The Application of the GlueVaR Measure in Risk Assessment on the Metal Market

Dominik Krężołek and Grażyna Trzpiot

Abstract The purpose of the study is the application of a new risk measure, called GlueVaR, into investment risk assessment. This measure is closely related to Value-at-Risk (VaR) and Conditional VaR (CVaR). In the literature describing theoretical background of VaR and CVaR certain properties of risk measures are highlighted. The first one is a the good risk measure has to be coherent, and the second one is that both VaR and CVaR belong to the class of distortion risk measures. As far as it is concerned, VaR is not a coherent risk measure because, it does not meet the subadditivity property. This unfulfilled property has a particular application in risk analysis, especially in extreme risk measurement. On the other hand, distortion risk measures are associated with an investor's risk attitude which is an individual attribute of any decision-maker. The research area chosen for this study is the metal market divided into two natural sub-markets: The precious metals and the non-ferrous metals market. Risk measures as VaR, CVaR and GlueVaR are calculated and the results are associated with the investor's attitude toward risk.

Dominik Krężołek
University of Economics in Katowice, Department of Demography and Economic Statistics, 1 Maja 50, 40-287 Katowice, Poland,
dominik.krezolek@ue.katowice.pl

and Grażyna Trzpiot
University of Economics in Katowice, Department of Demography and Economic Statistics, 1 Maja 50, 40-287 Katowice, Poland,
grazyna.trzpiot@ue.katowice.pl

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1 Concept of risk

Any activities related to investing require taking into consideration certain phenomena which sometimes are not directly observed in the market: Volatility, uncertainty and unpredictability. These phenomena are associated with investment risk and related to the problem discussed in this paper. Volatility represents dispersion observed within the level of prices/returns of an asset. Uncertainty refers to the ignorance of reality and how it affects the decision-making process. Finally, unpredictability is associated with the uncertainty about the future level of prices/returns. All these features reflect the mood among investors. Therefore, one can define risky investment as uncertain, unknown and diverging from the expectation.

Economic and financial disturbances observed in the market, the political situation in some significant zones in the world or even natural phenomena significantly affects the level of undertaken risk. Given the market risk, its level often derives from investors behavior and how they assess the reality. The reality is usually different from the assumptions of statistical models. One of these is the assumption of normality. In the area of financial time series of returns it is possible to mention certain specific features, like leptokurtosis, significant level of autocorrelation, clustering, fat tails in empirical distributions, asymmetry, etc., which reject the normality assumption. Therefore, researchers and scientists have to seek for new theoretical solutions responding reality. In this paper we focus on some specific type of risk - extreme risk. Extreme risk (catastrophic risk) is related to an event with low probability of occurrence, but if it does take place then can produce large losses (Jajuga (2008))Extreme risk is often defined as Low Frequency, High Severity (LFHS). See 1.

Table 1 Regular versus extreme risk

Loss	Low probability	High probability
Small	-	regular risk
Large	extreme risk	-

As derives from this definition, extreme risk is related to its negative perception. Theoretical methods used for modelling and examining such risk include two popular approaches. As shown by Embrechts et al. (Embrechts et al (1997)), the first one is based on the analysis of the distribution of maxima (Generalized

Extreme Value) and the second one is based on the peaks over threshold (Generalized Pareto Distribution), but these methods are omitted in this paper. Extreme risk analyzed in this article should be understood more generally. We consider such risk as related to the event that the probability of occurrence is significantly different from the expected one. Empirical distributions describing such phenomena are within the family of heavy-tailed distributions.

2 Coherent and distortion risk measures

Despite the type of risk we have to define the risk measure. Let $\mathbb{X}$ be the set of all random variables defined for a given probability space $(\Omega, \mathscr{A}, P)$. A risk measure ρ is a mapping from $\mathbb{X}$ to $\mathbb{R}$, which means the mapping from the set of random variables to the line represented by the real numbers:

$$\mathbb{X} \to \rho(X) \in \mathbb{R} \tag{1}$$

Therefore $\rho(X)$ is defined as a real value for each $X \in \mathbb{R}$. As the risk measures is defined, certain conditions for this measure have to be met. Artzner et al. (Artzner et al (1999)) defined some axioms describing an acceptable risk measure: positive homogeneity, subadditivity, monotonicity and translation invariance. All these axioms need hold for coherent risk measures. It is worth to mention that some authors replace the assumption of positive homogeneity and subadditivity by the convexity condition. Not going into details, from an investor's point of view all these axioms are of big importance, but a subadditivity deserves particular attention. This axiom states that the risk of the portfolio is equal or less than the sum of individual risks of its components. Following the definition of subadditivity one may associate it with diversification. Therefore, a good risk measure should fullfill all four axioms.

