 7\lambda_3^2 - 3\lambda_1(2\lambda_2 + \lambda_3)} \ \right) \ , \tag{5.34}$$

$$M_{H_5}^2 = M_{H_6}^2 = \frac{1}{6}v^2 \left(3\lambda_1 - \lambda_2 + 4\lambda_3 + \right. \\ \left. + \sqrt{9\lambda_1^2 + \lambda_2^2 + \lambda_2\lambda_3 + 7\lambda_3^2 - 3\lambda_1(2\lambda_2 + \lambda_3)} \ \right) \ . \tag{5.35}$$

Note, that the neutral Higgs boson H_1 is identified with the massless neutral Goldstone boson G^0, which is absorbed to give mass to the massive Z boson through the Higgs mechanism. In the above expressions, H_2 is denoted with $\tilde{H}$ to indicate the fact that after rotating the neutral mass matrix $\mathcal{M}_N$ with $\mathcal{R}_3$, which is parameter independent, H_2 does not mix with the other Higgs bosons.

5.4 Couplings to Gauge Bosons

We have derived the couplings to the gauge bosons in the same way as described in section 3.4. They are given in the appendix 8.3. The couplings are given with all momenta taken as in-going. The Higgs couplings to the fermions were not calculated for this specific case of the model, as this goes beyond the scope of this thesis.

Here, it is not obvious, which of the neutral Higgs bosons is the SM-like one, since the couplings depend on the rotation matrix elements $\mathcal{R}_N$ and $\mathcal{R}_C$, and $\mathcal{R}_N$ is parameter dependent. Thus a different neutral Higgs boson could happen to be identified as the SM-like Higgs for different combinations of the parameters appearing in the potential. Thus some couplings from the list in appendix 8.3 will vanish accordingly.

6 Case D: (ω^2,1,1)

The procedure in this case is identical to case C, described in chapter 5. Therefore, only the results of the calculations are presented. The Higgs doublets are given by

$$\Phi_j = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_j + i\psi_j \\ \Re(v_j) + h_j + i\left[a_j + \Im(v_j)\right] \end{pmatrix} , \qquad j \in \{1,2,3\} \tag{6.1}$$

with the vevs

$$v_1 = \omega^2 v' , \qquad v_2 = v_3 = v' , \qquad \text{with} \qquad \omega = \exp(2\pi i/3),\ v' = \frac{1}{\sqrt{3}} v . \tag{6.2}$$

6.1 Minimum Conditions

The minimum conditions lead to

$$m^2 = (3\lambda_0 + \lambda_2 - \lambda_3)(v')^2 . \tag{6.3}$$

6.2 Mass Matrices

The matrix elements of the symmetric 6×6 charged and neutral Higgs mass matrices can be cast into the form

$$\begin{aligned}(\mathcal{M}_C)_{1,1} &= (\mathcal{M}_C)_{2,2} = (\mathcal{M}_C)_{3,3} = (\mathcal{M}_C)_{4,4} = \\ &= (\mathcal{M}_C)_{5,5} = (\mathcal{M}_C)_{6,6} = \frac{1}{3} v^2(-\lambda_2 + \lambda_3)\end{aligned} \tag{6.4}$$

$$(\mathcal{M}_C)_{1,2} = (\mathcal{M}_C)_{1,3} = (\mathcal{M}_C)_{4,5} = (\mathcal{M}_C)_{4,6} = \frac{1}{12} v^2(-\lambda_2 + \lambda_3) \tag{6.5}$$

$$(\mathcal{M}_C)_{1,4} = (\mathcal{M}_C)_{2,5} = (\mathcal{M}_C)_{2,6} = (\mathcal{M}_C)_{3,5} = (\mathcal{M}_C)_{3,6} = 0 \tag{6.6}$$

$$(\mathcal{M}_C)_{1,5} = (\mathcal{M}_C)_{1,6} = \frac{1}{4\sqrt{3}} v^2(\lambda_2 - \lambda_3) \tag{6.7}$$

$$(\mathcal{M}_C)_{2,3} = (\mathcal{M}_C)_{5,6} = \frac{1}{6} v^2(\lambda_2 - \lambda_3) \tag{6.8}$$

$$(\mathcal{M}_C)_{2,4} = (\mathcal{M}_C)_{3,4} = \frac{1}{4\sqrt{3}} v^2(-\lambda_2 + \lambda_3) , \tag{6.9}$$

and

$$(\mathcal{M}_N)_{1,1} = \frac{1}{6}v^2(\lambda_0 + \lambda_1 + 4\lambda_3) \tag{6.10}$$

$$(\mathcal{M}_N)_{1,2} = (\mathcal{M}_N)_{1,3} = \frac{1}{6}v^2(-2\lambda_0 + \lambda_1 - \lambda_2 + 2\lambda_3) \tag{6.11}$$

$$(\mathcal{M}_N)_{1,4} = \frac{1}{2\sqrt{3}}v^2(\lambda_0 + \lambda_1) \tag{6.12}$$

$$(\mathcal{M}_N)_{1,5} = (\mathcal{M}_N)_{1,6} = (\mathcal{M}_N)_{2,5} = (\mathcal{M}_N)_{3,6} = -\frac{1}{2\sqrt{3}}v^2\lambda_3 \tag{6.13}$$

