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Cosmic ray physics with the KASCADE-Grande observatory

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Abstract. The existence of a knee at a few PeV in the all-particle cosmic ray energy spectrum has been well established by several experiments but its physical origin has eluded researches for a long time. It is believed that keys to disentangle the mystery could be found in the spectrum and the composition of cosmic rays between 1 PeV and 1 EeV. A first detailed look into the elemental chemical abundances of cosmic rays in this energy regime was provided by both the KASCADE and the KASCADE-Grande experiments. Their measurements opened the door to a wealth of new data on the subject, which led to the discovery of new structures in the all-particle energy spectrum and the confirmation of knee-like features in the spectra of individual mass groups, as well as the observation of an unexpected ankle-like structure at around 100 PeV in the flux of the light component of cosmic rays. In this contribution, early findings with the KASCADE-Grande experiment will be reviewed and then a short update on the analyses currently performed with the data of the observatory will be presented.

KEYWORDS: KASCADE-Grande, cosmic rays, air showers

1. The KASCADE-Grande experiment

1.1 Experimental set-up

KASCADE-Grande was a ground-based air-shower observatory dedicated to investigate the energy spectrum, the elemental composition and the arrival direction of cosmic rays in the energy range from 10¹⁵ to 10¹⁸ eV with the aim of studying the origin of the knee in the all-particle spectrum of cosmic rays [1]. The experiment was located at the Karlsruhe Institute of Technology, Campus North (49.1° N, 8.4° E, 110 m a.s.l.) in Kalrsruhe, Germany, and was active from 2003 up to 2012. KASCADE-Grande was an extension of the original KASCADE experiment (see Fig. 1, left), which began to operate in 1993.

KASCADE was designed to investigate extensive air showers (EAS) at energies around the knee, specifically, in the interval from 100 TeV to 80 PeV [2]. The main components of this experiment were the central detector (CD), with its hadron calorimeter and multiwire proportional chambers, the underground muon tracking detector (MTD) and the field array, which was composed of 252 detector stations placed on a squared grid of $200 \times 200 \text{ m}^2$ with a spacing of 13 m. The latter was used to measure independently electrons ($E_e > 5 \text{ MeV}$) and muons ($E_\mu > 230 \text{ MeV}$) in the shower using shielded and unshielded scintillation detectors, respectively. The field array observables were the total electron number (N_e) and the truncated muon number (N_{μ}^{tr}) , which represents the amount of muons at distances between 40 and 200 m from the EAS core. One of the early studies performed with these data consisted of a detailed unfolding analysis which led, for the first time, to the measurement of the spectra of five representative elemental groups in cosmic rays in the energy range from 10^{15} eV to 10^{17} eV [3]. In this regard, the most interesting result from KASCADE was to show that the knee is the result of individual breaks in the fluxes of the light to medium primary particles, Z < 6 (c.f. Fig. 1, right). Such conclusion was observed to be independent of the low- or high-energy hadronic interaction model used to interpret the data, in spite of the strong dependence of the relative abundances with the model that was found in the studies [3, 4].

The KASCADE measurements also revealed that the knee positions depend on the composition of the incoming particle: first bend the light component and then the medium one in the PeV cosmic ray flux [3, 4]. In conventional acceleration and propagation models, the knee positions are charge dependent [5]. Assuming $E_H \sim 3 \times 10^{15}$ eV [3] for the position of the knee of the proton, then a break in the iron spectrum at around $Z_{Fe} \cdot E_H \sim 10^{17}$ eV was expected. Therefore, the measurement of such structure was recognized as an important clue to constrain the models about the origin and propagation of cosmic rays. This region was, however, outside the sensitivity range of the KASCADE experiment. Therefore, with the aim of searching for the iron knee, KASCADE was upgraded to the new KASCADE-Grande experiment.

The main upgrade consisted of adding a bigger system of detectors called Grande [1], which extended the cosmic ray measurements up to 10^{18} eV. Grande was a 0.5 km^2 array composed of 37 plastic scintillator stations spaced by an average distance of 137 m. The Grande detectors were sensitive to charged particles ($E_{ch} > 3 \text{ MeV}$) in the EAS. As a part of the upgrade a small octagonal cluster, Piccolo, formed by eight scintillator stations spaced 20 m from each other, was integrated to provide an external trigger for the KASCADE and Grande arrays. Several radio (43 – 74 MHz) and microwave (3.4 – 4.2 GHz) antennas were added later, forming the LOPES [6] and the CROME experiments [7], respectively, which were used to explore and open new forms of EAS detection.

