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Online Estimation of Vehicle Driving Resistance Parameters with Recursive Least Squares and Recursive Total Least Squares*

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Abstract

Contribution: The contribution of this paper is a recursive generalized total least-squares (RGTLS) estimator that offers exponential forgetting and treats data with unequally sized and correlated noise.
Application: RGTLS is used for estimation of vehicle driving resistance parameters. A vehicle longitudinal dynamics model and available control area network (CAN) signals form appropriate estimator inputs and outputs.
Results: We present parameter estimates for the vehicle mass, two coefficients of rolling resistance, and drag coefficient of one test run on public road. Moreover, we compare the results of the proposed RGTLS estimator with two kinds of recursive least-squares (RLS) estimators.
Discussion: While RGTLS outperforms RLS with simulation data, the recursive least squares with multiple forgetting (RLSMF) estimator [1] provides superior accuracy and sufficient robustness through orthogonal parameter projection with experimental data.

1. Introduction

- Energy-efficient trajectory planning, range prediction, and recuperation strategies are highly reliant on accurate models of the vehicle total driving resistance.
- Parametric models are commonly used with unknown parameters that vary at different rates such as the vehicle mass, coefficients of rolling resistance, and drag coefficient.

2. Total Least Squares

- Total least squares is a data fitting method for the unconstrained perturbation problem (1) that is known as errors-in-variables (EIV) model.
- The input data $A \in \mathbb{R}^{m\times n}$ and output data $B \in \mathbb{R}^{m\times d}$ are sums of noise-free data $\overline{A}$, $\overline{B}$ and measurement noise $\widetilde{A}$, $\widetilde{B}$, respectively.
- The unknown parameters are denoted by $X \in \mathbb{R}^{n\times d}$.

$$AX \approx B, \quad A = \overline{A} + \widetilde{A}, \quad B = \overline{B} + \widetilde{B}. \tag{1}$$

Figure. 1 visualizes the difference between LS and TLS. While LS corrects the data vertically and assumes that A is exactly known, TLS performs perpendicular data corrections.

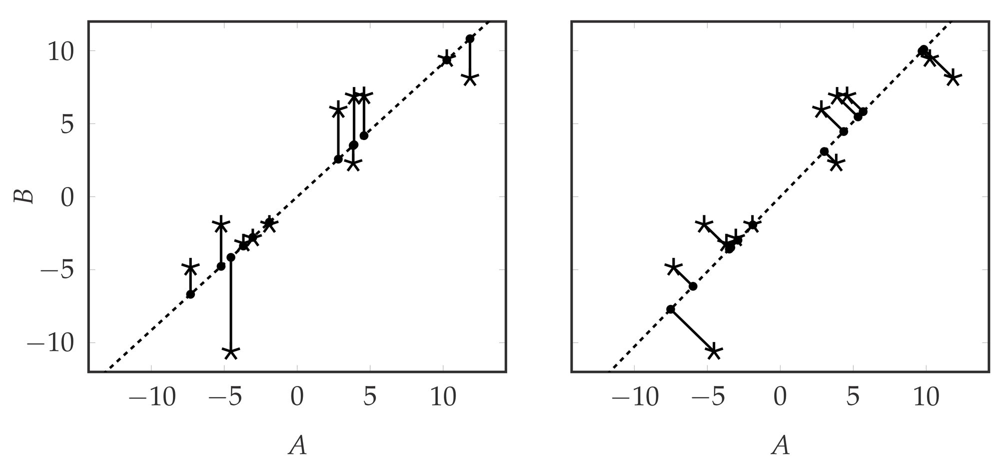

Figure 1: *Data fitting with* LS *on the left and* TLS *on the right.* $\star$ *shows the data* $[A\ B]$, $\cdot$ *shows approximations* $[\widehat{A}\ \widehat{B}]$, ⋯ *shows the estimated model and* — *shows the corrections* $[\Delta A\ \Delta B]$.

