Transition from walking to running of a bipedal robot to optimize energy efficiency

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The transition from walking to running gaits in bipedal locomotion is well known from humans. One explanation for this transition is a higher energy efficiency of running gaits at higher velocities. In this paper we use a five-link planar model of a robot to investigate the transition from walking to running based on energy efficiency. For this purpose a physically motivated cost function regarding static as well as dynamic costs is introduced. Periodic walking and running gaits are generated by means of numerical optimization to find the optimal gait of a human-like model in a range from 1.5 to 2.5 m/s. At the transition velocity walking and running require the same cost. Both gaits are investigated to identify the underlying mechanisms. The computed results correspond very well to reports from biomechanics which indicates that the model is suitable for the investigation of human locomotion as well as the generation of optimal gaits for humanoid robots.

Keywords: bipedal locomotion; energy efficiency; optimization.

1. Introduction

Human-like locomotion of bipedal robots faces two main challenges: a controller for stable gaits on the one hand and energy efficiency on the other hand. Whereas the stable control strategy is an obvious prerequisite for a walking or running humanoid robot, minimization of the consumed energy cannot be neglected either since it affects the robot's actuator design, range and therewith applicability.

Different models have been suggested to investigate the transition from walking to running. The most simple one lumps the total mass in a particle situated in the hip and renders the leg function by massless rods for walking and springs for running respectively.[1,2] These models can reproduce the characteristics of running or walking for certain parameter and velocity ranges. The transition however has not been investigated satisfactorily.

Another approach is the utilization of large, complex models inspired by biomechanics.[3,4] These models implement detailed skeletons actuated by many muscle groups and are able to reproduce experimental data satisfactorily. Due to the complexity of the model, however, they require much computing time which limits the applications in parameter studies. Furthermore, the complexity makes it difficult to identify underlying mechanisms.

We propose a compromise in terms of complexity to model the relevant mechanics on the one hand and nonetheless enable a mechanical interpretation of the underlying effects. Thus we introduce a simple five-link model with distributed mass capable of running. Energy efficiency, which is considered to be a main influence triggering the transition between walking and running,[3,5] is optimized for this model.

The paper is structured as follows. The models for walking and running are developed in Sec. 2. In Sec. 3 numerical optimization is used for generating energy efficient periodic gaits. In Sec. 4 periodic walking and running gaits from 1.5 to 2.5 m/s are compared from the energetic point of view and explained at the transition velocity. The results are discussed and the paper is concluded in Sec. 5.

2. Methodology

First the mechanical model is derived. The gaits are described by hybrid models consisting of continuous phases and switching between them. A nonlinear controller capable of tracking reference trajectories is applied using input-output linearization. The trajectory tracking system is stabilized by an additional linear controller.

The mechanical model is underactuated meaning the I/O-linearization preserves inner degrees of freedom, the zero dynamics, which cannot be influenced by the linear controller. Just like the controlled system, the zero dynamics is hybrid in nature. Investigation of the hybrid zero dynamics suggests a low-dimensional model for periodic solutions which is utilized in numerical optimization.

2.1. *Mechanical model*

A planar model of a five-link robot with rigid links for trunk, thighs and shanks connected by actuated revolute joints is introduced. The model has point feet, meaning it cannot transmit torque between the legs and the ground directly. Thus the stance leg is linked to the ground by an unactuated ideal revolute joint in the single support phase (Fig 1a)).

To simulate the two different gaits, two configurations of the mechanical model corresponding to the different phases of locomotion are considered:

Walking consists of one phase, the single support phase, and an instantaneous impact switching from one step to the next when the swing leg touches down at the end of the step.

Running consists of two phases: the single support phase which is analogous to walking, and the flight phase where the model is detached from the ground. Switching from single support to flight is described by the lift-off; from flight to single support by the impact of the foot.

The model for the flight phase is described by seven generalized coordinates, three absolute coordinates $q_{a,f}$ and four body coordinates q_b, whereas the stance phase requires only one absolute angle $q_{a,s}$. The generalized coordinates are then $q = [q_a \; q_b]^T$ (indices s/f: stance/flight phase).

The different continuous phases and discrete transitions are first treated separately and then combined to the hybrid models in Fig. 1bc).

