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Research Article

Cosmic Microwave Background as a Thermal Gas of SU(2) Photons: Implications for the High-z Cosmological Model and the Value of H_0

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Presently, we are facing a 3σ tension in the most basic cosmological parameter, the Hubble constant H_0. This tension arises when fitting the Lambda-cold-dark-matter model (ΛCDM) to the high-precision temperature-temperature (TT) power spectrum of the Cosmic Microwave Background (CMB) and to local cosmological observations. We propose a resolution of this problem by postulating that the thermal photon gas of the CMB obeys an SU(2) rather than U(1) gauge principle, suggesting a high-z cosmological model which is void of dark-matter. Observationally, we rely on precise low-frequency intensity measurements in the CMB spectrum and on a recent model independent (low-z) extraction of the relation between the comoving sound horizon r_s at the end of the baryon drag epoch and H_0 ($r_s H_0 = \text{const}$). We point out that the commonly employed condition for baryon-velocity freeze-out is imprecise, judged by a careful inspection of the formal solution to the associated Euler equation. As a consequence, the above-mentioned 3σ tension actually transforms into a 5σ discrepancy. To make contact with successful low-z ΛCDM cosmology we propose an interpolation based on percolated/depercolated vortices of a Planck-scale axion condensate. For a first consistency test of such an all-z model we compute the angular scale of the sound horizon at photon decoupling.

1. Introduction

Since the pioneering work by Yang and Mills [1] on the definition of a local four-dimensional, classical, and minimal field theory, which is based on the nonabelian gauge group SU(2), much progress has been made in elucidating the role of topologically stabilized and (anti)-self-dual field configurations in building the nonperturbative ground state and influencing the properties of its excitations [2–8]. In particular, the deconfining phase is subject to a highly accurate thermal ground state estimate [9, 10], being composed of so-called Harrington-Shepard (anti)calorons [11]. This (cosmologically relevant) ground state invokes both an adjoint Higgs mechanism [12–15], rendering two out of three directions of the SU(2) algebra massive (free thermal quasiparticles), and a $U(1)_A$ chiral anomaly [2, 3, 5, 6], giving mass to the Goldstone mode induced by the associated dynamical breaking of this global symmetry. Radiative corrections to thermodynamical quantities, evaluated on the level of free thermal (quasi)particles, are minute and well under control [9, 10]. Note that this is in contrast to the large effects of radiative corrections attributed to the effective QCD action at zero temperature in [16, 17] which are exploited as potential inducers of vacuum energy in the cosmological context in [18–22]. However, it was argued in [23, 24] that QCD condensates, which contribute to the trace anomaly of the energy-momentum tensor (as implied by the effective action), do not act cosmologically.

Postulating that thermal photon gases obey an SU(2) rather than a U(1) gauge principle, the SU(2) Yang-Mills scale can be inferred from low-(radio)frequency spectral intensity measurements, for example [25], of the Cosmic Microwave Background (CMB) [26], prompting the name $SU(2)_{CMB}$. Below we will use the name $SU(2)_{CMB}$ synonymously for the

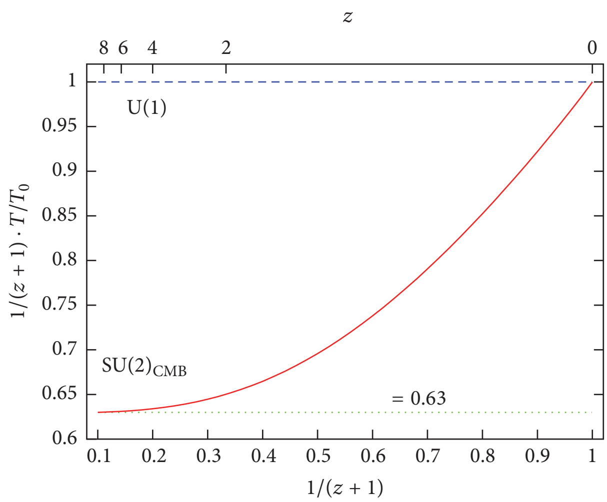


FIGURE 1: The T-z scaling relation $T/(T_0(z+1))$ in $SU(2)_{CMB}$ (solid). Note the emergence of $T/T_0 = 0.63(z+1)$ for $z \gtrsim 9$ (dotted). The conventional U(1) theory for thermal photon gases associates with the dashed line. Data taken from [27] after slight and inessential correction.