3. Recursive Total Least-squares Algorithm

- The recursive singular value decomposition (RSVD) Algorithm. 1 is based on the SVD ($Z = USV^\top$) update algorithm of Gu and Eisenstat [2], but skips the update of U entirely.
- We take advantage of the previous SVD matrices $S(t-1)$ and $V(t-1)$ when new data arrives in Algorithm. 1 line 2.
- Algorithm. 2 is iteratively executed throughout the data set from time step $t = 1$ to $t = m$.
- The nested RSVD in Algorithm. 2 line 9 allows a recursive TLS solution.

input: $A(t)$, $B(t)$, $S(t-1)$, $V(t-1)$, $\mathfrak{v}$, λ
1 *batch*
2 $z = [A(t)\ B(t)]^\top$
$a = V(t-1)^\top \cdot z$
$M = \begin{bmatrix} \lambda \cdot S(t-1) \\ a^\top \end{bmatrix}$
$JK = \mathrm{qr}(z - V(t-1)\cdot a)$
$v = \sqrt{\det(K^\top K)}$
if $v < \mathfrak{v}$ *then*
3 $PNQ^\top = \mathrm{svd}(M) \quad S(t) = N_{1:q,1:q}$
4 *else*
5 $PS(t)Q^\top = \mathrm{svd}(M, \text{'econ'})$
6 $V(t) = V(t-1)\cdot Q$
output: $S(t)$, $V(t)$
7 **Algorithm 1:** RSVD

for $t \leftarrow 1$ *to* m *do*
input: $A(t)$, $B(t)$, $S(t-1)$, $V(t-1)$, $(A^\top A)(t-1)$, $\mathfrak{v}$, λ
8 *function call*
input: $A(t)$, $B(t)$, $S(t-1)$, $V(t-1)$, $\mathfrak{v}$, λ
9 Algorithm. 1 *output*: $S(t)$, $V(t)$
10 $V(t) := \begin{array}{cc} n & d \\ \left[\begin{matrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{matrix}\right] & \begin{matrix} n \\ d \end{matrix} \end{array}$
$\widehat{X}(t) = -V_{12}V_{22}^{-1}$
$(A^\top A)(t) = ((A^\top A)(t-1)\cdot \lambda^2) + A(t)^\top A(t)$
$\mathrm{cov}(\widehat{X}(t)) \approx (1 + \|\widehat{X}(t)\|_2^2)\sigma^2 ((A^\top A)(t) - t\cdot\sigma^2\cdot I)^{-1}$
$\sigma^2 \approx \dfrac{S(t)_{q,q}}{t}$
output: $\widehat{X}(t)$, $\mathrm{cov}(\widehat{X}(t))$, $S(t)$, $V(t)$, $(A^\top A)(t)$

Algorithm 2: RTLS

4. Vehicle Longitudinal Dynamics Model

- We use the force equilibrium (2) between the tractive force F on the left-hand side and the sum of rolling resistance, climbing resistance, aerodynamic drag, and acceleration resistance, that is known as total driving resistance.
- In (2), α, v, ρ, g, and a_x are the road-grade, velocity, air density, gravity acceleration, and longitudinal acceleration respectively.

$$F = m\cdot g\cdot\cos\alpha(f_{\mathrm{r}0} + f_{\mathrm{r}1}\cdot v) + m\cdot g\cdot\sin\alpha + c_\mathrm{x}\frac{\rho}{2}A\cdot v^2 + (m + m_{\mathrm{rot}})a_\mathrm{x}. \tag{2}$$

CAN signals of F, α, v, ρ, and a_x form adequate input and output data to feed the MISO model

$$\underbrace{\begin{bmatrix} A_{11} & \dots & A_{1n} \\ \vdots & \vdots & \vdots \\ A_{m1} & \dots & A_{mn} \end{bmatrix}}_{\text{inputs}} \underbrace{\begin{bmatrix} X_1 & \dots & X_n \end{bmatrix}^\top}_{\text{parameters}} \approx \underbrace{\begin{bmatrix} B_1 \\ \vdots \\ B_m \end{bmatrix}}_{\text{output}}. \tag{3}$$

5. Results

Figure. 2 shows simulation results of a three input one output model with random data, a step in X_2 and noisy input and output measurements.

