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Neutrino Mass Sum Rules

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Abstract. Neutrino mass sum rules are an important class of predictions in flavour models relating the Majorana phases to the neutrino masses. This leads, for instance, to enormous restrictions on the effective mass as probed in experiments on neutrinoless double beta decay. While up to now these sum rules have in practically all cases been taken to hold exactly, we will go here beyond that. While the effect of the renormalisation group running can be visible, the qualitative features do not change. This changes somewhat for model dependent corrections which might alter even the qualitative predictions but only for large corrections and a high neutrino mass scale close to the edge of the current limits. This finding backs up the solidity of the predictions derived in the literature apart from some exceptions, and it thus marks a very important step in deriving testable and robust predictions from neutrino flavour models.

1. Introduction and Setup

Neutrino mass sum rules are relations between neutrino masses and Majorana phases and a common, testable prediction in flavour models trying to describe the origin of flavour in terms of (discrete) family symmetries. In [1–3] a general parametrisation for a neutrino mass sum rule, s, was given

$$s \equiv c_1 \left(m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1}\right)^d \mathrm{e}^{\mathrm{i}\Delta\chi_{13}} + c_2 \left(m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2}\right)^d \mathrm{e}^{\mathrm{i}\Delta\chi_{23}} + m_3^d \stackrel{!}{=} 0\,, \tag{1}$$

where m_i are the light, active neutrino masses and ϕ_i the Majorana phases. All the other coefficients are discrete parameters given by one of the twelve known sum rules, cf. table 1. For a perturbed sum rule in [3] we also defined a normalised sum rule, $\hat{s}$, and the first order perturbation of it, $\delta\hat{s}$, by

$$\hat{s} \equiv \frac{s}{m_n^d} \text{ and } \delta\hat{s} \equiv \frac{\delta s}{m_n^d}\,, \tag{2}$$

where m_n is a normalisation chosen in such a way that there is no artificial enhancement by a factor $m_i/m_j \gg 1$.

2. Selected Results

Some sum rules allow only for one of the two possible neutrino mass orderings. This can be most easily understood from a geometrical interpretation of the sum rule as a closed triangle in a complex plane with one of the angles labelled α [1]. In sum rule 2, for instance, one can then easily show that inverted ordering is excluded [2] because at tree level

$$(\cos\alpha)^{\text{tree}} = \frac{m_1^2 - 4m_2^2 - m_3^2}{4m_2 m_3} < -\frac{1}{4}\left(3\frac{m_2^2}{m_3^2} + 1\right) < -1 \Rightarrow \lightning\,, \tag{3}$$

where the m_i are the neutrino masses. The renormalisation group (RG) corrections to $\cos\alpha$ in the Minimal Supersymmetric Standard Model [2],

$$\delta(\cos\alpha)^{\text{RG}} \approx -\underbrace{\frac{y_\tau^2}{192\pi^2}}_{>0}\underbrace{\frac{2.8m_1^2 - 0.4m_2^2 + 0.1m_3^2}{m_2 m_3}}_{>0}\underbrace{\log\frac{M_S}{M_Z}}_{>0} < 0\,, \tag{4}$$

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Table 1. Summary table of the sum rules taken from [2, 3], cf. Eq. (1).

Sum rule	References	c_1	c_2	d	$\Delta\chi_{13}$	$\Delta\chi_{23}$
1	[4–13]	1	1	1	π	π
2	[14]	1	2	1	π	π
3	[4, 7–11, 15–34]	1	2	1	π	0
4	[35, 36]	1/2	1/2	1	π	π
5	[37]	$\frac{2}{\sqrt{3}+1}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	1	0	π
6	[4–6, 38–40]	1	1	−1	π	π
7	[4, 15–17, 41–55]	1	2	−1	π	0
8	[56–59]	1	2	−1	0	π
9	[60]	1	2	−1	π	$\pi/2, 3\pi/2$
10	[61, 62]	1	2	1/2	$\pi, 0, \pi/2$	$0, \pi, \pi/2$
11	[63]	1/3	1	1/2	π	0
12	[64]	1/2	1/2	−1/2	π	π

make things worse. Here, y_τ is the τ Yukawa coupling, M_S and M_Z are the seesaw and the weak scale respectively. In the overwhelming part of the parameter space the RG corrections have the wrong sign. For the very few cases where the sign is correct one would still need very extreme parameter choices, which are already disfavoured, see also [65].

For general corrections forbidden orderings can be reconstituted [3]. For sum rule 2 forbidden ordering could be reconstituted by a 30% correction and a neutrino mass scale of 0.05 eV, which is on the edge of being disfavoured by cosmology. For smaller mass scales the necessary corrections grow quickly, see Table 3 in [3] and indeed in most cases (except for sum rule 10) the newly allowed regions are not very big and practically excluded by the cosmological bounds on the neutrino mass scale.

3. Summary

Neutrino mass sum rules are a very powerful tool to test and discriminate flavour models [1, 66]. This was confirmed in the systematic studies of RG corrections [2] and general perturbations [3] where it was shown that many essential predictions still valid at least qualitatively.

In the future experimental information on the neutrino mass scale, their mass ordering, neutrinoless double beta decay, but also information about new physics at the TeV scale like supersymmetry will be a crucial test to understand if a neutrino mass sum rule is realised in nature.

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