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by Matthias Frese

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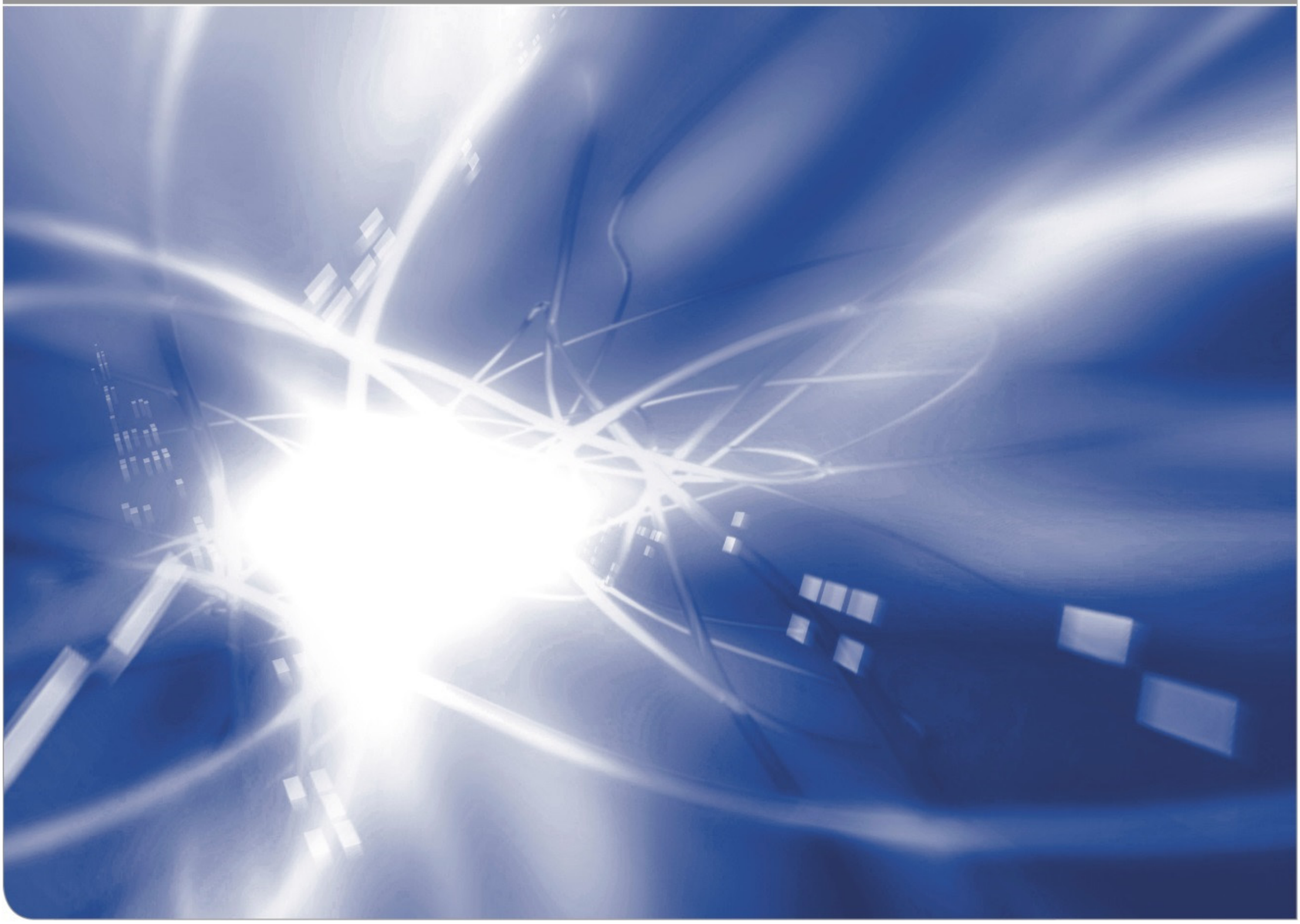

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A validation method for computationally determined strength values

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Abstract. The article presents a validation method for computationally determined glulam strength values. Using numerically modelled glulam, featuring brittle tensile failure, different load configurations were examined in a computationally performed study. The corresponding bending tests show variations in the bending strength depending on the applied configuration. From physical perspective, these strength variations are caused by the interaction between the actual natural strength variability, changes in the load configuration and/or the member volume and the brittle material behaviour. A targeted comparison between the apparent strength variations, obtained from the computational procedure on the one hand and a related analytical method on the other hand, leads to almost identical results. The comparison proves therefore that the computational approach reflects the principles of the size effect according to the Weibull theory with sufficient accuracy. The study generally exemplifies that related analytical methods, if available, are a beneficial tool to verify and validate computational procedures.