In addition to coherency in the literature we can find some other properties of risk measures. Wang (Wang (1996)) introduces distortion risk measures defined as follows. Let $\mathbb{X}$ be the set of all random variables defined for a given probability space $(\Omega, \mathscr{A}, P)$. Let g be a non-decreasing and injective function defined as $g : [0,1] \to [0,1]$ where $g(0) = 0$ and $g(1) = 1$ (Yaari (1987)). The function g is called the distortion function. For a given random variable X the distortion the risk measure associated with distortion function g is of the form:

$$\rho_g(X) = \int_0^\infty g\left[S_X(x)\right] dx \tag{2}$$

where $S_X(x)$ is called the decumulative distribution function or survival function.

Both these classes of risk measures have been defined to assess the validity of use in practice of certain popular risk measures: Value-at-Risk (VaR) and mathematical expectation beyond VaR, called Conditional Value-at-Risk (CVaR). This risk measure is used in practice as a standard tool to assess the risk of an investment. For a given time horizon and a certain fixed probability level α the VaR defines a loss that is exceeded over this specified time horizon with the probability $(1-\alpha)$.

Despite the popularity of this measure of risk in a practical use, VaR is not coherent because it does not meet the subadditivity assumption. VaR is subadditive for random variables which are elliptically distributed (McNeil et al (2005)), so in terms of other statistical distributions it does not measure the risk. Szegö (Szegö (2002)), referring to Rockafellar and Uryasev (Rockafellar and Uryasev (2002)), indicates additional disadvantages of VaR as a measure of risk:

1. VaR may provide discrepant results at different confidence levels.
2. Non-convexity, which does not allow use VaR in optimization problems.
3. A reduction of VaR may lead to stretch the tail exceeding VaR.
4. Value-at-Risk presents many extremes leading to unstable ranking of VaR.

An alternative risk measure against VaR is CVaR which is defined as an average quantile ranging from the $\alpha-$quantile to the maximum value of a random variable X.

Both VaR and CVaR can be understood as two particular cases of distortion risk measures. The advantage of CVaR against VaR is that the first one fullfills the subadditivity assumption and, in addition, measures the average level of loss in the most adverse cases, whereas VaR represents only the minimum loss. The value of CVaR is usually higher than the value of VaR and the selection of risk measure depends on the investor's attitude toward risk.

3 A new family of risk measures - GlueVaR risk measures

Taking into account any risk measure, its selection is based on underlying investor's risk attitude. Belles-Sempera et al. (Belles-Sampera et al (2014)) proposed a new family of risk measures based on VaR and CVaR — the GlueVaR risk measure. For a fixed confidence levels this family contains risk measure lying somewhere between values of VaR and CVaR and reflecting investor's attitude toward risk. The family of GlueVaR risk measures is usually defined in terms of distortion function. The distortion function of GlueVaR risk measure is the four-parameter function of the form:

$$\eta_{\beta,\alpha}^{\omega_1,\omega_2} = \begin{cases} \left[\frac{\omega_1}{1-\beta} + \frac{\omega_2}{1-\alpha}\right] u & if\ 0 \le u < 1-\beta \\ \omega_1 + \left[\frac{\omega_2}{1-\alpha}\right] u & if\ 1-\beta \le u < 1-\alpha \\ 1 & if\ 1-\alpha \le u \le 1 \end{cases} \tag{3}$$

where α and β represent the confidence levels such that $0 < \alpha \le \beta < 1$ and the two remaining parameters ω_1 and ω_2 represent weights such that $\frac{\beta-1}{\beta-\alpha} \le \omega_1 \le 1$ and $\omega_1 + \omega_2 \le 1$.

A very interesting feature of the new risk measure is that it can be rewritten as a linear combination of VaR at the level α, CVaR at the level α and CVaR at the level β $(0 < \alpha \le \beta < 1)$:

$$GlueVaR_{\beta,\alpha}^{\omega_1,\omega_2}(X) = \omega_1 CVaR_\beta + \omega_2 CVaR_\alpha + (1-\omega_1-\omega_2) VaR_\alpha \tag{4}$$

For fixed parameters ω_1 and ω_2 it is possible to define special cases of the GlueVaR risk measure:

1. If $\omega_1 = 0$ and $\omega_2 = 0$ then the GlueVaR reduces to VaR at the level α.
2. If $\omega_1 = 0$ and $\omega_2 = 1$ then the GlueVaR reduces to CVaR at the level α.
3. If $\omega_1 = 1$ and $\omega_2 = 0$ then the GlueVaR reduces to CVaR at the level β.

