# A Generalized Approach to the Analysis and Control of Modular Multilevel Converters

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# Abstract

A general method to derive the control strategy for arbitrary modular multilevel topologies is presented. The number of internal currents is derived from the topology. A strategy for arm symmetrization with arbitrarily controllable powers is given.

## 1. Modular multilevel converters

Modular multilevel converters (MMCs) consist of a number of arms that connect between external terminals. Each arm consists of an inductor and a number of cells connected in series. The cells are two port devices which contain an energy storage and can present different voltages at their terminals. For certain DC-based topologies cells which can only present positive or zero voltages may be feasible. For a stable operation the energy in the storage must be controlled.

For the purpose of general control considerations the series connected cells can be modeled as a controllable voltage source because the cells are modulated in a way that in average a chosen voltage appears across all cells of one arm. Modulation schemes to achieve this are described in various publications, e.g. [1], [2], and are not part of this paper.

This paper presents a unified approach for the analysis of different kinds of modular multilevel topologies. Previously, each topology was described in isolation, e.g. [3], [4]. An approach to a unified description of MMCs is presented in [5]. This paper improves on this by giving an explicit algorithm to derive the control scheme from the schematics of the inverter in a purely analytical way. Examples will be shown for the well known DC-3AC (M2C), matrix (M3C) and Hexverter [6] variants as well as more complex topologies like a 3AC-5AC converter or a Nonverter (see section 5.4).

The relationship between the arm voltages and the external currents is not obvious. The method de-



Fig. 1: Statcom prepared for generalized algorithm

scribed in this paper presents an analytical way to derive control strategies for these currents and the stored energies.

# 2. Prerequisites

All arms must be identical and not coupled magnetically. The inverter itself can contain only arms. At every junction of arms an external voltage is connected. Every external voltage exists between the arm junction and a star point. All external voltages are known.

The prerequisites are met by the well known topologies (Statcom, M2C, M3C, Hexverter).

# 3. Algorithm

To analyze the inverter the arm and external sources are numbered and given a direction. The direction of the external voltages is chosen from the converter to the star point (SP). For the Statcom configuration this is shown in fig. 1.

From this schematic the relationship between the external and the arm currents can be seen. Let  $i_a$  be the vector of arm currents and  $i_e$  the vector of external currents. A matrix M' can then be derived so that

$$i_{\rm e} = M' \cdot i_{\rm a}.$$

If we assume the star points of all systems to be on the same potential and that  $v_e$  is the vector of the external voltages and  $v_a$  is the vector of arm voltages then the negative transposed of M' also gives the relationship between the arm voltages and the external voltages as

$$v_{\rm a} = -M'^{\mathsf{T}} \cdot v_{\rm e}.$$

To create M' these simple rules can be applied:

- Number of columns *n* is number of arms
- Number of rows m is number of external sources
- If an external source is not directly connected to the arm a place in the matrix is 0.
- If an arm points towards the node the external source is connected to, the place in the matrix is 1.
- If an arm points away from the node the external source is connected to, the place in the matrix is -1.

Doing this for the Statcom results in

$$M'_{\rm SC} = \begin{pmatrix} -1 & 0 & 1\\ 1 & -1 & 0\\ 0 & 1 & -1 \end{pmatrix}.$$

In general M' does not have full rank. Therefore M' is extended by as many normalized rows orthogonal to every other row as possible. This new Matrix is called M. The vectors of external currents and voltages are extended by the same amount of entries. These additional currents are decoupled from the external currents and can be used as degrees of freedom. The additional voltages drive these currents.

 $M_{\rm SC}^\prime$  has rank 2. Therefore one internal current is added leading to

$$M_{\rm SC} = \begin{pmatrix} -1 & 0 & 1\\ 1 & -1 & 0\\ 0 & 1 & -1\\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}.$$

If each arm is controlled according to  $v_{\rm a} = -M^{\intercal}v_{\rm e}$  there will be no voltage drop across the inductors and the currents remain constant. To control the

currents the arm voltages have to deviate from this steady state voltage by  $\Delta v_{\rm a}.$  Then the change of arm current is

$$\dot{Li_{a}} = -\Delta v_{a}$$

The changes of the external currents therefore are

$$\dot{Li_{\rm e}} = -M \cdot \Delta v_{\rm a}.$$

Replacing the arm voltages with the external voltages leads to

$$\dot{Li_{e}} = MM^{\intercal} \cdot \Delta v_{e}.$$
 (1)

Since  $MM^{\mathsf{T}}$  is generally not a sparse matrix the system is highly coupled. The idea is now to apply a transformation to decouple the system. Then each transformed current  $i_{\mathrm{T},x}$  is influenced by only one decoupled voltage  $v_{\mathrm{T},x}$ . For the decoupling the external voltages and currents are transformed with a Matrix T:

$$i_{\rm T} = T \cdot i_{\rm e}$$
 (2)

$$v_{\rm T} = T \cdot v_{\rm e}$$
 (3)

This is the usual approach also used in [7] or [4]. Here T is derived systematically:

Inserting eq. (2) and eq. (3) into eq. (1) and leftmultiplying with T gives

$$\dot{Li_{\rm T}} = T \cdot M M^{\mathsf{T}} \cdot T^{-1} \Delta v_{\rm T}.$$

This is easily controllable if each current depends only on the corresponding voltage. This means Tmust be chosen so that  $TMM^{T}T^{-1}$  is a diagonal matrix.