1 INTRODUCTION

1.1 General

Mechanical properties of timber products, determined by means of computer simulations, need to be suitably validated prior to actual application for structural purposes. The origin for this demand is evident. The development of a code for a complex computer simulation is accompanied by minor and major errors. It is almost inevitable to commit errors during the transfer of a conceptual model (that is the idea to achieve an aim by abstract steps) to a computational model. Beside of this, it is possible that a computational model, although correctly programmed, finally does not really comply with the intended physical event to be represented.

The complexity of the code increases with the number of its commands. Hence, errors may occur during the computational modelling of the structure and the mechanical behaviour of an engineered wood product (as close to reality as possible or necessary), the simulation of a virtual test (for different configurations in order to obtain certain mechanical properties) and the determination of the desired properties (particularly strength and modulus of elasticity). During code development and ongoing amendments in the code, those errors have to be detected, identified and eliminated systematically. After these steps, contradictory, not plausible or illogical outcomes of the computational model must not exist.

1.2 Aim of the study

The article focuses therefore on a validation method rather than on the description of the computational model used and would like to connect the "engineering demand" for validated simulation results with the concept of verification and validation in scientific computing or computational science and engineering. In doing so, the article carefully and cautiously intends – as the author does not claim to be very familiar with the theoretical framework of verification and validation in scientific computing – linking intuitive ideas of checking the computational model used in the present study and former studies (Frese et al. 2009/2010a/2010b/2012) with the general concept of verification and validation in scientific computing. Therewith, it is the aim to increase the credibility in the computational model and its algorithms and to establish finally confidence in its results. This article recapitulates and refines ideas from recent publications on size and load configuration effects in glulam structures (Frese and Blaß 2015; Frese 2016a/b).

1.3 Verification & Validation in scientific computing

According to definitions in textbooks and publications on the fundamentals of verification and validation in scientific computing, in software-based structural analysis or other areas of simulation (Schlesinger 1979; Roache 1998; Babuska and Oden 2004; Rabe et al. 2008, Oberkampf and Roy 2010; VDI 2014; Dijkstra 1970) one has to distinguish between the meaning of verification and validation. Verification means the activities which prove correctness (within certain limits) of the calculations constituting the way from a conceptual model to a computational model. The right programming is a crucial point in this regard. Validation means those activities showing that the computational model complies with the reality of a physical event with sufficient or user-specified accuracy; thereby, validation concerns the question whether a computational model is suitable for a certain exercise to be solved. Comparisons between computational and experimental results play an important role in the validation. Verification and validation can be seen as a process creating ongoing confirmations about the accuracy of computational results. However, the sum of such confirmations will never be a waterproof evidence for the correctness of a computational model (Dijkstra 1970).

1.4 Concept and structure of the study

The idea for this study arose from the attempt to show whether computationally determined load configuration factors agree with analytically determined ones, which are already published (Isaksson 2003; Johnson 1953; Colling 1986a/b). In general, such factors consider the influence of the manner of loading of a structural member on the apparent strength while the material quality does not change. As both approaches, computational and analytical, reflect the behaviour of brittle materials, an agreement between corresponding factors would actually have to exist.

In order to show this assumed agreement by means of computational and analytical approaches the following concept was determined: Based on an available set of load configurations, already analytically solved and prepared for evaluation by the above-mentioned authors, four different configurations were selected and one of them was taken as reference. This reference case is the standard four point bending test according to the testing standard EN 408 (2010). The remaining three configurations constitute suitable variations fulfilling also conditions for a sensitivity analysis.

Glulam with a specifically modelled strength is the material basis for the computer simulations and the analytical considerations. Varying the volume of a modelled glulam member, seventy thousand tension tests were first simulated to create a database for the determination of the shape parameter *k* and the exponent of the Weibull distribution, respectively. It is used for the calculation of the analytical load configuration factors. Three thousand computer simulations were conducted on each modelled bending member of the four load configurations. Therewith, computational configuration factors were determined. The study closes with a comparison between the computational and analytical factors.

An important feature of the applied method is that the absolute strength level is not validated by means of experimental results. Such kind of validation was already stated in Blaß et al. 2008. The strength level may remain unknown in the present study since the final comparison between computational and analytical results is based on relative variations. In this context, the analytically calculated variations are a kind of objective and unerring reference (Oberkampf and Roy 2010).

