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Effective photon mass from black-hole formation

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Abstract

We compute the value of effective photon mass m_γ at one-loop level in QED in the background of small (10^{10} g $\lesssim M \ll 10^{16}$ g) spherically symmetric black hole in asymptotically flat spacetime. This effect is associated with the modification of electron/positron propagator in presence of event horizon. Physical manifestations of black-hole environment are compared with those of hot neutral plasma. We estimate the distance to the nearest black hole from the upper bound on m_γ obtained in the Coulomb-law test. We also find that corrections to electron mass m_e and fine structure constant α at one-loop level in QED are negligible in the weak gravity regime.

1. Introduction

In Minkowski space the photon acquires an effective (thermal) mass if it propagates through a (neutral) plasma of electrons and positrons held at high enough temperature, i.e. $T \gg m_e$. The effective photon mass m_γ turns out to be proportional to the temperature T of the electron–positron plasma at one-loop level [1] (see also [2,3]). This effect is exponentially suppressed if the plasma is cold, i.e. $T \ll m_e$ [4–6]. This occurs because most of electrons and positrons are in the ground state at low temperature. This leads in turn to suppression of photon–electron and photon–positron scattering events with respect to the photon–photon scattering which is the higher-loop effect [7]. In summary, we have

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$$m_\gamma^2 \approx e^2 T^2 \begin{cases} 1/6\,, & T \gg m_e\,, \\ 4(m_e\beta)^{\frac{1}{2}}\,\exp(-m_e\beta)/(2\pi)^{\frac{3}{2}}\,, & T \ll m_e \end{cases} \tag{1}$$

at one-loop approximation in quantum electrodynamics (QED), where $\beta \equiv 1/T$ is the inverse temperature.

There is a comparably recent idea of the assignment of readings of the macroscopic thermometer with the so-called Wick squared operator [8,9]. This is also known as the local temperature operator. Specifically, if one treats a scalar field model with the conformal coupling to gravity, then one can find that $\langle \hat{\Phi}^2(x)\rangle = T^2/12$ in a thermal state characterised by the temperature T. This was also generalized and treated in curved spacetimes [10–13]. Certain applications in flat space were studied in [14].

Considering a scalar non-interacting field model with mass $m = m_e$, one can obtain

$$\langle \hat{\Phi}^2(x)\rangle \approx \frac{1}{2}T^2 \begin{cases} 1/6\,, & T \gg m_e\,, \\ 2(m_e\beta)^{\frac{1}{2}}\,\exp(-m_e\beta)/(2\pi)^{\frac{3}{2}}\,, & T \ll m_e\,, \end{cases} \tag{2}$$

for the renormalized value of the Wick squared operator in the thermal state described by the temperature T. Thus, a quantitative discrepancy arises between the effective photon mass squared and $\langle \hat{\Phi}^2(x)\rangle$ at low temperatures. In fact, these quantities are diverse both physically and mathematically.

Nevertheless, it is tempting to conjecture that $m_\gamma^2 \propto \langle \hat{\Phi}^2(x)\rangle$ holds *qualitatively* at high temperatures at the α-order approximation in quantum electrodynamics. If one takes this relation for granted, then one can predict that the photon acquires an effective mass, for instance, in the background of small Schwarzschild black holes, i.e. $T_H \gg m_e$ and $M \gg M_{\rm Pl}$, where $T_H = M_{\rm Pl}^2/(8\pi M)$ is the Hawking temperature and $M_{\rm Pl} = (\hbar c/G)^{\frac{1}{2}}$ is the Planck mass. For these black holes, the size of the event horizon is $r_H = 2MG/c^2 \ll 3\times 10^{-14}$ m.[1]

Indeed, if we consider eternal Schwarzschild geometry with a black hole of mass M, then physical vacuum corresponds to the Hartle–Hawking state. Far away from the black hole, i.e. $r \gg r_H$, we have $\langle \hat{\Phi}^2(x)\rangle_H \approx T_H^2/12$ for the scalar field model conformally coupled to gravity [16]. If the black hole has formed through the gravitational collapse, then one might expect the photon possesses an effective mass decreasing with distance as $1/r$, because $\langle \hat{\Phi}^2(x)\rangle_U \propto T_H^2(2M/r)^2$ in the Unruh state [16]. We have actually surmised recently this dependence of m_γ on the distance to the black hole from a different perspective [17].

In this paper we analytically derive the effective photon mass m_γ at one-loop level in QED in asymptotically flat spacetime with a small spherically symmetric black hole ($M_{\rm Pl} \ll M \ll 10^{21} M_{\rm Pl}$). We find that the above relation between the effective photon mass m_γ and the expectation value of the Wick squared operator $\langle \hat{\Phi}^2(x)\rangle$ *does* qualitatively hold in the high-temperature limit, i.e. $T_H \gg m_e$, for the Hartle–Hawking and Unruh state.