2.1.1. *Continuous phases*

The continuous phases for single support and flight are described by their respective equations of motion. These are derived using the Euler-Lagrange-Formalism and transformed to the *Mixed-Partial-Feedback-Linearized* normal form.[6] This leads to the state space differential equation

$$\dot{x} = f(x) + g(x)\, v \; . \tag{1}$$

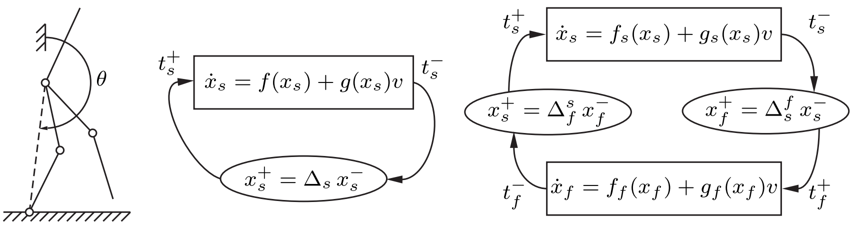
with the input v which is the vector of angular accelerations in the actuated joints. The inverse dynamics supplies the corresponding joint torques u.

2.1.2. *Discrete transitions*

There are two discrete transitions for running – lift-off and touchdown, the later one modeled as impact – and one for walking – the impact at the end of the step.

The lift-off is straightforward: the coupling of the foot to the ground is released demanding vanishing normal force in the ground contact and smoothness in all state space variables.

The impact however requires a more detailed description:[6] assuming a perfectly inelastic collision of the foot with the ground, the velocities change instantaneously. The impact map is depicted as $x^+ = \Delta\, x^-$.



a) stance phase b) walking c) running

Fig. 1. a) Stance phase model and hybrid models for b) walking and c) running.

2.2. *Controller*

Applying input-output linearization to the *MPFL* normal form eq. (1), a nonlinear controller is set up to track reference trajectories for the actuated joint angles q_b. The reference trajectories are parameterized by Bézier polynomials of degree M as

$$h_d(\alpha_k, s) = \sum_{k=0}^{M} \alpha_k \binom{M}{k} (s)^k (1-s)^{M-k}, \ s_s = \frac{\theta - \theta^+}{\theta^- - \theta^+}, \ s_f = \frac{t_f}{T_f} \ . \quad (2)$$

θ^- and θ^+ are the values of θ (Fig. 1a)) at the beginning t_s^+ and the end t_s^- of the stance phase and T_f is the flight duration. These definitions normalize s to the interval $[0, 1]$ in both phases; α_k are the Bézier coefficients.

Introducing the deviation from the reference trajectories $y = h = q_b - h_d$ as output of the system the controller is[6]

$$v = L_g L_f h^{-1} \left(\ddot{y} - L_f^2 h\right) \ . \quad (3)$$

To ensure attractivity of the input-output linearized system an additional linear controller $\ddot{y} = f(y, \dot{y})$ is introduced. The controlled system for walking is illustrated in Fig. 2.

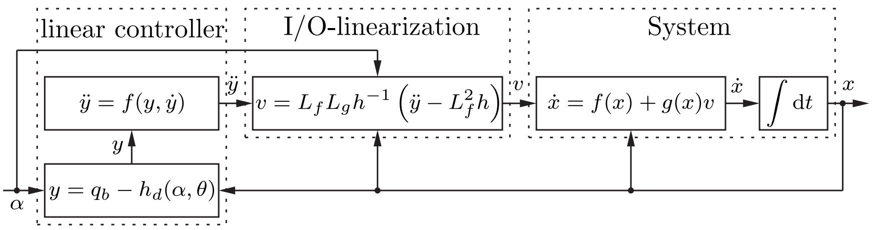


Fig. 2. Input-output linearized system with additional linear controller.

2.3. *Hybrid Zero Dynamics*

Since the system is modeled as underactuated, even perfect tracking of the reference trajectories in the joint angles does not determine the absolute orientation of the robot. Therefore, the rotation of the total system remains as an inner degree of freedom, the zero dynamics, of the model which the linear controller does not observe and hence cannot control. The evolution of the zero dynamics yields the desired walking or running motion.