implied cosmological model. To investigate the consequences of this postulate towards the equation of state radiative corrections are entirely negligible [9]. When subjecting local energy conservation in a Friedmann-Lemaître-Robertson-Walker (FLRW) universe to this equation of state the numerical temperature (T)-redshift (z) relation ($T(z)$) of the CMB follows; see Figure 1 [27, 28], where a comparison with the conventional U(1) photon gas is shown. The curvature of $T/(T_0(z+1))$ ($T_0 = 2.725$ K denoting today's CMB temperature) at low z is due to the influence of the SU(2) Yang-Mills mass scale on the equation of state. In [28] an argument is given why recent observational "extractions" of $T(z)$, which claim no deviations from the conventional behavior $T(z) = T_0(z+1)$, are circular. One has $T/T_0 = 0.63(z+1)$ at high z and therefore a lower slope compared to the conventional case. In an approximation, where recombination at z_* is subjected to thermodynamics, the decoupling condition is $\Gamma_{Th}(T_*) = H(z_*)$ where Γ_{Th} denotes the Thomson photon-electron scattering rate at the decoupling temperature $T_* \sim 3000$ K. We have $(\Omega_{0,b} + \Omega_{0,DM})/\Omega_{0,b} \sim 6.5 \equiv R_{m,1}$ where $\Omega_{0,b}$ and $\Omega_{0,DM}$ denote the respective ratios of today's energy densities in baryons and cold dark matter to the critical density. Since $z_{*,SU(2)_{CMB}}/z_{*,\Lambda CDM} \sim 1/0.63$ this roughly matches $(1/0.63)^3 \sim 4 \equiv R_{m,2}$. If a strong matter domination can be assumed during recombination then $R_{m,1}$ should be equal to $R_{m,2}$ but, due to matter-radiation equality occurring at $z \sim 1080$ in $SU(2)_{CMB}$, this assumption is not quite met, explaining the mild discrepancy between $R_{m,1}$ and $R_{m,2}$. Still, we take this rough argument and the desired minimality of the cosmological model as motivations to omit cold dark matter in the high-z cosmological model which operates down to recombination and well beyond it.

Concerning the number of massless neutrinos N_ν, a conservative input is used: $N_\nu = 3$ [29]. This high-z model, composed of $SU(2)_{CMB}$, baryonic matter, and massless neutrinos ($N_\nu = 3$), is sufficient to predict the sound horizon r_s at the end of the baryon drag epoch which, in turn, can be confronted with the r_s-H_0 relation, recently extracted from local cosmological observations [30], to determine the value of H_0. The value of r_s, as computed in a high-z model, rather sensitively depends on the definition of redshift z_{drag} for baryon-velocity (v_b) freeze-out. Usually, z_{drag} is identified with the maximum position of the so-called drag visibility function D_{drag} [31, 32]. However, inspecting the solution v_b of the corresponding Euler equation, given as a functional of D_{drag}, one concludes that this definition applies only in the limit of zero peak width. Realistic results for the ionization fraction χ_e, obtained by numerical integration of the according Boltzmann hierarchy (`recfast` [33]), imply that the width of this peak extends over several hundred units of redshift in both cases ΛCDM and $SU(2)_{CMB}$. As a consequence, a more precise definition of z_{drag} is in order which associates with the left flank of D_{drag}. Therefore, we will in the following refer to this corrected redshift for the freeze-out of v_b as $z_{lf,drag}$. Our value $r_s(z_{lf,drag}) \sim 1660$, after intersection with the r_s-H_0 relation of [30], determines the value of H_0 in good agreement with the value obtained in [34]. Also, we would like to point out that, as a consequence of the corrected baryon-velocity freeze-out condition, the value of H_0 in ΛCDM, obtained by this method, is now at a 5σ discrepancy with the value published in [34].

To be able to compute the CMB power spectra, our consistent high-z $SU(2)_{CMB}$ cosmological model of (3) needs to be connected to the observationally well cross-checked ΛCDM low-z parametrization of the universe's composition. To facilitate such an interpolation, a candidate real scalar field φ representing the dark sector is the so-called Planck-scale axion (PSA) condensate [35–37] which rests on chiral symmetry breaking within the Planckian epoch and the axial anomaly invoked by deconfining thermal ground states of Yang-Mills theories. Notice that the only Yang-Mills theory exhibiting the deconfining phase from today to well beyond recombination is $SU(2)_{CMB}$. A model, where φ undergoes coherent and damped oscillations at late times such as to effectively represent ΛCDM, is falsified by the redshift z_q, where the universe's expansion starts to accelerate, being too high. This prompts the idea that interpolation between $SU(2)_{CMB}$ at high z and ΛCDM at low z is achieved by the U(1) topologically stabilized solitonic configurations (vortices) of the PSA condensate occurring in percolated form (due to a Berezinskii-Kosterlitz-Thouless phase transition following their very creation during a nonthermal phase transition at very high z) down to intermediate z where a depercolation transition partially liberates them to effectively represent a pressureless vortex gas. Whether or not the cores of depercolated PSA vortices properly serve as dark-matter halos in spiral galaxies to explain the observed flattening of rotation curves and the lensing signatures of bullet galaxies is an open question. Likewise, it is not yet guaranteed that this new cosmological model, which exhibits radiation domination and baryon freeze-out prior to photon decoupling, explains the observed angular power spectra of the CMB.

Table 1: Cosmological parameter values employed in the computations and their sources, taken from [28].