We shall also show that the analogy between the hot plasma and the environment of a small black hole formed through the gravitational collapse is incomplete in that the black-hole environment cannot support plasmon-like and plasmino-like excitations. However, a point-like electric charge can be partially screened due to the modification of the electric permittivity and the magnetic permeability of the vacuum in the black-hole background.

[1] We assume such black holes exist in nature which were formed through gravitational collapse under extreme conditions present in early universe [15].

Throughout this paper the fundamental constants are set to unity, $c = G = k_{\rm B} = \hbar = 1$, unless stated otherwise.

2. Effective photon mass

To compute the photon self-energy at one-loop level in the background of evaporating Schwarzschild black hole, we need to have the free fermion propagator $S(x,x')$. Since the Dirac equation $\big(i\nabla\!\!\!/ - m_e\big)\psi(x) = 0$ can also be written as $\big(\Box + m_e^2\big)\psi(x) = 0$, it is enough, however, to deduce the scalar propagator $G(x,x')$. Indeed, the propagator $S(x,x')$ can then be obtained by acting on $G(x,x')$ by the operator $i\nabla\!\!\!/ + m_e$ (e.g., see [18]).

2.1. Scalar Wightman function

As pointed out above, we need to compute the scalar two-point function in spacetime with the Schwarzschild black hole of mass M. We start with a massive scalar field

$$\big(\Box + m_e^2\big)\Phi(x) = 0\,, \tag{3}$$

and look for positive frequency modes in the following form

$$\Phi_{kjm}(x) = \frac{1}{(4\pi\omega)^{\frac{1}{2}}}\frac{e^{-i\omega t}}{r}R_{kl}(r)Y_{lm}(\theta,\phi)\,, \tag{4}$$

where $\omega = (k^2 + m_e^2)^{\frac{1}{2}}$ and $Y_{lm}(\theta,\phi)$ are the spherical harmonics. Substituting (4) in the scalar field equation (3), we obtain

$$\frac{d^2}{dr_*^2}R_{kl}(r) + f(r)\left(\frac{\omega^2}{f(r)} - \frac{l(l+1)}{r^2} - m_e^2 + \frac{f'(r)}{r}\right)R_{kl}(r) = 0\,, \quad f(r) = 1 - \frac{r_H}{r}\,, \tag{5}$$

where $r_* = r + r_H \ln(r/r_H - 1)$ is the Regge–Wheeler radial coordinate and the prime stands for differentiation with respect to r. There are two types of radial modes, namely the ingoing and outgoing one. We denote these as $\overleftarrow{R}_{\omega l}(r)$ for the ingoing modes and $\vec{R}_{\omega l}(r)$ for the outgoing modes. The Wightman two-point function, e.g. in the Boulware (B) state, is then

$$\langle\hat{\Phi}(x)\hat{\Phi}(x')\rangle_B = \sum_{lm}\int\frac{d\omega}{4\pi\omega}\frac{e^{-i\omega\Delta t}}{rr'}Y_{lm}(\Omega)Y_{lm}^*(\Omega')\Big(\vec{R}_{\omega l}(r)\vec{R}^*_{\omega l}(r') + \overleftarrow{R}_{\omega l}(r)\overleftarrow{R}^*_{\omega l}(r')\Big), \tag{6}$$

where $\Delta t = t - t'$ by definition. The sum over m can be performed and yields

$$\sum_{m=-l}^{m=+l} Y_{lm}(\Omega)Y_{lm}^*(\Omega') = \frac{2l+1}{4\pi}P_l(\cos\Theta)\,, \tag{7}$$

where $\cos\Theta \equiv \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi')$ and $P_n(x)$ is the Legendre polynomial.

It is hardly possible to solve the radial mode equation (5) analytically, but one can always do that numerically. However, employing results of [16,19,20], we obtain in the limit of vanishing mass of the scalar field ($m_e \to 0$) that

$$\vec{K}_\omega(\mathbf{x},\mathbf{x}') \equiv \frac{1}{4\pi r r'}\sum_{l=0}^{+\infty}(2l+1)\vec{R}_{\omega l}(r)\vec{R}^{*}_{\omega l}(r')P_l(\cos\Theta)$$
$$\approx \frac{\Delta^{\frac{1}{2}}(\rho)\sin(\omega\rho)}{4\pi\omega\rho(f(r)f(r'))^{\frac{1}{2}}}\begin{cases} 4\omega^2 - \frac{(f(r)f(r'))^{\frac{1}{2}}}{rr'}\Gamma_\omega\,, & r\to 2M\,,\\ \frac{(f(r)f(r'))^{\frac{1}{2}}}{rr'}\Gamma_\omega\,, & r\gg 2M\,,\end{cases} \tag{8}$$

