Efforts for aligning the Brazilian Height System to the International Height Reference System

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Abstract

The International Height Reference System (IHRS) consolidated the idea of a world height system (WHS) according to the IAG Resolution 1/2015. Since 2000, the majority of activities related to the modernization of the Brazilian Height System (BHS) has referred to a physical meaning for the heights and have linked two existing vertical datums in the country to a same equipotential surface. Today, these activities relate to the BHS and its realization by the Brazilian Vertical Reference Network (BVRN) agrees with the IHRS precepts. In this context, the main developed activities related to BHS/BVRN and achieved goals are described in this work.

1 Introduction

The Global Geodetic Observing System (GGOS) of the International Association of Geodesy (IAG) was established in July 2003. It integrates several contributions of Geodesy for quantifying global changes in space and time with accuracy and reliability. The United Nations (UN) in its Asian Regional Cartographic Conference at Bangkok recommended the GGOS adoption in November 2012. Three main themes were established by GGOS: (1) unified height system; (2) geohazards monitoring; and (3) sea level change, variability, and forecasting.

The United Nations Global Geospatial Information Management (UN-GGIM), aimed at "a global geodetic reference frame for sustainable development" and recognizing the importance of the coordinated approach in Geodesy by the IAG/GGOS, established the key elements of the Global Geodetic Reference Frame (GGRF) as realization of the Global Geodetic Reference System (GGRS) (A/RES/69/266), February 26^{th}, 2015 (UN, 2017).

The IAG Resolution 1 (July 2015) established the criteria of definition and realization of an International Height Reference System (IHRS) in the geopotential space. The basic elements are the specification of a geopotential number, given by

$$C_P = -\Delta W_P = W_0 - W_P \tag{1.1}$$

as primary vertical coordinates and the global reference level specified by the geopotential value $W_0 = 62636853.4 m^2 s^{-2}$. The IAG Resolution 2 (July 2015) established the Global Absolute Gravity Reference System for initiating the replacement of the International Gravity Standardization Net 1971 (IGSN71) and the latest International Absolute Gravity Base Standardization Network.

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Following the efforts for establishing a reference system suitable for quantifying global changes, the IAG delimitated that "the GGRS comprises terrestrial and celestial components. The terrestrial component is a common reference for the geometry and the gravity field of the Earth, where a physical point P has a corresponding coordinate X, potential of the Earth's gravity field W, physical height H, and gravity vector g" (IAG, 2016). Hence, it is possible to conclude that the GGRS results from the integration of the ITRS and the IHRS. This emphasizes the present importance of establishing the IHRF as a set of stations distributed in all continents as a basis for integrating all the national vertical reference frames for the same definition given by the IHRS.

In 1997 the SIRGAS (Geocentric Reference System for the Americas) established its Working Group III – Vertical Datum (WG-III), charged of a unified height system and frame in the context of South America, Central America and the Caribbean. Since then, several activities have been developed for establishing standards related to the realization of the SIRGAS Vertical Reference Network (SVRN) with physical meaning based on geopotential numbers (VeReS, 2002). The road map for establishing the SVRN was exposed by Drewes et al. (2002). Nevertheless, the basic strategies for connecting vertical reference networks were proposed by de Freitas et al. (2002a) inspired by the previous works of Heck and Rummel (1990) and Lehmann (2000). Luz (2008) developed a deep diagnostic of the BHS/BVRN and the aspects for integrating the BHS/BVRN to a WHS were discussed by Ferreira (2008, 2011).

Given these points, in this paper we present the foundations and analysis of the main activities towards a modernization of the BHS and BVRN. Furthermore, new tendencies related to the guidelines for modern vertical systems and networks, and the Brazilian insertion in the WG-III of the SIRGAS project are presented.