As results from the equation (4) the selection of risk measure depends on the weights ω_1 and ω_2. These weights may help to define a particular investor in terms or his attitude toward risk as (Belles-Sampera et al (2016)):

1. highly conservative, if $\omega_1 = 1$ and $\omega_2 = 0$,
2. conservative, if $\omega_1 = 0$ and $\omega_2 = 1$,
3. less conservative, if $\omega_1 = 0$ and $\omega_2 = 0$.

Hence, for given confidence levels α and β and for certain preferred levels of weights ω_1 and ω_2 reflecting the investor's attitude toward risk, the appropriate risk measure within the new family of risk measures may be selected. An interesting feature of the GlueVaR risk measures is that they allow for subadditivity depending on the associated weights. As the GlueVaR may be defined as a linear combination of VaR and CVaR, and as the CVaR is a coherent risk measure, the subadditivity of GlueVaR holds, if the weight $(1-\omega_1-\omega_2)$ corresponding to VaR in equation (4) is equal to zero. More general, the GlueVaR defined by equation (4) is subadditive, if $\frac{\beta-1}{\beta-\alpha} \le \omega_1 \le 1$ and $\omega_1 + \omega_2 \le 1$.

4 Empirical analysis on the metal market

Modern financial markets are characterized by the a high level of volatility and unpredictability. Investors are trying to reduce risks by creating portfolios composed of different types of assets, including those from commodity markets. Currently, the most popular commodities are energy resources, industrial and precious metals or agricultural commodities. It is worth mentioning that the prices (and returns) of such commodities, due to the fact of being quoted on the commodity exchanges, are volatile and unpredictable as well. Among the factors which determine this volatility we can mention some macroeconomic factors, supply-demand (or market) factors, economic and demographic development or even the geo-political situation. If a commodity market is of interest, there are some factors not directly related to the economy but having substantial impact - e.g. the weather factors (especially those that arise unexpectedly, not typical for a given area but causing huge financial losses).

In this paper we focus on one of the most important commodity markets, the metals market. Taking into account the use of metals we can easily make a simple division into two groups: Industrial (non-ferrous) metals and precious metals. Non-ferrous metals are widely used in many sectors of the economy and, therefore, are usually called strategic metals (their use is of great strategic importance for the industries, scientific areas, industrial production and the economy). Among the areas of applications we can identify the construction industry (including infrastructure), automotive, aerospace and medicine as well. The second group are the precious metals, which are very often too explicitly associated with jewellery. This link is obviously correct, however it is not exhaustive. Precious metals, in addition to jewellery, are also used for the production of coins and in industry. Gold is widely used in medicine, where it is used for the production of different types of devices or as a compound in medicaments (i.e. in medicines supporting anti-cancer treatments or in cosmetology). The use of gold is also common in electronics (high electrical conductivity) and foodservice (i.e. gold leaf or the Goldwasser - herbal liqueur). Similar applications can be indicated for silver, platinum or palladium. As we can see, significantly related to the economy and hence, necessary it's in-depth exploration, is the metal market.

Looking at the metal markets from the analytical point of view, there are few research papers dealing with the analysis of metals markets in the same way as the capital market is usually analysed. The most popular approaches refer to fundamental or macroeconomic analysis. For the analysis of investment risk and the possibility of taking into account metals in investment portfolios we should look at this problem in a similar way as in the classical capital investments. The key point in the assessment of effectiveness of investments is the diversification approach, proposed by Markovitz in 1952 (Markowitz (1952)). In the construction of a portfolio we should include those components which are low or negative correlated between each other. The correct selection of these assets allows the investor to protect himself against potential declines in prices of one group of assets, through the lack of changes or increases in prices for another alternative group of assets. Analyzing the economic and financial situation in the world within the period of 2006 - 2015 on increasing investor's interest in alternative forms of investment, including those in the metal market (especially precious metals). Similarly, industrial metals are of interest for which demand in emerging markets is still rising. Therefore, it seems to be extremely reasonable to diversify the investment portfolios assets derived from alternative markets.