1.2 EAS reconstruction

For each event, Grande and KASCADE provided independent measurements of the total number of charged particles, N_{ch} , and the total number of muons, N_{μ} , respectively [1]. These observables are calculated from individual fits to the lateral particle density functions (LDF) measured with the corresponding set-ups. The shower core position and the arrival direction of the EAS are estimated solely

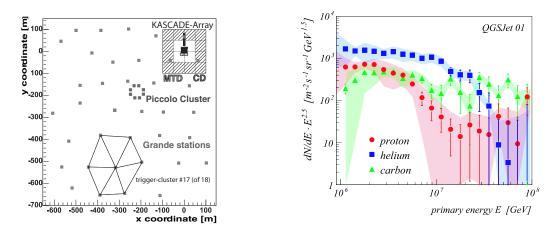


Fig. 1. *Left panel*: The KASCADE-Grande experiment. Small squares represent the Grande stations. The KASCADE array is seen at the upper right hand of the figure. Close to the center of KASCADE, both the MTD and CD are shown. KASCADE detectors are arranged in 16 clusters (big squares). The outer 12 clusters contain the muon detectors. One example of a Grande-trigger is shown. It consisted of 6 activated stations in hexagonal configuration plus a central one. *Right panel*: Energy spectra for the light elemental mass groups of PeV cosmic rays obtained from EAS data at KASCADE using the unfolding technique [3].

from the Grande data. In particular, the arrival direction of EAS is derived via χ^2 -minimization by comparing the arrival times measured by the Grande detectors with the ones expected from a curved shower front. From direct comparisons of EAS independently reconstructed with the KASCADE and Grande arrays, it was found a detector angular resolution of ~ 0.7°, a core location accuracy of ~ 5 m and a total charged particle number resolution $\leq 15\%$, all values in full agreement with expectations from Monte Carlo simulations [1]. The muon size accuracy reconstruction, on the other hand, can reach 20% at low energies, but decreases at high energies according to MC predictions [1], and shows a small dependence with the hadronic interaction model. For the Grande analyses, N_{μ} was corrected for this systematic bias, as the main features of the corresponding systematic errors are well understood.

2. Main achievements with the KASCADE-Grande detector

2.1 The all-particle energy spectrum

In the first step of the Grande analysis, the all-particle energy spectrum between $E = 10^{16}$ and 10^{18} eV was reconstructed by exploiting the correlation between the N_{ch} and N_{μ} observables [8]. The energy assignment was done event-by-event according to a MC derived formula obtained from CORSIKA simulations using FLUKA [9] and QGSJET-II-02 [10] as low and high energy hadronic interaction models, respectively. The formula depends on N_{ch} and the so called k parameter, which takes into account the mass sensitivity in the energy assignment. The k parameter is defined in terms of the ratio of the shower size to the total muon number:

$$k = \frac{\log_{10}(N_{ch}/N_{\mu}) - \log_{10}(N_{ch}/N_{\mu})_H}{\log_{10}(N_{ch}/N_{\mu})_{Fe} - \log_{10}(N_{ch}/N_{\mu})_H},$$
(1)

with

$$\log_{10}(N_{ch}/N_{\mu})_{H,Fe} = c_{H,Fe} \cdot \log_{10}N_{ch} + d_{H,Fe},$$
(2)

where $c_{H,Fe}$ and $d_{H,Fe}$ are calibration constants.

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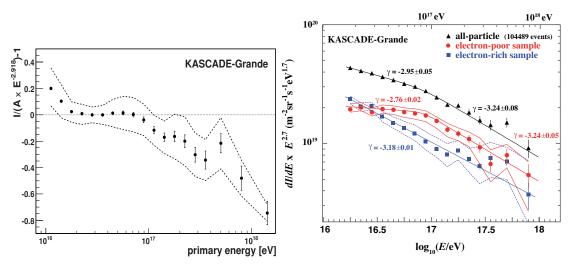


Fig. 2. *Left panel*: The unfolded all-particle energy spectrum of KASCADE-Grande multiplied by a power law to better recognize its main structures. The systematic error band is shown with dotted lines [8]. *Right panel*: All-particle flux and energy spectra of the light and heavy mass groups of cosmic rays obtained with KASCADE-Grande data using the *k* parameter technique [11]. In the above plots, QGSJET-II-02 was used as the baseline model.