Since  $MM^{\intercal}$  is symmetrical it can be diagonalized with an orthogonal Matrix A consisting of the normalized and orthogonalized eigenvectors of  $MM^{\intercal}$ . Therefore T is chosen as  $A^{\intercal}$ .

For the Statcom  $M_{\rm SC}M_{\rm SC}^{\rm T}$  has the eigenvalues 0,3,3,1 with the eigenvectors

$$\begin{pmatrix} 1\\1\\1\\0 \end{pmatrix}, \begin{pmatrix} -1\\1\\0\\0 \end{pmatrix}, \begin{pmatrix} -1\\0\\1\\0 \end{pmatrix}, \begin{pmatrix} 0\\0\\0\\1 \end{pmatrix}.$$

Normalizing and orthogonalizing those leads to the transformation matrix

$$T_{\rm SC} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0\\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{\sqrt{6}}{3} & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

It can be seen that this is basically the Clarketransformation. This means that the well known Clarke-transformation based control schemes can be applied.

To simplify the further formulas the product  $T \cdot M$  is defined as S'.

One row of S' consists only of zeros. This row and the corresponding transformed values are removed. This leads to the system matrix S giving the relationship between arm values and transformed values:

$$i_{\mathbf{a}} = S^{-1} \cdot i_{\mathbf{t}}$$
$$v_{\mathbf{a}} = -S^{\mathsf{T}} \cdot v_{\mathbf{t}}$$

For the Statcom this system matrix is

$$S_{\rm SC} = \begin{pmatrix} \sqrt{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{6}}{2} & -\frac{\sqrt{6}}{2} \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}.$$

#### 3.1. Current control

The diagonalized values  $i_{\rm t}$  and  $v_{\rm t}$  then lead to effective inductances

$$L_{\text{eff}}\dot{i}_{t} = \Delta v_{\text{T}}$$
 with  $L_{\text{eff}} = L(SS^{\intercal})^{-1}$ .

Since the star points of the different systems are not necessarily connected some of the transformed currents cannot appear. These can be identified from T by finding rows in which all currents of a system appear with the same factor. Here instead of the current the voltage between the star points of the corresponding systems can be controlled.

To control the external currents, the setpoints of those currents are transformed with T. This results in setpoints in the transformed coordinates. The transformed currents can then be controlled to these setpoints by using the transformed voltage differences. The voltage differences can then be transformed into arm voltage differences with  $S^{T}$ . This transformation scheme is shown in fig. 2;



Fig. 2: Transformation scheme for the current control

#### 3.2. Energy control

To control the energy balancing between the arms a vector of transformed powers, i.e. products of a transformed voltage and a transformed current,  $p_t$  must be chosen. The average values of the powers must be independently controllable. That means that it is possible to use for example the product of two voltages with one internal current as long as the average of the product of the voltages is zero. I.e.  $v_{t1}i_{t3}$  and  $v_{t2}i_{t3}$  can both be used if  $avg(v_{t1}v_{t2})$ .

Since the energy distribution between the arms needs to be controlled, as many controllable powers as there are arms are needed. One of those powers is the difference of active powers of the external sources. It controls the total energy stored in all arms. Therefore we have to chose one less power than there are arms for  $p_t$ . The powers to control the energy distribution should, if possible, be selected as not having influence on the external sources. Therefore the internal currents in combination with all kinds of voltages are sensible choices. The possible choices depend on the topology and the specific mode of operation. Care must be taken to chose powers which can be independently controlled. This can be problematic especially if different systems with the same frequency occur.

For the Statcom

$$p_{\rm tSC} = \begin{pmatrix} i_{\rm t3} \cdot v_{\rm t1} \\ i_{\rm t3} \cdot v_{\rm t2} \end{pmatrix}$$

is chosen. Here  $i_{tn}$  and  $v_{tn}$  are the n-th element of the vector of transformed currents or voltages.

The power  $p_{\rm a}$  in each arm is given by

$$p_{\mathbf{a}} = (-S^{\mathsf{T}} \cdot v_{\mathbf{t}}) \circ (S^{-1} \cdot i_{\mathbf{t}})$$

with  $\circ$  being the elementwise product.