2 COMPUTATIONAL METHODS, MATERIALS AND RESULTS

2.1 Computational model

The modelling of the glulam material and the test simulation were performed with a computational model, the so-called Karlsruhe Rechenmodell. This computational model enables in general simulations of bending, tension and compression tests on glulam specimens and structures, respectively. In this study, the programme configurations – in its latest version – for bending and tension tests are relevant for the computational analysis. The code of the computational model is composed of ANSYS commands that are available in the corresponding ANSYS processors.

The glulam structure is discretised and a matrix of strength and stiffness values empirically represents the reality of the actual physical and mechanical properties as precisely as possible. Physical uncertainties influencing probabilistic distribution functions of the response variables are minimised. However, they can not completely be removed. The finite element method is used for computing the stress states in the discretised glulam structure. A local brittle tensile failure in the outermost laminations constitutes the failure of the complete loaded structure. After structural failure, the programme automatically evaluates the load carrying capacity and a corresponding strength value.

2.2 Virtual material used for the simulations and the analytical approach

The elementary principle in the concept of modelling glulam with the Karlsruhe Rechenmodell are discretised stiffness and strength values. These mechanical properties are constant within a volume of a 150 mm long lamination section. The main influencing factors on the glulam bending ($f_{m,g}$) and tensile strength ($f_{t,g}$), respectively, are the discretised tensile strength of the boards (f_t) and the tensile strength of the finger joints ($f_{t,j}$) connecting single boards to laminations. Fig. 1 exemplifies this influence for the ultimate bending capacity. Fig. 2 conveys the technical background of the discretised strength values and its original experimental examination; $f_{t,min}$ denotes the minimum tensile strength inside a modelled board.

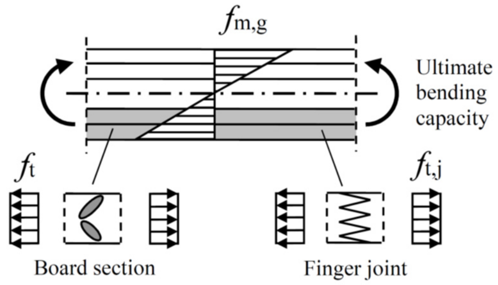

Figure 1: Leading influencing factors on the bending capacity of glulam: discretised tensile strength of board sections (f_t) and of finger joints ($f_{t,j}$) which are effective in the tension zone

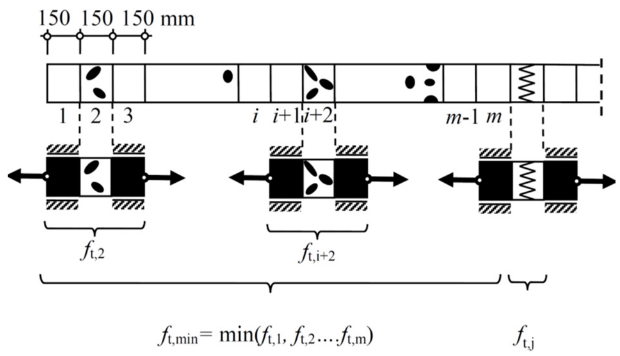

Figure 2: Conceptual modelling of the discretised tensile strength along a lamination; single tensile strength values are originally obtained from actual tension tests as exemplified for the board sections 2 and i+2 and the adjacent finger joint

The material, used throughout the study, features certain distributions of the discretised minimum tensile strength for board sections ($f_{t,min}$) and of the finger joint tensile strength. Table 1 contains the corresponding statistics and Fig. 3 shows the cumulative frequency distributions in detail.

Table 1: Statistics of the modelled tensile strength values

Section type	N	Symbol	Min	5^{th} P.	Median N/mm²	Max	Mean	SD^{c}	CV^{d} %
Board section	2000[a]	$f_{t,min}$	13.7	32.2	47.3	94	48.2	11.0	22.8
Finger joint	2000[b]	$f_{t,j}$	24.5	30.5	43.3	94.9	44.7	10.4	23.2

[a] Number of minimum values of 2000 modelled boards
[b] Number of modelled finger joints
[c] Standard deviation
[d] Coefficient of variation

Based on these two frequency distributions, the 5^{th} percentiles (5^{th} P.) amount to 32.2 N/mm² for board sections and 30.5 N/mm² for finger joints. On average, the ratio of modelled finger joint sections to modelled board sections (m) is $1/m_{mean}$ = 1/29.84. Therewith, the total number of the discretised tensile strength values, necessary for two thousand boards with finger joints, amounts to $2000 \cdot (29.84+1) = 61680 \approx 61687$. Fig. 4 shows the histogram of all discretised tensile strength values representing the virtual material for the simulations and the thereof depending analytical approach. This representation conveys an imagination of the capabilities of the Karlsruhe Rechenmodell in continuously generating timber specific properties for Monte Carlo simulations.