Parameter	Value	Source
H_0 ($SU(2)_{CMB}$)	$(73.24 \pm 1.74)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$	[34]
H_0 (ΛCDM)	$(67.31 \pm 0.96)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$	TT + low P, [38]
T_0	2.725 K	[39]
$\Omega_{\gamma,0}h^2$	2.46796×10^{-5}.	Based on $T_0 = 2.725$ K
$\Omega_{b,0}h^2$	0.02222 ± 0.99923	TT + low P[38]
$\Omega_{\mathrm{CDM},0}h^2$	0.1197 ± 0.0022	TT + low P, [38]
η_{10}	6.08232 ± 0.06296	Based on $\Omega_{\gamma,0}h^2$, TT + low P[38]
Y_P	0.252 ± 0.041	TT, [38]
N_{eff}	3.15 ± 0.23	Abstract, [38]

This work is organized as follows. In Section 2 we explain our high-z cosmological model $SU(2)_{CMB}$, introduced in [28], and compare it with the conventional ΛCDM cosmology. The modification of decoupling conditions due to finite widths visibility functions is discussed in Section 3. Based on this, we perform the computation of r_s and confront it with the r_s-H_0 relation of [30] to determine the value of H_0. Subsequently, in Section 5 we investigate whether coherent and damped oscillations of the PSA field can realistically represent ΛCDM at low z, with a negative result. According to [28] we are thus led to propose an interpolation between high-z $SU(2)_{CMB}$ and low-z ΛCDM in terms of percolated PSA vortices which, at some intermediate redshift z_p, partially undergo a depercolation transition. Such a model is demonstrated to be consistent with the extremely well observed angular scale of the sound horizon at photon decoupling [38]. Finally, we summarize our results and provide an outlook on how the new model can be tested further by confrontation with the power spectra of various CMB angular correlation functions.

2. Definition of Cosmological Model $SU(2)_{CMB}$

In a flat FLRW universe, a cosmological model is given in terms of the z-dependence of the Hubble parameter

$$H(z) = H_0 \sqrt{\sum_i \Omega_i(z)}, \tag{1}$$

where H_0 is today's cosmological expansion rate and $\Omega_i(z) = f_i(z)\Omega_{i,0}$. Here $\Omega_{i,0}$ is the fraction of the energy density $\rho_{i,0}$ of fluid i to the critical density $\rho_{c,0}$ today. The function $f_i(z)$ is determined by energy conservation subject to fluid i's equation of state. From now on we work in supernatural units ($c = \hbar = k_B = 1$) where Newton's constant G has units of inverse mass squared. Table 1 lists the parameter values used subsequently.

2.1. The Conventional ΛCDM Model. In the conventional high-z ΛCDM model $H(z)$ is given as

$$H(z) = H_0 \left[(\Omega_{b,0} + \Omega_{\mathrm{CDM},0})(z+1)^3 + \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right)\Omega_{\gamma,0}(z+1)^4 \right]^{1/2}. \tag{2}$$

Here nonrelativistic matter decomposes into baryonic (b) and cold dark matter (CDM). The radiation component contains photons with two polarizations, two relativistic vector modes with three polarizations each, and N_{eff} flavours of massless neutrinos with two polarizations each. $\Omega_{\gamma,0}$ is today's fraction of photonic energy density to critical energy density (for details see [38]).

2.2. Modifications of ΛCDM towards $SU(2)_{CMB}$. In high-z $SU(2)_{CMB}$ the Hubble parameter is given as

$$H(z) = H_0 \left[\Omega_{b,0}(z+1)^3 + 4 \cdot (0.63)^4 \left(1 + \frac{7}{32}\left(\frac{16}{23}\right)^{4/3} N_\nu\right)\Omega_{\gamma,0}(z+1)^4 \right]^{1/2}. \tag{3}$$

In this case, only baryonic matter is present. We reiterate that both models, (2) and (3), need to be supplemented by a dark sector to yield successful low-z ΛCDM cosmology; see (32). The radiation sector is modified due to a different number of relativistic degrees of freedom and due to the $SU(2)_{CMB}$ high-z temperature-redshift relation $T(z)$; for details see [27, 28].

3. The End of Recombination

The comoving sound horizon r_s at redshift z is defined as

$$r_s(z) = \int_z^\infty dz' \frac{c_s(z')}{H(z')}, \tag{4}$$

whereby c_s denotes the sound velocity in the primordial baryon-electron-photon plasma, given as

$$c_s \equiv \frac{1}{\sqrt{3(1+R)}}. \tag{5}$$

The function $R(z)$ is determined by 3/4 of the ratio of energy densities in baryons and photons. In ΛCDM we have

$$R(z) \equiv 111.019 \frac{\eta_{10}}{z+1}, \tag{6}$$

whereas in $SU(2)_{CMB}$ one obtains

$$R(z) \equiv 111.019 \frac{\eta_{10}}{(0.63)^4 (z+1)}. \tag{7}$$

The values of η_{10} can be read off Table 1.

3.1. Conventional Freeze-Out. The final stages of recombination can be characterized in a twofold way. One considers either (i) photon temperature freeze-out, which is relevant for the peak structure in the temperature-temperature (TT) angular power spectrum of the CMB or (ii) baryon-velocity freeze-out, which is detectable in the matter correlation function (galaxy counts). Concerning case (i), the conventional criterion, which fixes the redshift z_*, reads

$$\tau(z_*) = \sigma_T \int_0^{z_*} \mathrm{d}z \frac{\chi_e(z)\, n_e^b(z)}{(z+1)\, H(z)} = 1, \tag{8}$$

where σ_T denotes the total cross section for Thomson scattering, χ_e is the ionization fraction (calculated with `recfast`), and n_e^b refers to the density of free electrons just before hydrogen recombination, given as

$$n_e^b(z) = 410.48 \cdot 10^{-10} \eta_{10} (1 - Y_P)(z+1)^3\ \mathrm{cm}^{-3}. \tag{9}$$