For the Statcom this results in

$$p_{\rm aSC} = \begin{pmatrix} p_{\rm aSC,1} \\ p_{\rm aSC,2} \\ p_{\rm aSC,3} \end{pmatrix}$$

with

$$p_{aSC,1} = -\frac{2i_{t1}v_{t1}}{3} - \frac{i_{t3}v_{t1}}{3}\sqrt{6}$$

$$p_{aSC,2} = -\frac{i_{t1}v_{t1}}{6} + \frac{i_{t1}v_{t2}}{6}\sqrt{3} + \frac{i_{t2}v_{t1}}{6}\sqrt{3} - \frac{i_{t2}v_{t2}}{2}$$

$$+ \frac{i_{t3}v_{t1}}{6}\sqrt{6} - \frac{i_{t3}v_{t2}}{2}\sqrt{2}$$

$$p_{aSC,3} = -\frac{i_{t1}v_{t1}}{6} - \frac{i_{t1}v_{t2}}{6}\sqrt{3} - \frac{i_{t2}v_{t1}}{6}\sqrt{3} - \frac{i_{t2}v_{t2}}{2}$$

$$+ \frac{i_{t3}v_{t1}}{6}\sqrt{6} + \frac{i_{t3}v_{t2}}{2}\sqrt{2}$$

For the purpose of the energy control all powers except those in  $p_t$  are assumed to be disturbances and thus to be zero. With this modification  $p_a$  can be written as  $p_a = X \cdot p_t$ . If the powers chosen for  $p_t$  are suitable, X will have a maximal rank.

For the Statcom this leads to

$$X_{\rm SC} = \begin{pmatrix} -\frac{\sqrt{6}}{3} & 0\\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{6}}{6} & \frac{\sqrt{2}}{2} \end{pmatrix}.$$

To ease the control transformed energies are controlled instead of the arm energies. The transformation is chosen in a way that results in each transformed energy being influenced by exactly one power from  $p_t$  or by the power difference of the external systems.

A suitable transformation matrix  $T_p$  has as many columns as there are arms and one row consisting only of ones and as many linearly independent rows orthogonal to it as possible. The rows are further chosen such that the product  $T_p \cdot X$  is a diagonal matrix except for the first row which is zero. The first row must be zero because the entries of  $p_t$  do not influence the external power by design and therefore cannot change the total energy of the arms.

For the Statcom one possible transformation is

$$T_{\rm PSC} = \begin{pmatrix} 1 & 1 & 1 \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}.$$



Fig. 3: Demonstration of energy balancing for Statcom

The rows orthogonal to the total-energy-row represent an imbalance between the energies in the different arms. As such they have to be controlled to an average of zero.

The imbalance part of  $T_{\rm p}$  can be found as the Moore-Penrose-Pseudoinverse of X.

When transforming the arm energies in this way each imbalance is controlled by exactly one of the components of  $p_t$ . This enables a decoupled control of the balancing of the arm energies.

Since each of the transformed energies depends only upon one transformed power and all of these transformed powers are chosen to be independently controllable the imbalance of the energy distribution is controllable and therefore a stable operation is possible.

### 4. Simulation results

All simulations are done with OpenModelica 1.12. For the arms a continuous model consisting of an inductor of 1 mH, a resistor of  $100 \text{ m}\Omega$  and a variable voltage source is used. All controllers are continuous PI-controllers.

The energy balancing of the cells is of special importance for this class of inverters. Therefore as a demonstration of the controllability of the energy distribution an asymmetry is created in the cell energies which is then removed.



Fig. 4: simplified schematic of an M2C

Figure 3 shows a simulation of an energy controller deliberately creating and then removing an energy imbalance in the arms of a Statcom. For the whole duration an effective reactive current of 200 A is flowing. The connected voltage system has an effective phase-to-phase voltage of 400 V.

### 5. Further examples

To show the applicability of the algorithm simulation results of further topologies are shown. Special attention should be given to the as of yet undescribed topologies of the Nonverter and the AC3-AC5-Matrix converter.

To save space simplified schematics will be used for the topologies: arms are shown only as lines with an arrow and external sources are implicit at every connection of arms and therefore left out. Nodes numbered with the same colour are connected to the same voltage system.

For the purpose of the simulations each inverter has an input and one or more output systems. The input system is controlled by the total energy controller with a power factor of one. The input system is always the system containing node 1. Each output system is controlled to a current  $I_{out,eff} = 50$  A with a power factor of one.

The first AC system of each inverter has a frequency of 50 Hz, the second of 100 Hz and the third of 75 Hz. Different frequencies are used to simplify the control of the energy balancing. To use equal frequencies more care must be taken when selecting the balancing powers.

All AC systems have a phase-to-phase voltage of 400 V.



