

## Simulation of electromagnetic fields scattered from arbitrary shaped electric conductors

A. Marek<sup>1</sup>, K.A. Avramidis<sup>1</sup>, S.M. Coppelstone<sup>5</sup>, N.S. Ginzburg<sup>4</sup>, S. Illy<sup>1</sup>, J. Jelonek<sup>1,2</sup>,  
J. Jin<sup>1</sup>, S.V. Mishakin<sup>4</sup>, A.-S. Müller<sup>3</sup>, P. Ortwein<sup>5</sup>, M. Thumm<sup>1,2</sup>

<sup>1</sup>IHM, <sup>2</sup>IHE, <sup>3</sup>IBPT, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, alexander.marek@kit.edu

<sup>4</sup>Institute of Applied Physics, Russian Academy of Sciences (IAP- RAS), Nizhny Novgorod, Russia

<sup>5</sup>Institute of Aerodynamics and Gas Dynamics, University of Stuttgart, Stuttgart, Germany

### Introduction

An advanced tool (KarLESSS) for the simulation of full 3D electromagnetic scattering problems with arbitrarily shaped, large perfect conductors is under development. High accuracy in the simulation is achieved by applying a full-wave simulation method based on the electric field integral equation (EFIE) [1] which is solved numerically using a Galerkin-type method of moments (MoM) [2]. The tool was originally developed for the analysis of quasi-optical mode converters in high power gyrotrons. In this paper, additional areas of application of KarLESSS will be demonstrated. After a brief introduction of the theory in the first section, simulation results of a quasi-optical input-output system for an amplifier operating in the millimeter-wave range are shown. Afterwards, cold simulations of a helically corrugated resonator of a gyro-amplifier [3] are presented.

### Theory

The components of the electric field are described with the EFIE as follows:

$$\begin{aligned} \frac{-i}{\omega\mu} \mathbf{n}(\mathbf{r}) \times \mathbf{E}_i &= \\ &= n(\mathbf{r}) \times \iint_S d\mathbf{r}' G(\mathbf{r}, \mathbf{r}') [\mathbf{J}(\mathbf{r}') + \frac{1}{k^2} \nabla' \nabla' \cdot \mathbf{J}(\mathbf{r}')] \end{aligned} \quad (1)$$

where  $\mathbf{J}$  is the unknown electrical surface current,  $\mathbf{E}_i$  is the known incident field and  $G$  is the free space Green's function. In the MoM approach, the currents are expanded into a sum of weighted basis functions  $\mathbf{f}$ . Applying this expansion into (1), using the linearity of the equation and taking an inner product of the equation with the basis functions leads to a solution for which the problem can be formulated as a matrix equation.

For a 3D problem it is known to approximate a surface by a series of triangles. Therefore, basis functions for a surface divided into such a series of triangles are required. The RWG basis functions [4] are the lowest order set of functions fulfilling this requirement. To improve the accuracy and performance, higher order basis functions [5] are used in KarLESSS. In addition, surfaces are approximated by curved triangular patches. The curved triangles are parametrized with a local B-Splines interpolation [6].

The calculation of the impedance matrix with the MoM results in a dense matrix. This is caused by the non-local kernel of the EFIE, the Green's function. Consequently, the memory requirements and calculation time for the impedance matrix are of the order  $O(N^2)$ , where  $N$

denotes the number of unknowns. Therefore, it is a challenge to solve the EFIE for highly over-sized scatterers such as quasi-optical systems of high-power gyrotrons. To solve this problem, a speed-up of the calculation of scattering and radiation problems based on the adaptive cross approximation (ACA) was first proposed in [7]. With the ACA, the impedance matrix can be compressed in a purely algebraic manner. No information about the integral kernel is required and the ACA can be used for different sets of basis functions as well as for different Green's functions. The premise of the ACA is that interactions of well-separated parts of the geometry result in a rank-deficient impedance matrix. Therefore, the dense impedance matrix consists of many numerically rank-deficient sub-blocks. These sub-blocks of low rank can be compressed efficiently with the ACA:

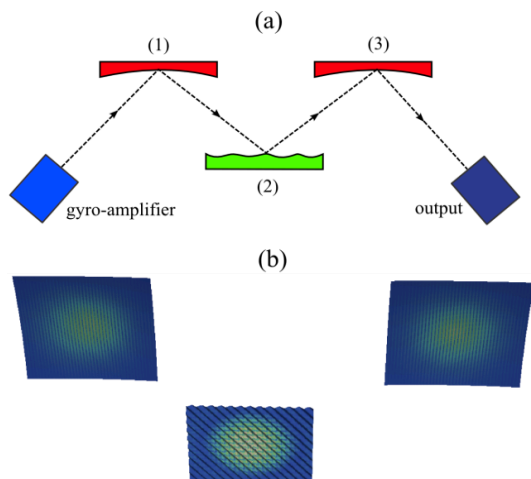
$$\mathbf{Z}^{m \times n} \approx \mathbf{U}^{m \times r} \mathbf{V}^{r \times n} \quad (2)$$

where  $r$  is the rank of the approximate matrix. If  $r$  is sufficiently small, the approximated matrix requires less memory and can be calculated faster than the full matrix. The advantage of this algorithm compared to other compression algorithms is that it requires only a partial knowledge of the original matrix. This is achieved by an iterative calculation of the approximated matrix (see [7] for details). Therefore, the memory requirements as well as the required calculation time can be reduced. In addition, the calculation of matrix-vector products with the approximated matrix is faster than for the uncompressed matrix. That accelerates the iterative solver (based on a generalized minimal residual method) which is used for solving the system of linear equations.

