Estimation of GRACE-like geopotential models

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Abstract

The availability of time-dependent global gravity field models like EIGEN-6C or EIGEN-6C2 allows the generation of GRACE-like geopotential Earth models. The paper introduces an approach to generate those GRACE-like models using time-dependent global gravity field models. Such GRACE-like models can then be used to estimate, for example, the Terrestrial Water Storage (TWS) for the months where GRACE data are not available. The paper gives the necessary derivation of such GRACE-like models from the time-dependent global gravity field models. GRACE-like models, created by using the time-dependent global gravity field models at the same months where GRACE data are available, are compared to the original GRACE models. The results proved that the GRACE-like models give comparable values to the original GRACE models and no loss of spectrum power occurs.

1 Introduction

The widely known satellite mission Gravity Recovery and Climate Experiment (GRACE) has collected global gravity observations since the year 2002. It offers a perfect possibility to monitor the mass redistribution within the Earth's system. This is directly related to the variation in terrestrial water storage (Wahr et al., 2004; Tapley et al., 2004).

The GRACE mission has been designed for a nominal mission lifetime of five years, which lasted for 15 years now. Some months are dropped from the GRACE data. The GRACE spacecrafts orbit has decayed by about 150 km, starting in May 2017 (http://www.csr.utexas.edu/grace/operations/configuration.html). The GRACE follow-on mission is being designated. Accordingly, there is a need for filling in the GRACE data gaps.

The paper presents an approach to create GRACE-like geopotential models by using time-dependent global gravity field models. Among other important usages of the created GRACE-like models, they can be used to fill in the gaps of the original GRACE data.

The nowadays available time-dependent global gravity field models are outlined and discussed. The necessary derivation of computing the monthly average GRACE-like models from the time-dependent global gravity field models is given. The GRACE-like models have been created at the same months where original GRACE data are available. A comparison between the original GRACE and the GRACE-like models is made and widely discussed.

2 Time-dependent gravity field models

A number of time-dependent global gravity field models exist nowadays. We will focus here on two models, namely the EIGEN-6C model (Förste et al., 2011) and the EIGEN-6C2 (Förste et al., 2012). For these global gravity field models, the lower-degree harmonic coefficients, up to degree and order 50, are functions of a few time-dependent parameters.

Erschienen bei KIT Scientific Publishing
Schriftenreihe des Studiengangs
Geodäsie und Geoinformatik 2018,1

DOI Einzelbeitrag:
10.5445/KSP/1000080201

Festschrift zur Verabschiedung von
Prof. Dr.-Ing. Dr. h.c. Bernhard Heck
(Schw)Ehre, wem (Schw)Ehre gebührt

DOI Festschrift:
10.5445/KSP/1000080324

2.1 EIGEN-6C gravity field model

The EIGEN-6C global gravity field model (Förste et al., 2011) combines the following data sets:

- 6.5 years of LAGEOS (SLR) and GRACE (GPS-SST and K-band range rate) data from the time span 1.1.2003 till 30.6.2009.
- 6.7 months of GOCE data (Satellite gradiometry only) from the time span 1.11.2009 till 30.6.2010.
- DTU2010 global gravity anomaly data set (Andersen, 2010) obtained from altimetry and gravimetry as surface data.

The EIGEN-6C model has a set of time-variable parameters ($gfct$, $trnd$, $acos$, $asin$) up to degree and order 50. To compute the values of the time variable fully normalized spherical harmonic coefficients $G_{6C}(t)$ using the EIGEN-6C model at a certain epoch t, one needs to apply both the drift parameter $trnd$ as well as the periodic terms $acos$ and $asin$ as follows (Förste et al., 2011)

$$\begin{aligned} G_{6C}(t) =& gfct(t_\circ) + trnd \times (t - t_\circ) + \\ & + \sum_{i=1}^{2} \left\{ asin(i) \sin\left[\frac{2\pi(t - t_\circ)}{p(i)}\right] + \right. \\ & \left. + acos(i) \cos\left[\frac{2\pi(t - t_\circ)}{p(i)}\right] \right\} , \end{aligned} \quad (2.1)$$

where $gfct(t_\circ)$ is the value of the fully normalized harmonic coefficient $G_{6C}(t_\circ)$ at the reference epoch $t_\circ$ ($t_\circ$ = 1.1.2005), and

$$\begin{aligned} p(1) &= 1.0\,\text{year}, \\ p(2) &= 0.5\,\text{year}. \end{aligned} \quad (2.2)$$