The analysis of the system can be reduced to the analysis of the zero dynamics if they are attractive which is ensured by the linear controller and if they are invariant with respect to the impact. This imposes further conditions on the coefficients α_k. Applying a perfect linear controller $q_b \equiv h_d$ to the system in Fig. 2a), the hybrid zero dynamics of the stance phase

$$z_s = \begin{bmatrix} \theta \\ L \end{bmatrix} , \qquad \dot{z}_s = \begin{bmatrix} \kappa_{1,s}(\theta)\ L \\ \kappa_{2,s}(\theta) \end{bmatrix} \tag{4}$$

remains, where $\kappa_{1,s}$ and $\kappa_{2,s}$ are functions of the absolute angle θ only and L is the conjugate angular momentum. Eliminating the time derivatives in Eq. (4) and integrating yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}L} = \frac{\kappa_{1,s}(\theta)\ L}{\kappa_{2,s}(\theta)} \qquad \Rightarrow \qquad (L^-)^2 = (L^+)^2 + \int_{\theta^+}^{\theta^-} \frac{\kappa_{2,s}(\theta)}{\kappa_{1,s}(\theta)} \mathrm{d}\theta\ . \tag{5}$$

Moreover, Westerveld *et al.*[6] describe a method to derive the impact map for the angular momentum, meaning the dependency $L^+ = \delta_s(L^-)$ can be determined directly. This means the reference trajectory corresponding to stable, periodic walking or running gaits can be evaluated using Gaussian quadrature in Eq. (5).

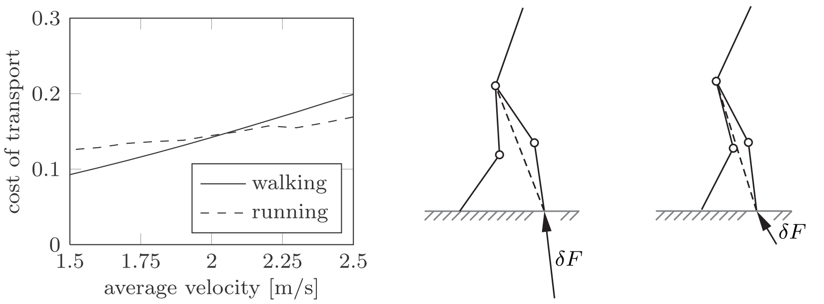
The zero dynamics for the flight phase can be derived analogously.

3. Optimization

Equation (5) in combination with the reference trajectories Eq. 2 allows for computing of the most energy efficient gait using parameter optimization. For this purpose *MATLAB*'s `fmincon` solver with the included *SQP* algorithm is utilized. As objective function the dimensionless cost of transport

$$cot = \frac{\sum_{i=1}^{4} \int_{t^+}^{t^-} \max(c_P\, u_i^2 + u_i\, \dot{q}_{b,i}, 0)\mathrm{d}t}{m_{tot}\ g_0\ \ell_{step}} \tag{6}$$

defined as the quotient of used energy in one step and weight $m_{tot} g_0$ times step length ℓ_{step} is introduced. This definition for the cost of transport assumes electric motors as actuators and calculates the supplied electric work



a) comparison: *cot* b) impact: walking c) impact: running

Fig. 3. a) Comparison of cost of transport for walking and running; and configurations at the time of impact for b) walking and c) running with impact force vector δF.

assuming no energy can be recuperated in generator mode. The lengths, masses and inertias of the robot's links are set to match an average man with $h = 1.8$ m and $m_{tot} = 80$ kg.[7]

4. Results

Figure 3a) shows the cost of transport for optimized periodic gaits with average velocities in a range from 1.5 to 2.5 m/s with Bézier polynomials of degree $M_s = 6$ in the stance phase and $M_f = 3$ in the flight phase.