and

$$\overleftarrow{K}_\omega(\mathbf{x},\mathbf{x}') \equiv \frac{1}{4\pi r r'}\sum_{l=0}^{+\infty}(2l+1)\overleftarrow{R}_{\omega l}(r)\overleftarrow{R}^{*}_{\omega l}(r')P_l(\cos\Theta)$$
$$\approx \frac{\Delta^{\frac{1}{2}}(\rho)\sin(\omega\rho)}{4\pi\omega\rho(f(r)f(r'))^{\frac{1}{2}}}\begin{cases} \frac{(f(r)f(r'))^{\frac{1}{2}}}{rr'}\Gamma_\omega\,, & r\to 2M\,,\\ 4\omega^2 - \frac{(f(r)f(r'))^{\frac{1}{2}}}{rr'}\Gamma_\omega\,, & r\gg 2M\,,\end{cases} \tag{9}$$

where $\rho \equiv (2\sigma(\mathbf{x},\mathbf{x}'))^{\frac{1}{2}}$, $\sigma(\mathbf{x},\mathbf{x}')$ is the three-dimensional geodetic interval for the ultrastatic or optical metric $\bar{g}_{\mu\nu} = g_{\mu\nu}/f(r)$, $\Delta(\mathbf{x},\mathbf{x}')$ is the Van Vleck determinant [21] and

$$\Gamma_\omega \equiv \sum_{l=0}^{+\infty}(2l+1)|B_{\omega l}|^2 \approx 27\omega^2M^2 \tag{10}$$

in the DeWitt approximation [19].

The scalar two-point function in the case when the outgoing and ingoing modes are "heated up" to inverse temperatures β_1 and β_2, respectively, is

$$W_{\beta_1,\beta_2}(x,x') = \vec{W}_{\beta_1}(x,x') + \overleftarrow{W}_{\beta_2}(x,x') \tag{11}$$
$$\approx \int_0^{+\infty} d\omega\left(\frac{\cos\left(\omega\Delta t + i\,\frac{\omega\beta_1}{2}\right)}{4\pi\omega\sinh\left(\frac{\beta_1\omega}{2}\right)}\vec{K}_\omega(\mathbf{x},\mathbf{x}') + \frac{\cos\left(\omega\Delta t + i\,\frac{\omega\beta_2}{2}\right)}{4\pi\omega\sinh\left(\frac{\beta_2\omega}{2}\right)}\overleftarrow{K}_\omega(\mathbf{x},\mathbf{x}')\right).$$

The Hartle–Hawking state corresponds to $\beta_1 = \beta_2 = \beta = 2\pi/\kappa$, where κ is a value of the surface gravity on the horizon $r = r_H$ [22]. The Boulware state follows from the Hartle–Hawking one if we set $\beta_1 = \beta_2 = +\infty$ [23]. The physical state for the black holes formed through the gravitational collapse is known as the Unruh state [24]. This corresponds to $\beta_1 = \beta$ and $\beta_2 = +\infty$.

We now define the commutator function

$$C(x,x') = \vec{C}(x,x') + \overleftarrow{C}(x,x') \tag{12}$$

which will be used below, where

$$\vec{C}(x,x') = \vec{W}_{\beta_1}(x,x') - \vec{W}_{\beta_1}(x',x) \approx \int_0^{+\infty}\frac{d\omega}{4\pi\omega}\left(e^{-i\omega\Delta t} - e^{+i\omega\Delta t}\right)\vec{K}_\omega(\mathbf{x},\mathbf{x}')\,, \tag{13a}$$

$$\overleftarrow{C}(x,x') = \overleftarrow{W}_{\beta_2}(x,x') - \overleftarrow{W}_{\beta_2}(x',x) \approx \int_0^{+\infty}\frac{d\omega}{4\pi\omega}\left(e^{-i\omega\Delta t} - e^{+i\omega\Delta t}\right)\overleftarrow{K}_\omega(\mathbf{x},\mathbf{x}')\,. \tag{13b}$$

It is worth noting that the commutator functions defined in the above manner do not depend on the temperatures.[2] In general, this occurs because the commutator of the field operators plays a role of the algebraic structure of the algebra of local field operators. This structure is independent of Fock space representations of the algebra. Hence, it remains the same, for instance, independent of whether one treats the Boulware or Unruh state.

2.2. Spinor Feynman propagator

In general, fermion anti-commutation function $C_f(x, x')$ is related with the scalar commutator function as follows

$$C_f(x, x') = \vec{C}_f(x, x') + \overleftarrow{C}_f(x, x') = \big(i\nabla\!\!\!\!/ + m_e\big)C(x, x'), \tag{14}$$

where

$$\vec{C}_f(x, x') = \vec{S}_{\beta_1}(x, x') + \vec{S}_{\beta_1}(x', x), \tag{15a}$$
$$\overleftarrow{C}_f(x, x') = \overleftarrow{S}_{\beta_2}(x, x') + \overleftarrow{S}_{\beta_2}(x', x). \tag{15b}$$