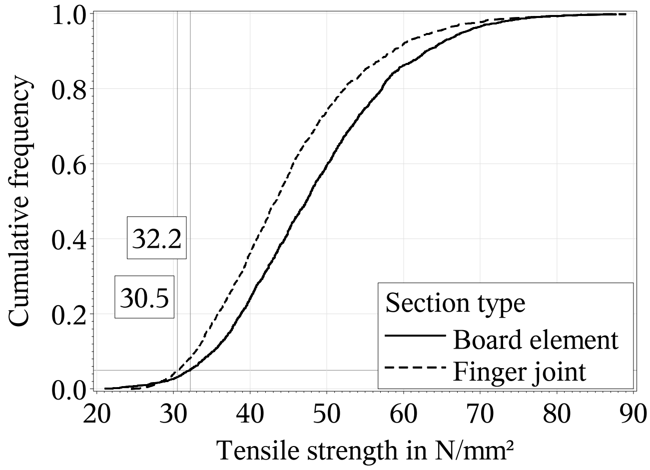

Figure 3: Cumulative frequency distribution of the discretised tensile strength $f_{t,min}$ and $f_{t,j}$

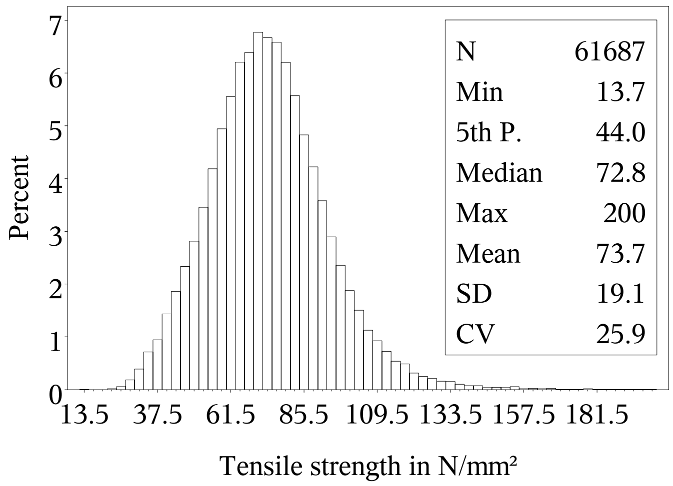

Figure 4: Relative frequency of the discretised strength of sections for two thousand boards and corresponding finger joints

2.3 Load configurations

Fig. 5 shows four relations between the selected load configurations, the corresponding moment diagrams and the formulas for the effective bending strength ($f_{m,eff}$); therein, W denotes the section modulus. These four cases are the basis for the computational examination and the analytical determination of load configurations. Case 0 is the reference case and cases I to III represent the variations. These load configurations were selected in agreement with the analytical calculation procedure which was already available. The corresponding formulas for the analytical approach were published by Isaksson (2003) and Johnson (1953). Testing the numerical outcome of the computational model against the results of the analytical approach is, therefore, a valuable comparison to support the credibility of the computational model and the confidence in its outcome.

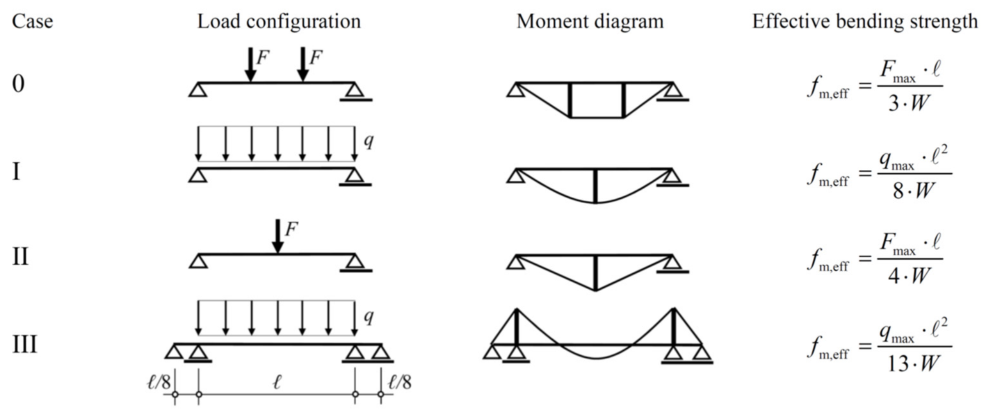

Figure 5: Load configurations, moment diagrams and calculation of the effective bending strength

Case III should reflect a bending member with fixed supports at both ends and uniformly distributed load. However, due to the restricted possibilities of incorporating model amendments in the Karlsruhe Rechenmodell an alternative was chosen. Instead of fixed supports at both ends, short spans were added at both member ends and two pairs of reaction forces, each including a distance of $\ell/8$, are realising the fixation.