$$(\mathcal{M}_N)_{2,2} = (\mathcal{M}_N)_{3,3} = \frac{1}{6}v^2(4(\lambda_0 + \lambda_1) + \lambda_3) \tag{6.14}$$

$$(\mathcal{M}_N)_{2,3} = \frac{1}{12}v^2(8\lambda_0 - 4\lambda_1 + 4\lambda_2 - 5\lambda_3) \tag{6.15}$$

$$(\mathcal{M}_N)_{2,4} = (\mathcal{M}_N)_{3,4} = -\frac{1}{2\sqrt{3}}v^2(2\lambda_0 - \lambda_1 + \lambda_2 - \lambda_3) \tag{6.16}$$

$$(\mathcal{M}_N)_{2,6} = (\mathcal{M}_N)_{3,5} = \frac{1}{4\sqrt{3}}v^2\lambda_3 \tag{6.17}$$

$$(\mathcal{M}_N)_{4,4} = \frac{1}{2}v^2(\lambda_0 + \lambda_1) \tag{6.18}$$

$$(\mathcal{M}_N)_{4,5} = (\mathcal{M}_N)_{4,6} = 0 \tag{6.19}$$

$$(\mathcal{M}_N)_{5,5} = (\mathcal{M}_N)_{6,6} = \frac{1}{2}v^2\lambda_3 \tag{6.20}$$

$$(\mathcal{M}_N)_{5,6} = -\frac{1}{4}v^2\lambda_3 \ . \tag{6.21}$$

6.3 Diagonalization of the Mass Matrices

The neutral 6×6 rotation matrix $\mathcal{R}_N$ is parametrized as

$$\mathcal{R}_N = \mathcal{R}_1\mathcal{R}_2\mathcal{R}_3 \ , \tag{6.22}$$

where

$$\mathcal{R}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & -b & -c & -d \\ 0 & 0 & b & a & -d & c \\ 0 & 0 & c & d & a & -b \\ 0 & 0 & d & -c & b & a \end{pmatrix} \ , \tag{6.23}$$

and

$$\mathcal{R}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & p & -q & -r & -s \\ 0 & 0 & q & p & s & -r \\ 0 & 0 & r & -s & p & q \\ 0 & 0 & s & r & -q & p \end{pmatrix} \ , \tag{6.24}$$

with

$$a^2 + b^2 + c^2 + d^2 = 1 \tag{6.25}$$
$$p^2 + q^2 + r^2 + s^2 = 1 \tag{6.26}$$
$$a, b, c, d, p, q, r, s \in \mathbb{R} \ , \tag{6.27}$$

and

$$\mathcal{R}_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -\frac{1}{2\sqrt{26}} & \frac{3}{\sqrt{26}} & 0 & \frac{1}{2}\sqrt{\frac{13}{6}} & \sqrt{\frac{2}{39}} & \sqrt{\frac{2}{39}} \\ -\sqrt{\frac{3}{26}} & -\frac{1}{2}\sqrt{\frac{3}{26}} & 0 & 0 & 2\sqrt{\frac{2}{13}} & -\frac{5}{2\sqrt{26}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}\sqrt{\frac{3}{2}} \end{pmatrix} . \tag{6.28}$$

The charged rotation matrix $\mathcal{R}_C$ is parametrized as

$$\mathcal{R}_C = \begin{pmatrix} \mathcal{U}_C^* & 0 \\ 0 & \mathcal{U}_C \end{pmatrix} \mathcal{U}_\mathcal{P} \tag{6.29}$$

with

$$\mathcal{U}_\mathcal{P} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & i & 0 & 0 \\ 0 & 1 & 0 & 0 & i & 0 \\ 0 & 0 & 1 & 0 & 0 & i \\ 1 & 0 & 0 & -i & 0 & 0 \\ 0 & 1 & 0 & 0 & -i & 0 \\ 0 & 0 & 1 & 0 & 0 & -i \end{pmatrix} \tag{6.30}$$

and

$$\mathcal{U}_C = \begin{pmatrix} \frac{1}{6}(-3i - \sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1+i\sqrt{3}}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1+i\sqrt{3}}{2\sqrt{6}} & \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} \end{pmatrix} . \tag{6.31}$$

The charged Higgs masses are given by

$$M^2_{H^\pm_{2,3}} = \frac{1}{2} v^2 (-\lambda_2 + \lambda_3) \ . \tag{6.32}$$

Note, that, again, the charged Higgs boson $H_1^\pm$ is identified with the massless charged Goldstone boson $G^\pm$, which gives mass to the charged $W^\pm$ boson through the Higgs mechanism.

The neutral Higgs masses are given by

$$M^2_{\tilde{H}} := M^2_{H_2} = \frac{2}{3} v^2 (3\lambda_0 + \lambda_2 - \lambda_3) \ , \tag{6.33}$$

$$M^2_{H_3} = M^2_{H_4} = \frac{1}{6} v^2 \left(3\lambda_1 - \lambda_2 + 4\lambda_3 - \right. \\ \left. - \sqrt{9\lambda_1^2 + \lambda_2^2 + \lambda_2\lambda_3 + 7\lambda_3^2 - 3\lambda_1(2\lambda_2 + \lambda_3)} \ \right) , \tag{6.34}$$

$$M^2_{H_5} = M^2_{H_6} = \frac{1}{6} v^2 \left(3\lambda_1 - \lambda_2 + 4\lambda_3 + \right. \\ \left. + \sqrt{9\lambda_1^2 + \lambda_2^2 + \lambda_2\lambda_3 + 7\lambda_3^2 - 3\lambda_1(2\lambda_2 + \lambda_3)}\ \right) \quad . \tag{6.35}$$

Note, that, just as in case C (see section 5.3 for the relevant part), the neutral Higgs boson H_1 is identified with the massless neutral Goldstone boson G^0, which is absorbed to give mass to the massive Z boson through the Higgs mechanism. H_2 is again denoted with $\tilde{H}$ for the same reason as in section 5.3.