Fig. 5: Demonstration of energy balancing for M2C

#### 5.1. M2C

This is the inverter shown in fig. 4. A method to control this type of inverter is described in [4].

From the schematic the matrix

$$M_{\rm M2C} = \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 1 & 1\\ 1 & 0 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & -1\\ -\frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} & 0\\ -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$$

can be created. The rows added for the internal currents are highlighted in red.

Further application of the algorithm leads to a controllable system. Here, as it does with the statcom, the Clarke-transformation appears naturally.

The controllability of the energy distribution is shown in fig. 5. An energy imbalance has been controlled.

#### 5.2. M3C

This type of inverter is shown in fig. 6. A known method of controlling it is described in [7].

From the schematic the matrix  $M_{\rm M3C}$  can be created analogously to the Statcom and M2C cases.

Applying the described algorithm enables the control of this class of converter. Here special care has



Fig. 6: simplified schematic of an M3C



Fig. 7: Demonstration of energy balancing for M3C. For increased clarity only the arms one to five are shown. The arms not shown are similar to arm three.

to be taken when orthogonalizing the eigenvectors. The eigenvectors corresponding to the values of the AC-systems have the same eigenvalues. Therefore there are various ways of orthogonalization. The most sensible choice is to orthogonalize the eigenvectors into into two sets that each apply a clarke transformation to one of the voltage systems. This choice makes it possible to use the standard clarketransformation-based control schemes.

Figure 7 shows a demonstration of the control of the energy balancing.

#### 5.3. Hexverter

Figure 8 shows the simplified schematics of a Hexverter. The control of this topology is described in [5]. While this topology is controllable using the approach from this paper, the two voltage systems cannot be separated. Therefore each of the con-



Fig. 8: simplified schematic of a Hexverter



Fig. 9: Demonstration of energy balancing for the Hexverter

trollable currents is a combination of values from both voltage systems. This makes the application of control methods, that rely on clarke- and parktransformations, difficult.

Energy control is possible as shown in fig. 9.

#### 5.4. Nonverter

The Nonverter is the extension of the Hexverter concept to nine arms and three voltage systems. It



Fig. 10: simplified schematic of a Nonverter



Fig. 11: Demonstration of energy balancing for Nonverter



Fig. 12: simplified schematic of an AC3-AC5-Matrix

consists of nine arms in a circular arrangement. Its simplified schematics are shown in fig. 10. It enables the exchange of energy between all three voltage systems. This general concept can be further extended to an arbitrary number of voltage systems by using a ring with more arms.

The algorithm has the same drawbacks as for the Hexverter: Each of the controllable currents is a combination of currents of all three voltage systems. The control quality shown here could be improved by using more sophisticated control schemes.

Using the PI-control of this paper it is possible to show the controllability of the energy distribution for the Nonverter as is shown in fig. 11.



Fig. 13: Demonstration of energy balancing for AC3-AC5-Matrix. Only five of the 15 arms are shown. The others are similar to arm 3.

#### 5.5. AC3-AC5-Matrix

The AC3-AC5-Matrix-converter is the extension of the M3C to five phase systems. Here the systems do get separated by the transformation. Care must be taken to properly transform the five-phase system. Its eigenvalue occurs four times. To transform the system into two orthogonal values, i.e. the five phase equivalent to the clarke transformation, the nonzero parts of two of the four eigenvectors have to be chosen as  $\Re\{v\}$  and  $\Im\{v\}$  with

and

$$v_i = \mathrm{e}^{\frac{i}{5}2\pi\mathrm{j}}.$$

 $v = \frac{5}{2}(v_0 v_1 v_2 v_3 v_4)^{\mathsf{T}}$ 

This formula is applicable to any number of phases when adjusting the constants. Therefore any  $M \times N$  Matrix-converter can be controlled.

Figure 13 shows the controllability of the energy balancing for the AC3-AC5-Matrix-converter.

### 6. Limitations

The algorithm can only be applied if the structure of the converter is suitably symmetric.

Figure 14 shows the simplified schematics of a topology for which the algorithm fails. The failure is obvious from the transformation matrix: It does not



Fig. 14: Schematic of a topology that cannot be used with the described algorithm

contain a line for the star point voltage between the two systems. This means that the voltage between the star points cannot be controlled and instead is part of several of the transformed voltages. Since there cannot be a current between the star points the idea of one transformed voltage directly controlling one transformed current then breaks down.

## 7. Conclusion

The presented method enables the semi-automatic creation of control strategies for modular multilevel converters of various topologies. This allows the description and usage of topologies which were deemed too complicated to control until now. Because of the strictly algorithmic way in which the method works no "engineering intuition" is needed and new topologies can be explored with little investment of time and effort.

It has been shown that the presented algorithm can be applied to a wide variety of types of modular multilevel converters. This includes well known as well as new ones.

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