Following the remarks presented earlier, we focus in this paper on the analysis of the risk of investment in terms of extreme changes (extreme events) observed within the returns of investments in metals. As a measure of risk we use the VaR, ES and the GlueVaR risk measure. The last measure is computed for a different levels of weights corresponding to those extreme events. These weights reflect the investor's attitudes towards risk.

Due to the financial and economic crises observed in the first decade of of the 21st century, investors have been forced into searching other possibilities to invest capital which - despites the generally observed decline - would generate positive returns (Krężołek (2012)). The analysis is based on daily log-returns of spot closing prices of certain metals quoted on the London Metal Exchange from January 2006 to June 2015. The set of assets includes gold, silver, platinum, palladium, aluminium, copper, lead, nickel, tin and zinc. The quantile-based risk measures such as VaR, CVaR and GlueVaR have been calculated, for quantile 0.95 and 0.99, using empirical and theoretical distributions: normal, $t-$Student and $\alpha-$stable. Figures 1 and 3 present levels of prices and figs. 2 and 4 the log-returns for gold and copper.

Both metals show significant fluctuations in price levels which of course affect the volatility in log-returns. Moreover, if log-returns are considered, we can find interesting characteristics which are very typical for time series observed within popular financial assets: Clustering of variance, high volatility,

Fig. 1 Spot prices of gold

long memory effect, etc. In table 2 certain descriptive statistics of log-returns are presented.

Table 2 Descriptive statistics of log-returns

Metal/Statistics	Mean	Standard deviation	Kurtosis	Skewness	Min	Max
Gold	0.00033	0.01283	**5.18772**	**-0.33920**	-0.08879	0.10392
Silver	0.00023	0.02295	**7.31618**	**-1.09103**	-0.17322	0.13926
Platinum	0.00004	0.01504	**5.16419**	**-0.54847**	-0.10445	0.09254
Palladium	0.00039	0.02068	**4.93499**	**-0.66108**	-0.16556	0.09995
Aluminium	**-0.00013**	0.01515	1.39880	-0.15917	-0.07437	0.05913
Copper	0.00011	0.01958	3.34357	-0.03873	-0.10400	0.11880
Lead	0.00020	0.02340	2.65170	-0.20177	-0.12850	0.12675
Nickel	**-0.00006**	0.02446	2.51394	-0.01349	-0.13605	0.13060
Tin	0.00031	0.01990	5.04123	-0.13282	-0.11435	0.14253
Zinc	0.00002	0.02162	1.86285	-0.18064	-0.10832	0.09135

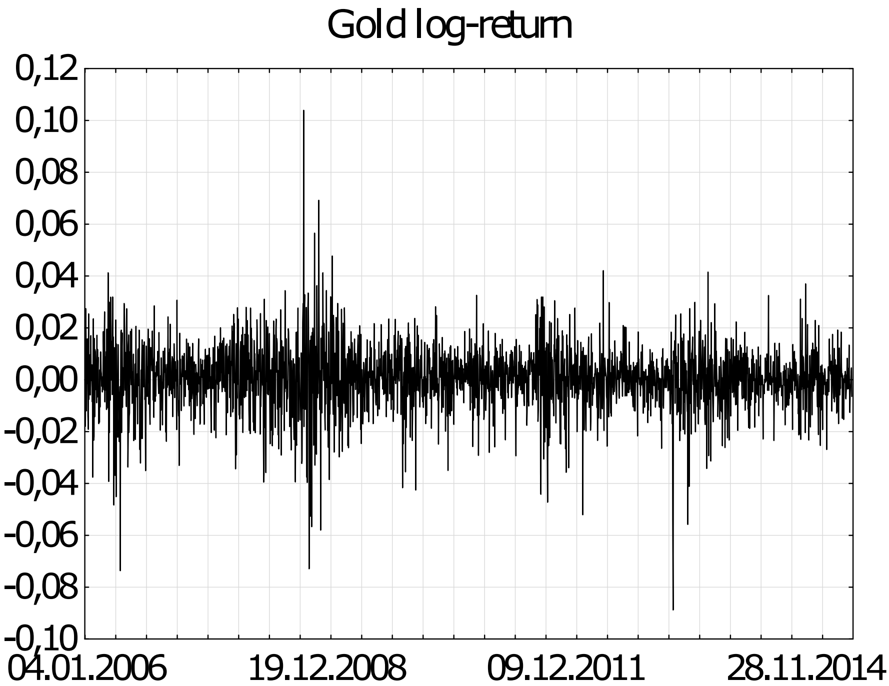

Fig. 2 Log-returns of gold

According to these results the empirical distributions for precious metals (gold, silver, platinum, and palladium) demonstrate higher level of both kurtosis and negative skewness compared to the other metals. As a consequence one may assume that the empirical distributions are not normal. Goodness-of-fit tests (Anderson-Darling and Cramer-von Misses) confirmed this hypothesis[1]. As an alternative, the $t-$Student and $\alpha-$stable distributions have been fitted to the data. The estimated parameters of the $\alpha-$stable distribution are presented in table 3.