The result of this analysis showed that the reconstructed all-particle energy spectrum does not follow a single power-law with a significance of 2.1 σ . In particular, the analysis revealed the presence of a *hardening* around 10¹⁶ eV and a weak knee-like feature at $10^{16.92\pm0.09}$ eV in the spectrum, the latter is characterized, by a change in spectral index from $\gamma = -2.92 \pm 0.02$ to $\gamma = -3.39 \pm 0.07$ (see Fig. 2, left).

2.2 Cosmic ray composition

In order to take a first look into the composition of cosmic rays, the k parameter was used to classify event-by-event the data into a light and a heavy mass group [11]. The procedure was simple and consisted in the application of a cut on the data based on the k parameter and guided by FLUKA/QGSJET-II-02 simulations. The cut was applied between the MC predictions for C and Si primary nuclei in the parameter space of k vs E. Here, the energy calibration proceeded in the same way that in the case of the study of the all-particle energy spectrum.

Using the above analysis, the energy spectra for the heavy and light primaries were reconstructed in the energy range from ~ 10 PeV to 1 EeV. The energy spectrum for the heavy primaries showed a knee-like feature at about $10^{16.92\pm0.04}$ eV (with a statistical significance of 3.5σ that the spectrum is not described by a single power law), which was produced by a change in the spectral index from $\gamma = -2.76 \pm 0.02$ to $\gamma = -3.24 \pm 0.05$ (c.f. Fig. 2, right) [11]. As the heavy component was shown to be dominant at around 100 PeV, this structure was soon recognized to be the origin of the kneelike feature in the all-particle energy spectrum located at roughly the same energy. Since the whole procedure depends on EAS simulations, cross-checks were carried out by repeating the previous analysis with several high-energy hadronic interaction models. The results showed that the structures are model independent but that the relative abundances vary significantly with the model [12].

An unfolding analysis, like that employed in KASCADE, was also performed on the Grande N_{ch} and N_{μ} data to reconstruct the energy spectrum of different elemental mass groups [13]. The analysis, which used QGSJET-II-02 as a baseline, confirmed the existence of a knee-like feature in the iron spectrum. The structure was found around 80 PeV close to the location where the heavy knee (described in the above paragraph) was found (see Fig. 3, left). With the position of the iron-knee at

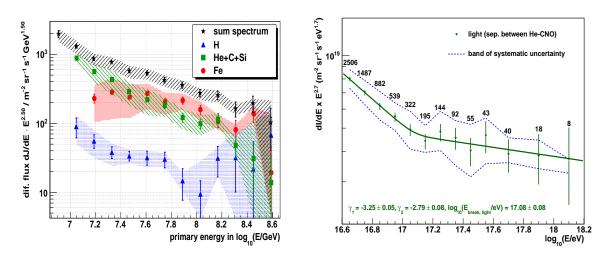


Fig. 3. *Left panel*: The all-particle energy spectrum and spectra of individual mass groups obtained from KASCADE-Grande data using an unfolding procedure [13]. *Right panel*: The energy spectrum for light primaries of KASCADE-Grande according to the analysis of [14] based on the *k* technique.

~ 80 PeV and that for the proton-knee between 3 PeV and 5 PeV a rigidity-dependent picture for the position of the knees of the elemental primary cosmic rays seems to be preferred. In addition to the above results, the unfolding analysis also yielded some hints that the hardening in the total spectrum of cosmic rays around $E \sim 10^{16}$ eV was due to the gap between the positions of the individual knees of the light (Z < 2) and medium (Z = 3 - 8) mass components in the primary flux and the heavy one.

With regard to the light mass group of cosmic rays, a further study was devoted to investigate this component with more detail using the Grande data [14]. For this research the technique with the *k* parameter was also employed but a larger effective area was used and more events were included than in previous analyses. With regard to the light/heavy mass group separation, a cut was applied between the expected mean *k* values for He and C primary nuclei. The work led to an unexpected finding: an ankle-like feature at $10^{17.08\pm0.08}$ eV in the flux of the light component of cosmic rays with a statistical significance of 5.8σ that the flux is not described by a single power law (see Fig. 3, right). This ankle is characterized by a change of slope from $\gamma = -3.25 \pm 0.05$ to $\gamma = -2.79 \pm 0.08$ and might indicate an early transition of galactic to extragalactic cosmic rays.