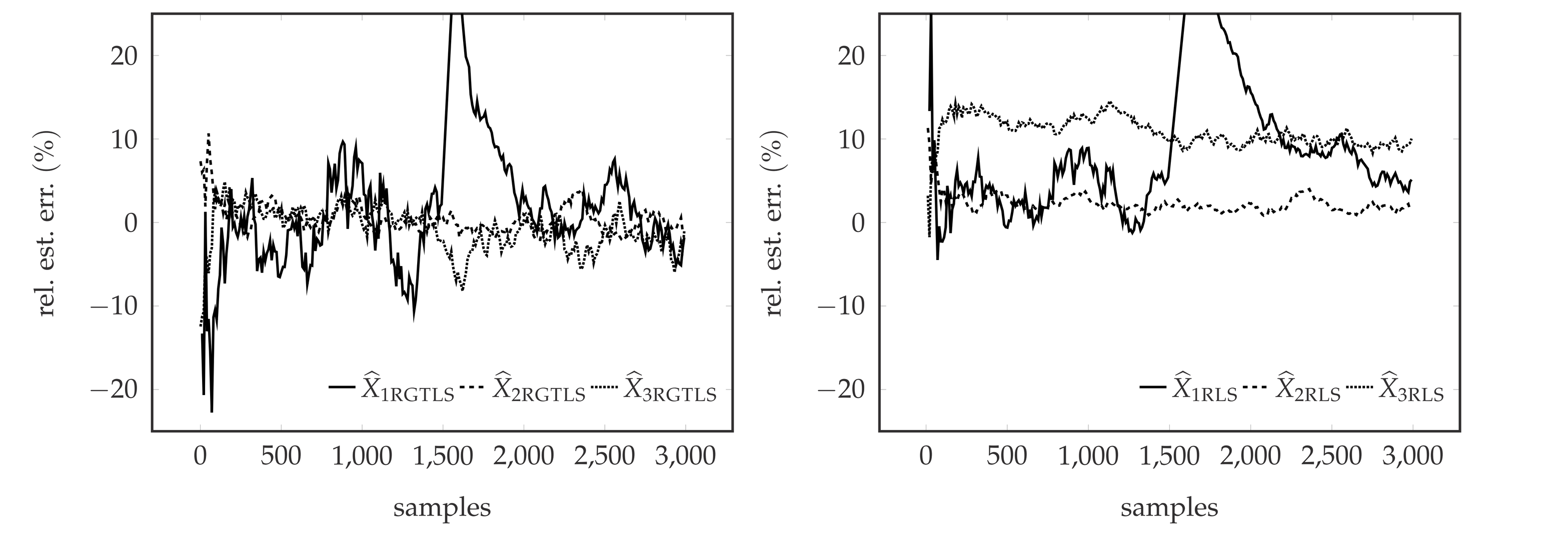

Figure 2: *Relative estimation error of* RGTLS *with* $W = cov([\widetilde{A}\ \widetilde{B}])$ *on the left and* RLS *on the right.* RGTLS *clearly outperforms* RLS *when prior knowledge of the noise covariance* W *is present.*

Figure. 3 gives results of two distinct coefficients of rolling resistance from a test run with a grand touring sports car.