Here Y_P denotes the helium mass fraction in baryons (see Table 1). Concerning case (ii), the conventional criterion is defined as

$$\tau_{\mathrm{drag}}(z) = \sigma_T \int_0^{z} \mathrm{d}z' \frac{\chi_e(z')\, n_e^b(z')}{(z'+1)\, H(z')\, R(z')} = 1. \tag{10}$$

3.2. Corrected Freeze-Out. We now show that conditions (8) and (10) are imprecise due to the finite widths of the respective visibility functions. To see this, we have to analyze the formal solution of the Boltzmann hierarchy for the temperature perturbation and of the Euler equation for v_b [31, 32, 40]. Since the argument is similar for both cases we focus on the latter only. The Euler equation reads

$$\dot{v}_b = \frac{\dot{z}}{z+1} v_b + k\Psi + \dot{\tau}_{\mathrm{drag}} (\Theta_1 - v_b), \tag{11}$$

where k is the comoving wave number (omitted as a subscript in the following), Θ_1 denotes the (relative) dipole of the temperature anisotropy [41], and Ψ represents the Newtonian gravitational potential. The overdot demands differentiation with respect to conformal time. Transforming the conformal time to a redshift dependence, the solution of (11) is

$$\begin{aligned} \frac{v_b(z)}{z+1} &= \lim_{Z \nearrow \infty} \int_z^Z \mathrm{d}z' \\ &\quad \cdot \frac{e^{-\tau_{\mathrm{drag}}(z',z)}}{H(z')(z'+1)} \left(\dot{\tau}_{\mathrm{drag}}(z')\, \Theta_1(z') + k\Psi(z') \right) \\ &\sim \lim_{Z \nearrow \infty} \int_z^Z \mathrm{d}z' D_{\mathrm{drag}}(z', z)\, \Theta_1(z'). \end{aligned} \tag{12}$$

Here τ_{drag} is defined as

$$\tau_{\mathrm{drag}}(z', z) \equiv \int_z^{z'} \mathrm{d}z'' \frac{\dot{\tau}_{\mathrm{drag}}(z'')}{H(z'')}, \tag{13}$$

and the visibility function $D_{\mathrm{drag}}(z', z)$ is represented by

$$D_{\mathrm{drag}}(z', z) \equiv \frac{e^{-\tau_{\mathrm{drag}}(z',z)}\, \dot{\tau}_{\mathrm{drag}}(z')}{H(z')(z'+1)}. \tag{14}$$

In order to study freeze-out the function Θ_1 in (12) is considered slowly varying. Therefore, the variability of the integral solely depends on D_{drag} within its peak region. In both cases ΛCDM and SU(2)$_{\mathrm{CMB}}$ function D_{drag} exhibits a broad peak in dependence of z' whose shape and maxima do not depend on z; see Figure 2. Note that (10) describes the maxima $z'_{\mathrm{max,drag}}$ of $D_{\mathrm{drag}}(z', z)$. However, due to the finite width the integral in (12) is not saturated at $z = z_{\mathrm{max,drag}}$ but rather ceases to vary for $z < z_{\mathrm{lf,drag}}$ where lf denotes the maxima of the z' derivative of D_{drag}. Therefore, $z_{\mathrm{lf,drag}}$ defines the freeze-out point more realistically than $z_{\mathrm{max,drag}}$. According to Figure 2's caption the values of $z_{\mathrm{drag}}, z_{\mathrm{lf,drag}}$ deviate substantially. Namely,

$$\begin{aligned} z_{\mathrm{drag}} &= 1813, \\ z_{\mathrm{max,drag}} &= 1789, \\ z_{\mathrm{lf,drag}} &= 1659 \\ &\quad (\mathrm{SU}(2)_{\mathrm{CMB}}), \\ z_{\mathrm{drag}} &= 1059, \\ z_{\mathrm{max,drag}} &= 1046, \\ z_{\mathrm{lf,drag}} &= 973 \\ &\quad (\Lambda\mathrm{CDM}). \end{aligned} \tag{15}$$

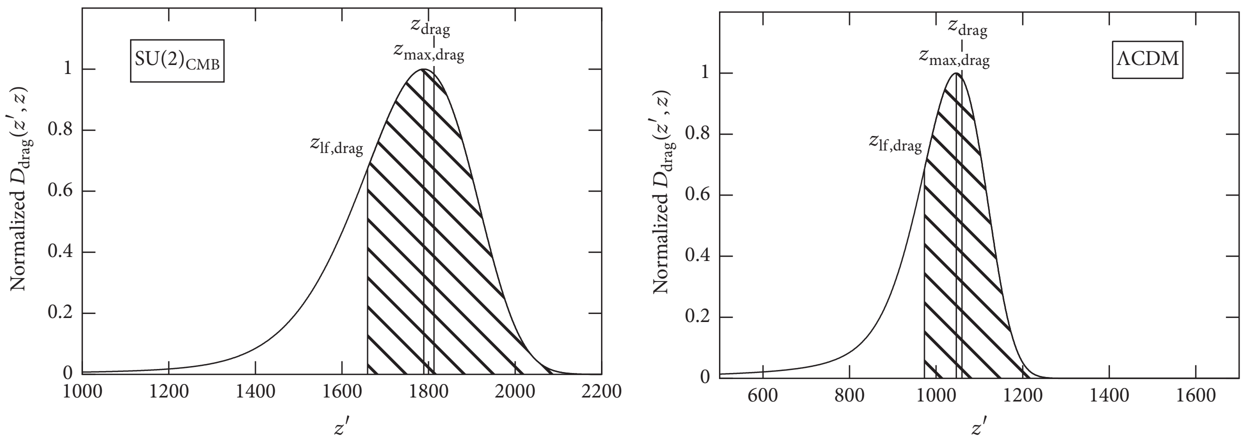
An analogous discussion applies to photon temperature freeze-out with the following results (see [28]):