2.2 EIGEN-6C2 gravity field model

The EIGEN-6C2 global gravity field model combines GRACE data for 7.8 years, LAGEOS-1/2 SLR data for 25 years, GOCE data for 350 days and surface data. The surface data consists of altimetry-derived data in oceans as well as terrestrial data on land. Three surface data sets were included in EIGEN-6C2 model. Two of them are products of the Danish National Space Institute DTU Space. They are:

- Data set 1: An update of the DTU10 global gravity anomaly data set (Andersen, 2010) which was obtained from altimetry over the oceans.
- Data set 2: The DTU10 geoid data over the oceans, obtained from DTU10 MSSH and DOT data.
- Data set 3: Geoid heights over the continents generated from EGM2008 (Pavlis et al., 2008, 2012).

The EIGEN-6C2 model has also a set of time-variable parameters (namely, $gfct$, $trnd$, $acos$, $asin$) up to degree and order 50. To compute the values of the time variable fully normalized spherical harmonic coefficients $G_{6C2}(t)$ using the EIGEN-6C2 model at a certain epoch t, one needs to apply both the drift parameter $trnd$ as well as the periodic terms $acos$ and $asin$ as follows (Förste et al., 2012)

$$\begin{aligned} G_{6C2}(t) =& gfct(t_\circ) + trnd \times (t - t_\circ) + \\ & + \sum_{i=1}^{3} \left\{ asin(i) \sin\left[\frac{2\pi(t - t_\circ)}{p(i)}\right] + \right. \\ & \left. + acos(i) \cos\left[\frac{2\pi(t - t_\circ)}{p(i)}\right] \right\} , \end{aligned} \quad (2.3)$$

where $gfct(t_\circ)$ is the value of the fully normalized harmonic coefficient $G_{6C2}(t_\circ)$ at the reference epoch $t_\circ$ ($t_\circ$ = 1.1.2005), and

$$\begin{aligned} p(1) &= 1.0\,\text{year}, \\ p(2) &= 0.5\,\text{year}, \\ p(3) &= 18.6129\,\text{year (only for } \bar{C}_{20}). \end{aligned} \quad (2.4)$$

It is worth mentioning that the third period $p(3)$ is the lunar period, which affects only the $\bar{C}_{20}$ harmonic coefficient. This means that the summation appearing in the expression of computing $G_{6C2}(t)$, Eq. (2.3), for all other harmonic coefficients rather $\bar{C}_{20}$ is performed only up to $i = 2$.

3 Creating monthly average GRACE-like models

The monthly average GRACE-like models are created by generating the monthly average fully normalized harmonic coefficients for the same months as the GRACE data. This is achieved by performing the following integration

$$\bar{G}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} G(t)\, dt\,, \quad (3.1)$$

where t_1 and t_2 are the epochs of the starting and ending of each of the original GRACE-months. In the following sections, we will perform the integration appearing in Eq. (3.1) for both EIGEN-6C and EIGEN-6C2 models, generating GRACE-like-EIGEN-6C and GRACE-like-EIGEN-6C2 models, respectively.

3.1 Creating GRACE-like-EIGEN-6C model

To generate the GRACE-like-EIGEN-6C model, we insert Eq. (2.1) into Eq. (3.1) and perform the required integration analytically. This immediately leads to the following expression for the monthly average fully normalized harmonic coefficients $\bar{G}_{6C}(t_1,t_2)$ for the GRACE-like-EIGEN-6C model

$$\begin{aligned}\bar{G}_{6C}(t_1,t_2) = & \frac{1}{t_2-t_1}\left[gfct(t_\circ)\times t + \right.\\ & \left. + \, trnd \times \left(\frac{t^2}{2} - t\,t_\circ\right)\right]_{t_1}^{t_2} + \frac{1}{2\pi(t_2-t_1)}\cdot \\ & \cdot \sum_{i=1}^{2}\left[-asin(i)\,p(i)\cos\left(\frac{2\pi(t-t_\circ)}{p(i)}\right) + \right.\\ & \left. + \, acos(i)\,p(i)\sin\left(\frac{2\pi(t-t_\circ)}{p(i)}\right)\right]_{t_1}^{t_2}.\end{aligned} \tag{3.2}$$

The expressions between the square brackets appearing in Eq. (3.2) are numerically computed between the lower t_1 and upper t_2 integration limits, and $p(i)$ is given by Eq. (2.2).