6.4 Couplings to Gauge Bosons

We have derived the couplings to the gauge bosons in the same way as described in section 3.4. They are given in the appendix 8.4. The couplings are given with all momenta taken as in-going. The Higgs couplings to the fermions were not calculated for this specific case of the model, as this goes beyond the scope of this thesis.

Here, just as in the relevant section in case C (see section 5.4), it is not obvious, which of the neutral Higgs bosons is the SM-like one, since the couplings depend on the rotation matrix elements $\mathcal{R}_N$ and $\mathcal{R}_C$, and $\mathcal{R}_N$ is parameter dependent. Therefore, again, a different neutral Higgs boson can happen to be identified as the SM-like Higgs for different combinations of the parameters appearing in the potential, and again some couplings from the list in appendix 8.4 will vanish accordingly.

7 Conclusions and New Goals

In this thesis, the minimum conditions, the physical Higgs masses, the Higgs couplings to gauge bosons and the effects induced on these by the symmetry of the potential, i.e. parameter independent diagonalization of the mass matrices, equal masses of different Higgs bosons (this also increases the chances of the considered Higgs being a DM candidate) and the absence of certain couplings, were determined for all four cases of global minima of the $\Delta(54)$ family of discrete symmetry groups in the 3HDM. The possible Dark Matter candidates were found for two of these cases. In one case (ch. 4), in addition to the above, the $\Delta(54)/\mathbb{Z}_3$ symmetry was reduced to a $\mathbb{Z}_2$ symmetry, and the couplings of the Higgs bosons to fermions were calculated. A Python program was written, which computes the decays of the different Higgs bosons in the investigated special scenarios of case B of the 3HDM that is symmetric under the $\mathbb{Z}_2$ symmetry. The decays of the Higgs bosons were then analyzed by discussing and comparing their branching ratios.

Future possible goals are:

(1) the computation of the trilinear and quartic Higgs self-couplings

(2) the derivation of the Higgs couplings to fermions for the $\Delta(54)$ family of discrete symmetry groups

(3) after performing items (1) and (2), calculating all possible branching ratios

(4) repeating the analysis for a CP-violating potential

8 Appendix

8.1 Case A: (1,0,0) - Couplings to Gauge Bosons

All momenta are taken as in-going.

$$A_2H_3Z \qquad \frac{1}{2}g(k^\mu_{A_2} - k^\mu_{H_3})\sec(\theta_W) \tag{8.1}$$

$$A_2H_2^-W^+ \qquad \frac{1}{2\sqrt{2}}g(k^\mu_{A_2} - k^\mu_{H_2^-}) \tag{8.2}$$

$$A_2H_3^-W^+ \qquad -\frac{1}{2\sqrt{2}}g(k^\mu_{A_2} - k^\mu_{H_3^-}) \tag{8.3}$$

$$A_2H_2^+W^- \qquad \frac{1}{2\sqrt{2}}g(k^\mu_{A_2} - k^\mu_{H_2^+}) \tag{8.4}$$

$$A_2H_3^+W^- \qquad -\frac{1}{2\sqrt{2}}g(k^\mu_{A_2} - k^\mu_{H_3^+}) \tag{8.5}$$

$$A_3H_2Z \qquad -\frac{1}{2}g(k^\mu_{A_3} - k^\mu_{H_2})\sec(\theta_W) \tag{8.6}$$

$$A_3H_2^-W^+ \qquad \frac{1}{2\sqrt{2}}g(k^\mu_{A_3} - k^\mu_{H_2^-}) \tag{8.7}$$

$$A_3H_3^-W^+ \qquad \frac{1}{2\sqrt{2}}g(k^\mu_{A_3} - k^\mu_{H_3^-}) \tag{8.8}$$

$$A_3H_2^+W^- \qquad \frac{1}{2\sqrt{2}}g(k^\mu_{A_3} - k^\mu_{H_2^+}) \tag{8.9}$$

$$A_3H_3^+W^- \qquad \frac{1}{2\sqrt{2}}g(k^\mu_{A_3} - k^\mu_{H_3^+}) \tag{8.10}$$

$$\gamma H_2^-H_2^+ \qquad ig(k^\mu_{H_2^-} - k^\mu_{H_2^+})\sin(\theta_W) \tag{8.11}$$

$$\gamma H_3^-H_3^+ \qquad ig(k^\mu_{H_3^-} - k^\mu_{H_3^+})\sin(\theta_W) \tag{8.12}$$