2 Overview of main activities for modernizing the BHS

2.1 Altimetry-gauge-leveling approach

Since 1994, the Coordination of Geodesy in the Brazilian Institute of Geography and Statistics (IBGE) and the Post-Graduation Program in Geodesy at the Federal University of Paraná (UFPR) decided to cooperate for supporting the modernizations of BHS and its BVRN. The first activities have focused on the instrumentation and reactivation of sea level observations in the Brazilian Vertical Datum at Imbituba (BVD-I), Southern Brazil. A multi parametric experiment was established for determining the geocentric position of the BVD-I (de Freitas et al., 1999) followed by the determination of local effects for modeling the interaction of ocean-continents (de Freitas et al., 2002b). Strategies for connecting vertical networks in South America were proposed in the context of the SIRGAS project mainly by considering modernization of vertical networks in the continent and adoption of strategies for connecting them (e. g. Drewes et al., 2002; de Freitas et al., 2002a; Luz et al., 2002). Following these efforts related to the BVD-I, the task force, composed by IBGE and UFPR, decided to recover the local vertical reference network around the BVD-I. The main aim was to understand the historical level references and their relationship with the apparent observed position. As discriminator for the sea level rising was considered an available satellite altimetry time series since 1992 in near beans for recovering local trends (see Dalazoana, 2006; Dalazoana et al., 2007; Luz et al., 2009b).

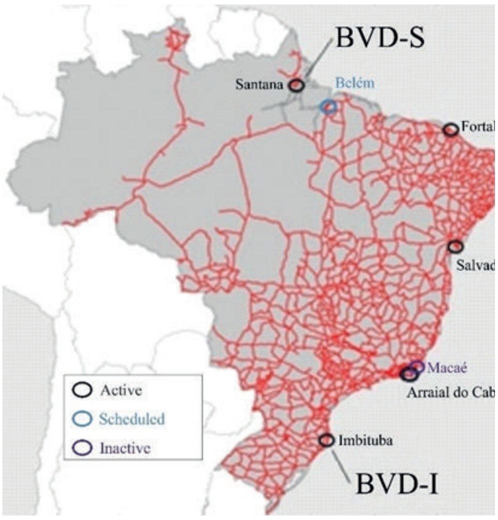

Figure 2.1: The BVRN (red dots) and the Permanent Geodetic Tide Gauge Network (RMPG) identified on the map by their respective names, from south to north, Imbituba, Arraial do Cabo, Macaé, Salvador, Fortaleza, Belém and Santana, respectively.

Luz (2008) established in his doctoral thesis, (which was supported by UFPR, Geodetic Institute of Karlsruhe (GIK), and Deutsches Geodätisches

Forschungsinstitut (DGFI)), a deep analysis of the BVRN by recovering historical information about its evolution. His results are supported in comparison to former sequential adjustment and a simultaneous least square adjustment of the whole network (Preliminary Height Global Adjustment – AAGP). Several needs were detected and propositions were established including strategies for the analysis of temporal evolution and its new realization in the geopotential space. The external control of the incorporation of data from satellite altimetry and the Permanent Geodetic Tide Gauge Network (RMPG) presented in Fig. 2.1 was also discussed.

Furthermore, Luz (2008) has used gravity and leveling data provided by IBGE in order to identify the main difficulties on the computation of geopotential differences. A subset of more recent lines was identified, in which virtually all benchmarks (BMs) had direct and homogeneous gravimetric information. A computer program was developed specifically for the integration of gravity information into the leveling data and the organization of the network of internodal geopotential differences (Luz et al., 2007; Luz et al., 2009a). With this sub-network of BVRN, it was possible to simulate various scenarios of the lack of gravity over BMs, assessing the interpolated values through least squares collocation using a script provided by SIRGAS project. The simulations indicated an overestimation of the quality of interpolation based on least squares collocation, especially in the scenario of inadequate distribution of the reference gravity values. We discussed the influence of spatial and temporal heterogeneities in the adjustment of internodal geopotential differences through the analysis of the effects of the partitioning strategy adopted in the AAGP of the BVRN, which IBGE computed the heights at that time and stored their values in its geodetic database (BDG). The simultaneous adjustment of the same network originally partitioned in AAGP showed excessive distortions in the height values as well as correcting a problem arising from spatial-temporal heterogeneities of the BVRN near the BVD-I. The partial assessment of AAGP has provided insights for the organization of a sub-network connecting three tide-gauges from RMPG (i.e., Imbituba, Macaé, and Salvador), with the aim of establishing a reference for the study of the sea surface topography (SSTop) effects by using data from satellite altimetry. A configuration was designed so that the tracks of the most recent altimeter missions were virtually co-linear to those RMPG tide-gauges. In order to homogenize the reference levels of the observations of these TG-tracks, reference tracks were chosen in the open ocean far from small depth areas where satellite altimetry observations present poor quality. Along these tracks, the global SSTop solution from the DGFI was examined, whose results for the TG-tracks showed inconsistencies regarding the reference tracks at their crossing points, especially around the Abrolhos plateau. These inconsistencies were interpreted as a possible residual effect due to the plateau, spreading to the neighboring crossings during the SSTop filtering.