To generate the monthly average GRACE-like-EIGEN-6C model, Eq. (3.2) has been employed using the EIGEN-6C model for the same months as the original GRACE data.

3.2 Creating GRACE-like-EIGEN-6C2 model

To generate the GRACE-like-EIGEN-6C2 model, one needs to insert Eq. (2.3) into Eq. (3.1) and performs the required integration analytically. This immediately leads to the following expression for the monthly average fully normalized harmonic coefficients $\bar{G}_{6C2}(t_1,t_2)$ for the GRACE-like-EIGEN-6C2 model

$$\begin{aligned}\bar{G}_{6C2}(t_1,t_2) = & \frac{1}{t_2-t_1}\left[gfct(t_\circ)\times t + \right.\\ & \left. + \, trnd \times \left(\frac{t^2}{2} - t\,t_\circ\right)\right]_{t_1}^{t_2} + \frac{1}{2\pi(t_2-t_1)}\cdot \\ & \cdot \sum_{i=1}^{3}\left[-asin(i)\,p(i)\cos\left(\frac{2\pi(t-t_\circ)}{p(i)}\right) + \right.\\ & \left. + \, acos(i)\,p(i)\sin\left(\frac{2\pi(t-t_\circ)}{p(i)}\right)\right]_{t_1}^{t_2}.\end{aligned} \tag{3.3}$$

Here the expressions between the square brackets appearing in Eq. (3.3) are numerically computed between the lower t_1 and upper t_2 integration limits, and $p(i)$ is given by Eq. (2.4). It should be mentioned again that the summation appearing in Eq. (3.3) for all monthly average fully normalized harmonic coefficients except $\bar{C}_{20}$ is performed only up to $i = 2$.

Expression (3.3) has been used to generate the monthly average GRACE-like-EIGEN-6C2 model using the EIGEN-6C2 model for the same months as the original GRACE data.

4 Comparison with GRACE model

Figure 4.1 shows the comparison between the spherical harmonic coefficients of the GRACE and GRACE-like-EIGEN-6C models for March 2009. The absolute differences between GRACE and GRACE-like-EIGEN-6C models range between zero and 3.95×10^{-9} with an average of 3.28×10^{-12} and a standard deviation of about 7.67×10^{-11}. The lower-left panel of Fig. 4.1 shows that the absolute differences between the GRACE and GRACE-like-EIGEN-6C models are always significantly small (and range mainly between 10^{-11} and 10^{-13} except for the very low harmonics). The lower-right panel of Fig. 4.1 shows very small relative errors. This indicates that the GRACE-like-EIGEN-6C model behaves similar to the original GRACE model.

Figure 4.2 shows the comparison between the GRACE and GRACE-like-EIGEN-6C2 models for March 2009. Here, the absolute differences between the GRACE and GRACE-like-EIGEN-6C2 models range between zero and 4.00×10^{-9} with an average of

3.39×10^{-12} and a standard deviation of about 7.77×10^{-11}. The lower-left panel of Fig. 4.2 shows that the absolute differences between the GRACE and GRACE-like-EIGEN-6C2 models are always significantly small (and range mainly between 10^{-11} and 10^{-13} except for the very low harmonics). The lower-right panel of Fig. 4.2 shows very small relative errors. This indicates that the GRACE-like-EIGEN-6C2 model also behaves similar to the original GRACE model.

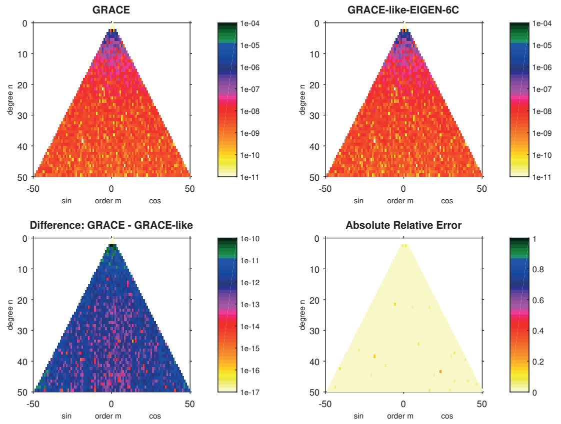

Figure 4.1: Comparison between the GRACE and GRACE-like-EIGEN-6C models for March 2009.