$$H_1W^-W^+ \qquad \frac{1}{2}ig^2v \tag{8.13}$$

$$H_1ZZ \qquad \frac{1}{2}ig^2v\sec^2(\theta_W) \tag{8.14}$$

$$H_2H_2^-W^+ \qquad -\frac{1}{2\sqrt{2}}ig(k^\mu_{H_2} - k^\mu_{H_2^-}) \tag{8.15}$$

$$H_2H_3^-W^+ \qquad -\frac{1}{2\sqrt{2}}ig(k^\mu_{H_2} - k^\mu_{H_3^-}) \tag{8.16}$$

$$H_2H_2^+W^- \qquad \frac{1}{2\sqrt{2}}ig(k^\mu_{H_2} - k^\mu_{H_2^+}) \tag{8.17}$$

$$H_2H_3^+W^- \qquad \frac{1}{2\sqrt{2}}ig(k^\mu_{H_2} - k^\mu_{H_3^+}) \tag{8.18}$$

$$H_3H_2^-W^+ \qquad \frac{1}{2\sqrt{2}}ig(k^\mu_{H_3} - k^\mu_{H_2^-}) \tag{8.19}$$

$$H_3H_3^-W^+ \qquad -\frac{1}{2\sqrt{2}}ig(k^\mu_{H_3} - k^\mu_{H_3^-}) \tag{8.20}$$

$$H_3H_2^+W^- \qquad -\frac{1}{2\sqrt{2}}ig(k^\mu_{H_3} - k^\mu_{H_2^+}) \tag{8.21}$$

$$H_3H_3^+W^- \qquad \frac{1}{2\sqrt{2}}ig(k^\mu_{H_3} - k^\mu_{H_3^+}) \tag{8.22}$$

$$H_2^-H_2^+Z \qquad \frac{1}{2}ig(k^\mu_{H_2^-} - k^\mu_{H_2^+})\cos(2\theta_W)\sec(\theta_W) \tag{8.23}$$

$$H_3^-H_3^+Z \qquad \frac{1}{2}ig(k^\mu_{H_3^-} - k^\mu_{H_3^+})\cos(2\theta_W)\sec(\theta_W) \tag{8.24}$$

8.2 Case B: (1,1,1) - Couplings to Gauge Bosons

All momenta are taken as in-going.

$$A_2H_2Z \qquad -\frac{1}{2}g(k^\mu_{A_2} - k^\mu_{H_2})\sec(\theta_W) \tag{8.25}$$

$$A_2H_2^-W^+ \qquad \frac{1}{2}g(k^\mu_{A_2} - k^\mu_{H_2^-}) \tag{8.26}$$

$$A_2H_2^+W^- \qquad \frac{1}{2}g(k^\mu_{A_2} - k^\mu_{H_2^+}) \tag{8.27}$$

$$A_3H_3Z \qquad -\frac{1}{2}g(k^\mu_{A_3} - k^\mu_{H_3})\sec(\theta_W) \tag{8.28}$$

$$A_3H_3^-W^+ \qquad \frac{1}{2}g(k^\mu_{A_3} - k^\mu_{H_3^-}) \tag{8.29}$$

$$A_3H_3^+W^- \qquad \frac{1}{2}g(k^\mu_{A_3} - k^\mu_{H_3^+}) \tag{8.30}$$

$$\gamma H_2^-H_2^+ \qquad ig(k^\mu_{H_2^-} - k^\mu_{H_2^+})\sin(\theta_W) \tag{8.31}$$

$$\gamma H_3^-H_3^+ \qquad ig(k^\mu_{H_3^-} - k^\mu_{H_3^+})\sin(\theta_W) \tag{8.32}$$

$$H_1W^-W^+ \qquad \frac{1}{2}ig^2v \tag{8.33}$$

$$H_1ZZ \qquad \frac{1}{2}ig^2v\sec^2(\theta_W) \tag{8.34}$$

$$H_2H_2^-W^+ \qquad -\frac{1}{2}ig(k^\mu_{H_2} - k^\mu_{H_2^-}) \tag{8.35}$$

$$H_2H_2^+W^- \qquad \frac{1}{2}ig(k^\mu_{H_2} - k^\mu_{H_2^+}) \tag{8.36}$$

$$H_3H_3^-W^+ \qquad -\frac{1}{2}ig(k^\mu_{H_3} - k^\mu_{H_3^-}) \tag{8.37}$$

$$H_3H_3^+W^- \qquad \frac{1}{2}ig(k^\mu_{H_3} - k^\mu_{H_3^+}) \tag{8.38}$$

$$H_2^-H_2^+Z \qquad \frac{1}{2}ig(k^\mu_{H_2^-} - k^\mu_{H_2^+})\cos(2\theta_W)\sec(\theta_W) \tag{8.39}$$

$$H_3^-H_3^+Z \qquad \frac{1}{2}ig(k^\mu_{H_3^-} - k^\mu_{H_3^+})\cos(2\theta_W)\sec(\theta_W) \tag{8.40}$$

8.3 Case C: (ω,1,1) - Couplings to Gauge Bosons

All momenta are taken as in-going. The rotation matrices $\mathcal{R}_N$ and $\mathcal{R}_C$ were defined in eqns. (5.22) and (5.29), respectively.