The studies carried out by Luz (2008) helped to define the procedures for integrating leveling, gravity, sea level, and satellite altimetry data, considering the challenges presented IBGE decided to concentrate efforts on new approaches. Old-fashioned gravity data without sure positioning or gravity data (available only as gravity anomalies) as well as the observed heterogeneities in quality and/or absence of quality control, positional information in different GRFs imposed the search by new strategies. In this sense a stronger cooperation in the Brazilian context. These last aspects were considered in a recent doctoral thesis (Silva, 2017) by introducing the integration of a 10yr GNSS time series of the geocentric position of the BVD-I from 2007 to 2016 with the corresponding tide gauge time series. The obtained up crustal velocity determined by GNSS at BVD-I was ˇ3.02 ± 0.34 mm/yr. The obtained relative trend of the Mean Sea Level by tide gauge was 5.26 ± 0.11 mm/yr. The integration of these two trends pointed out a rising value of 2.24 ± 0.36 mm/yr in the MSL at BVD-I. This result agrees very well with the value of 2.23 ± 0.42 mm/yr obtained from Jason 2 satellite altimetry in offshore stable beans near to BVD-I.

Ferreira (2008) presented an experiment for determining the geopotential in the BVD-I, which showed a lack of geodetic information necessary for its connection with other vertical networks as preconceived for a WHS. The referred purpose is fundamental for the future Vertical Datum SIRGAS (DV-SIRGAS). In the BVD-I region, several BMs have been lost. In order to improve the distribution of data in this region, a study was conducted on the behaviour of a lagoon system covering an approximate surface of 600 km^2 in the contiguous region of the DVB-I (Fig. 2.2). The central idea was to regard the mean level of the lagoon system as a smoothed indicator for a natural equipotential surface close to the local MSL, which could be understood as a materialization of the local "geoid" (or quasi-geoid).

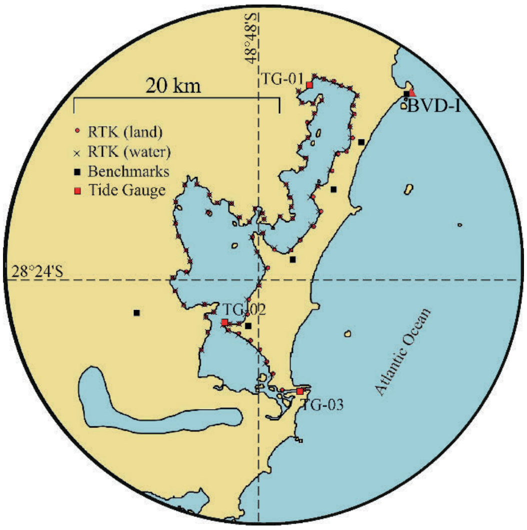

Figure 2.2: Imarui lagoon system and Brazilian Vertical Datum.

A local geodetic network with about two hundred points where observed gravity and precise position with Global Positioning System (GPS) was established on the lagoon perimeter (about 150 *km* long) and nearby the BVD-I (Fig. 2.2). Some of these points coincide with existing BMs belonging to leveling lines departing from the BVD-I. Three tide gauges that recorded the heights of the water level in the lagoon system over a period of approximately three months were also employed (Fig. 2.1). In this lagoon system, it was possible to adjust one equipotential surface from the mean lake levels. From the observations and known parameters from Global Geopotential Models (GGMs) it was possible to realize another robust estimation of the shift between the BVD-I and a global equipotential reference surface. Thus, the availability of information necessary to the knowledge of the gravity field in the BVD-I was increased. Then, it was possible to determine a provisory value for the geopotential in the BVD-I and to estimate a value for the SSTop. Because the difficulties for modernizing the BHS/BVRN come from identified problems in previous works, UFPR and with GIK started. Then, aiming to introduce new paradigms related the development of BHS/BVRN in the geopotential space, other data sources and paradigms were considered mainly to avoid indirect effects on the data base coming from different GRFs and for overcoming problems in regions with poor data coverage. Techniques such as Residual Terrain Modeling (RTM) and fixed solution of the Geodetic Boundary Value Problem (GBVP) became usual in the SIRGAS and UFPR/IBGE cooperation context.