Table 3 shows that the empirical distributions of all assets are not normal. The parameter $\hat{\alpha}$ which estimates the thickness of the tail of the distribution differs from 2 (For a normal distribution $\alpha = 2$. If $\alpha < 2$ the distribution is fat-tailed). These results show certain similarities among the analysed assets, which are not directly observed. Therefore, a cluster analysis has been performed to

[1] Due to the volume of paper some results, not directly related to the topic, are omitted

Fig. 3 Spot prices of copper

find out if this hypothesis is supported by the data. The following data and methods have been used:

1. Data: log-returns of daily log-returns.
2. Distance measure: squared Euclidean distance.
3. Linkage criterion: Ward's criterion.

Figure 5 shows the result of clustering.

The dendrogram in fig. 5 two groups of metals: Precious and non-ferrous metals. We mention that this grouping is based only on log-returns of metals spot prices. The main part of this analysis is the assessment of risk using quantile-based risk measures. Assuming two confidence levels $\alpha = 0.95$ and $\beta = 0.99$ and equal weights for each risk measures VaR_α, $CVaR_\alpha$ and $CVaR_\beta$ representing three different attitudes toward risk $\omega_1 = \omega_2 = \frac{1}{3}$ we show the distance between levels of risk related to empirical distributions of all assets. Table 4 presents the numerical results.

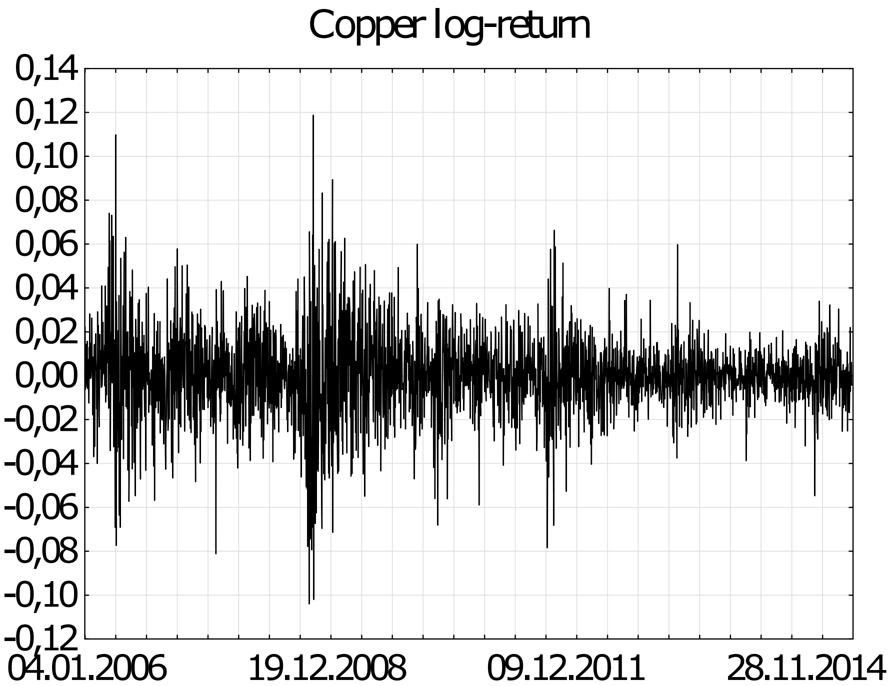

Fig. 4 Log-returns of copper

The bold values highlight the most risky metals in terms of risk measures used. If we look at the values of $GlueVaR^{\omega_1,\omega_2}_{\beta,\alpha}$ for fixed parameters, we can easily find that its values are closer to $CVaR_\beta$ than to $CVaR_\alpha$. That means that, using such weights, an investor's attitude toward risk might be defined more like conservative than highly conservative.