2.4 Size effects according to Weibull theory

Equation (1) shows the ratio between two different strengths expressed by the ratio of its unequal volumes to the power of k. The equation corresponds to the Weibull theory. σ_1 refers to V_1, σ_2 to V_2 and $1/k$ is the exponent of the Weibull distribution.

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_1}{V_2}\right)^k \tag{1}$$

Fig. 6 describes the meaning of equation (1): both tension members feature the same failure probability P_f although the stress level and the volumes, respectively, are different from each other ($\sigma_1 > \sigma_2$, $V_1 < V_2$). This connection enables describing strength variations analytically.

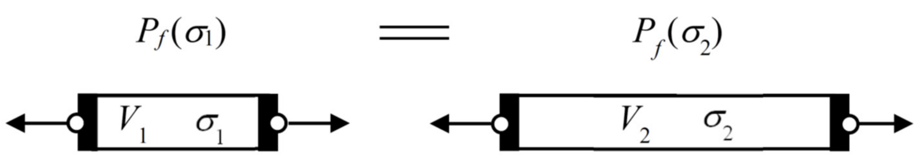

Figure 6: Equal failure probabilities P_f in different volumes under individual and constant stresses

2.5 Preliminary computer simulations

The knowledge of the k-value is essential for the analytical calculation of apparent strength differences for deviating load configurations. The k-value has an affinity with the coefficient of variation for the strength. From equation (1), it is evident that a very small k-value leads to a σ_2/σ_1-ratio of almost 1; hence, a changing volume hardly affects the apparent strength. However, a high k-value leads to a pronounced σ_2/σ_1-ratio in case of changing volumes.
For this study, the k-value is determined with simulated tension tests on modelled glulam (Fig. 7). In doing so, the modelled members experience constant tensile stress. The virtual material used, already characterised in section 2.2, features a characteristic tensile strength of about 28 N/mm²; this value refers to the tensile strength of a 5400 mm long member with reference size V_1. This size is given with $\alpha = \beta = 1$ and is constantly valid for usual lamination widths (100 - 160 mm). By increasing the

volume in the computational procedure – effectively by increasing the member length and hardly by the member depth – from about 1/36 up to 10 times the reference length, corresponding tensile strength values were determined; an analysis of the results showed that the independent variation of β (between the two limits 1/5 and 1) does not really influence the strength level. The simulation results of the five independently examined depths were, therefore, merged resulting in five thousand strength values for each examined length. Table 2 contains the statistics of the strength values for the limit configurations V_2/V_1 = 1/36 and 10/1 and the reference case V_2/V_1 = 1/1; Fig. 8 shows the corresponding cumulative distribution functions of the simulated strength values; the equations (2) to (4) exemplify the calculation of the corresponding σ_2/σ_1-ratios. While the smallest volume shows a ratio of 1.41, the largest examined volume leads to a ratio of 0.806. According to equation (5), the V_2/V_1-ratio could also be replaced by a ratio of different lengths (ℓ).

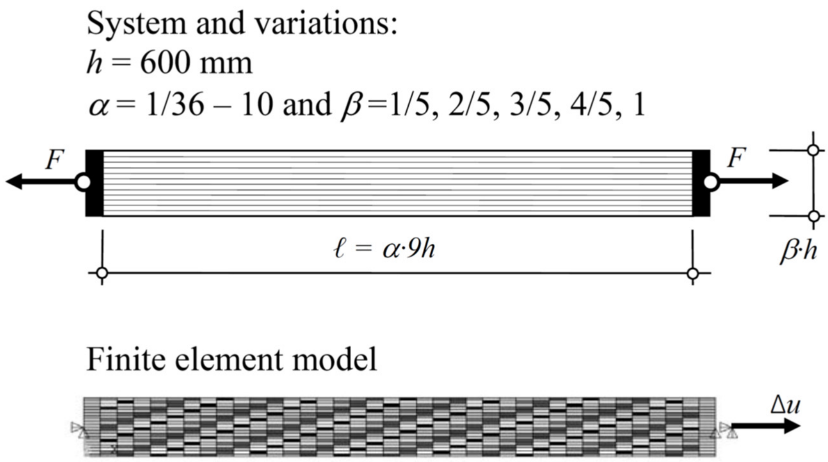

Figure 7: Constantly stressed member (top) and computational model (bottom)

