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A high-quality narrow passband filter for elastic SV waves via aligned parallel separated thin polymethylmethacrylate plates

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We designed a high-quality filter that consists of aligned parallel polymethylmethacrylate (PMMA) thin plates with small gaps for elastic SV waves propagate in metals. Both the theoretical model and the full numerical simulation show the transmission spectrum of the elastic SV waves through such a filter has several sharp peaks with flawless transmission within the investigated frequencies. These peaks can be readily tuned by manipulating the geometry parameters of the PMMA plates. Our investigation finds that the same filter performs well for different metals where the elastic SV waves propagated. [http://dx.doi.org/10.1063/1.4994597]

Since the existence of the forbidden band gaps in phononic crystals[1,2] (PnCs), these metamaterials have been thought to be a promising material within the design of isolators and filters[3–6] for acoustic and elastic waves. Both the width and location of the forbidden band gaps are highly connected to various aspects of PnCs such as the material properties of the base material and the inclusions, as well as geometrical shapes and spatial distribution of inclusions. The forbidden band gaps in PnCs tend to be relatively narrow and the waves within the forbidden band gaps are usually located in the high frequency region. This has led to the increased research in order to increase the width of forbidden band gaps[7–10] and reduce the working frequency[11–13] of PnCs.

Up to date, besides the prevailing PnCs in the forms of 1D layered structures and 2D composite materials, there has also been attention paid to the PnCs based on basic mechanical components including beams,[14–16] thin plates,[17–21] and membranes.[22,23] The dynamic behaviors of these basic mechanical components have been well investigated and are widely used in real applications, compared to the more complicated metamaterials. Therefore, phononic crystallization of these basic mechanical elements is very beneficial to the practical applications of PnCs. The existing PnCs typically perform well as broad passband filters, due to their narrow forbidden band gaps. However, the PnCs acting as filters for elastic waves with a narrow passband[3,24] are not largely studied, particularly the ones based on the basic mechanical elements without complicated material modulation.[24]

In this paper, a narrow passband filter for elastic SV waves propagate in metals was designed based on thin plates made of a single material. The filter is composed of aligned parallel PMMA

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plates, which are separated by small gaps, and are seamlessly bonded at the two ends to the metals where the SV waves propagated. A schematic of this filter is shown in FIG. 1. The thickness and length of the plates are denoted by h and L, and the gap between adjacent plates is denoted by a, which is much smaller that h. The propagation of SV waves is along the direction indicated by the long arrows and the material particles vibrate along the short arrows, as shown in FIG. 1 (a). The material properties are denoted by E, μ, ν, and ρ, which correspond to Young modulus, shear modulus, Poisson ratio, and mass density. In the classical thin plate theory, the governing equation for flexural waves in thin plates under a harmonic excitation is expressed as:[25]

$$D\frac{\partial^4 w}{\partial x^4} - \rho h \omega^2 w = 0 \tag{1}$$

Within this expression, ω is the angular frequency, w is the flexural displacement, and $D = Eh^3/12(1-\nu^2)$ is the bending stiffness of the plate. General solutions to Eq.(1) consist of two parts: a travelling wave and a wave attenuating as it progresses.[25] Since the periodicity of geometries of this filter along the vertical direction, a unit cell[26] as seen in FIG. 1 (b) is adopted to analyze its transmission spectrum. Under a normal incidence of a plane elastic SV wave, the wave fields in the unit cell can be expressed as:

$$w = \begin{cases} e^{ik_T^0 x} + Re^{-ik_T^0 x}, x < 0 \\ Ae^{ik_F^1 x} + Be^{-ik_F^1 x} + Ue^{k_F^1 x} + Ve^{-k_F^1 x}, 0 < x < L \\ Te^{ik_T^0 (x-L)}, x > L \end{cases} \tag{2}$$

where i is the imaginary unit. The parameters R, A, B, U, V, and T are the coefficients of displacements for the reflected wave, the forward and backward waves in the plate and the transmitted wave. In the present study, values for the base material are characterized by the superscript 0 and 1 for the plates. $k_T = \omega/c_T$ and $k_F = \omega/c_F$ are the wave numbers of transverse and flexural waves, in which $c_T = \sqrt{E/2\rho(1+\nu)}$ and $c_F = \sqrt[4]{Eh^2\omega^2/12\rho(1-\nu^2)}$ are the corresponding wave speeds.