Considering theoretical distributions we have compared the results with those obtained for empirical data. For estimating VaR at the confidence level 0.95 it is better to use a normal or Student $t-$distribution, whereas for estimating the remaining risk measures (at the confidence level 0.99) it is better to use fat-tailed distributions ($\alpha-$stable). Numerical results for gold and copper are presented in tables 5 and 6.

As presented in section 3, the new risk measure GlueVaR is strongly dependent on the confidence levels and weights given to VaR and CVaR. The assumption of subadditivity for GlueVaR, to hold the weight corresponding

Table 3 Estimated* parameters of α-stable distribution

Metal/Parameter	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	$\hat{\sigma}$
Gold	**1.71056**	**-0.22901**	0.00024	0.00736
Silver	**1.65639**	**-0.24341**	0.00023	0.01233
Platinum	**1.70803**	**-0.24142**	-0.00009	0.00840
Palladium	**1.69346**	**-0.22391**	0.00029	0.01169
Aluminium	1.83632	-0.05684	-0.00005	0.00971
Copper	1.65241	-0.03794	0.00007	0.01092
Lead	1.72842	-0.16144	0.00002	0.01385
Nickel	1.77505	0.02572	-0.00002	0.01492
Tin	1.56237	-0.18879	-0.00010	0.01019
Zinc	1.73598	-0.00393	0.00017	0.01305

* Maximum Likelihood Method

to the non-subadditive risk component of GlueVaR (i.e. VaR) should meet the

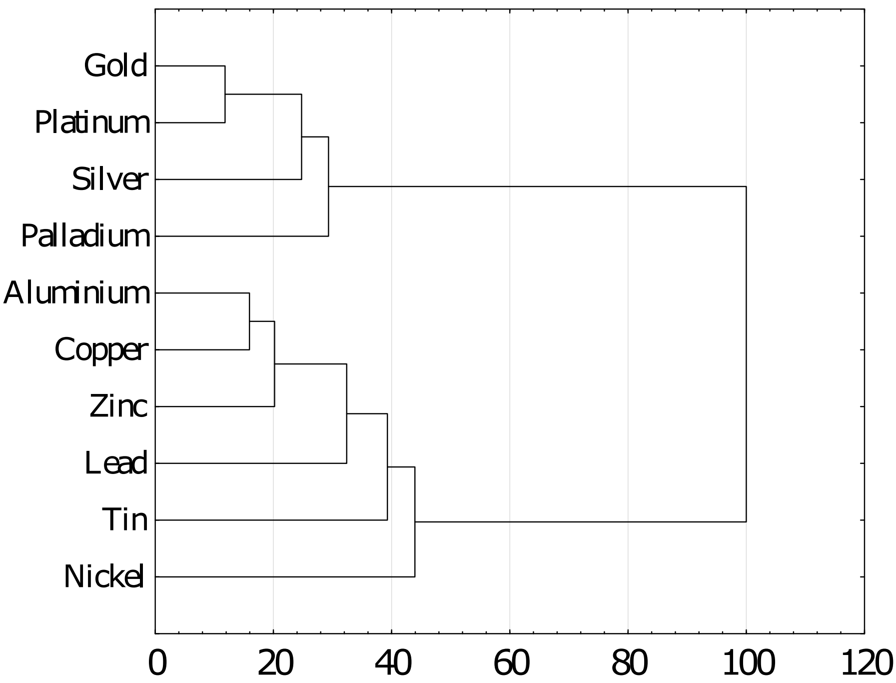

Fig. 5 Dendrogram for prices log-returns of metals

relation $(1-\omega_1-\omega_2)=0$. Belles-Sampera et al. showed that GlueVaR is subadditive if both weights ω_1 and ω_2 lie on the line segment in a coordinate system described by the two points: $A=(\omega_1,\omega_2)=(0,1)$ and $B=(\omega_1,\omega_2)=(1,0)$ for fixed values of α and β $(0<\alpha\leq\beta<1)$. Moreover, the position of a particular point on this line represents an investor's attitude toward risk. The nearer to the point A, the less conservative attitude toward risk.

As mentioned in section 3, the GlueVaR might be defined as a linear combination of VaR and CVaR for fixed tolerance levels and for fixed weights. The weights reflect an investor's attitude towards risk. If the assumption of subadditivity is required, the weights should satisfy the equality $(1-\omega_1-\omega_2)=0$ which means the elimination of the VaR measure from the final calculation of the GlueVaR. With this assumption and for a given theoretical distribution, the values of the GlueVaR have been calculated and compared to the empirical ones. All the results for gold and copper are presented in tables 7-8 and on the figure 6.