2.3 Test of hadronic interaction models

Another interesting line of research inside the KASCADE-Grande collaboration has been model testing due to the importance of the hadronic interaction models for the interpretation of the EAS data. The KASCADE-Grande observations are suitable to this task on the grounds of the diversity of measured EAS properties, the high-statistics and the good precision of the data collected with the experiment. Some tests already performed with the KASCADE-Grande data have revealed interesting discrepancies between the experiment and the predictions of the current high-energy hadronic interaction models. One of them, for example, has shown that the maxima of the measured distributions of the muon production height of EAS (for zenith angles below 18°) occur at altitudes lower than QGSJET-II-02 expectations [15]. There is also another important test [16] that has provided hints that this discrepancy also appears when comparing the measured data with the predictions of the QGSJET-II-04 [17] and EPOS-LHC [18] models. Finally, it is worth to mention the result of a recent study [19], which showed that the models QGSJET-II-02 and EPOS 1.99 [20] as well as their post-LHC versions do not reproduce the actual LDF of charged particles in the KASCADE-Grande

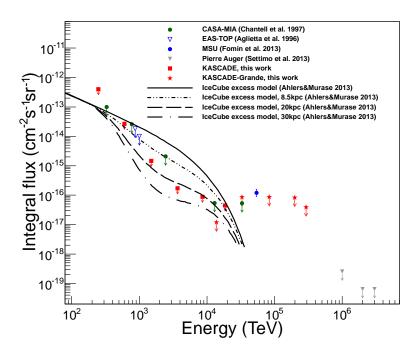


Fig. 4. Upper limits to the integral flux of gamma-rays from KASCADE-Grande (red stars) [21] and KAS-CADE (red squares) [22] data. Results are compared with limits from other experiments and with theoretical curves from an IceCube excess model [23], in this regard, from bottom to top, the first three lines correspond to source distances of 30, 20 and 8.5 kpc, respectively.

data. The consequence of this is that the all-particle energy spectrum reconstructed from the charged particle density of measured EAS at 500 m from the shower axis, S(500), is shifted to higher energies in comparison with the result obtained with the *k* parameter using the N_{ch} and N_{μ} shower observables.

3. Update of EAS analysis at the KASCADE-Grande observatory

Although the data-taking period with KASCADE-Grande has already finished, several analyses are still ongoing with the collected data inside the Collaboration. Some of them will be shortly summarized in the present section.

3.1 Limits to the integral flux of astrophysical gamma rays

Using the KASCADE-Grande data, upper limits at 90% CL on the fraction of the gamma-ray to the cosmic-ray integral flux have been recently obtained for primary energies from 10 PeV up to 300 PeV [21]. The analysis was performed by selecting muon-poor air shower events from the measured data using a cut based on MC simulations generated with FLUKA and QGSJET-II-02, applying statistical methods and taking into account the efficiency of the experiment to γ -ray detection. As an example, at $E_{\gamma} = 13.8$ PeV, the study yielded $I_{\gamma}/I_{CR} < 1.88 \times 10^{-5}$, while for $E_{\gamma} = 292$ PeV the result was $I_{\gamma}/I_{CR} < 2.48 \times 10^{-2}$. It is worth to point out that for $E_{\gamma} = 50 - 300$ PeV, the limits so derived are the first ones of their kind to be estimated.

In addition, upper limits to the diffuse integral flux of gamma-rays were also calculated with the KASCADE-Grande detector at 90% CL for the E_{γ} interval presented above [21]. The results are shown in Fig. 4 along with limits from previous experiments, including an updated result from the KASCADE observatory in the range $E_{\gamma} = 200 \text{ TeV} - 20 \text{ PeV}$ [22]. The latter is a crucial result, as it

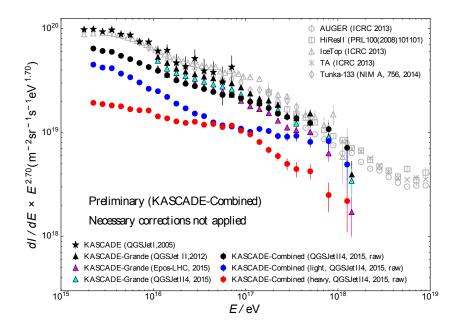


Fig. 5. The raw energy spectra of the all-particle (circles), light and heavy mass groups (blue and red circles, respectively) of cosmic rays reconstructed from the KASCADE-Grande combined analysis compared with the unfolded all-particle fluxes obtained with the KASCADE and KASCADE-Grande stand-alone analyses using QGSJET-II-02 (triangles) [24]. In the combined analysis the following post-LHC model QGSJET-II-04 was used for calibration purposes.

might put some constraints on the origin of the ICECUBE neutrino excess. According to the model of [23], the hadronic interactions producing this excess would also create a diffuse high-energy gammaray flux whose intensity would depend on the distance to the astrophysical sources. From Fig. 4, we see that in the framework of the previous model the KASCADE data seems to reject galactic sources within 20 kpc from the Earth [21].