The six unknowns (R, A, B, U, V, and T) are determined by the z-averaged continuity conditions of displacement, rotation angle, and shear force at $x = 0$ and $x = L$.[26] Thus, six equations are established

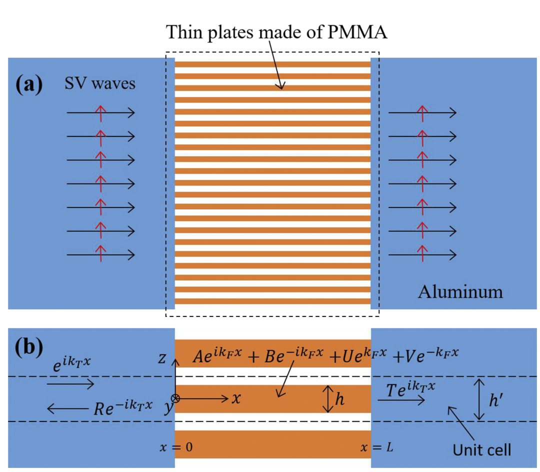

FIG. 1. Schematic of (a) the filter made by aligned parallel thin PMMA plates for elastic SV waves propagating in metals and (b) the unit cell used to analyze the transmission spectrum of this filter.

for the six unknowns as follows:

$$\begin{cases} 1+R = A+B+U+V \\ 1-R = \dfrac{k_F^1}{k_T^0}\left(A-B-iU+iV\right) \\ 1-R = \dfrac{D\left(k_F^1\right)^3}{\mu_0 h' k_T^0}\left(A-B+iU-iV\right) \\ T = Ae^{ik_F^1 L} + Be^{-ik_F^1 L} + Ue^{k_F^1 L} + Ve^{-k_F^1 L} \\ T = \dfrac{k_F^1}{k_T^0}\left(Ae^{ik_F^1 L} - Be^{-ik_F^1 L} - iUe^{k_F^1 L} + iVe^{-k_F^1 L}\right) \\ T = \dfrac{D\left(k_F^1\right)^3}{\mu_0 h' k_T^0}\left(Ae^{ik_F^1 L} - Be^{-ik_F^1 L} + iUe^{k_F^1 L} - iVe^{-k_F^1 L}\right) \end{cases} \quad (3)$$

Solving the above equations provides the transmission and the reflection coefficients. Since the PMMA is much softer than metals, the rotation angle of the thin PMMA plate at the two ends is smaller. As a matter of fact, the rotation angles at the two junctions ($x = 0$ and $x = L$) can be assumed to be zero. Figure 2 shows the analytical transmission coefficient changed with frequencies, with $L = 0.05$m, $h = 0.005$m, and $h' = 0.0055$m. The results based on the conditions where the rotation angle equaled zero at the two junctions are also plotted. It is indicated that the two different conditions for rotation angles at $x = 0$ and $x = L$ give nearly the same solution, which is in good agreement with the theoretical expectation. For a comparison, the results for the design in which the plates are made of the same material as the base material are also illustrated. It is clear that the transmission spectrum of the current design shows several sharp peaks within the investigated frequencies. This is not the case with the transmission spectrum of the plates composed of the same material as the base material. This suggests that the incident SV wave can be transmitted through the filter perfectly at these frequencies while inhibiting other waves. This design, in theory, is able to be used as a high-quality narrow passband filter for elastic SV waves propagating in metals. It is noted that, in the present work, we have not considered the oblique incidence because the situation and calculation will become complicated.

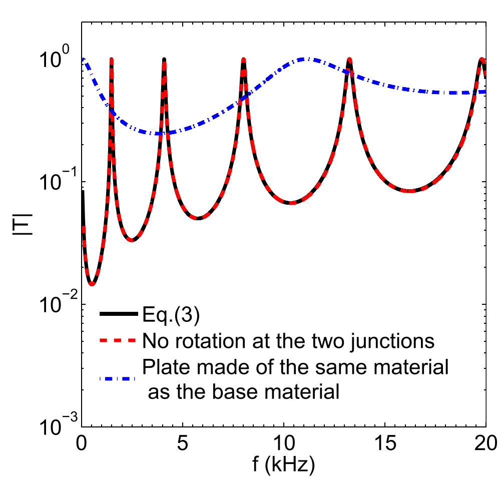

FIG. 2. Analytical transmission spectrum for the designed filter under a normal incidence of a plane elastic SV wave. For a comparison, the transmission spectrum of the design where the PMMA plates were replaced by plates made of the same material as the base material is also plotted.

FIG. 3. The numerical model used in this paper.