$$\begin{aligned} z_* &= 1694, \\ z_{\mathrm{max},*} &= 1694, \\ z_{\mathrm{lf},*} &= 1555 \\ &\quad (\mathrm{SU}(2)_{\mathrm{CMB}}), \\ z_* &= 1090, \\ z_{\mathrm{max},*} &= 1072, \\ z_{\mathrm{lf},*} &= 988 \\ &\quad (\Lambda\mathrm{CDM}). \end{aligned} \tag{16}$$

4. The Value of H_0

Subjecting the freeze-out redshifts of (15) to (4) under consideration of (2) and (3) yields

$$\begin{aligned} r_s(z_{\mathrm{drag}}) &= (129.22 \pm 0.52)\ \mathrm{Mpc} \quad (\mathrm{SU}(2)_{\mathrm{CMB}}), \\ r_s(z_{\mathrm{lf,drag}}) &= (137.19 \pm 0.45)\ \mathrm{Mpc} \quad (\mathrm{SU}(2)_{\mathrm{CMB}}), \end{aligned}$$



(a) (b)

Figure 2: Normalised function $D_{\mathrm{drag}}(z', z)$, defined in (14), if $z \leq z_{\mathrm{max,drag}}$ for $\mathrm{SU(2)_{CMB}}$ (a) and ΛCDM (b). Redshift $z_{\mathrm{lf,drag}}$ is defined as the position of the maximum of $\mathrm{d}D_{\mathrm{drag}}/\mathrm{d}z'$ (position of left flank of D_{drag}) whereas $z_{\mathrm{max,drag}}$ denotes the position of the maximum of D_{drag}. The value of z_{drag}, defined in (10), essentially coincides with $z_{\mathrm{max,drag}}$: $z_{\mathrm{drag}} = 1813 \sim z_{\mathrm{max,drag}} = 1789$ for $\mathrm{SU(2)_{CMB}}$ and $z_{\mathrm{drag}} = 1059 \sim z_{\mathrm{max,drag}} = 1046$ for ΛCDM. This should be contrasted with $z_{\mathrm{lf,drag}} = 1659$ for $\mathrm{SU(2)_{CMB}}$ and $z_{\mathrm{lf,drag}} = 973$ for ΛCDM. The hatched area under the curve determines the freeze-out value of $v_b/(z+1)$.

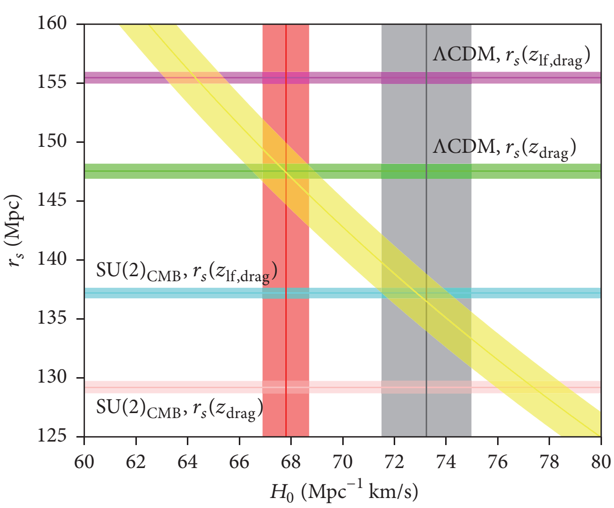


Figure 3: The $r_s(z_{\mathrm{lf,drag}})$-$H_0$ relation (curved band) of [30] in confrontation with the high-z predictions of $r_s(z_{\mathrm{lf,drag}})$ and $r_s(z_{\mathrm{drag}})$ in ΛCDM and $\mathrm{SU(2)_{CMB}}$ (horizontal bands) of (17). Vertical bands indicate the values of H_0 extracted in [38] (low) and in [34] (high). Note that there is a ~3σ tension. However, a ~7σ discrepancy exists between the H_0 values of $(64.3 \pm 1.1)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $(72.9 \pm 1.2)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ associated with the intersections of $r_s(z_{\mathrm{lf,drag}})$ in ΛCDM and in $\mathrm{SU(2)_{CMB}}$, respectively, with the $r_s(z_{\mathrm{lf,drag}})$-$H_0$ relation. Taking $H_0 = (73.24 \pm 1.7)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ from [34] the discrepancy between this value and $(64.3 \pm 1.1)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is about 5σ.