3.2 KASCADE and KASCADE-Grande combined analysis

Returning to the composition and spectral studies, the KASCADE and KASCADE-Grande results described so far were obtained from independent analyses, each of them with its own uncertainties and energy range. Although, both results are in agreement in the overlapping region, it is desirable to have a unique spectrum along the whole KASCADE and KASCADE-Grande energy intervals. For this reason, a recent analysis was focused on combining measurements from the KAS-CADE and KASCADE-Grande arrays on the same shower event in a single reconstruction procedure [24]. The advantages of this new approach are the following: it eliminates systematic differences due to distinct reconstruction procedures, it also increases the effective area, besides it improves the accuracy of the results and, finally, it provides the spectra and the composition of cosmic rays over the combined energy range. Using the *k* parameter together with the EAS data reconstructed from this combined analysis, a preliminary result on the all-particle energy spectrum over three energy decades ($E = 10^{15} - 10^{18}$ eV) was obtained using two post-LHC hadronic interaction models, QGSJET-II-04 and EPOS-LHC. This spectrum presented the same individual features observed in the spectra reconstructed with the stand-alone KASCADE and KASCADE-Grande reconstructions (see, for example, Fig. 5).

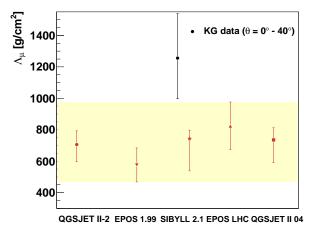


Fig. 6. Muon attenuation length of EAS data measured with the KASCADE-Grande experiment (upper point) compared with predictions of several pre-LHC and post-LHC high-energy hadronic interaction models (lower points) [26].

Preliminary spectra of the light and heavy mass groups were also obtained over the whole KAS-CADE and Grande energy ranges, as seen in Fig. 5, using measured data processed with the combined analyses [24]. The analysis was also based on the standard reconstruction relying on the *k* parameter method and used the QGSJET-II-04 hadronic interaction model for calibration of the corresponding formulae. The results confirmed the existence of the features previously observed with the individual analyses. The combined analyses have been recently repeated using EPOS-LHC and even SIBYLL 2.3 [25]. From such results, it was observed that the abundances of the light and heavy components are still model dependent. The results will be published elsewhere.

3.3 Measurement of the attenuation length of shower muons

An analysis that is currently in the process of being published studies the attenuation of muons in the Earth's atmosphere [26]. This is especially interesting for cross-checking hadronic interaction models. It makes use of the constant intensity cut (CIC) method that assumes that the intensity of cosmic rays is isotropic in zenith angle, θ [27]. The method consists of selecting EAS data from different zenith angles by means of cuts applied at fixed frequencies. Actually, data selected along curves with the same integral intensity should correspond to events with the same primary energy but different shower sizes due to the increasing atmospheric depth at high zenith angles. This dependence can be parameterized in terms of a typical exponential absorption formula, which is characterized by an attenuation length that changes with the nature of the shower particles. Using this technique, the attenuation length of several components of the EAS can be measured, e.g., of the total muon number. The studies performed using data of KASCADE-Grande with $\theta < 40^\circ$ and energies between $\sim 10^{16}$ eV and 10^{17} eV have shown that the actual muon attenuation length of N_{μ} is bigger than the predictions of MC simulations. Preliminary results are shown in Fig. 6. Final results will be presented soon.

3.4 The KASCADE Cosmic-ray Data Center: KCDC

The KASCADE-Grande Collaboration is developing a webportal that aims to provide easy and open access to reconstructed events measured with the KASCADE experiment, including detailed information on the experiments' layout and example analyses [28]. The data can be accessed through the webportal: https://kcdc.ikp.kit.edu/. Interested colleagues and the public are invited to visit KCDC.

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