Integrated measures of household risk in financial plans optimized under a consumption rate constraint

Radoslaw Pietrzyk and Pawel Rokita

Abstract Measures of risk suited to the life-long financial plan of a household differ from other popular risk measures. When discussing a risk variable that would incorporate exposures to all significant types of risk, a natural choice for a financial institution would be value (e.g. of a portfolio), and, for enterprises, it would be, for instance, net income or cash flow. Based on these natural risk variables, integrated risk measures for financial institutions and investors are thus Value at Risk (VaR) or Expected Shortfall (ES) or related measures. For enterprises, these may be Earnings at Risk (EaR) and Cash Flow at Risk (CFaR). A measure of risk suited to the specificity of a household should, in turn, address threats to the accomplishment of its life objectives (to be more precise – life objectives of its members) and must take into account its life cycle. In this article, we present some proposals of new downside risk measures that fulfill these conditions. The measures are suited to the household financial planning framework proposed in other works by the authors of this paper and their concept is presented within this framework, but the very idea of the measures is

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much more universal and should be possible to be applied (maybe after some adjustments) in many other optimization models of financial plans.

1 Introduction

Life long financial planning for households entails dealing with risk of many different types (compare, e.g., Mitchell et al, 2002). So far, there has not been developed any household-specific integrated measure of risk. Such a risk measure would be very useful in household financial planning.

Some proposals for such measures are presented in the article. To be able to better explain the concept, an example of a household financial planning model is also discussed here briefly. The proposed measures may be used with other models of a financial plan of a household, but they require that the future financial situation of the household, which would be a result of the plan application, is expressed as a bunch of term structures of cumulated net cash flow. Each term structure corresponds to a scenario. If the financial plan's model is continuous, it should be discretized in two respects. Firstly, the resulting trajectories of the cumulated net cash flow should be changed into discrete term structures. Secondly, the joint distribution of all of the risk factors that are taken into account in the model should be expressed in the form of a discrete number of joint scenarios. A scenario is, thus, a value of a random vector. The general idea of the proposed risk measures is in fact invariant both to the model that underlies the generation of the values of the random vector and to the model which is behind the corresponding trajectories of cumulated net cash flow.

To propose a risk measure, it is necessary to define a general criterion of the success and failure of the financial plan of a household. Dependent on what is understood by a success, a risk variable may be specified and a risk measure based on this variable may be constructed. It seems reasonable to think that a criterion of the success of the life-long financial plan is whether all life objectives of the household members are accomplished. In financial terms, realization of life objectives may be expressed as realization of the financial goals of the household (Pietrzyk and Rokita, 2015c). In the model that is used here, it is assumed that the financial goals declared by the household members are constraints that must be met. Under the assumptions of this model, a measure of failure is thus the financial shortfall which must be covered to accomplish all goals under some less advantageous scenarios of the future. In addition, one may assume some extra costs of contingency financing in case of shortfalls. Also some default threshold may be assumed. Exceeding of this threshold by the cumulated shortfall would mean bankruptcy of the household. All three risk measures proposed in this article are based on the cumulated net cash flow of the household.

The information about the risks of a financial plan may be used in different stages of household finance management. The two most obvious ones are the stage of plan optimization and of plan revision. Moreover, the result of risk measurement may be also used for comparing financial plans.

On the stage of plan optimization, the information about risk may be implicitly present in the model with no need of measuring it. In the model proposed by Feldman et al (2014) and Pietrzyk and Rokita (2015b), which is used here as a basis for construction of the proposed risk measures, no risk measure is directly used in the very optimization procedure. The risk is taken into account, but this effect is obtained by means of including a bunch of possible scenarios of future states of nature into the optimization procedure, rather than measuring the risk. Another approach might consist in seeking for a tradeoff between minimization of some a priori defined risk measure and, for instance, some aggregate that reflects the expected standard of living. This would, however, require a specification of the objective function in the optimization task that is a function of the two aforementioned categories, namely – the risk measure and a variable that contains the aggregated information about the expected standard of living. Nevertheless, even if the objective function is lacking these properties, risk measurement makes sense after all. This is mainly because of the need of plan evaluation with respect to risk, even after optimization.