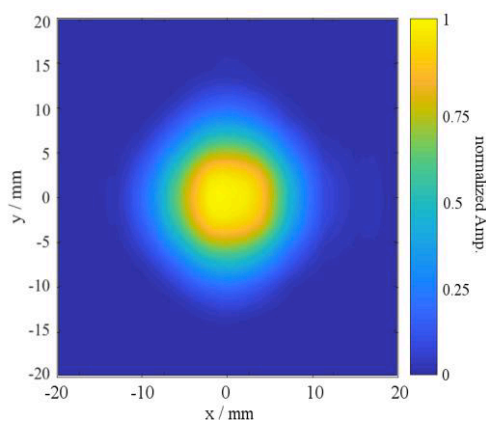
### Quasi-Optical Input-Output System

A problem very similar to the simulation of the quasi-optical systems used in high-power gyrotrons, is the simulation of an input-output system for a gyro-amplifier (see Fig. 1 a)). The system is designed for a center frequency of 260 GHz and consists of parabolic and sinusoidal corrugated mirrors. Shown is the simulation of the output channel. For the simulation, a linear polarized  $HE_{1,1}$ -beam is radiated from the position of the gyro-amplifier. After it is reflected on the mirrors 1, 2 and 3, the field distribution is analyzed at the plane of the output-port (see Fig. 2 b)).

In addition to the separate simulation of the input and output channels, a simultaneous simulation of both channels was performed. The resulting surface currents on the mirrors of the output channel are shown in Fig. 1 b).



**Fig. 1.** Part of a mirror system for a gyro-amplifier's I/O system. a) Schematic draw. b) Calculated currents on the mirrors (a.u.)



**Fig. 2.** Simulated field distribution for a frequency of 260 GHz at the position of the output port

### Helically Corrugated Resonator

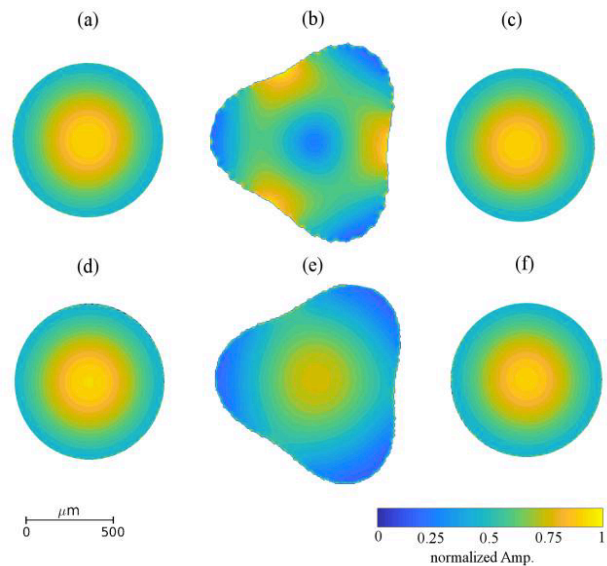
To show the flexibility of KarLESSS, field simulations, without an electron beam, of the interaction circuit of a helically corrugated gyro-amplifier [3] at 260 GHz are performed. The profile of the wall is described by:

$$r(\varphi, z) = r_0 + l \sin\left(\frac{2\pi}{d}z - 3\varphi\right) \quad (3)$$

where  $r_0$  defines the average of the waveguide which is about 1 mm. The depth of the corrugation  $l$  is about 0.1 mm. The period of the corrugation  $d$  is also about 1 mm. At the begin and at the end of the circuit, tapers with a length of 3 periods transform the mode of a circular waveguide into eigen modes of the corrugated waveguide. In total, the structure has a length of 54 periods. The threefold right-handed corrugation couples the right-handed  $TE_{2,1}$ -mode with the forward-propagating left-handed  $TE_{-1,1}$ -input mode, whereas a right-handed  $TE_{1,1}$ -input mode should be almost unperturbed. In Fig. 3 the field distributions before (1st column), inside (2nd column) and after the regions with corrugations (3rd column) are shown. The upper row shows the simulated field distributions for a  $TE_{-1,1}$ -mode as input and in the lower row for a  $TE_{1,1}$ -input mode. As expected, the field distribution of the  $TE_{1,1}$ -mode is almost unperturbed

while the  $TE_{-1,1}$ -mode is significantly modified. In both cases, the mode is nicely back-transformed to a cylindrical waveguide mode after the corrugations.

In the presentation, further cold analyses of the helically corrugated resonator, i.e. a simulation of the dispersion relation, will be presented.



**Fig. 3.** Simulated field distributions for a left-handed  $TE_{-1,1}$  (a, b, c) and a right-handed  $TE_{1,1}$  input mode (d, e, f). Left column: entry of resonator, Center column: in the resonator, Right column: end of resonator

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