Table 2: Statistics of the tensile strength obtained from simulated tests represented in Fig. 7

V_2/V_1	Number	Min	5^{th} P.	Median	Max	Mean	SD	CV
				N/mm²				%
1/36	5000	20.5	40.0	55.0	132	56.1	11.2	20.0
1/1	5000	13.2	28.3	36.3	54.1	35.9	4.19	11.7
10/1	5000	11.4	22.8	29.4	36.2	28.9	3.28	11.3

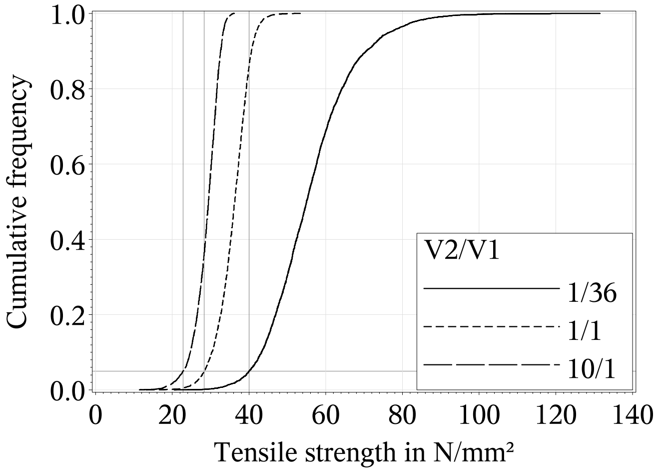

Figure 8: Selected cumulative frequency distributions of the tensile strength

$$\frac{V_2}{V_1} = \frac{1}{36} \rightarrow \frac{\sigma_2}{\sigma_1} = \frac{40}{28.3} = 1.41 \text{ (smallest volume)} \tag{2}$$

$$\frac{V_2}{V_1} = \frac{1}{1} \rightarrow \frac{\sigma_2}{\sigma_1} = \frac{28.3}{28.3} = 1 \text{ (reference case)} \tag{3}$$

$$\frac{V_2}{V_1} = \frac{10}{1} \rightarrow \frac{\sigma_2}{\sigma_1} = \frac{22.8}{28.3} = 0.806 \text{ (largest volume)} \tag{4}$$

$$\frac{V_2}{V_1} \approx \frac{\ell_2}{\ell_1} \tag{5}$$

2.6 Determination of the *k*-value

Fig. 9 shows the smooth course which averages all of the computationally obtained σ_2/σ_1-ratios. This course features a *k*-value given by equation (6). Since this *k*-value is based on 5th percentiles for the tensile strength, it is valid for a failure probability of 5 %.

$$k = 1/10.8 = 0.0926 \tag{6}$$

2.7 Computationally determined load configuration factors

2.7.1 Finite element models and computer simulations

Fig. 10 depicts simplified representations of the finite element models (cf. Fig. 7 below) for the reference case 0 and the variations I to III. These models correspond with the load configurations in Fig. 5. Specific ANSYS commands were used to work these models into the modular system of the Karlsruhe Rechenmodell accordingly. The load transfer, originally caused by a stepwise displacement (Δu) onto the modelled glulam body, is realised by a purposeful arrangement of link (vertical

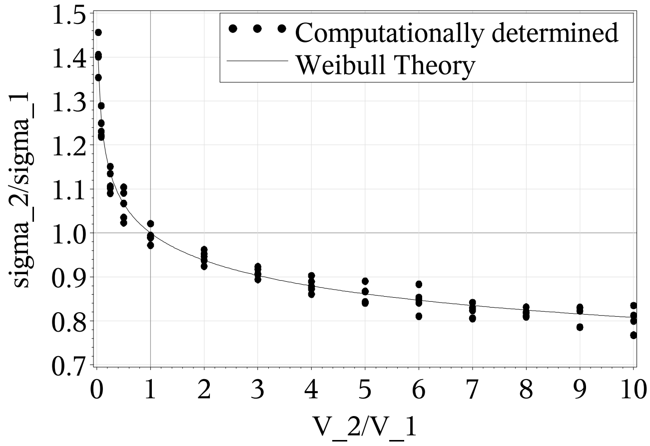

Figure 9: Strength ratios depending on the volume in terms of Equation (1)

members) and beam elements (horizontal members if unloaded). Thereby, lateral supports ensure horizontal stability of the modelled loading equipment. The uniformly distributed load (cases I and III) is reproduced with sufficient accuracy by six concentrated single loads.