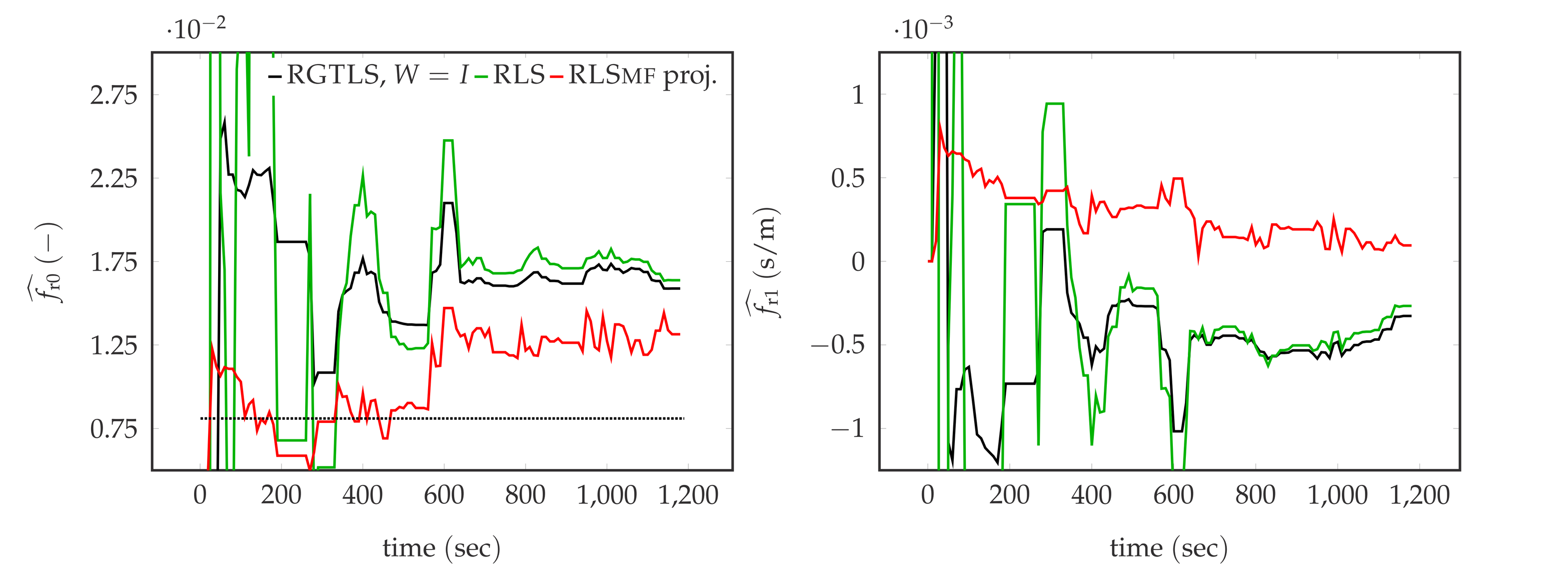

Figure 3: *Estimated coefficient of rolling resistance* $\widehat{f_{\mathrm{r}0}}$ *on the left and* $\widehat{f_{\mathrm{r}1}}$ *on the right.* - *shows the value of* $f_{\mathrm{r}0}$ *from rolling resistance measurements. Parameter projection is required for* RGTLS *and* RLS *to prevent negative values on the right in this real world example.*

6. Conclusions

- The presented RGTLS estimator with exponential forgetting can treat basic TLS and the extensions WTLS and GTLS in real-time by appropriate setting of the weighting matrix W.
- If prior knowledge of the noise covariance matrix is present, RGTLS outperforms RLS.
- RGTLS and RLS fail in the estimation of particular driving resistance parameters such as the coefficients of rolling resistance. A parameter projection scheme is required in the future.

References

[1] A. Vahidi, A. Stefanopoulou, and H. Peng. "Recursive least squares with forgetting for online estimation of vehicle mass and road grade: theory and experiments". In: *Vehicle System Dynamics* 43.1 (2005), pp. 31–55.
[2] M. Gu and S.C. Eisenstat. "A stable and fast algorithm for updating the singular value decomposition". In: *New Haven: Yale University Department of Computer Science. RR-939* (1993).