2.2 Regional gravity field approach

Ferreira (2011), in his doctoral thesis, worked on the GBVP. He carried out his research with the support of UFPR and GIK regarding the possibilities of practical applications of the GBVP. Among the existing GBVPs solutions, he focused the fixed GBVP, which is compatible with the technologies emerging from GNSS, which gives the boundary surface, i.e., the Earth's surface. The employed solution in his work was based on the Brovar's type, originally applied for solving the Molodenskii's boundary value problem. This approach allows us to solve the problem of the oblique derivative by adding suitable correction terms. The solution is compatible with current techniques for smoothing the external gravity field namely: remove-restore technique, the topographic-isostatic masses reduction; RTM, the high resolution GGMs; integration over a spherical cap by modifying the Hotine's kernel. All discussions were in the context of the modernization of the BHS considering that a modern height system allows the determination of heights in relation to the vertical datum everywhere in a country employing the GNSS technology. Two case studies, one in the Federal State of Baden-Württemberg, Germany, and another in Paraná State, Brazil, showed that the problem in determining the gravity field in Brazil is the omission error in gravity data. Accuracy analysis showed an insufficient discretization of the gravity field used for determining the geoid in the state of Paraná. Assuming that the commission error in the gravity values is random, an absolute error expected in the quasi-geoid model due to omission error is in the order of decimeter. But, for the relative sense, the evaluation of this error is 0.2 *ppm* for distances beyond the resolution of the model, i.e., 4.6 *km*. As a contribution, the ellipsoidal correction was determined to the boundary value of the order $O(e^4)$.

2.3 Estimation of BVRF offset related to a WHS by disturbing potential approach

A simple and practical method for direct transformation of normal-orthometric heights, currently employed in Brazil in normal height system, (which has physical meaning) was proposed in Ferreira et al. (2016). By exploring the mathematical definition of both height systems, the difference between normal height H^n and normal-orthometric height H^{no}, can be easily expressed as:

$$\delta H = H^n - H^{no} \approx \frac{T_g - T_P}{\bar{\gamma}'} + \frac{\delta U}{\bar{\gamma}'} \tag{2.1}$$

where, T_g and T_P refer to the disturbing potentials at the geoid and the Earth's surface respectively, and δU is the unknown normal potential difference between the zero reference for the $H^{no}(U_0^{no})$ and the geoid U_g. The relation (2.1) can be further simplified and re-written as:

$$\delta H = \frac{\gamma_0}{\bar{\gamma}'} \cdot (N - \zeta) - \frac{T_P}{\bar{\gamma}'} \cdot \left(1 - \frac{\gamma_0}{\gamma_Q}\right) + \frac{\delta U}{\bar{\gamma}'} \tag{2.2}$$

where γ_0 and γ_Q are the normal gravity at the reference ellipsoid and at the telluroid respectively; N is the geoidal height; and ζ is the height anomaly. Note that the difference $N - \zeta$ is the same as the difference between normal height and orthometric height and can be computed, for example, as proposed in Sjöberg (2010).