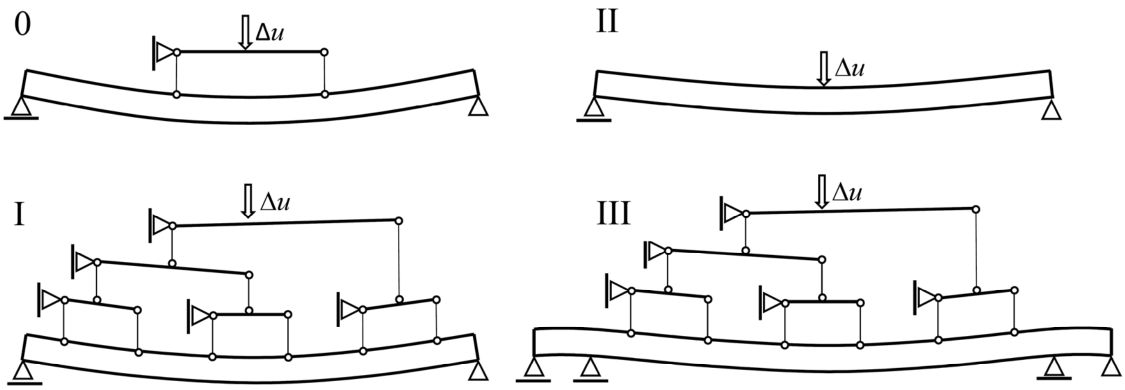

Figure 10: Finite element models used for representation of the selected load configurations

For each case, the corresponding bending test was executed three thousand times as Monte Carlo simulation. Therewith, the individual material properties in the glulam body varied from test to test while the quality level of the material remains the same within the simulated test series. Immediately after tensile failure had occurred in a modelled element of an outermost tension lamination, the effective bending strength was calculated from the actual loading ($F_{\max}$ resp. $q_{\max}$) present at failure. The corresponding equations used for the single cases are stated in Fig. 5.

2.7.2 Simulation results

Table 3 contains the statistics of the strength results. The 5th percentiles (5th P.) amount to 32.85 (case 0), 33.80 (I), 38.00 (II) and 42.60 N/mm² (III). Fig. 11 shows a comparison between the corresponding cumulative frequency distributions. Judging these results in the sense of a sensitivity analysis, the Karlsruhe Rechenmodell possesses a technical sensitivity for marginal variations regarding the stressed volume and the stress distribution within this volume. The four strength results are plausible from empirical view: The lowest values are achieved for modelled glulam bodies with wide areas of high stressed outermost laminations (case 0: between the two concentrated single loads and case I: around the vertex of the quasi-parabolic moment diagram). The highest values were realised for bodies where only single points of the outermost laminations are highly stressed in tension. Even the ascending order of the 5th percentiles is logic since the stressed volume decreases with increasing case number.

Table 3: Statistics of the bending strength from simulated tests represented in Fig. 10

Case	Number	Min	5th P.	Median N/mm²	Max	Mean	SD	CV %
0	3000	14.8	32.85	44.2	68.8	44.2	6.82	15.4
I	3000	18.9	33.80	45.9	68.1	45.7	6.93	15.1
II	3000	16.1	38.00	52.5	82.7	52.4	8.72	16.7
III	3000	24.6	42.60	58.1	86.7	57.8	8.89	15.4

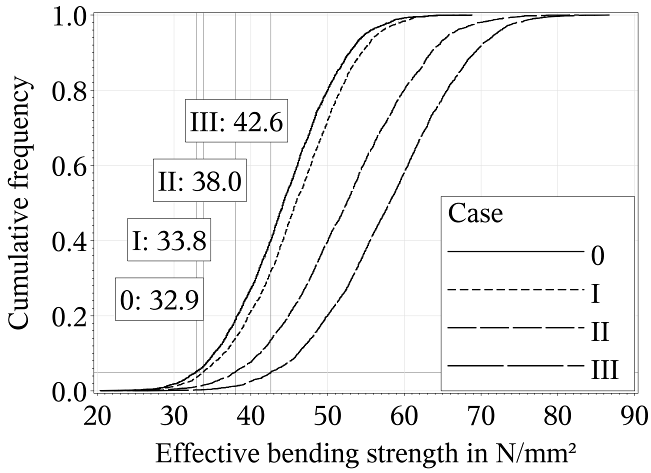

Figure 11: Comparison between the cumulative frequency distributions for the examined cases

2.7.3 Calculation of the load configuration factors

Based on the computationally determined 5th percentiles (Table 3) and case 0 as reference, the numerical load configuration factors $f_{0/\mathrm{I,comp}}$, $f_{0/\mathrm{II,comp}}$ and $f_{0/\mathrm{III,comp}}$ amount to 1.027, 1.16 and 1.29 for the cases I, II and III, respectively. The equations (7) to (9) exemplify its calculations.