No automated procedure may optimize a bunch of financial goals of the household. This is a consequence of the non-separability of preferences and, consistently, of the lack of their perfect transitivity (Arrow and Intriligator, 1993). Only household members themselves may decide to reduce, postpone or abandon some of their financial goals. Defining the plan optimization task, one must include the financial goals as a set of constraints, rather than decision variables. The model returns an optimal financial plan given the constraints, but it may be hardly feasible under some less advantageous scenarios. This would mean that the best plan for a given set of goals is indeed very risky. The risk is realized in the form of a deep financial shortfall in scenarios that are worse than expected. This might even lead to bankruptcy. To avoid this, the household might choose to construct a plan with less ambitious goals. If automated optimization is not the final step of the plan choice process, the household members need instruments for the evaluation and comparison of different plans. Integrated measures of risk may serve as one kind of such instruments.

In the existing personal finance literature, various risk factors were taken into account. The ones that suggest themselves immediately are those related to the survival process of an individual or a joint survival process of a couple, factors of market risk (financial prices, interest rates), factors connected with income situation or health condition. The classical life cycle model by Yaari (1965), being a consumption model for an individual, allowed for uncertainty about the length of life. There are also generalizations to the case of a two-person household (married couple) – compare Hurd (1999), Kotlikoff and Spivak (1981), Brown and Poterba (2000). Other sources of risk addressed in literature are, for example, returns on assets (Bodie et al, 2004; Richard, 1975; Merton, 1971), stochastic labor income (Geyer et al, 2009) or health condition (Scholz and Seshadri, 2012). Besides risk factors that have impact on the ability to accomplish goals, there are also factors influencing the goals themselves, that is – their size or time when they should be realized, which is mainly caused by changing needs of the household (compare Pietrzyk and Rokita, 2015c).

As there are many types of risk and risk factors that might be taken into consideration in a financial planning model, the measure of risk to be used for plan revision or for comparison between plans should allow for all these risk types at the same time.

2 Outline of the household financial plan optimization task

As it is explained in more details by Pietrzyk and Rokita (2015c) and Pietrzyk and Rokita (2015b), the objective function of a household financial plan optimization task, called there value function of the household, is based on expected discounted utility of consumption and bequest. Equation 1 presents the general concept of the value function.

This function is based on the expected discounted utility concept. Its two basic components are utility of consumption (see equation. 2) and the utility that results from being able to bequeath some amount of residual wealth to one's descendants or other heirs, here called "utility of bequest" (see equation. 3). The future term structure of consumption through the whole life cycle, the length of life, as well as the amount of wealth available at the end of life (the estate to be bequeathed), depend on a set of risk factors. These risk factors are modelled as discrete random variables. For some pairs of risk factors, it may be necessary to assume that they are dependent. Others may be treated as independent without any significant loss of precision. The random vector of risk factors is incorporated into the model in the following way. The value function takes into account only some chosen ranges of the possible values of its elements. These ranges are called "marginal ranges of concern" here. Each element of the random vector (each risk factor) may have its own upper and lower bounds of its individual marginal range of concern, but the widths of the ranges should depend on the risk aversion of the household members. The higher the risk aversion, the higher the width of the marginal range of concern. The hypercube composed of all marginal ranges of concern is called the "range of concern". Only such values of the risk factor vector that belong to the range of concern are further used. Each considered value of the random vector is called a "scenario". Values of the discounted utility of consumption and discounted utility of bequest are calculated for each scenario. These values, weighted by joint probabilities of corresponding scenarios, are summed to obtain the expected discounted utility of a given financial plan. The procedure of financial plan selection is based on maximization of this expected discounted utility.