$$\begin{aligned} r_s\left(z_{\mathrm{drag}}\right) &= (147.33 \pm 0.49)\,\mathrm{Mpc} \quad (\Lambda\mathrm{CDM}), \\ r_s\left(z_{\mathrm{lf,drag}}\right) &= (154.57 \pm 3.33)\,\mathrm{Mpc} \quad (\Lambda\mathrm{CDM}). \end{aligned} \tag{17}$$

In Figure 3, these (H_0 independent) values of the sound horizon are confronted with the r_s-H_0 relation of [30]. Note the good agreement between the values of H_0 implied by $r_s(z_{\mathrm{lf,drag}})$ in $\mathrm{SU(2)_{CMB}}$ and the extraction performed in [34]. On the other hand, $r_s(z_{\mathrm{drag}})$ reproduces the value of H_0 published in [38] which exhibits a 3σ tension compared to [34]. However, according to Figure 3, the more realistic freeze-out value $z_{\mathrm{lf,drag}}$ in ΛCDM entails

$$H_0 = (64.5 \pm 1)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{18}$$

Thus, in ΛCDM there actually is a 5σ discrepancy between the value of H_0 quoted in [34] and obtained by confrontation of r_s with the r_s-H_0 relation of [30].

5. Planck-Scale-Axion Field and Interpolation of High-z with Low-z Cosmology

Here we would like to analyze cosmological models which link low-z ΛCDM with high-z $\mathrm{SU(2)_{CMB}}$. We assume a dark sector which originates from a real, minimally coupled scalar field, a pseudo Nambu-Goldstone mode of dynamical chiral symmetry occurring at the Planck scale [35, 36], whose potential is due to the chiral $\mathrm{U(1)}_A$ anomaly invoked by (anti)calorons of the deconfining, thermal ground state of Yang-Mills theories [1–3, 6, 42–44]. This prompts the name Planck-scale axion (PSA). The only Yang-Mills theory, which is deconfining well above recombination, is $\mathrm{SU(2)_{CMB}}$ because otherwise there would not be just one species of photons.

The radiatively protected potential for the axion condensate φ, arising due to the thermal ground state of $\mathrm{SU(2)_{CMB}}$ [43, 44], reads as follows:

$$V(\varphi) = \left(\kappa\Lambda_{\mathrm{CMB}}\right)^4 \cdot \left(1 - \cos\frac{\varphi}{m_P}\right), \tag{19}$$

where $\Lambda_{\rm CMB} \sim 10^{-4}$ eV, κ is a dimensionless factor of order unity, and the reduced Planck mass reads

$$m_P \equiv \frac{1.22 \times 10^{19}}{\sqrt{8\pi}} \text{ GeV} = (8\pi G)^{-1/2}. \quad (20)$$

With a canonical kinetic term for φ the according equation of motion is

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{d}{d\varphi}V(\varphi) = 0, \quad (21)$$

where an overdot signals the derivative with respect to cosmological time.

In a first attempt at a ΛCDM - SU(2)$_{\rm CMB}$ interpolation we assume spatially homogeneous φ-field dynamics subject to ΛCDM constraints at low z. It turns out, however, that such a model predicts a value of z_q, defined as the zero of the deacceleration parameter

$$q(z) \equiv \frac{z+1}{2\widehat{H}^2}\left(\widehat{H}^2\right)' - 1, \quad (22)$$

of about $z_q \sim 3$ which is much higher than the realistic value ~0.7 obtained in ΛCDM. Therefore, as a second proposal we abolish the energy density arising from *spatially homogeneous* configurations of the field φ. Rather, we conceive the dark-matter sector in ΛCDM as a piece of energy density due to depercolated topological solitons (vortices) of the field φ which percolate instantaneously into a dark-energy like piece at some redshift z_p such that $z_{\rm re} \ll z_p \ll z_{\rm lf,drag}$. The origin of such a vortex percolate, with hierarchically ordered core sizes, could be due to Hagedorn transitions of Yang-Mills theories in the early universe which are accompanied by Berezinskii-Kosterlitz-Thouless transitions in the axionic sector. Today's value of Ω_Λ would then be interpreted in terms of not-yet depercolated vortices. Indeed, in such an interpolation between ΛCDM and SU(2)$_{\rm CMB}$ a value of $z_p \sim 155$ can be fitted to the angular size of the sound horizon at photon decoupling. At $z_{\rm lf,drag}$ the extra contribution to dark-energy amounts to ~0.65% of the baryonic energy density which is consistent with SU(2)$_{\rm CMB}$.

5.1. Spatially Homogeneous, Coherent Oscillations. Here we discuss a cosmological model where the interpolation between ΛCDM and SU(2)$_{\rm CMB}$ is attempted by a spatially homogeneous PSA field undergoing damped and coherent oscillations at late times. This models a pressureless component (cold dark matter) and component with negative pressure (dark-energy). Notice that these two components represent fluids that are not separately conserved.

