Online Estimation of Vehicle Driving Resistance Parameters with Recursive Least Squares and Recursive Total Least Squares*

*This work was accomplished in cooperation with the Energy Management Complete Vehicle Department at Dr. Ing. h. c. F. Porsche AG, Weissach, Germany

Stephan Rhode and Frank Gauterin

Institute of Vehicle System Technology
Chair of Vehicle Technology

Abstract

Contribution: The contribution of this paper is a recursive generalized total least-squares (RGTLS) estimator that offers exponential forgetting and treats data with unequally sized and correlated noise.

Application: RGTLS is used for estimation of vehicle driving resistance parameters. A vehicle longitudinal dynamics model and available control area network (CAN) signals form appropriate estimator inputs and outputs.

Results: We present parameter estimates for the vehicle mass, two coefficients of rolling resistance, and drag coefficient of one test run on public road. Moreover, we compare the results of the proposed RGTLS estimator with two kinds of recursive least-squares (RLS) estimates.

Discussion: While RGTLS outperforms RLS with simulation data, the recursive least squares with multiple forgetting (RLSmt) estimator [1] provides superior accuracy and sufficient robustness through orthogonal parameter projection with experimental data.

1. Introduction

Energy-efficient trajectory planning, range prediction, and recuperation strategies are highly relevant on accurate models of the vehicle total driving resistance.

Parametric models are commonly used with unknown parameters that vary at different rates such as the vehicle mass, coefficients of rolling resistance, and drag coefficient.

2. Total Least Squares

Total least squares is a data fitting method for the unconstrained perturbation problem (1) that is known as errors-in-variables (EIV) model.

The input data \( A \in \mathbb{R}^{n \times m} \) and output data \( B \in \mathbb{R}^{n \times d} \) are sums of noise-free data \( \tilde{A}, \tilde{B} \) and measurement noise \( \Delta A, \Delta B \), respectively.

The unknown parameters are denoted by \( X \in \mathbb{R}^{p \times d} \).

\[
AX \approx B, \quad A = \tilde{A} + \Delta A, \quad B = \tilde{B} + \Delta B.
\]

Figure 1 visualizes the difference between LS and TLS. While LS corrects the data vertically and assumes that \( A \) is exactly known, TLS performs perpendicularly data corrections.

3. Recursive Total Least-squares Algorithm

The recursive singular value decomposition (RSVD) Algorithm 1 is based on the SVD \((Z = USV^T)\) update algorithm of Gu and Eisenstat [2], but skips the update of \( U \) entirely.

We take advantage of the previous SVD matrices \( (S(t-1) = V(t-1) \) when new data arrives in Algorithm 1 line 2.

Algorithm 2 is iteratively executed throughout the data set from time step \( t = 1 \) to \( t = m \).

The nested RSVD in Algorithm 2 line 9 allows a recursive TLS solution.

4. Vehicle Longitudinal Dynamics Model

We use the force equilibrium (2) between the tractive force \( F_T \) on the left-hand side and the sum of rolling resistance, climbing resistance, aerodynamic drag, and acceleration resistance, that is known as total driving force.

\[
F = \alpha_0 \cdot \cos(\phi_a) + l_{f}(\delta_{fr}) + m \cdot g \cdot \sin(\beta) + c_d \frac{A}{2} \rho V^2 + (m + m_r) \ddot{x}.
\]

CAN signals of \( F, \alpha, \phi, \delta_{fr} \) and \( \alpha_0 \) form adequate input and output data to feed the MISO model.

\[
\begin{bmatrix}
A_11 & \cdots & A_{1d} \\
\vdots & \ddots & \vdots \\
A_{m1} & \cdots & A_{md}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
\vdots \\
X_d
\end{bmatrix}
\approx
\begin{bmatrix}
B_1 \\
\vdots \\
B_d
\end{bmatrix}.
\]

5. Results

Figure 2 shows simulation results of a three input one output model with random data, a step in \( X_2 \) and noisy input and output measurements.

Figure 3 gives results of two distinct coefficients of rolling resistance from a test run with a grand touring sports car.

6. Conclusions

The presented RGTLS estimator with exponential forgetting can treat basic TLS and the extensions RTLS and GTLS in real-time by appropriate setting of the weighting matrix \( W \).

If prior knowledge of the noise covariance matrix is present, RGTLS outperforms RLS.

RLTS and RLS fail in the estimation of particular driving resistance parameters such as the coefficients of rolling resistance. A parameter projection scheme is required in the future.

References


Figure 1: Data fitting with LS on the left and TLS on the right. * shows the data \([A B], *\) shows approximations \([AB], \) shows the estimated model and * shows corrections \([A \Delta B] \).

Figure 2: Relative estimation error of RGTLS with \( W = \text{cov}(\hat{A} \hat{B}) \) on the left and RLS on the right. RGTLS clearly outperforms RLS when prior knowledge of the noise covariance \( W \) is present.

Figure 3: Estimated coefficient of rolling resistance \( f_{roll} \) on the left and \( f_{roll} \) on the right. * shows the value of \( f_{roll} \) from rolling resistance measurements. Parameter projection is required for RGTLS and RLS to prevent negative values on the right in this real world example.