$$V\left(c_0, v, \kappa; \mathbf{Z}\right) = \sum_{z_j^*=LB\left(z_j\right)}^{UB\left(z_j\right)} \sum_{z_k^*=LB(z_k)}^{UB(z_k)} p_{jk}^* \cdots \sum_{z_l^*=LB(z_l)}^{UB(z_l)} p_l^* \cdots \sum_{z_m^*=LB(z_m)}^{UB(z_m)} \sum_{z_k^*=LB(z_n)}^{UB(z_n)} p_{mn}^* \cdots \left(\alpha V_C + \beta V_B\right) \tag{1}$$

where:

$$V_C = \left(\sum_{t=0}^{T_B^*} \frac{1}{\left(1+r_C\right)^t} u\left(C\left(t; \mathbf{Z}^*\right)\right)\left(\gamma(t) + \delta\left(t\right)\right)\right) \tag{2}$$

$$V_B = \frac{1}{\left(1+r_B\right)^{T_B^*}} u\left(B\left(T_B^*; \mathbf{Z}^*\right)\right) \tag{3}$$

c_0, v, κ – decision variables: c_0 – consumption rate at start of the plan, v – proportion of household retirement investment contribution assigned to private pension plan of Person 1 (pension plan contribution proportion of Person 2 is just $1-v$), κ – vector of own-means-proportions for financing of all financial goals planned by the household;

$\mathbf{Z}$ – random vector of risk factors,
$z_i \equiv \mathbf{Z}(i)$ – i-th element of vector $\mathbf{Z}$ (amongst elements z_i of the vector $\mathbf{Z}$, there are also times of death of the two main household members, i.e., D_1 – time of death of Person 1, D_2 – time of death of Person 2),
$\mathbf{Z}^* \equiv [z_1^* \ldots z_n^*]$ – a particular realization of the random vector $\mathbf{Z}$,
z_i^* – a particular value of the i-th element of vector $\mathbf{Z}$ (amongst particular realizations z_i^*, there are also realizations of times of death, i.e., D_1^* and D_2^*),
$LB(z_i)$ – lower bound of the range of z_i scenarios,
$UB(z_i)$ – upper bound of the range of z_i scenarios,
$p_{ij}^* \equiv P\left(z_i = z_i^*, z_j = z_j^*\right)$ – joint probability of particular realizations of risk factors z_i and z_j if they are treated as dependent in the model,
$p_i^* \equiv P(z_i = z_i^*)$ – probability of a particular realization of risk factor z_i if it is treated in the model as independent from other risk factors in the model (e.g., in the equation (1), risk factors z_j and z_k are dependent of each other but independent of the rest, z_l is independent of any other, z_m and z_n are dependent on each other and independent of any of the other ones),
T_B^* – time of household end ($T_B^* \equiv \max\{D_1^*, D_2^*\}$) for a given scenario $\mathbf{Z}^*$,
α – propensity-to-consume parameter,
β – bequest preference parameter,
$u(.)$ – utility of consumption,
$C(t;\mathbf{Z}^*)$ – consumption at a moment t and under a scenario $\mathbf{Z}^*$,
$B(T_B^*;\mathbf{Z}^*)$ – bequest at the end of the scenario $\mathbf{Z}^*$(at the time T_B^*, under the scenario $\mathbf{Z}^*$),
$\gamma(t)$, $\delta(t)$ – functions reflecting subjective severity of premature death and longevity, respectively, based on life-length risk aversion parameters (γ^*, δ^*) declared by the household,
r_C – rate used for discounting of the utilities of consumption,
r_B – rate used for discounting of the utility of bequest.

Optimization of the household financial plan consists in maximizing the function V(.) given by the formula 1 under several constraints. The budget constraint is the most obvious one. But in the model by Feldman, Pietrzyk, and Rokita (2014) and Pietrzyk and Rokita (2015b,c), there is also the constraint that all financial goals are met, as well as a constraint of minimum acceptable consumption. Whereas the value function of the household is based on consumption and bequest, verification of optimization results and measuring risk of the plan is based on net cash flows. To be more precise, as it has already been mentioned, the considered financial category is then cumulated net cash flow (i.e., cumulated surplus or cumulated shortfall).