To validate the feasibility of the designed filter, a full numerical simulation of its performance using the Finite Element Method (FEM) is conducted. Figure 3 illustrates the numerical model used. The middle rectangle represents the PMMA plate and the connecting two parts are the base material. In the numerical simulations, the PMMA plate is perfectly bonded to the base blocks and plain-strain quadrilateral elements are used to discretize the entire domain. The perfectly matched layers (PML) at both ends act as anechoic terminations to avoid reflections from the ends. In order to mimic the periodicity in the vertical direction, the periodic boundary conditions are applied along the red lines. The horizontal displacement is fixed at the two ends in order to avoid rigid motion. A uniform vertical displacement is applied on the left solid blue line to generate a plane SV wave (see red arrows). The vertical displacement at the right dashed line is obtained to calculate the transmission coefficient. Properties of the PMMA and several other metals have been investigated are listed in Table I.

Figure 4 shows the calculated transmission spectrum of this filter for the aluminum matrix. The size of the thin PMMA plate is $L = 0.05$m, $h = 0.005$m, and $h' = 0.0055$m. For a comparison, the analytical transmission spectrum is also plotted in this figure. The results shows that the numerical transmission spectrum has several sharp peaks, as predicted by the theoretical model. When the frequency is low, the numerical results match the analytical solutions. However, the locations of the numerical calculated peaks shift as the frequency increases. This is likely caused by the assumptions used in the theoretical model break down at high frequencies. In a word, both the numerical simulations and the analytical analysis indicate that this design is able to serve as a high-quality narrow passband filter for elastic SV waves.

TABLE I. The material properties used in the present numerical simulations.

	E(GPa)	ν	ρ(kg/m^3)
PMMA	0.53	0.37	1180
Aluminum	70.0	0.35	2700
Steel	210.0	0.29	7800
Copper	210.0	0.25	8500

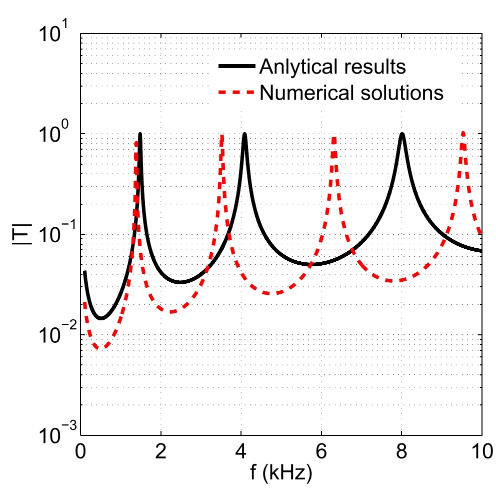

FIG. 4. The calculated transmission spectrum of the designed filter based on the full numerical simulation via FEM, and the comparison with the analytical results.

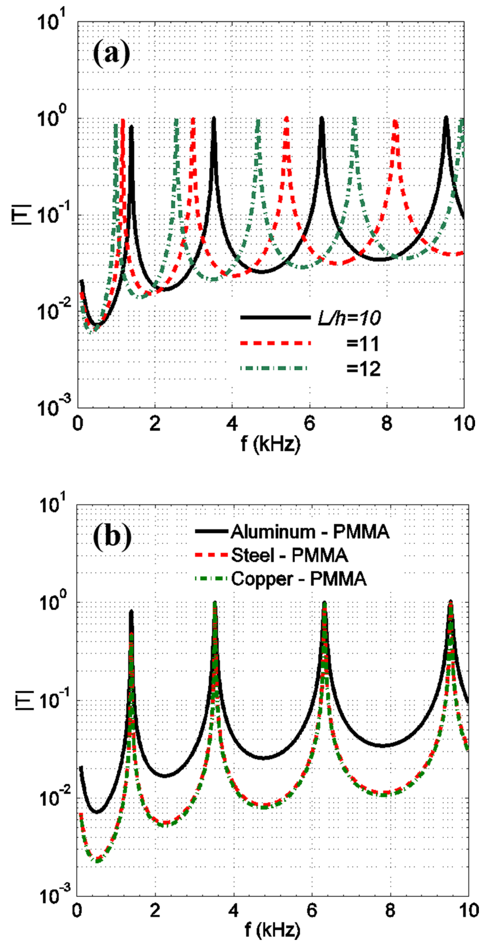

FIG. 5. Transmission spectrum of this filter (a) with plates of different lengths, (b) used in different base materials. In (a), the supporting base material is Aluminum, the thickness of plates is fixed, and the length of plates is changed. In (b), the geometry of the PMMA plate is L = 0.05m, h = 0.005m, and h' = 0.0055m.