$$\gamma H_2^- H_2^+ \qquad -ig(k^\mu_{H_2^+} - k^\mu_{H_2^-})\sin(\theta_W) \tag{8.41}$$

$$\gamma H_3^- H_3^+ \qquad -ig(k^\mu_{H_3^+} - k^\mu_{H_3^-})\sin(\theta_W) \tag{8.42}$$

$$H_3 H_4 Z \qquad -\frac{1}{2}g(k^\mu_{H_3} - k^\mu_{H_4})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,3} - (\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,4}\right] \tag{8.43}$$

$$H_3 H_5 Z \qquad -\frac{1}{2}g(k^\mu_{H_3} - k^\mu_{H_5})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,3} - (\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,5}\right] \tag{8.44}$$

$$H_3 H_6 Z \qquad -\frac{1}{2}g(k^\mu_{H_3} - k^\mu_{H_6})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,3} - (\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,6}\right] \tag{8.45}$$

$$H_3 H_2^- W^+ \qquad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_2^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\right)\left((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5}\right)\right] \tag{8.46}$$

$$H_3 H_3^- W^+ \qquad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_3^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\right)\left((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\right)\right] \tag{8.47}$$

$$H_3 H_2^+ W^- \qquad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_2^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\right)\left((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\right)\right] \tag{8.48}$$

$$H_3 H_3^+ W^- \qquad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_3^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\right)\left((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3}\right)\right] \tag{8.49}$$

$$H_3 \tilde{H} Z \qquad -\frac{1}{2}g(k^\mu_{\tilde{H}} - k^\mu_{H_3})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2}(\mathcal{R}_N^T)_{j+3,3}\right] \tag{8.50}$$

$$H_3 W^- W^+ \qquad -\frac{1}{12}ig^2 v\left[\sqrt{3}(\mathcal{R}_N^T)_{1,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,3} - 3(\mathcal{R}_N^T)_{4,3}\right] \tag{8.51}$$

$$H_3 Z Z \qquad -\frac{1}{12}ig^2 v\sec^2(\theta_W)\left[\sqrt{3}(\mathcal{R}_N^T)_{1,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,3} - 3(\mathcal{R}_N^T)_{4,3}\right] \tag{8.52}$$

$$H_4 H_5 Z \quad -\frac{1}{2} g(k^\mu_{H_4} - k^\mu_{H_5}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,4} - (\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,5}\right] \tag{8.53}$$

$$H_4 H_6 Z \quad -\frac{1}{2} g(k^\mu_{H_4} - k^\mu_{H_6}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,4} - (\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,6}\right] \tag{8.54}$$

$$H_4 H_2^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_2^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\right)\left((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5}\right)\right] \tag{8.55}$$

$$H_4 H_3^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_3^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\right)\left((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\right)\right] \tag{8.56}$$

$$H_4 H_2^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_2^+}) \sum_{n=1}^{3} \left[\left(i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\right)\left((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\right)\right] \tag{8.57}$$

$$H_4 H_3^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_3^+}) \sum_{n=1}^{3} \left[\left(i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\right)\left((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3}\right)\right] \tag{8.58}$$

$$H_4 \tilde{H} Z \quad -\frac{1}{2} g(k^\mu_{\tilde{H}} - k^\mu_{H_4}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2}(\mathcal{R}_N^T)_{j+3,4}\right] \tag{8.59}$$

$$H_4 W^- W^+ \quad -\frac{1}{12} i g^2 v \left[\sqrt{3}(\mathcal{R}_N^T)_{1,4} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,4} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,4} - 3(\mathcal{R}_N^T)_{4,4}\right] \tag{8.60}$$

$$H_4 Z Z \quad -\frac{1}{12} i g^2 v \sec^2(\theta_W) \left[\sqrt{3}(\mathcal{R}_N^T)_{1,4} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,4} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,4} - 3(\mathcal{R}_N^T)_{4,4}\right] \tag{8.61}$$

$$H_5 H_6 Z \quad -\frac{1}{2} g(k^\mu_{H_5} - k^\mu_{H_6}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,5} - (\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,6}\right] \tag{8.62}$$

$$H_5 H_2^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5} - k^\mu_{H_2^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,5} + (\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5}\right)\right] \tag{8.63}$$

$$H_5 H_3^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5} - k^\mu_{H_3^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,5} + (\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\right)\right] \tag{8.64}$$

$$H_5 H_2^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5} - k^\mu_{H_2^+}) \sum_{n=1}^{3} \left[\left(i(\mathcal{R}_N^T)_{n,5} + (\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\right)\right] \tag{8.65}$$

$$H_5 H_3^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5} - k^\mu_{H_3^+}) \sum_{n=1}^{3} \left[\left(i(\mathcal{R}_N^T)_{n,5} + (\mathcal{R}_N^T)_{n+3,5} \right) \left((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3} \right) \right] \tag{8.66}$$

$$H_5 \tilde{H} Z \quad -\frac{1}{2} g(k^\mu_{\tilde{H}} - k^\mu_{H_5}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,5} (\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2} (\mathcal{R}_N^T)_{j+3,5} \right] \tag{8.67}$$

$$H_5 W^- W^+ \quad -\frac{1}{12} i g^2 v \left[\sqrt{3}(\mathcal{R}_N^T)_{1,5} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,5} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,5} - 3(\mathcal{R}_N^T)_{4,5} \right] \tag{8.68}$$