3 Dealing with time value of money if the impact of cash-flow timing is not obvious

When analyzing potential threats to the financial liquidity of the household, particularly if caused by a deep shortfall, whose recovery may require time and efforts, classical discounting of cash flows may not be the most comprehensive way to reflect the time value of money. In the model, comparing cash flows from different periods is facilitated in two ways. The first is that all cash flows are expressed in real terms, assuming some long-run inflation rate forecast. This would, however, be insufficient because the time value of money is dependent also on other components of interest rates than just inflation. On the other hand, discounting of all cash flows at some interest rate would obviously attach a higher weight to the cash flows of the nearest future and a low weight to the ones that are more distant in time from the present. This is fully appropriate in the overwhelming majority of situations in finance, but not when applied to weighting the shortfalls in the life-cycle of a household. This is because the severity of shortfalls, especially of deep shortfalls, is not decreasing with age. Moreover, it is not even monotonic. A negative cumulated net cash flow is most problematic for very young and for elderly persons. How should this property be taken in account? For all individual clients, the nominal interest rate of loan is usually equal or may be dependent on collateral, for instance, but it will rather not depend on age or similar individual characteristics. Nonetheless, the creditworthiness of a person who is very young or very old will be evaluated as lower than the creditworthiness of the clients who are in their mid-career period. The effective cost of a credit or loan offered to a less creditworthy person will, consistently, be higher due to higher collateral or because of a requirement that the customer has some third-party endorsers, even if the loan is granted at the same nominal cost as for the more creditworthy client. All of these additional costs, including time and additional effort needed to obtain financing in a very young or old age, is here called "burdensomness cost". In the model, to take account of the non-monotonic nature of the relationship between age and the burdensomness of shortfall financing, all negative cumulated cash flow values

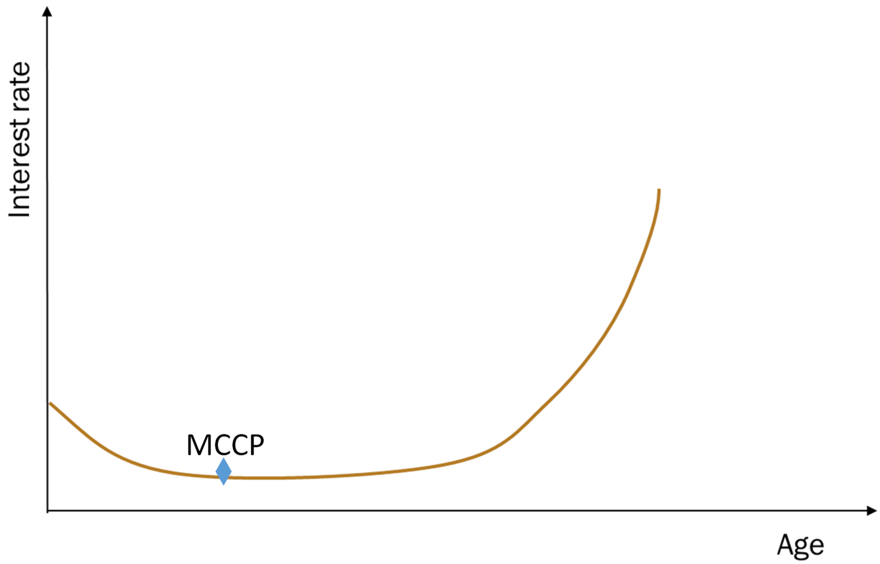

Fig. 1 Correction of credit costs with household age taken into account

are charged additionally at a rate of contingency financing, which is composed of the following elements:

$$\eta_{c_t} = \eta_t + \lambda_t + \zeta_t + \xi_t, \tag{4}$$

where η_t is market benchmark interest rate (in real terms) at t, λ_t is credit spread (risk premium plus bank margin) for regular financing, ζ_t is additional margin for contingency financing at t, and ξ_t is burdensomeness charge (dependent on household age) at t.

An example of the shape of the relationship between household age and contingency financing rate is shown in Fig. 1.

Having added the additional charge for contingency financing to the negative cumulated net cash flows, no limitations on the size of cumulated shortfall is imposed here. The only exception from this rule is the third of the proposed measures of risk, that is – household default probability.

Summing up, cumulated net cash flow is calculated in the following way:

- real values of subsequent net cash flows are added,

- in periods when the cumulated net cash flow is negative, an additional charge, equal to interest for the annual contingent loan, is added.