As we can see in fig. 6, the more conservative an investor's attitudes are towards risk, the higher the discrepancies between empirical and theoretical estimates of the GlueVaR risk measure. The smallest differences were observed for the $\alpha-$stable distribution. This results directly from the properties of this class of probability models. The $\alpha-$stable distributions allow for describing data with outliers or extreme observations. Therefore, they are widely used for modeling financial data. Similar results were obtained for the other metals.

Table 4 Estimated risk measures for empirical distributions

Metal	$VaR_{0.95}$	$CVaR_{0.95}$	$CVaR_{0.99}$	$GlueVaR^{\frac{1}{3},\frac{1}{3}}_{0.99,0.95}$
Gold	0.01964	0.02747	0.04040	0.02917
Silver	**0.03429**	**0.04661**	**0.06660**	**0.04917**
Platinum	0.02152	0.03181	0.05119	0.03484
Palladium	0.03106	0.04368	0.06449	0.04641
Aluminium	0.02473	0.03265	0.04442	0.03394
Copper	0.03108	0.04501	0.06880	0.04830
Lead	0.03694	0.05181	0.07680	0.05519
Nickel	**0.03884**	**0.05524**	**0.08248**	**0.05885**
Tin	0.02929	0.04482	0.07454	0.04955
Zinc	0.03609	0.04819	0.06610	0.05013

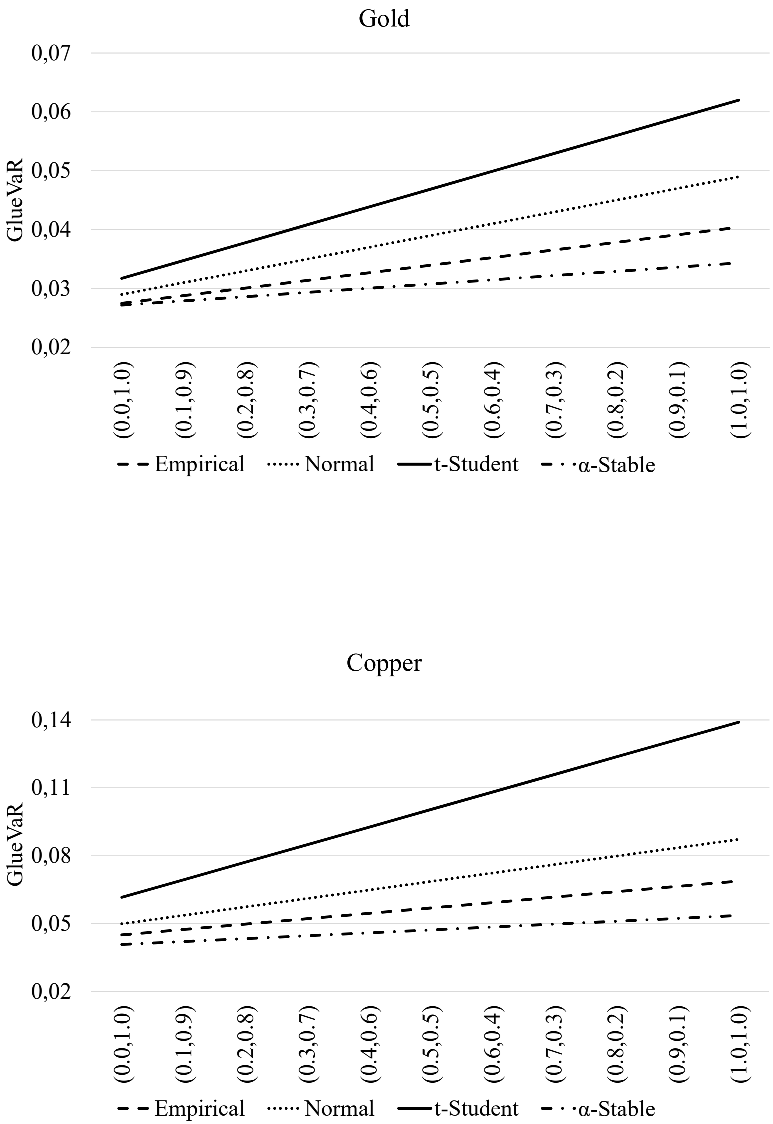

Fig. 6 GlueVaR risk measure for different weights (Gold - left, Copper - right)