$$H_5 Z Z \quad -\frac{1}{12} i g^2 v \sec^2(\theta_W) \left[\sqrt{3}(\mathcal{R}_N^T)_{1,5} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,5} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,5} - 3(\mathcal{R}_N^T)_{4,5} \right] \tag{8.69}$$

$$H_6 H_2^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6} - k^\mu_{H_2^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,6} + (\mathcal{R}_N^T)_{n+3,6} \right) \left((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5} \right) \right] \tag{8.70}$$

$$H_6 H_3^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6} - k^\mu_{H_3^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,6} + (\mathcal{R}_N^T)_{n+3,6} \right) \left((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6} \right) \right] \tag{8.71}$$

$$H_6 H_2^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6} - k^\mu_{H_2^+}) \sum_{n=1}^{3} \left[\left(i(\mathcal{R}_N^T)_{n,6} + (\mathcal{R}_N^T)_{n+3,6} \right) \left((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2} \right) \right] \tag{8.72}$$

$$H_6 H_3^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6} - k^\mu_{H_3^+}) \sum_{n=1}^{3} \left[\left(i(\mathcal{R}_N^T)_{n,6} + (\mathcal{R}_N^T)_{n+3,6} \right) \left((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3} \right) \right] \tag{8.73}$$

$$H_6 \tilde{H} Z \quad -\frac{1}{2} g(k^\mu_{\tilde{H}} - k^\mu_{H_6}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,6} (\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2} (\mathcal{R}_N^T)_{j+3,6} \right] \tag{8.74}$$

$$H_6 W^- W^+ \quad -\frac{1}{12} i g^2 v \left[\sqrt{3}(\mathcal{R}_N^T)_{1,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,6} - 3(\mathcal{R}_N^T)_{4,6} \right] \tag{8.75}$$

$$H_6 Z Z \quad -\frac{1}{12} i g^2 v \sec^2(\theta_W) \left[\sqrt{3}(\mathcal{R}_N^T)_{1,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,6} - 3(\mathcal{R}_N^T)_{4,6} \right] \tag{8.76}$$

$$H_2^- H_2^+ Z \quad -i\frac{1}{2} g \cos(2\theta_W)(k^\mu_{H_2^+} - k^\mu_{H_2^-}) \sec(\theta_W) \tag{8.77}$$

$$H_2^- \tilde{H} W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_2^-}) \sum_{n=1}^{3} \left[\left(-i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2} \right) \left((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5} \right) \right] \tag{8.78}$$

$$H_3^- H_3^+ Z \quad -i\frac{1}{2} g \cos(2\theta_W)(k^\mu_{H_3^+} - k^\mu_{H_3^-}) \sec(\theta_W) \tag{8.79}$$

$$H_3^-\tilde{H}W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_3^-}) \sum_{n=1}^{3} \Big[\Big(-i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\Big)\Big((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\Big)\Big] \tag{8.80}$$

$$H_2^+\tilde{H}W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_2^+}) \sum_{n=1}^{3} \Big[\Big(i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\Big)\Big((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\Big)\Big] \tag{8.81}$$

$$H_3^+\tilde{H}W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_3^+}) \sum_{n=1}^{3} \Big[\Big(i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\Big)\Big((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3}\Big)\Big] \tag{8.82}$$

$$\tilde{H}W^-W^+ \quad -\frac{1}{12}ig^2v\Big[\sqrt{3}(\mathcal{R}_N^T)_{1,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,2} - 3(\mathcal{R}_N^T)_{4,2}\Big] \tag{8.83}$$

$$\tilde{H}ZZ \quad -\frac{1}{12}ig^2v\sec^2(\theta_W)\Big[\sqrt{3}(\mathcal{R}_N^T)_{1,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,2} - 3(\mathcal{R}_N^T)_{4,2}\Big] \tag{8.84}$$

8.4 Case D: (ω^2,1,1) - Couplings to Gauge Bosons

All momenta are taken as in-going. The rotation matrices $\mathcal{R}_N$ and $\mathcal{R}_C$ were defined in eqns. (6.22) and (6.29), respectively.

$$\gamma H_2^- H_2^+ \quad -ig(k^\mu_{H_2^+} - k^\mu_{H_2^-})\sin(\theta_W) \tag{8.85}$$

$$\gamma H_3^- H_3^+ \quad -ig(k^\mu_{H_3^+} - k^\mu_{H_3^-})\sin(\theta_W) \tag{8.86}$$

$$H_3H_4Z \quad -\frac{1}{2}g(k^\mu_{H_3} - k^\mu_{H_4})\sec(\theta_W)\sum_{j=1}^{3}\big[(\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,3} - (\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,4}\big] \tag{8.87}$$

$$H_3H_5Z \quad -\frac{1}{2}g(k^\mu_{H_3} - k^\mu_{H_5})\sec(\theta_W)\sum_{j=1}^{3}\big[(\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,3} - (\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,5}\big] \tag{8.88}$$

$$H_3H_6Z \quad -\frac{1}{2}g(k^\mu_{H_3} - k^\mu_{H_6})\sec(\theta_W)\sum_{j=1}^{3}\big[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,3} - (\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,6}\big] \tag{8.89}$$

$$H_3H_2^-W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_2^-}) \sum_{n=1}^{3} \Big[\Big(-i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\Big)\Big((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5}\Big)\Big] \tag{8.90}$$