The procedure may be illustrated with the following formula 5:

$$CSp_0 = Sp_0, \quad CSp_t = \begin{Bmatrix} CSp_{t-1} + Sp_t & \text{if } CSp_{t-1} >= 0 \\ CSp_{t-1} \cdot (1 + \eta_{c_t}) + Sp_t & \text{if } \; CSp_{t-1} < 0 \end{Bmatrix}, \tag{5}$$

where Sp_t is the net cash flow (in real terms) at t, called here surplus, and CSp_t is the cumulated surplus at t.

Does it mean that no traditional discounting is used in the model?
Not exactly – compare with equation 1, in which utility of consumption and bequest is discounted.

4 Measures of risk of the financial plan of the household

As it has already been mentioned, the underlying model of the household financial plan is based on cash flows and one of its major outcomes is the term structure of the cumulated net cash flows of the household. The proposed risk measures are all suited for this way of description of the financial situation of the household. The nature of the problem implies a negative concept of risk rather than a neutral, which, in turn, elicits the choice of downside risk measures. The following three concepts of measures of risk of the financial plan of the household are discussed: Residual Wealth at Risk, Lifetime Cumulated Net Cash Flow at Risk and Household Default Probability. As the models proposed by Pietrzyk and Rokita (2015c,b) are discrete and based on a finite number of survival scenarios, estimation of the measure is also performed for a grid of discrete multivariate scenarios, belonging to a hypercube formed by confidence intervals for all of the considered risk factors.

An important requirement the measures should meet is to take into account phases of household life-cycle and to analyse the financial situation of the household in many moments. It would be far from satisfactory to measure threats to household finance just at the end of some period (compare Pietrzyk and Rokita, 2015a). Generally speaking, the measure should grasp possibilities of suboptimal shapes of the whole term structure (trajectory) of the household net cash flow process.

However, here arises a question about the definition of suboptimal or disadvantageous dynamic behavior of the cash flows. By disadvantageous we mean a behaviour that causes a threat to accomplishing the life objectives of the household. For a household financial plan, there is, yet, a number of possible criteria one may use to determine whether the plan is successful or not. This may be preservation or improvement of the life standard, realization of some financial goals (in addition to regular consumption), smoothing of the consumption path throughout the whole life cycle, leaving a bequest, etc. To that, minimization of risk itself may be also on the list, which makes things even more complicated. It is hard to find a risk measure that reflects threats undermining all of these sources of financial plan success to the same extent. This is why several integrated risk measures are proposed here.

Each of these puts different emphasis on some of the aforementioned aspects. They may be classified with respect to different criteria, like the risk variable, construction of the very measure, or the benchmark used in its definition. All of the proposed measures are based on the same risk variable. It is cumulated net cash flow. A reservation should be made, however, that in some variants this is cumulated net cash flow at household end and, in some others, these are cumulated net cash flows at each moment of the planning period, or a kind of aggregate.

As far as the construction of the measure is concerned, three types of measure are used. The first are volatility measures. The second are downside risk measures expressed as quantile of risk variable distribution. The third are downside risk measures expressed as probability.

The last criterion that is used here places focus on the benchmark. Using combinations of these three criteria, the following general systematization of the proposed risk measures is presented (Pietrzyk and Rokita, 2015a):

- Measures based on residual wealth:
  - Residual Wealth at Risk
  - Residual Wealth Volatility
  - Residual Wealth Aspiration Level
- Measures indicating problems during whole life-cycle:
  - Measures indicating threats to household financial plan realization:
    - Lifetime Cumulated Net Cash Flow at Risk
    - Incremental Shortfall
    - Shortfall Scenario Probability

- Measures of household default risk:
  - Household Default Probability

4.1 Residual Wealth at Risk (RWaR)

This measure is very similar to Cash Flow at Risk (CFaR), with the difference that cumulated net cash flow is the risk variable here and that realization of the considered random variable may refer to different points in time (household end may be different in different scenarios). The course of actions to obtain the measure is as follows:

1. In each scenario, there is calculated a deviation from the applied benchmark of cumulated net cash flow at household end,
2. A left-tail quantile of deviations, corresponding to a pre-set tolerance level (small significance level), is determined on the basis of the scenarios.