To compute one-loop contribution to the photon self-energy, one needs to find the Feynman propagator $S(x, x')$. This can be expressed through the anti-commutation function [25]. Specifically, we have

$$S(\omega|\mathbf{x}, \mathbf{x}') = \int \frac{d\omega'}{2\pi}\frac{iC_f(\omega'|\mathbf{x}, \mathbf{x}')}{\omega - \omega' + i\varepsilon} - n_{\beta_1}(\omega)\vec{C}_f(\omega|\mathbf{x}, \mathbf{x}') - n_{\beta_2}(\omega)\overleftarrow{C}_f(\omega|\mathbf{x}, \mathbf{x}'), \tag{16}$$

where $\varepsilon \to +0$, the integral is over all ω' lying in $\mathbf{R}$,

$$n_\beta(\omega) = \frac{1}{e^{\beta\omega} + 1}, \tag{17}$$

and $C_f(\omega|\mathbf{x}, \mathbf{x}')$ is the Fourier transform over time of the anti-commutator function, i.e.

$$C_f(\omega|\mathbf{x}, \mathbf{x}') \equiv \int d\Delta t\, e^{+i\omega\Delta t} C_f(x, x') = \int d\Delta t\, e^{+i\omega\Delta t}\big(i\nabla\!\!\!\!/_x + m_e\big)C(x, x'),$$

where $C(x, x')$ is the scalar commutator given in (12).

2.3. Photon self-energy at one-loop level

In terms of the photon Feynman propagator, we have up to the α^2-order in the perturbation theory

$$G_\alpha^{\mu\nu}(x, x') = G^{\mu\nu}(x, x') - 4\pi\alpha \int dx_1 dx_2 (g(x_1)g(x_2))^{\frac{1}{2}} \times G^{\mu\lambda}(x, x_1)\,\mathrm{Tr}\Big(\gamma_\lambda S(x_1, x_2)\gamma_\rho S(x_2, x_1)\Big)\, G^{\rho\nu}(x_2, x') + \mathrm{O}\big(\alpha^2\big), \tag{18}$$

where we expect up to the α^2-order that

$$\Big(\Box_x \delta^\mu_\lambda + \big(m_\gamma^2\big)^\mu_\lambda\Big) G_\alpha^{\lambda\nu}(x, x') = \frac{ig^{\mu\nu}(x)}{(-g(x))^{\frac{1}{2}}}\,\delta\big(x - x'\big), \tag{19}$$

[2] This is true for non-interacting theories or in the leading order approximation of the perturbation theory.

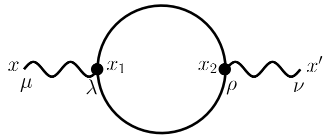

Fig. 1. One-loop vacuum polarization diagram.

and $G^{\mu\nu}(x, x')$ satisfies this equation at the zeroth order in the fine structure constant α. To find the effective photon mass squared, we thus need to compute

$$\big(m_\gamma^2\big)^\mu_\lambda G^{\lambda\nu}(\omega|\mathbf{x}, \mathbf{x}') = -4\pi\alpha \int d\mathbf{y}\sqrt{-g(\mathbf{y})}\, K^\mu_\lambda(\omega, \mathbf{x}, \mathbf{y}) G^{\lambda\nu}(\omega|\mathbf{y}, \mathbf{x}') + \mathrm{O}\big(\alpha^2\big)\,, \tag{20}$$

where by definition

$$K^\mu_\lambda(\omega, \mathbf{x}, \mathbf{y}) = \int \frac{d\Omega}{2\pi}\,\mathrm{Tr}\Big(\gamma^\mu S(\Omega|\mathbf{x}, \mathbf{y})\gamma_\lambda S(\Omega - \omega|\mathbf{y}, \mathbf{x})\Big)\,. \tag{21}$$

Since the black hole is small, we are working in the high-temperature limit. Consequently, one is allowed to use the hard thermal loop approximation [2,3]. In other words, we omit the electron mass m_e in the fermion correlation function as well as the frequency ω as being negligible with respect to the temperature parameter T_H. Therefore, it is legitimate to employ the correlation function found above in the limit $m_e \to 0$ to obtain the effective photon mass in the temperature regime $T_H \gg m_e$ and $T_H \gg \omega$.

Minkowski space One can employ the Feynman propagator given in (16) to compute the effective photon mass at one-loop level in the high-temperature limit. This is achieved through setting $\beta_1 = \beta_2 = 1/T < \infty$ and substituting $M = 0$ in (8) and (9). The vanishing mass of the black hole implies that $\Gamma_\omega = 0$, because $\Gamma_\omega \approx 27\omega^2 M^2$ in the DeWitt approximation [19].

Our computation of the photon mass in the hot plasma will be non-standard if we work in the spherical coordinates. Nevertheless, we still have the result $m_\gamma^2 = e^2 T^2/6$ as the theory is covariant.