Figure 5 (a) shows the effect of geometries of the PMMA plates on the transmission spectrum of this filter. The results show that the corresponding peaks shift from high frequencies to low frequencies as the plate length increases. At higher frequencies, this shift becomes greater. This suggests that the working frequency of this filter can be easily tuned through the changing of geometries of the PMMA plates. The effect of properties of the base material on the transmission capability of this filter is also investigated, and results are plotted in FIG. 5 (b). Results indicate that the same filter would work well for different base materials where the elastic SV waves propagate. We expect that the present design would be beneficial in the development and application of high-quality, narrow pass-band filters for elastic waves. Our present study specifically addresses a thin PMMA plates based filter with simple configurations for elastic SV waves, which has not been previously studied at length.

In conclusion, a narrow passband filter made with aligned parallel PMMA thin plates for elastic SV waves propagate in metals was designed. Both the analytical model and the full numerical simulation verified the feasibility of the designed filter. The working frequencies of this filter can be easily adjusted by changing of geometries of the thin PMMA plates. The same filter works well for different metals where the elastic SV waves propagate. This filter has potential benefits in controlling SV waves, such as mechanical computation, etc.

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[1] R. H. Olsson III and I. El-Kady, Measurement Science and Technology **20**(1) (2009).
[2] Y. Pennec, J. O. Vasseur, B. Djafari-Rouhani, L. Dobrzynski, and P. A. Deymier, Surface Science Reports **65**(8), 229 (2010).
[3] A. Khelif, P. A. Deymier, B. Djafari-Rouhani, J. O. Vasseur, and L. Dobrzynski, Journal of Applied Physics **94**(3), 1308 (2003).
[4] Y. Pennec, B. Djafari-Rouhani, J. O. Vasseur, A. Khelif, and P. A. Deymier, Physical Review E **69**(4) (2004).
[5] H.-W. Dong, Y.-S. Wang, and C. Zhang, Ultrasonics **76**, 109 (2017).
[6] P. Zhang and A. C. To, Applied Physics Letters **102**(12) (2013).
[7] M. I. Hussein, K. Hamza, G. M. Hulbert, and K. Saitou, Waves in Random and Complex Media **17**(4), 491 (2007).
[8] S. Halkjaer, O. Sigmund, and J. S. Jensen, Zeitschrift Fur Kristallographie **220**(9-10), 895 (2005).
[9] S. Hedayatrasa, K. Abhary, M. Uddin, and C.-T. Ng, Journal of the Mechanics and Physics of Solids **89**, 31 (2016).
[10] Y. M. Soliman, M. F. Su, Z. C. Leseman, C. M. Reinke, I. El-Kady, and R. H. Olsson III, Applied Physics Letters **97**(19) (2010).
[11] Z. Liu, X. Zhang, Y. Mao, Y. Y. Zhu, Z. Yang, C. T. Chan, and P. Sheng, Science **289**(5485), 1734 (2000).
[12] Y. Ding, Z. Liu, C. Qiu, and J. Shi, Physical Review Letters **99**(9) (2007).
[13] Y. Gu, X. Luo, and H. Ma, Journal of Applied Physics **105**(4) (2009).
[14] L. Ding, H.-P. Zhu, and T. Yin, Journal of Sound and Vibration **332**(24), 6377 (2013).
[15] Y. Q. Guo and D. N. Fang, Journal of Applied Mechanics-Transactions of the Asme **81**(1) (2014).
[16] H. Khales, A. Hassein-Bey, and A. Khelif, Journal of Vibration and Acoustics-Transactions of the Asme **135**(4) (2013).
[17] M. Mazzotti, M. Miniaci, and I. Bartoli, Ultrasonics **74**, 140 (2017).
[18] H. Shu, W. Liu, S. Li, L. Dong, W. Wang, S. Liu, and D. Zhao, Journal of Vibration and Control **22**(7), 1777 (2016).
[19] S. Hedayatrasa, K. Abhary, and M. Uddin, Ultrasonics **57**, 104 (2015).
[20] J. C. Hsu and T. T. Wu, Journal of Mechanics **23**(3), 223 (2007).
[21] Y. Pennec, B. Djafari-Rouhani, H. Larabi, J. O. Vasseur, and A. C. Hladky-Hennion, Physical Review B **78**(10) (2008).
[22] Z. Yang, J. Mei, M. Yang, N. H. Chan, and P. Sheng, Physical Review Letters **101**(20) (2008).
[23] T. Y. Huang, C. Shen, and Y. Jing, Journal of the Acoustical Society of America **139**(6), 3239 (2016).
[24] Il K. Lee, H. Min Seung, and Y. Y. Kim, Journal of Sound and Vibration **355**, 86 (2015).
[25] C. C. Mow and Y. H. Pao, "The diffraction of elastic waves and dynamic stress concentrations" (1971).
[26] X. Su and A. N. Norris, Journal of the Acoustical Society of America **139**(6), 3385 (2016).