Table 5 Risk measures - theoretical vs. empirical results for gold

DISTRIBUTION	$VaR_{0.95}$	$CVaR_{0.95}$	$CVaR_{0.99}$	$GlueVaR^{\frac{1}{3},\frac{1}{3}}_{0.99,0.95}$
EMPIRICAL	0,01964	0,02747	0,04040	0,02917
NORMAL	**0,01969**	0,02898	0,04897	0,03255
t-STUDENT	0,01862	0,03171	0,06199	0,03744
α-STABLE	0,02223	**0,02717**	**0,03433**	**0,02791**

Table 6 Risk measures - theoretical vs. empirical results for copper

DISTRIBUTION	$VaR_{0.95}$	$CVaR_{0.95}$	$CVaR_{0.99}$	$GlueVaR^{\frac{1}{3},\frac{1}{3}}_{0.99,0.95}$
EMPIRICAL	0,03108	0,04501	0,06880	0,04830
NORMAL	0,03025	0,04992	0,08722	0,05580
t-STUDENT	**0,03149**	0,06163	0,13901	0,07737
α-STABLE	0,03247	**0,04079**	**0,05359**	**0,04228**

Table 7 GlueVaRfor empirical and theoretical distributions for different weights - GOLD

ω_1	ω_2	Empirical	Normal	$t-$Student	$\alpha-$Stable
0.0	1.0	0.02747	0.02898	0.03171	0.02717
0.1	0.9	0.02876	0.03098	0.03474	0.02789
0.2	0.8	0.03005	0.03298	0.03776	0.02861
0.3	0.7	0.03135	0.03498	0.04079	0.02932
0.4	0.6	0.03264	0.03697	0.04382	0.03004
0.5	0.5	0.03393	0.03897	0.04685	0.03075
0.6	0.4	0.03523	0.04097	0.04987	0.03147
0.7	0.3	0.03652	0.04297	0.05290	0.03218
0.8	0.2	0.03781	0.04497	0.05593	0.03290
0.9	0.1	0.03910	0.04697	0.05896	0.03362
1.0	0.0	0.04040	0.04897	0.06199	0.03433

5 Conclusions and remarks

In the research presented in this we have described a new family of risk measures, called GlueVaR, and we have presented its application to risk measurement on the metal market. This commodity market is not very popular with researchers although it might be considered as an alternative to classical investments areas. We have shown that the typical risk assessment tools used on financial markets can also be used effectively on alternative markets. The analysis has been conducted using GlueVaR risk measure which are directly related to the popular quantile-based risk measures: VaR and CVaR. Both of these measures determine the value of loss for "extreme" events. An interesting

Table 8 GlueVaR for empirical and theoretical distributions for different weights - COPPER

ω_1	ω_2	Empirical	Normal	$t-$Student	$\alpha-$Stable
0.0	1.0	0.04501	0.04992	0.06163	0.04079
0.1	0.9	0.04739	0.05365	0.06936	0.04207
0.2	0.8	0.04977	0.05738	0.07710	0.04335
0.3	0.7	0.05215	0.06111	0.08484	0.04463
0.4	0.6	0.05453	0.06484	0.09258	0.04591
0.5	0.5	0.05691	0.06857	0.10032	0.04719
0.6	0.4	0.05929	0.07230	0.10806	0.04847
0.7	0.3	0.06166	0.07603	0.11579	0.04975
0.8	0.2	0.06404	0.07976	0.12353	0.05103
0.9	0.1	0.06642	0.08349	0.13127	0.05231
1.0	0.0	0.06880	0.08722	0.13901	0.05359

feature of this family of GlueVaR risk measures is that it can be defined as a linear combination of VaR and CVaR for given confidence levels and for given weights. The confidence level represents the probability of occurrence of some catastrophic event whereas the weights indicate, how important such an event is for the investor. Thus, a particular investor is able to decide consciously about the acceptable level of risk.

Referring to the analysis, we used both empirical and theoretical distributions (normal, $t-$Student and $\alpha-$stable) to calculate appropriate risk measures. The selection of distribution was dictated by the characteristics of log-returns of the prices of the analysed metals. The results show that if the probability of an unwanted event is not very low (i.e. 0.95), then the corresponding risk measure should be calculated using the normal distribution. Otherwise, fat-tailed distributions are more appropriate. Summarizing our observations we can say that the family of GlueVaR risk measures is an interesting and effective tool for assessing risk both in terms of subadditivity as well as because of the possibility of taking into account an individual investor's attitude toward risk.

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