This measure of risk can be described by the following formula:

$$P(CSp_{T_B} \leq XCSp_{T_B} - RWaR) = q, \tag{6}$$

where $XCSp_{T_B}$ is the benchmark of cumulated net cash flow at household end, CSp_{T_B} the cumulated net cash flow at household end, $RWaR$ the Residual Wealth at Risk and q the $RWaR$ tolerance level (a small significance level).

4.2 Lifetime Cumulated Net Cash Flow at Risk (LCNCFaR, LCaR)

In search of a measure that encompasses all threats throughout the whole life time of the household to a higher extent, a new proposal is formulated. Its concept assumes that both the moment when a shortfall is encountered and the length of period during which the household remains in the situation of a negative cumulated net cash flow are important and should be taken into account. If, in any of the considered scenarios, there is a cumulated shortfall in whichever period, it is treated as a realization of risk. The proposition is to do the following:

In each of the considered scenarios, all cumulated shortfalls are summed (cumulating of cumulated shortfalls), From all scenarios, there is taken a quantile of the sum of cumulated shortfalls, corresponding to a predefined small tolerance level.

This measure of risk can be described by the following formulas:

$$P(CCSh \leq -LCaR) = q, \tag{7}$$

where q is $LCaR$ tolerance level (a small significance level).

The Lifetime Cumulated Net Cash Flow at Risk is a quantile of the sum of cumulated net cash flows throughout the whole life time, multiplied by (-1), so that it gives a positive value if the quantile is negative (a shortfall). The sum of cumulated shortfalls ($CCSh$) is defined for each scenario $\mathbf{Z}$, compare eq. 9. Cumulated shortfalls are defined in equation 8. The sum of cumulated surplus is calculated for all periods from the starting point of the plan until the end (T_B^*) of a given scenario ($\mathbf{Z}^*$):

$$CSh_t = \begin{Bmatrix} CSp_t \text{ if } CSp_t < 0 \\ 0 \quad \text{if } CSp_t \geq 0 \end{Bmatrix}., \tag{8}$$

$$CCSh^{(\mathbf{Z}^*)} = \left\{ \sum_{t=1}^{T_B^*} CSh_t^{(\mathbf{Z}^*)} \right\}, \tag{9}$$

where $CSh_t^{(\mathbf{Z}^*)}$ is the cumulated shortfall at a moment t, under a scenario $\mathbf{Z}^*$ and $CCSh^{(\mathbf{Z}^*)}$ the sum of cumulated shortfalls from the start of the plan until the end of the particular scenario $\mathbf{Z}^*$.

The measure takes into account the information about time and magnitude of shortfalls, but it does not include explicitly the information if the household is able to make up for the shortfall in subsequent periods or not.

4.3 Household Default Probability (HDP)

The Lifetime Cumulated Net Cash Flow at Risk presented above supplies a decision maker with no bankruptcy threat signals. The Household Default Probability, in turn, takes into account only these negative values of cumulated net cash flow which exceed some threshold above which the household is not able to increase its indebtness. The threshold is determined for each period and in each of the considered scenarios. The course of actions to construct the measure is as follows:

1. A default threshold is determined at the moment t_0 (e.g., credit worthiness of the household at t_0); then, the default threshold changes in time with financial situation of the household.
2. Scenarios in which cumulated shortfall exceeds the default threshold are identified as default scenarios.
3. Probabilities of default scenarios are summed.
4. Probability that any of the default scenario realizes is the measure of risk.

$$HDP = P(CSh < DTh), \tag{10}$$

where DTh is the default threshold.