$$H_3H_3^-W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_3^-}) \sum_{n=1}^{3} \Big[\Big(-i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\Big)\Big((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\Big)\Big] \tag{8.91}$$

$$H_3 H_2^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_2^+}) \sum_{n=1}^{3} \Big[\Big(i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\Big)\Big((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\Big)\Big] \tag{8.92}$$

$$H_3 H_3^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_3} - k^\mu_{H_3^+}) \sum_{n=1}^{3} \Big[\Big(i(\mathcal{R}_N^T)_{n,3} + (\mathcal{R}_N^T)_{n+3,3}\Big)\Big((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3}\Big)\Big] \tag{8.93}$$

$$H_3 \tilde{H} Z \quad -\frac{1}{2} g (k^\mu_{\tilde{H}} - k^\mu_{H_3}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,3}(\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2}(\mathcal{R}_N^T)_{j+3,3}\right] \tag{8.94}$$

$$H_3 W^- W^+ \quad -\frac{1}{12} i g^2 v \left[\sqrt{3}(\mathcal{R}_N^T)_{1,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,3} + 3(\mathcal{R}_N^T)_{4,3}\right] \tag{8.95}$$

$$H_3 Z Z \quad -\frac{1}{12} i g^2 v \sec^2(\theta_W) \left[\sqrt{3}(\mathcal{R}_N^T)_{1,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,3} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,3} + 3(\mathcal{R}_N^T)_{4,3}\right] \tag{8.96}$$

$$H_4 H_5 Z \quad -\frac{1}{2} g (k^\mu_{H_4} - k^\mu_{H_5}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,4} - (\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,5}\right] \tag{8.97}$$

$$H_4 H_6 Z \quad -\frac{1}{2} g (k^\mu_{H_4} - k^\mu_{H_6}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,4} - (\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,6}\right] \tag{8.98}$$

$$H_4 H_2^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_2^-}) \sum_{n=1}^{3} \Big[\Big(-i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\Big)\Big((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5}\Big)\Big] \tag{8.99}$$

$$H_4 H_3^- W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_3^-}) \sum_{n=1}^{3} \Big[\Big(-i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\Big)\Big((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\Big)\Big] \tag{8.100}$$

$$H_4 H_2^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_2^+}) \sum_{n=1}^{3} \Big[\Big(i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\Big)\Big((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\Big)\Big] \tag{8.101}$$

$$H_4 H_3^+ W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_4} - k^\mu_{H_3^+}) \sum_{n=1}^{3} \Big[\Big(i(\mathcal{R}_N^T)_{n,4} + (\mathcal{R}_N^T)_{n+3,4}\Big)\Big((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3}\Big)\Big] \tag{8.102}$$

$$H_4 \tilde{H} Z \quad -\frac{1}{2} g (k^\mu_{\tilde{H}} - k^\mu_{H_4}) \sec(\theta_W) \sum_{j=1}^{3} \left[(\mathcal{R}_N^T)_{j,4}(\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2}(\mathcal{R}_N^T)_{j+3,4}\right] \tag{8.103}$$

$$H_4 W^- W^+ \quad -\frac{1}{12} i g^2 v \left[\sqrt{3}(\mathcal{R}_N^T)_{1,4} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,4} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,4} + 3(\mathcal{R}_N^T)_{4,4}\right] \tag{8.104}$$

$$H_4ZZ \quad -\frac{1}{12}ig^2v\sec^2(\theta_W)\left[\sqrt{3}(\mathcal{R}_N^T)_{1,4}-2\sqrt{3}(\mathcal{R}_N^T)_{2,4}-2\sqrt{3}(\mathcal{R}_N^T)_{3,4}+3(\mathcal{R}_N^T)_{4,4}\right] \tag{8.105}$$

$$H_5H_6Z \quad -\frac{1}{2}g(k^\mu_{H_5}-k^\mu_{H_6})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,5}-(\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,6}\right] \tag{8.106}$$

$$H_5H_2^-W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5}-k^\mu_{H_2^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,5}+(\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,5}-i(\mathcal{R}_C^\dagger)_{n+3,5}\right)\right] \tag{8.107}$$

$$H_5H_3^-W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5}-k^\mu_{H_3^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,5}+(\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,6}-i(\mathcal{R}_C^\dagger)_{n+3,6}\right)\right] \tag{8.108}$$

$$H_5H_2^+W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5}-k^\mu_{H_2^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,5}+(\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,2}+i(\mathcal{R}_C^\dagger)_{n+3,2}\right)\right] \tag{8.109}$$

$$H_5H_3^+W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_5}-k^\mu_{H_3^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,5}+(\mathcal{R}_N^T)_{n+3,5}\right)\left((\mathcal{R}_C^\dagger)_{n,3}+i(\mathcal{R}_C^\dagger)_{n+3,3}\right)\right] \tag{8.110}$$

$$H_5\tilde{H}Z \quad -\frac{1}{2}g(k^\mu_{\tilde{H}}-k^\mu_{H_5})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,5}(\mathcal{R}_N^T)_{j+3,2}-(\mathcal{R}_N^T)_{j,2}(\mathcal{R}_N^T)_{j+3,5}\right] \tag{8.111}$$