3 Inconsistencies of the BHS/BVRN

As pointed out in sub-Section 2.1, the BVRN has spatio-temporal heterogeneities with large distortions because of its realization starting from south to north-east (1945-1969 and 1981-2005) and later from Central-West to the North (1970-1980). In order to investigate such inconsistencies due to the spatio-temporal propagation of the BVRN as well as due to errors in the leveling procedure, lack of actual gravity measurements, etc., potential tilts in the north-south and east-west direction as well as bias in the BVRN w.r.t. to a suitable geopotential model was forwarded. For instance, in Ferreira et al. (2016) a suitable geopotential model was considered by spectrally enhancing a GOCE (Gravity field and steady-state Ocean Circulation Explorer) based model, namely TIM-R4 (Pail et al., 2011), and a high-resolution model namely Earth Gravity Model 2008 - EGM2008 (Pavlis et al., 2012). This enhancement scheme was carried out aiming to take advantage of both, that is, the improvement in long wavelength determination provided by GOCE relatively to the previous gravity field mission GRACE (Gravity Recovery and Climate Experiment) and the short wavelength contents of the EGM2008, which is based on observed and topography-generated gravity values. The spectral enhancement was carried out using least-squares adjustment, thereby making the best use of the available data.

The results have shown that the enhanced geopotential model fits the GNSS-leveling with a root mean square error (RMSE) of 20.2 cm. The estimated bias of -0.40 ± 0.6 cm (w.r.t. the centroid of the network) implies that any future changes to the geopotential value W_0 (62636856.0 m^2s^{-2}) should be minor for the BHS w.r.t. to any potential WHS. Furthermore, the authors have estimated systematic effects in the north-south and the west-east directions of the order of -0.90 ± 0.08 and 2.27 ± 0.10 $cm/degree$, respectively. The tilts in both directions are in agreement with the Brazilian leveling specification of ± 4 $mm\sqrt{k}$ where k is the length of the leveled line in km, which is equivalent to approximately 4.2 $cm/degree$ at the equator.

More recently, Grombein et al. (2016) used high-frequency topography-generated gravity signals and a GOCE-based geopotential model to unify height systems, and among others, BHS was investigated. The authors have proposed spectrally enhanced geopotential models using an adapted Hanning window. They have found a bias of -1.1 ± 0.7 cm and tilts of the order of -1.20 ± 0.9 and 2.70 ± 0.11 $cm/degree$ at the north-south and east-west direction, respectively. Despite the difference in the methodologies and data, the results provided by Grombein et al. (2016) confirm the previous findings in Ferreira et al. (2016). This implies that, despite the issues in sub-Section 2.1, the BHS (through its BVRN) is consistent and provides the minimal requirements for any potential connection to WHS in the future. Improvements shall be achieved by considering the transformation of a normal-orthometric height system to normal height system using, for example, the option described in sub-Section 2.3.

4 Strategies for connecting two segments of the BVRN

Due to the challenge imposed by the Amazon River basin for traditional geodetic surveys, BVRN is materialized in two independent parts: the southern segment is linked to the Imbituba tide gauge and the northern part is linked to the Santana tide gauge (Fig. 2.1). The mouth of the Amazon River and its surrounding wetlands generate a large area without access for spirit leveling and conventional gravimetry. There is a minimum distance of about 330 *km* between the nearest benchmarks of the two above mentioned parts. The gravity field mission GOCE makes it possible to explore new solutions based on GGMs obtained from satellite data only. Digital elevation models (DEMs) allow an improvement in the spectral resolution of the GGMs based on Residual Terrain Modeling (RTM). Such an approach is an alternative to filling the information gaps in the GGMs by reducing the omission errors. The spectral improvement of the GGMs allows us to integrate the vertical datums in a more realistic way, and with a reduction of terrestrial gravity dependency. Some alternatives for connecting were initially explored by considering GGMs and improvements in their resolutions by the RTM technique (Montecino and de Freitas, 2014). The basis of the solution is given by:

$$\zeta_{RTM} = \frac{1}{\gamma_Q} \sum_k V_k \quad (4.1)$$

and

$$\zeta = \zeta_{GGM}^{N_{max}} + \zeta_{RTM}^{>N_{max}} . \quad (4.2)$$

In this solution, only a combination of GGM satellite-only data from the GOCE mission and the spectral contribution of the RTM were applied. Another solution was based on the integration of information from GOCE, EGM2008, and the RTM effect considering a spectral decomposition given by:

$$\zeta = \zeta_{GOCE}^{240} + \zeta_{EGM08}^{241-2190} + \zeta_{RTM}^{>N_{max}} . \quad (4.3)$$

The offset obtained shows that the Imbituba datum is located 1.32 *m* and 1.43 *m* below the Santana datum for the solutions GOCE+RTM and GOCE+EGM2008+RTM, respectively.