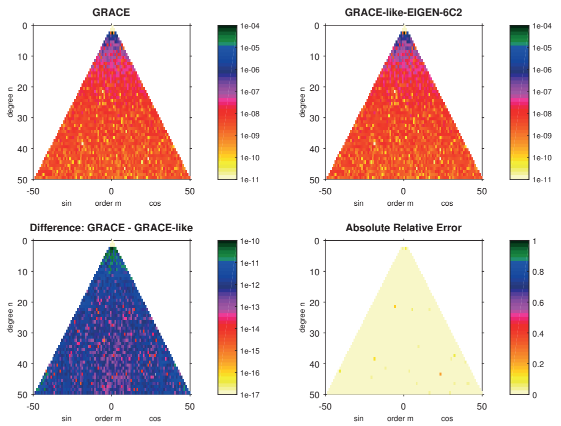

Figure 4.2: Comparison between the GRACE and GRACE-like-EIGEN-6C2 models for March 2009.

Comparing Fig. 4.1 and Fig. 4.2 shows that both GRACE-like-EIGEN-6C and GRACE-like-EIGEN-6C2 give a good approximation to the original GRACE model, with nearly the same degree of approximation. Figure 4.3 shows the degree variances for the GRACE and GRACE-like-EIGEN-6C2 models for March 2009. It shows that both curves are coincide. This means that there is no loss of spectrum power between the GRACE and GRACE-like-EIGEN-6C2 models. The degree variances for both GRACE and GRACE-like-EIGEN-6C2 models range between 9.64×10^{-16} and 2.34×10^{-7} with an average of 3.97×10^{-9} and a standard deviation of about 3.05×10^{-8}. It should be noted that a similar conclusion has also been drawn for the GRACE-like-EIGEN-6C model.

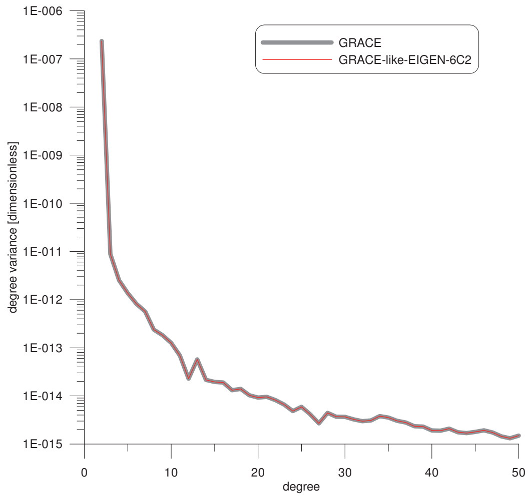

Figure 4.3: Degree variances for the GRACE and GRACE-like-EIGEN-6C2 models for March 2009.

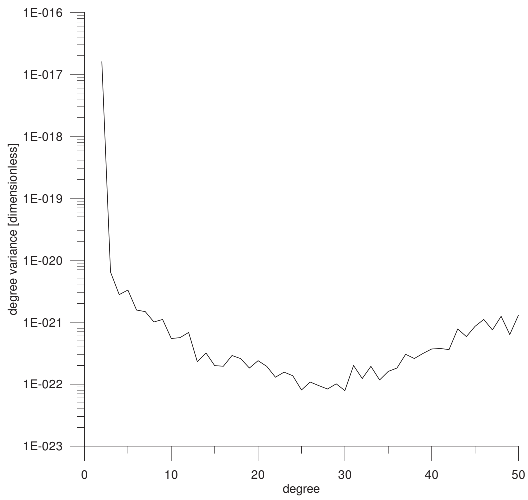

Figure 4.4: Difference of the degree variances between GRACE and GRACE-like-EIGEN-6C2 models for March 2009.

Figure 4.4 shows the differences of the degree variances between the GRACE and GRACE-like-EIGEN-6C2 models for March 2009. These differences range between 7.89×10^{-23} and 1.60×10^{-17} with an average of 3.28×10^{-19} and a standard deviation of about 2.29×10^{-18}. Comparing the range values for the degree variance (Fig. 4.3) and the range values for the degree variance differences (Fig. 4.4) confirms again that there is no loss of spectrum power between the GRACE and GRACE-like-EIGEN-6C2 models. Again, similar conclusion has been proved for the GRACE-like-EIGEN-6C model.

Figures 4.5 and 4.6 show the time series of the degree variances c_{10} and c_{20} for the GRACE and GRACE-like-EIGEN-6C2 models, respectively. They show again the comparable behaviour of the GRACE-like models to that of the original GRACE model, also with respect to time (please note the very small variations in the degree variance axis in both Figs. 4.5 and 4.6). Similar graphs have been created for the other degree variances up to degree 50.