$$H_5W^-W^+ \quad -\frac{1}{12}ig^2v\left[\sqrt{3}(\mathcal{R}_N^T)_{1,5}-2\sqrt{3}(\mathcal{R}_N^T)_{2,5}-2\sqrt{3}(\mathcal{R}_N^T)_{3,5}+3(\mathcal{R}_N^T)_{4,5}\right] \tag{8.112}$$

$$H_5ZZ \quad -\frac{1}{12}ig^2v\sec^2(\theta_W)\left[\sqrt{3}(\mathcal{R}_N^T)_{1,5}-2\sqrt{3}(\mathcal{R}_N^T)_{2,5}-2\sqrt{3}(\mathcal{R}_N^T)_{3,5}+3(\mathcal{R}_N^T)_{4,5}\right] \tag{8.113}$$

$$H_6H_2^-W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6}-k^\mu_{H_2^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,6}+(\mathcal{R}_N^T)_{n+3,6}\right)\left((\mathcal{R}_C^\dagger)_{n,5}-i(\mathcal{R}_C^\dagger)_{n+3,5}\right)\right] \tag{8.114}$$

$$H_6H_3^-W^+ \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6}-k^\mu_{H_3^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,6}+(\mathcal{R}_N^T)_{n+3,6}\right)\left((\mathcal{R}_C^\dagger)_{n,6}-i(\mathcal{R}_C^\dagger)_{n+3,6}\right)\right] \tag{8.115}$$

$$H_6H_2^+W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6}-k^\mu_{H_2^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,6}+(\mathcal{R}_N^T)_{n+3,6}\right)\left((\mathcal{R}_C^\dagger)_{n,2}+i(\mathcal{R}_C^\dagger)_{n+3,2}\right)\right] \tag{8.116}$$

$$H_6H_3^+W^- \quad \frac{g}{2\sqrt{2}}(k^\mu_{H_6}-k^\mu_{H_3^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,6}+(\mathcal{R}_N^T)_{n+3,6}\right)\left((\mathcal{R}_C^\dagger)_{n,3}+i(\mathcal{R}_C^\dagger)_{n+3,3}\right)\right] \tag{8.117}$$

$H_6\tilde{H}Z$	$-\frac{1}{2}g(k^\mu_{\tilde{H}} - k^\mu_{H_6})\sec(\theta_W)\sum_{j=1}^{3}\left[(\mathcal{R}_N^T)_{j,6}(\mathcal{R}_N^T)_{j+3,2} - (\mathcal{R}_N^T)_{j,2}(\mathcal{R}_N^T)_{j+3,6}\right]$ (8.118)
$H_6W^-W^+$	$-\frac{1}{12}ig^2v\left[\sqrt{3}(\mathcal{R}_N^T)_{1,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,6} + 3(\mathcal{R}_N^T)_{4,6}\right]$ (8.119)
H_6ZZ	$-\frac{1}{12}ig^2v\sec^2(\theta_W)\left[\sqrt{3}(\mathcal{R}_N^T)_{1,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,6} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,6} + 3(\mathcal{R}_N^T)_{4,6}\right]$ (8.120)
$H_2^-H_2^+Z$	$-i\frac{1}{2}g\cos(2\theta_W)(k^\mu_{H_2^+} - k^\mu_{H_2^-})\sec(\theta_W)$ (8.121)
$H_2^-\tilde{H}W^+$	$\frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_2^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\right)\left((\mathcal{R}_C^\dagger)_{n,5} - i(\mathcal{R}_C^\dagger)_{n+3,5}\right)\right]$ (8.122)
$H_3^-H_3^+Z$	$-i\frac{1}{2}g\cos(2\theta_W)(k^\mu_{H_3^+} - k^\mu_{H_3^-})\sec(\theta_W)$ (8.123)
$H_3^-\tilde{H}W^+$	$\frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_3^-})\sum_{n=1}^{3}\left[\left(-i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\right)\left((\mathcal{R}_C^\dagger)_{n,6} - i(\mathcal{R}_C^\dagger)_{n+3,6}\right)\right]$ (8.124)
$H_2^+\tilde{H}W^-$	$\frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_2^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\right)\left((\mathcal{R}_C^\dagger)_{n,2} + i(\mathcal{R}_C^\dagger)_{n+3,2}\right)\right]$ (8.125)
$H_3^+\tilde{H}W^-$	$\frac{g}{2\sqrt{2}}(k^\mu_{\tilde{H}} - k^\mu_{H_3^+})\sum_{n=1}^{3}\left[\left(i(\mathcal{R}_N^T)_{n,2} + (\mathcal{R}_N^T)_{n+3,2}\right)\left((\mathcal{R}_C^\dagger)_{n,3} + i(\mathcal{R}_C^\dagger)_{n+3,3}\right)\right]$ (8.126)
$\tilde{H}W^-W^+$	$-\frac{1}{12}ig^2v\left[\sqrt{3}(\mathcal{R}_N^T)_{1,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,2} + 3(\mathcal{R}_N^T)_{4,2}\right]$ (8.127)
$\tilde{H}ZZ$	$-\frac{1}{12}ig^2v\sec^2(\theta_W)\left[\sqrt{3}(\mathcal{R}_N^T)_{1,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{2,2} - 2\sqrt{3}(\mathcal{R}_N^T)_{3,2} + 3(\mathcal{R}_N^T)_{4,2}\right]$ (8.128)

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