In Minkowski space we can express (20) and (21) through the Cartesian coordinates and then perform the standard evaluations of the integrals. However, we can do the same when $M \neq 0$ far away from the event horizon $r \gg r_H$. Comparing then the right-hand sides of (20) in Minkowski space and Schwarzschild space far from the black hole, we can immediately obtain the photon mass m_γ due to the black hole in the weak gravity regime.

Schwarzschild space The scalar propagator can be computed exactly in Minkowski space. This is not the case in Schwarzschild space even in the limit of the vanishing electron mass m_e. Our expressions for $\vec{K}_\omega(\mathbf{x}, \mathbf{x}')$ and $\overleftarrow{K}_\omega(\mathbf{x}, \mathbf{x}')$ given in (8) and (9), respectively, are reliable whenever the points $\mathbf{x}$ and $\mathbf{x}'$ are close to each other.

The physical idea now is to notice that although one must integrate in (20) over all values of $\mathbf{y}$, the main contribution to the integral will be from the spacial region in the vicinity of the point $\mathbf{x}$. In other words, the virtual electron–positron pair depicted in Fig. 1 is a short-time or local event in spacetime.

This can also be exemplified by the computation of the photon self-energy in-between conducting plates in the Casimir set-up. The photon propagator in-between the conducting plates differs from that in Minkowski space due to the non-trivial boundary conditions satisfied by the electromagnetic four-potential operator on the plates. The two-loop contribution to the photon self-energy after renormalization is then non-trivial, because of the internal photon propagator in the loops [26]. In the coordinate representation of the loop integrals, one would need to integrate over the whole spacetime. However, the main contribution will be when the vertices are close to each other. For instance, the result will be independent of the contributions from the points outside of the plates, where the photon propagator differs from that in-between the plates. This is fully consistent with computations based on the effective action of the electromagnetic field (fermion degrees of freedom are integrated out) at one-loop level in which one merely needs to have the propagator at space–time points being close to each other [27].

This argument can also be supported by the empirical observations in the particle physics. Indeed, we have been successfully employing the Minkowski-space approximation in studying various processes in the particle colliders. However, the universe is non-flat at cosmological scales. According to the equivalence principle, nevertheless, there always exists a local Minkowski frame. Therefore, the description of the scattering processes in QFT is performed in the local Minkowski frame as if it is of the infinite extent. This is an adequate approximation whenever relevant physics is characterised by a length scale being much smaller than a characteristic curvature scale. In our case, this length is $l_c = R(R/r_H)^{1/2}$, where R is the distance to the black-hole centre.

Thus, we find that the (massless) scalar Feynman propagator in momentum space far away from the black-hole horizon is approximately given by

$$G_U(k,k') \approx \left(\frac{i}{k^2 + i\varepsilon} + 2\pi\left(\frac{27M^2}{4R^2}\right) n_\beta(\omega)\delta\big(k^2\big)\right)\delta\big(k-k'\big) \tag{22}$$

at $T_H \gg m_e$ and $T_H \gg \omega$ in the Unruh state, where $k = (\omega, \mathbf{k})$. It is worth noticing that $G_U(k,k')$ reduces to the ordinary scalar propagator in Minkowski space in the limit of the vanishing black-hole mass $M \to 0$ or $R \to \infty$. This does not happen to be the case for the eternal black hole described by the Hartle–Hawking state.

Having derived the propagator (22), we can now obtain m_γ at the one-loop approximation. For the black hole formed through the gravitational collapse, we find

$$m_\gamma^2\Big|^{\text{electron–positron}} \approx \frac{27\pi\alpha}{24}\,T_H^2\left(\frac{r_H}{R}\right)^2 + \mathrm{O}\left(\frac{T_H r_H^2}{R^3}\right) \tag{23}$$

far from the hole ($R \gg r_H$) in the high-temperature limit ($T_H \gg m_e$ and $M \gg M_{\rm Pl}$).[3] The formula (23) is the main result of our paper. It is worth noticing that m_γ does not directly depend on the black-hole mass M at one-loop level in the leading order of the weak gravity limit.[4] This does not mean $m_\gamma \neq 0$ for $M = 0$, because Eq. (23) has been computed under the assumption $M \neq 0$. Taking the limit $M \to 0$ and evaluating the integral (21) are not commuting operations. It is worth reminding that the renormalised stress tensor depends on M as $1/M^2R^2$, so that it is non-vanishing in the limit $M \to 0$.

[3] The leading-order correction to the first term in (23) is due to the action of $\not\nabla$ on the prefactor r_H^2/R^2. The next-to-leading term is of the order of r_H^3/R^4.

[4] Note that we have taken into account only electron–positron virtual pair to the photon self-energy. For instance, the same result holds for the virtual muon–antimuon pair, but then the black hole should be smaller $T_H \gg m_\mu \gg m_e$ for not having exponentially suppressed contribution of this pair. If $T_H \gg m_\mu$ holds, then m_γ^2 is 2 times larger.