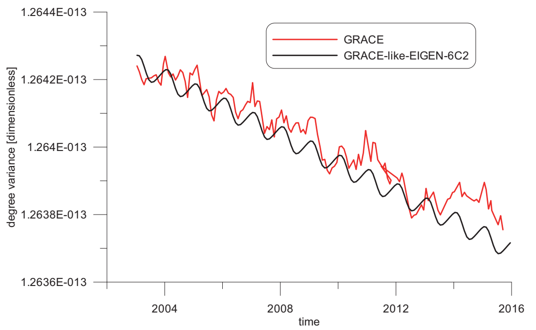

Figure 4.5: Time series of the degree variance c_{10} for the GRACE and GRACE-like-EIGEN-6C2 models.

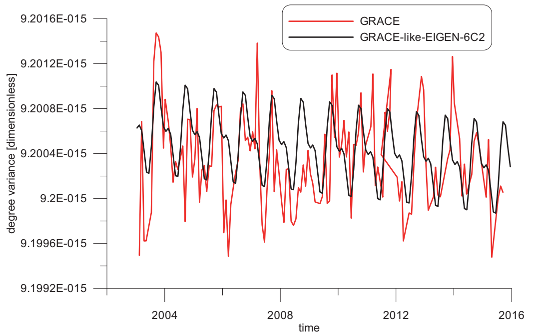

Figure 4.6: Time series of the degree variance c_{20} for the GRACE and GRACE-like-EIGEN-6C2 models.

5 Conclusion

The paper presents an approach to generate GRACE-like geopotential models. The developed approach uses the time-dependent global gravity field models to generate the GRACE-like models. Two time-dependent global gravity field models have been used in the current investigation; they are EIGEN-6C and EIGEN-6C2 models. The paper gave the necessary derivation of generating the GRACE-like models using these time-dependent global gravity filed models.

The results proved that the generated GRACE-like models, either using EIGEN-6C or EIGEN-6C2 time-dependent global gravity field models, behave similarly as the original GRACE model. It has also been shown that there is no loss of spectrum power when using the GRACE-like models instead of the original GRACE model.

References

Andersen, O. B. (2010): The DTU10 Gravity field and Mean sea surface (2010). *Second international symposium of the gravity field of the Earth (IGFS2)* Fairbanks, Alaska.

Förste, C., Bruinsma, S. L., Flechtner, F., Marty, J.-C., Lemoine, J.-M., Dahle, C., Abrikosov, O., Neumeyer, H., Biancale, R., Barthelmes, F., and Balmino, G. (2012): A new release of EIGEN-6: The latest combined global gravity field model including LAGEOS, GRACE and GOCE data from the collaboration of GFZ Potsdam and GRGS Toulouse. *American Geophysical Union, Fall Meeting, San Francisco, USA* December 3–7, 2012.

Förste, C., Bruinsma, S. L., Shako, R., Marty, J.-C., Flechtner, F., Abrikosov, O., Dahle, C., Lemoine, J.-M., Neumeyer, K. H., Biancale, R., Barthelmes, F., König, R., and Balmino, G. (2011): EIGEN-6 - A new combined global gravity field model including GOCE data from the collaboration of GFZ-Potsdam and GRGS-Toulouse. *Geophysical Research Abstracts* 13, EGU2011-3242-2, EGU General Assembly, 2011.

Pavlis, N. K., Holmes, S. A., Kenyon, S. C., and Factor, J. K. (2008): An Earth Gravitational Model to Degree 2160: EGM2008. *General Assembly of the European Geosciences Union* Vienna, Austria, April 13–18.

Pavlis, N. K., Holmes, S. A., Kenyon, S. C., and Factor, J. K. (2012): The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). *J Geophys Res* 117(B04406). DOI: 10.1029/2011JB008916.

Tapley, B. D., Bettadpur, S., Watkins, M., and Reigber, C. (2004): The gravity recovery and climate experiment: Mission overview and early results. *Geophysical Research Letters* 31(L09607):1–4. ISSN: 1944-8007. DOI: 10.1029/2004GL019920.

Wahr, J., Swenson, S., Zlotnicki, V., and Velicogna, I. (2004): Time-variable gravity from GRACE: First results. *Geophysical Research Letters* 31(L11501):1–4. ISSN: 1944-8007. DOI: 10.1029/2004GL019779.