$$f_{0/\mathrm{I,comp}} = \frac{f_{\mathrm{m,eff,I}}}{f_{\mathrm{m,eff,0}}} = \frac{33.8}{32.9} = 1.027 \tag{7}$$

$$f_{0/\mathrm{II,comp}} = \frac{f_{\mathrm{m,eff,II}}}{f_{\mathrm{m,eff,0}}} = \frac{38.0}{32.9} = 1.16 \tag{8}$$

$$f_{0/\mathrm{III,comp}} = \frac{f_{\mathrm{m,eff,III}}}{f_{\mathrm{m,eff,0}}} = \frac{42.6}{32.9} = 1.29 \tag{9}$$

2.8 Analytically determined load configuration factors

2.8.1 Calculation with Isaksson's formulas: Cases I and II

Considering the fullness parameters λ tabulated in Isaksson (2003), the following analytical load configuration factors $f_{0/\mathrm{I,ana}}$ and $f_{0/\mathrm{II,ana}}$ were found for the cases I and II. In agreement with the calculation of the computationally determined load configuration factors, case 0 is also taken as reference.

$$\lambda_0 = \frac{3k+1}{3(k+1)} = 0.3898 \tag{10}$$

$$\lambda_{\mathrm{I}} = \frac{k^3 + 0.345k^2 - 0.027k + 0.0013}{k^2(k+1)} = 0.2724 \tag{11}$$

$$\rightarrow f_{0/\mathrm{I,ana}} = \left(\frac{\lambda_0}{\lambda_{\mathrm{I}}}\right)^k = \left(\frac{0.3898}{0.2724}\right)^{1/10.8} = 1.034 \tag{12}$$

$$\lambda_{\mathrm{II}} = \frac{k}{k+1} = 0.08475 \tag{13}$$

$$\rightarrow f_{0/\mathrm{II,ana}} = \left(\frac{\lambda_0}{\lambda_{\mathrm{II}}}\right)^k = \left(\frac{0.3898}{0.08475}\right)^{1/10.8} = 1.152 \tag{14}$$

2.8.2 Calculation with Johnson's formulas: Case III

Using the corresponding graphical courses in Johnson (1953, p. 72), the analytical load configuration factor $f_{0/\mathrm{III,ana}}$ for case III could be estimated as follows.

$$\rightarrow f_{0/\mathrm{III,ana}} = \left(\frac{\lambda_0}{\lambda_{\mathrm{III}}}\right)^k \approx 1.37 / 1.09 = 1.26 \tag{15}$$

2.9 Comparison between the load configuration factors

The ratios (16) to (18) feature values of almost 1. Hence, no evident contradiction exists between the computational and analytical approach. The larger deviation of the ratio in case III (1.02 > 1.0) may be caused by the difference between the model used for the computational approach and the model used by Johnson (1953). While the computational approach uses a fixation realised by short spans and a pair of reaction forces at both ends of the bending member (Fig. 5 and 10, respectively), Johnson's analytical model represents a bending member fully fixed at both ends. This has not only a consequence for the moment diagram but also on the stressed volume.

$$\frac{f_{0/\mathrm{I,comp}}}{f_{0/\mathrm{I,ana}}} = \frac{1.027}{1.034} = 0.99 \tag{16}$$

$$\frac{f_{0/\mathrm{II,comp}}}{f_{0/\mathrm{II,ana}}} = \frac{1.16}{1.15} = 1.01 \tag{17}$$

$$\frac{f_{0/\mathrm{III,comp}}}{f_{0/\mathrm{III,ana}}} = \frac{1.29}{1.26} = 1.02 \tag{18}$$

3 CONCLUSIONS

Relative strength variations were quantified for purposefully selected load configurations with a computational procedure, already employed in former studies for determining the glulam bending strength. These strength variations were confirmed by an analytical procedure based on the size effect according to the Weibull theory. Assuming that the resulting analytical load configuration factors state more or less an objective and unerring reference, the agreement proves, to a certain extent, the suitability of the computational model for determining the glulam bending strength. Hence, the study and the following comparison between analytical and computational results strengthen the credibility of the computational model and the confidence in its outcome. Furthermore, marginal differences between the computationally examined load configurations 0 and I conditioned by minimum variations of the moment diagram also lead to a corresponding minor and logical change in the apparent strength. The study generally exemplifies that linking computational and analytical procedures may be a beneficial means to check the suitability of computational methods. Such checks should be applied if possible and corresponding means are available.

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