The Hubble equation reads

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \rho_b + \rho_r\right) \equiv \frac{8\pi G}{3}\rho_c. \quad (23)$$

Here ρ_r denotes radiation-like energy density including SU(2)$_{\rm CMB}$ (for $z \leq 9$ radiation energy density is severely suppressed in the cosmological model; for $z > 9$ the thermal ground state and the masses of the vector modes of SU(2)$_{\rm CMB}$ can be neglected) and three flavours of massless neutrinos; ρ_b is the energy density of baryons, in addition to the energy density $(1/2)\dot{\varphi}^2 + V(\varphi)$ associated with the spatially homogeneous PSA field φ which evolves temporally. Eqs. (21) and (23) can be cast into fully dimensionless equations by rescaling with powers of m_P in the following way:

$$\begin{aligned} V &= m_P^4 \widehat{V}, \\ \rho_i &= m_P^4 \widehat{\rho}_i \quad (i = b, r), \\ \varphi &= m_P \widehat{\varphi}, \\ H &= m_P \widehat{H}. \end{aligned} \quad (24)$$

In general, dimensionless quantities (after rescaling with the appropriate power of m_P) are indicated by the hat-symbol. After rescaling and in dependence of z (21) and (23) transmute into

$$\begin{aligned} &\widehat{\varphi}''\left[(z+1)\widehat{H}\right]^2 \\ &\quad + \widehat{\varphi}'\left[\frac{1}{2}(z+1)^2\left(\widehat{H}^2\right)' - 2(z+1)\widehat{H}^2\right] + \widehat{\frac{dV}{d\varphi}} = 0, \end{aligned} \quad (25)$$

$$\widehat{H}^2 = \frac{1}{3}\frac{\widehat{V} + \widehat{\rho}_{b,0}(z+1)^3 + \widehat{\rho}_r}{1 - (1/6)(z+1)^2\widehat{\varphi}'^2}, \quad (26)$$

where a prime demands z-differentiation. In (26) we approximate $\widehat{\rho}_r$ as

$$\begin{aligned} \widehat{\rho}_r &= \widehat{\rho}_{\gamma,0} \\ &\cdot \begin{cases} 0 & (z < 9) \\ 4(0.63)^3\left(1 + \dfrac{7}{32}\left(\dfrac{16}{23}\right)^{4/3} 3\right)(z+1)^4 & (z \geq 9). \end{cases} \end{aligned} \quad (27)$$

With the initial conditions

$$\begin{aligned} \widehat{\varphi}(z = z_i) &= \widehat{\varphi}_i, \\ \widehat{\varphi}'(z = z_i) &= 0 \end{aligned} \quad (28)$$

for sufficiently large z_i (no roll; in practice one safely can choose $z_i \sim 50$) the solution to (25) subject to (26) is unique. To fix the values of κ in (19) and $\widehat{\varphi}_i$ in (28) we demand

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 3m_P^4\widehat{H}_0^2 \quad (29)$$

and that Ω_Λ coincides with typical fit value $\Omega_\Lambda \sim 0.7$ obtained in ΛCDM cosmology [38]:

$$\Omega_\Lambda = \frac{m_P^4}{\rho_{c,0}}\lim_{z\searrow 0}\left(\widehat{V} - \frac{1}{2}\left((z+1)\widehat{H}\widehat{\varphi}'\right)^2\right) \sim 0.7. \quad (30)$$

Figure 4 shows the deacceleration parameter $q(z)$ for the model defined by (25), (26), (29), and (30). Obviously, this model is falsified by a much too high value of the zero z_q of $q(z)$.

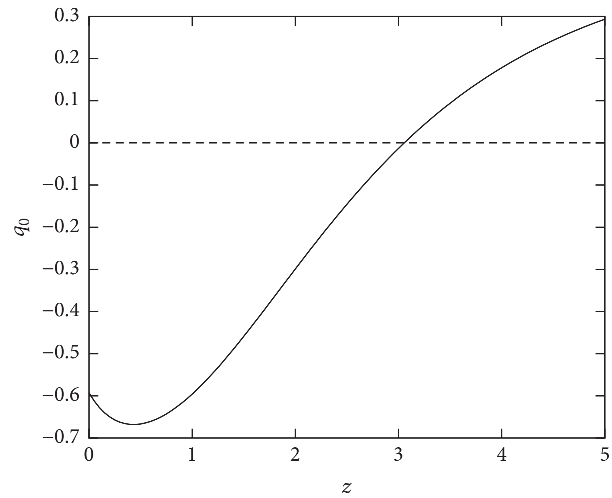


FIGURE 4: The deacceleration parameter $q(z)$ of (22) for the model defined by (25), (26), (29), and (30). Notice that the value of the zero z_q of $q(z)$ is $z_q \sim 3$. This is much higher than the realistic value $z_q \sim 0.7$ obtained in ΛCDM.

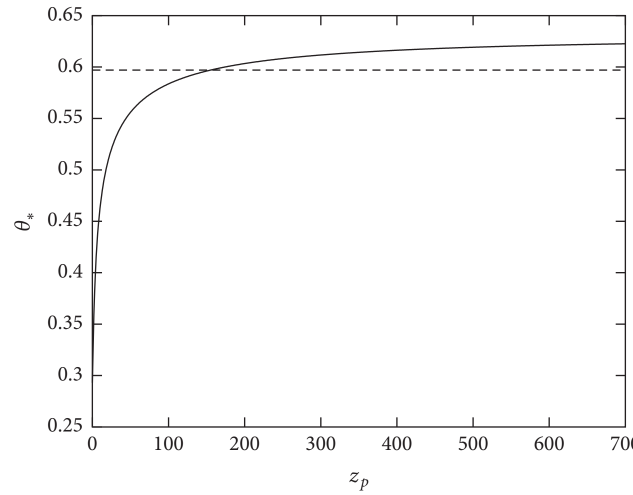


FIGURE 5: Function $\theta_*(z_p)$ for $\Omega_\Lambda = 0.7$, $\Omega_{\mathrm{DM},0} = 0.26$, $\Omega_{b,0} = 0.04$, $\Omega_{\gamma,0} = 4.6 \times 10^{-5}$, and $H_0 = 73.24\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ for the high-z $\mathrm{SU(2)_{CMB}}$ and low-z ΛCDM interpolating cosmological model considered. Also indicated is the value $\theta_* = 0.597^\circ$ (dashed line), fitted to the CMB TT power spectrum.