Following these attempts, a connection in the geopotential space was tried with a basis in spectral decomposition by using available gravimetry in the region of the mouth of Amazon River, as well as several GGMs with different spectral resolutions and the RTM technique. The basis of analysis was to determine the offset of each vertical reference surface (DVB-I and DVB-S) related to the global reference given by recovering the reference equipotential, which pointed out a discrepancy showed in Fig. 4.1 (Moreira and De Freitas, 2016).

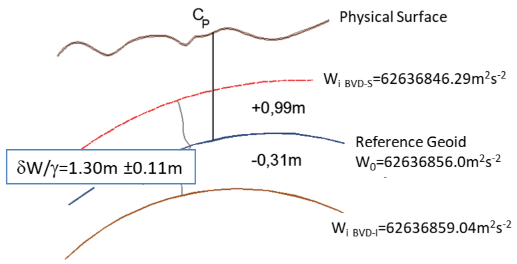

Figure 4.1: Connection of two segments of BVRN in the geopotential space at the Amazon River estuary region.

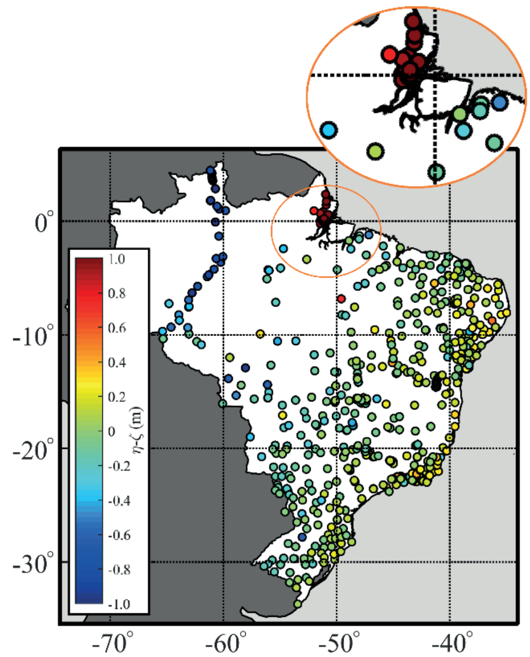

Figure 4.2: Observed offsets in GPS/Leveling stations in the two segments of BVRN.

Because a suspect of a systematic effect on the transported height values along the BVRN until the connection region, a new attempt was realized by considering an enhanced GGM obtained by combining GO_CONS DIR-R5 (Bruinsma et al., 2013) and EGM2008, and observations of co-located GPS and leveling benchmarks, which provide $N - \zeta$ (see Eq. (2.2)) on 638 sites for BVD-I and 16 sites for BVD-S (Fig. 4.2). The offset between the two segments, considering the reduction to the respective barycenter, was 1.416 m ($\pm$0.120 m) (de Freitas et al., 2016).

5 Concluding remarks and future prospects

The spatial and temporal heterogeneities of the BVRN, besides the lack of full physical meaning for its heights, are the major problems to be faced in its modernization process. Several activities have already been successfully developed since the 2000's; however, it is still necessary to accomplish some fundamental tasks. Because the new precepts of the IHRS/IHRF road mapped the modernization of BVRN and was established by considering the following standards:

- Realization based on physical heights in the form $H_P = f(C_P)$, where the geopotential number is the adjusted primary coordinate;
- BVRN connected to the SIRGAS-CON continuous GNSS stations;
- Integration with vertical reference networks of neighboring countries;
- Reference equipotential surface given by the W_0 IHRS value; and
- Referred to a reference epoch; i.e. to realize the determination of coordinates‘ velocities or adopt the variations observed in SIRGAS-CON or future GGRF stations by segments of BVRN.

The end of 2017 expects the conclusion of the first global adjustment of the BVRN in terms of geopotential number differences.

Today, a physical leveling link between the two segments of BVRN at Amazonian region in the geopotential space is in development. A set of about 40 new stations associated with gravity measurements covering a GPS leveling line will determine the disturbing potential in each station by fixed GBVP solution. These solutions will integrate new available gravity observations, high resolution GGM, and the RTM technique.

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