When constructing the measure, one needs to define a rule of default threshold determination. The proposition is that the threshold is calculated on the basis of household potential free cash flow (PFC), costs of contingency loan servicing and the household age. The default threshold changes over time. An important element that is taken into consideration when determining the default threshold are loans for contingency financing of cumulated shortfall, and a minimum possible payment for a given age (assuming that expected further life length at a given moment is treated as the maximum crediting period). The minimum possible payment (per period) for contingency loans cannot exceed the potential free cash flow. It must be emphasized that all payments resulting from planned loans are already deduced by definition from the potential free cash flow (compare eq. 11):

$$PFC_t = \; Ic_t - C_{min_t} + NIt_t + ND_t + CSp_t, \tag{11}$$

where
Ic_t is the income at a moment t, C_{min_t} is the minimum consumption at t, NIt_t is the net cash flow from investment at t and ND_t is the net cash flow from planned debts at t.

The default threshold at a given moment is set as the value of shortfall that cost of debt servicing, which is connected with this shortfall exceeds potential free cash flow during the next period.

In Fig. 2, there is an example of a scenario for which the cumulated surplus trajectory takes on negative values in several periods and the default threshold is exceeded in periods number 9 and 16. Of course, only one exceedance of

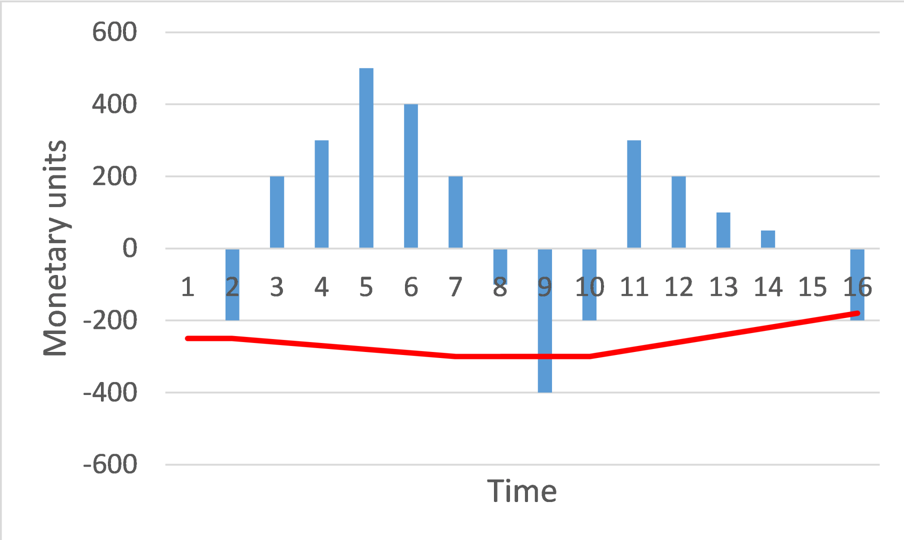

Fig. 2 A trajectory of cumulated surplus (real values, additionally modified by contingency financing charge) with the default threshold marked with the line.

the threshold is sufficient to recognize the whole scenario as a default one. This measure does not contain the information, by what amount of a shortfall the threshold is exceeded, nor how many times it would be exceeded, given a default scenario.

5 Summary

The construction of the three presented measures of risk is based on an assumption that the shape of the cumulated net cash flow term structure (trajectory) reflects the general financial situation of the household throughout its whole life, under a given realization of risk factors. The measures expose some chosen properties of the cumulated net cash flow process, given a distribution of risk factors. The methodology does not impose any particular distributional model of the risk factors, nor even any particular set of them. The concept is presented here by an example of a two-person household financial plan model, proposed by Feldman et al (2014) and Pietrzyk and Rokita (2015b), but it might be easily transferred to other models, if it is possible in the models to obtain the information about potential trajectories of cumulated net cash flow. In fact, the very concept of the risk measure which is presented in this article leaves aside the question of the source of the cumulated net cash flow trajectories that underlie these measures.

It should be pointed out that each of the presented measures addresses some other aspects of risk of the financial plan of the household. It would be very hard to incorporate the whole potentially significant information in one measure. In the first (*RWaR*), the stress is placed on the end of household. The second (*LCaR*) aggregates information about all potential shortfalls that may be encountered during the life of the household, as well as lengths of periods during which the household is in a situation of shortfall. The last (*HDP*) informs how likely it is that the household loses its financial liquidity.

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