For the Hartle–Hawking state we obtain the standard result for m_γ far away from the event horizon like in the hot physical plasma in Minkowski space. Hence, small eternal black holes would considerably influence photon kinematics. This is not a problem, because these black holes are not realizable through the gravitational collapse anyway.

In the Standard Model the electromagnetic field corresponds to the $U(1)$ gauge group under which the vacuum is invariant. This group is a subgroup of the spontaneously broken electroweak symmetry $SU(2)_L \times U(1)_Y$ with the electroweak phase transition occurring at the electroweak energy scale $M_{\rm EW} \approx 10^2$ GeV. The temperature parameter T_H for the small black holes is greater than $M_{\rm EW}$ if the black-hole mass $M \lesssim 10^{15} M_{\rm Pl}$. It is not obvious whether one can rely on (23) when the black-hole mass is smaller than $10^{15} M_{\rm Pl}$. However, we expect that (23) is still reliable at least far from the black hole, because all physical parameters are then small (see below). It is still not excluded that the phase transition may occur at the distance $r_{ew} \approx 10^{-19}$ m for black holes of mass M in the range $M_{\rm Pl} \ll M \ll 10^{16} M_{\rm Pl}$. Note that the phase transition should occur far from the horizon as $r_H \ll r_{ew}$. A similar observation was made long ago in [29,30].

We have focused on the black holes of mass $M \ll 10^{16}$ g for which $T_H \gg m_e$ and implicitly presumed that the quasi-equilibrium approximation holds, i.e. spacetime is quasi-static. For sufficiently small black holes, this approximation does not hold, because of the black-hole evaporation. This implies that m_γ given in Eq. (23) should be a reliable result for M in the range

$$10^{10}\ \text{g} \lesssim M \ll 10^{16}\ \text{g}\,, \tag{24}$$

where we have chosen the lower bound on M by requiring that the smallest black hole has at least a one-day lifetime (assuming the evaporation lasts till the complete disappearance of the black hole). This range corresponds to $3\times10^{14}\ \text{g} \lesssim M_0 \ll 10^{16}$ g of the initial mass of the primordial black holes.

3. Concluding remarks

The space–time structure significantly modifies when a black hole forms. The algebraic structure of a set of the quantum field operators also modifies. As a consequence, propagators of quantum fields have a different form in comparison with that in Minkowski space. Far away from the black holes, one might expect that quantum field theory becomes almost indistinguishable from its formulation in Minkowski spacetime. Indeed, quantum field theory formulated in Minkowski space is well tested and verified in the particle colliders, although there are a lot of gravitational sources in our universe which make the geometry of spacetime be of non-Minkowskian form at sufficiently large length scales.

Although it is legitimate to await of recovering Minkowskian quantum field theory far away from the black holes, there must be specific imprints of these gravitational sources in physical experiments performed on earth. In this paper, we have investigated these imprints of the small spherically symmetric black hole ($10^{10}\ \text{g} \lesssim M \ll 10^{16}$ g) on the effective photon mass at the α-order approximation in QED. Physically, it might be a consequence of the event-horizon formation which leads to the modification of the quantum field operators (as the field equations explicitly depend on the black-hole mass M). Assuming the process of the black-hole formation is unitary, the total quantum system (gravity and matter fields) evolves semi-classically if the backreaction of the quantum fields on the geometry is small. This is usually described by saying that the quantum fields occupy the Unruh state [24]. It inevitably implies the presence of the thermal-like correction in Eq. (22) yielding $m_\gamma \neq 0$.[5]

The Wick squared operator cannot always be interpreted as a macroscopic temperature squared [12]. Moreover, the effective photon mass m_γ for the Boulware state vanishes. However, the Wick squared $\langle\hat{\Phi}^2(x)\rangle_B$ is non-zero and negative. Specifically, $\langle\hat{\Phi}^2(x)\rangle_B \propto -T_H^2(2M/r)^4$ as this can be shown employing the Page approximation [20]. Therefore, the qualitative validity of the relation between m_γ^2 and $\langle\hat{\Phi}^2(x)\rangle$ is counter-intuitive for this state. It should be noted, however, that all divergencies in the evaluation of m_γ has been subtracted, such that m_γ vanishes in the Boulware state. The result of this renormalization is finite and depends on the parameter T_H. This is completely analogous to that of how one proceeds in the hot plasma in flat space. The Wick squared operator in turn also depends on how one renormalizes it. One usually defines this operator as follows

$$\hat{\Phi}^2(x) = \lim_{x'\to x}\left(\hat{\Phi}(x)\hat{\Phi}(x') - H(x,x')\hat{1}\right),$$

where $H(x,x')$ is the Hadamard parametrix. This definition is state-independent. It is worth noting that the Wick squared operator can also be written down as $\hat{\Phi}^2(x) = :\hat{\Phi}(x)\hat{\Phi}(x):$, where colons refer to the normal order product.