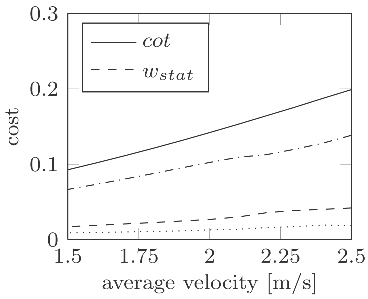
For the transition velocity of 2.05 m/s the two curves intersect. Regarding the composition of the cost of transport for walking (Fig. 4a)) and running (Fig. 4b)) in detail, the main difference appears in the factors

$$w_{stat} = cot - \sum_{i=1}^{4} \int_{t^+}^{t^-} \max(u_i\, \dot{q}_{b,i}, 0)\mathrm{d}t / \left(m_{tot}\, g_0\, \ell_{step}\right) \ , \tag{7a}$$

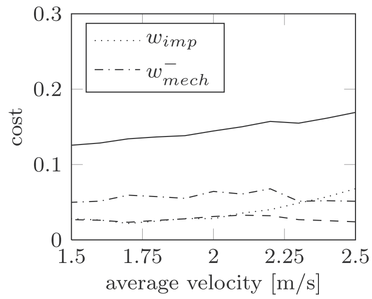
$$w_{imp} = \left(E_{kin}^{+} - E_{kin}^{-}\right) / \left(m_{tot}\, g_0\, \ell_{step}\right) \ , \tag{7b}$$

$$w_{mech}^{-} = \sum_{i=1}^{4} \int_{t^+}^{t^-} \min(u_i\, \dot{q}_{b,i}, 0)\mathrm{d}t / \left(m_{tot}\, g_0\, \ell_{step}\right) \ . \tag{7c}$$

The static work w_{stat} is the cost of transport minus the specific positive mechanical work. This part of the supplied electric energy does not contribute to the mechanical work and is lost due to the electric resistance. The impact loss is the specific change in kinetic energy E_{kin} before and after the impact. w_{mech}^{-} is the specific negative mechanical work used for braking during the gait.



a) walking



b) running

Fig. 4. Cost of transport compared to w_{stat} and w_{imp} for a) walking and b) running.

Figure 4a) shows that in walking, the specific static work increases with the average velocity, whereas the impact loss rises only very slightly. Figure 4b) shows the contrary correlation for running meaning the specific static work is almost constant over the regarded velocity range, but the impact loss increases significantly.

Comparing the magnitudes of both factors near the transition velocity of 2 m/s (Tab. 1) reveals two different strategies of the optimization algorithm: walking gaits try to minimize the amount of work lost in the impact whereas running gaits reduce specific static and negative mechanical work.

The gaits differ in another parameter as well: δ is the quotient of the angular momentum before and after impact (Tab. 1). The value is significantly higher for running which means the loss of angular momentum is lower although the impact losses w_{imp} are higher.

Figure 3bc) shows the configuration of both gaits at the time of impact. The impact force magnitude and direction differs considerably. Whereas the impact force is aligned to the impacting shank in case of walking it does not show this behavior for running where the horizontal and vertical components of the force are of similar magnitude. The impact force in running is also considerably smaller than for walking.

Both configurations look similar, though in walking the step length is

Table 1. Comparison of walking and running at 2 m/s

	cot	w_{stat}	w_{imp}	δ	u_{max}	ℓ_{step}	f_{step}
walking	0.1421	0.0577	0.0129	0.9443	666 Nm	0.55 m	3.7 Hz
running	0.1421	0.0279	0.0282	0.9836	248 Nm	0.47 m	4.3 Hz

bigger. This leads to higher hip joint accelerations since the bigger spread angle of the legs has to be passed in a similar time. In Tab. 1 the maximum hip torques u_{max}, the step lengths ℓ_{step} and frequencies f_{step} are listed.

5. Discussion and Conclusion

The results based on the optimization of cost of transport are in good accordance with the transition velocity of about 2 m/s reported in human locomotion.[8] The main characteristic of the observed gaits originates from the definition of the cost of transport as specific positive electrical power consumed by the actuators. This kind of cost function is not limited to electric motors but was also supposed to describe muscles since they consume energy when they generate a static force, too.[5] This especially separates the presented investigation from the approach to use only the torques squared.[9]

The results of this paper indicate that the presented model of a planar five-link robot with ideal revolute joints is suitable for the investigation of bipedal locomotion and the underlying mechanisms as well as the design and gait optimization of robots.

Further investigation of the underlying mechanisms of energy consumption in both walking and running gaits can be conducted with the presented model. Especially the differences of both models at the time of impact have not been explained completely.

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