5.2. Percolated and Unpercolated Vortices. Here the basic idea invokes the fact that a PSA field φ, due to nonthermal phase transitions of the Hagedorn type (e.g., there should be an $\mathrm{SU(2)}_e$ Yang-Mills theory of scale $\Lambda_e \gg \Lambda_{\mathrm{SU(2)_{CMB}}}$ going confining at $T \sim \Lambda_e$) is subject to $\mathrm{U(1)}_A$ winding and in this way creation of a density of percolated topological solitons (vortex percolate) with a hierarchical ordering of core sizes. Percolation could be understood as a Berezinskii-Kosterlitz-Thouless phase transition [45, 46]. Effectively, this percolate represents homogeneous, constant energy density. As the universe expands the vortex percolate is increasingly stretched, and at around some critical redshift $z_p \ll z_{\mathrm{lf,drag}}$ it releases a part of its solitons characterized by some specific core size. The ensuing vortex gas acts cosmologically like pressureless matter. Vortices of larger core sizes remain trapped in the percolate. For this scenario to be a consistent interpolation of $\mathrm{SU(2)_{CMB}}$ and ΛCDM we need to assure that $z_p \gg z_{\mathrm{re}} \sim 6$ [47, 48].

With the definition of (27) the cosmological model to be considered thus reads

$$\widehat{H}^2 = \frac{1}{3}\left(\widehat{\rho}_b + \widehat{\rho}_{\mathrm{DS}} + \widehat{\rho}_r\right), \quad (31)$$

where $\widehat{\rho}_{\mathrm{DS}}$ is the dark sector energy density, defined as

$$\widehat{\rho}_{\mathrm{DS}} = \widehat{\rho}_\Lambda + \widehat{\rho}_{\mathrm{CDM},0} \cdot \begin{cases} (z+1)^3 & (z < z_p) \\ (z_p+1)^3 & (z \geq z_p), \end{cases} \quad (32)$$

where $\widehat{\rho}_\Lambda$ and $\widehat{\rho}_{\mathrm{CDM},0}$ are today's values of the dark-energy and cold-dark-matter densities associated with (30) and the value quoted in Table 1, respectively.

In order to fix the value of z_p we confront the model of (31) and (32) with the observed angular scale θ_* of the sound horizon at CMB photon decoupling, occurring at $z_{\mathrm{lf},*}$. Theoretically, θ_* is given as

$$\theta_* = \frac{r_s(z_{\mathrm{lf},*})}{\int_0^{z_{\mathrm{lf},*}} (\mathrm{d}z/H(z))}. \quad (33)$$

To match $\theta_* = 0.597^\circ$, as extracted in [38] from the TT power spectrum, we require $z_p = 155.4$; see Figure 5. This yields a percentage of vacuum energy at CMB photon decoupling of about

$$\frac{\Omega_{\mathrm{DM},0}}{\Omega_{b,0}}\left(\frac{z_p+1}{z_{\mathrm{lf},*}+1}\right)^3 \sim 0.65\%. \quad (34)$$

The omission of vacuum energy in our $\mathrm{SU(2)_{CMB}}$ high-z cosmological model of (3) thus is justified for the interpolating model defined in (31) and (32).

6. Summary and Outlook

In the present work we have analyzed, based on a modified temperature-redshift relation for the CMB which, in turn, derives from the postulate that thermal photon gases are subject to an SU(2) rather than a U(1) gauge principle, a high-z cosmological model which is void of dark-matter and considers three species of massless neutrinos. Such a model predicts (after a reconsideration of baryon-velocity freeze-out) a value of the sound horizon r_s which, together with a model independent extraction of the r_s-H_0 relation from cosmologically local observations in [30], yields good agreement with the value of H_0 determined by low-z observations in [34]. The same r_s-H_0 relation predicts a low value of H_0 in standard ΛCDM cosmology which is at a 5σ discrepancy with the value given in [34].

Motivated by the above results, an interpolation between ΛCDM at low z and our new high-z model is called for. In a first attempt, we have investigated whether coherent and damped oscillations of a Planck-scale axion condensate can realistically accomplish this, with a negative result. With [28] we were thus led to propose an interpolation in terms of percolated PSA vortices which, at some intermediate z_p, partially undergo a depercolation transition. We have demonstrated this model to be consistent with the angular scale of the sound horizon at photon decoupling.

The new model needs to be tested against the various CMB angular spectra. Our hope is that radiative corrections in SU(2) Yang-Mills thermodynamics, which play out at low z, are capable of explaining the large-angle anomalies of the CMB [49].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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