We have recently found in [17] that the two-loop or, possibly, even higher-loop effect is dominant far from the small black holes if the wave-length λ_γ of the electromagnetic radiation is in the range $\lambda_e \ll \lambda_\gamma \ll \alpha^{1/2}\lambda_e(T_H/m_e)$, where λ_e is the Compton length of the electron. However, the one-loop dominance occurs whenever $\alpha^{1/2}\lambda_e(T_H/m_e) \ll \lambda_\gamma \ll l_c$, where l_c is a characteristic curvature scale.

3.1. Plasma-like environment of black hole

Quantum fluctuations of the electromagnetic and fermion field around the black holes reveal plasma-like properties. This can be characterised by the modification of the electric permittivity $\epsilon(\omega,k,R)$ and the magnetic permeability $\mu(\omega,k,R)$ of the vacuum. This is analogous to a similar phenomenon in the Casimir set-up [26,27]. Note that $\epsilon(\omega,k,R) = 1/\mu(\omega,k,R)$ in the limit $M \to 0$ as in the Minkowski vacuum, because the second term in (22) vanishes. The same holds for $M \neq 0$, but in the spatial infinity, i.e. at $R \to \infty$.

The normal hot plasma is characterised by two parameters, namely α and temperature T_H. The black-hole plasma-like environment is described by one more parameter which is of the order of $T_H(2M/R)$. Although the temperature parameter T_H is large with respect to m_e, the plasma-like environment is "cold" in the sense that the plasma-like frequency ω_p is small for $R \gg r_H$ with respect to T_H, i.e.

$$\omega_p \approx \left(\frac{27\pi\alpha}{36}\right)^{\frac{1}{2}} T_H\left(\frac{r_H}{R}\right) \ll T_H\,. \tag{25}$$

The photon propagator has the longitudinal and transverse part in the hot plasma [1]. The longitudinal part becomes a propagating degree of freedom (a collective mode mediated by the

[5] In a hot physical electron–positron plasma in Minkowski space, the thermal photon mass is a result of the photon interaction with the plasma particles. We could interpret $m_\gamma \neq 0$ as due to a hot plasma of Hawking electrons and Hawking positrons, but then we should accept a phase transition during black-hole formation. In fact, these particles are elements of the non-Minkowskian Fock space representation of the field operator algebra far away from the black hole, where one expects to have the ordinary representation [28].

plasma particles) known as plasmon for frequencies $\omega_l \sim \omega_p$, while the transverse part, photon, is dynamical for $\omega_t \geq \omega_p$. In the black-hole background, it implies that there should exist a plasmon-like excitation. However, the plasmon wavelength $\lambda_p \sim 10^2\, R$ is much bigger than R. In general, our approximation is only reliable for $\lambda_\gamma \ll R$ as pointed out above. Thus, there are no plasmon-like waves in the black-hole background.[6]

The physical plasma is opaque for electromagnetic waves with frequency ω less than the plasma frequency. Thus, these electromagnetic waves are reflected due to the collective response of the plasma particles. The poles in the photon propagator also disappear at $\omega < \omega_p$ close to the black hole. For instance, we find that the region $r_H \ll R \lesssim 1$ nm should be opaque for the light wave of length $\lambda_\gamma = 500$ nm. Therefore, we have $\lambda_\gamma \gg R$. In the hot plasma of the size R, one would expect merely a negligible damping of the wave amplitude. We expect the same effect for the small black holes due to the non-trivial manifestation of vacuum fluctuations. The reflection of the light waves from the plasma-like environment of the black hole should be an extremely rare event (if at all).

The plasma frequency in the normal plasma is a classical quantity, i.e. that does not explicitly depend on the Planck constant $\hbar$. Indeed, one has $\omega_p^2 = 4\pi e^2 n/m_e$ in the cold plasma, where n is a density number of the particles [31,32]. In the hot physical plasma $n \sim T^3$ and $m_e \sim T$ resulting in $\omega_p^2 \sim e^2 T^2$. In particular, we have $\omega_p^2 = e^2 T^2/9$ for the neutral electron–positron plasma (e.g., see [2]). However, our result (23) cannot be understood classically, because of the quantum nature of the parameter T_H ($\propto \hbar$).

Moreover, the plasma-like environment of small black holes far away from the horizon is effectively characterised by a new (local) temperature parameter

$$T_L = \frac{3\sqrt{3}}{16\pi}\, M_{\rm Pl}\, \frac{L_{\rm Pl}}{R} \tag{26}$$

which is much smaller than T_H far from the hole ($R \gg r_H$), where $L_{\rm Pl} = (\hbar G/c^3)^{\frac{1}{2}}$ is the Planck length. The numerical factor can deviate from its exact value as we have been working in the DeWitt approximation. The same scaling of the temperature from the distance has been recently found in [33] within a different framework.

3.2. Modified Coulomb law

One can employ our formula (23) to estimate the distance to the nearest small black hole from the upper bound on the photon mass. In the hot physical plasma, Coulomb's potential of a point-like electric charge is exponentially suppressed far from the charge as $\exp(-r/r_D)$, where $r_D = 1/m_D = (\sqrt{2}m_\gamma)^{-1}$ is the Debye radius. This phenomenon is known as the Debye screening (e.g., see [2,3] and [31,32] in the case of the hot and cold plasma, respectively). In our situation, the electromagnetic field effectively becomes a short range interaction.

We obtain from $m_\gamma \lesssim 10^{-14}$ eV [34] that the small black hole at that time could not be closer to the laboratory than R, where

$$R \approx 8.6\times 10^5\, R_\odot \left(\frac{10^{-18}\ \text{eV}}{m_\gamma}\right) \gtrsim 250\ \text{km}\,, \tag{27}$$

[6] It might be an effect of the absence of the plasmon's mediators. This appears to be in agreement with [28].

where $R_\odot \approx 2.95$ km is sun's gravitational radius. Neglecting any other possible contributions to the effective photon mass, then the stronger upper bound on m_γ, the farther small black hole should be from the laboratory.

It is worth emphasizing that the Debye screening of the charge due to the black hole cannot be complete (within our approximation), because the Debye radius is much bigger than R, specifically $r_D \gtrsim 8.8\times10^4$ km. The fact $R \ll r_D$ does not imply our approximation is unreliable. Indeed, the conducting shell used in [34] to test the Coulomb law has a size about 1 m which is much smaller than the distance to the black hole R.

In the physical plasma the Debye screening occurs due to the collective response of the plasma particles to the external electric charge. In our case, it is a vacuum polarization effect. In the absence of the black hole or very far away from it, the photon is almost massless at any order of the perturbation theory due to the gauge and Lorentz symmetry. Not too far from the black hole, spacetime isometry starts to significantly deviate from the Minkowskian one due to the black-hole horizon. As a consequence, the vacuum response to the electromagnetic field operator described by the electric permittivity and magnetic permeability modifies. This eventually results in the non-trivial photon dispersion relation. A similar effect occurs in-between the conducting plates, wherein, however, (low-energy) photons remain massless [26,27].

3.3. One-loop correction to electron mass m_e and fine structure constant α

The electron mass is also modified in the black-hole background. Following [35,36] (see also [2,3]), we obtain at one-loop level that

$$\delta m_e \approx \left(\frac{27\pi\alpha}{32}\right)^{\frac{1}{2}} T_H \left(\frac{r_H}{R}\right). \tag{28}$$

The correction to m_e is thus negligibly small with respect to m_e if $R \gg 4.4\times10^{-15}$ m. It is worth mentioning that classical estimate of the electron size is about 2.8×10^{-15} m. In the hot plasma, the thermal correction δm_e to the electron mass is much bigger than m_e. In our case, this correction to m_e is suppressed by the factor r_H/R. Hence, we have $m_e \gg \delta m_e$, although $T_H \gg m_e$. This implies there are no plasmino-like excitations in the background of the small evaporating black holes. This is fully consistent with of having no mediator due to which these collective modes could propagate.

The temperature-dependent correction to the fine structure constant α in the hot plasma has been derived in [37]. In the background of the small black hole we find

$$\alpha(M) \approx \alpha\left(1 + \frac{2\alpha}{3\pi}\left(\frac{27r_H^2}{16R^2}\right)\ln\left(\frac{M_{\rm Pl}^2}{8\pi M m_e}\right)\right). \tag{29}$$

The effective fine structure constant $\alpha(M)$ approaches α in the limit $M \to 0$. Its maximal value in the range $M_{\rm Pl} \ll M \ll 10^{21} M_{\rm Pl}$ slightly differs from α. Specifically, the deviation of $\alpha(M)$ from α is much smaller than 10^{-8} for those values of the black-hole mass. At the distance 1 m from the black hole, this deviation becomes 10^{-15} times smaller.

3.4. Black holes in analogue gravity

The effect we have derived in this paper is due to the interaction between photons and electrons/positrons and the presence of the small black hole. In the $\lambda\Phi^4$-model, the massless scalar particle acquires an effective mass m_Φ in the background of the black holes as well. Following [2], one can obtain

$$m_\Phi^2 \approx \frac{27\lambda}{384}\, T_H^2 \left(\frac{r_H}{R}\right)^2 \tag{30}$$

at one-loop level far away from the event horizon. We shall treat this theory in a forthcoming paper [38] near evaporating black holes, where one may expect a breakdown of the perturbation theory analogous to that observed in [17].

There is an analogue of black holes in a medium known as a dumb hole [39] (see also [40] for a comprehensive review of analogue gravity). Experimental evidences have been recently reported in favour of the dumb-hole evaporation which is supposed to be analogous to the black-hole evaporation [41,42].

For fluids in which phonons are self-interacting, one might expect a non-trivial dispersion relation for the phonon similar to that for the photon far from the small black hole. Specifically, an effective phonon mass might depend on the distance to the sonic horizon.

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