A new approach for assessing tropospheric delay model performance for safety-of-life GNSS applications

Szabolcs Rózsa

Department of Geodesy and Surveying, Budapest University of Technology and Economics, Hungary
E-Mail: rozsa.szabolcs@epito.bme.hu

Abstract

GNSS satellite signals suffer considerable delays while travelling through the troposphere. The delay caused, it can be separated into two different parts: the effect of gases in hydrostatic equilibrium and the effect of water vapour and condensed water present in the troposphere. In safety-of-life navigation applications of GNSS (e.g. positioning and navigation of aircrafts, autonomous vehicles, etc.) not only the accuracy of the positioning needs to be known, but the integrity of the positioning service should be evaluated, too. The integrity information means that the maximum positioning error at an extremely rare probability level (approximately 10^{-7}), called the protection level, must be determined. The widely adopted RTCA (Radio Technical Commission for Aeronautics) recommends the minimum operational performance standard (MOPS) for GNSS systems used in the aeronautics. According to this recommendation, the maximum total tropospheric delay error in the zenith direction is 0.12 m in terms of standard deviation. However, this model neglects both the geographical and seasonal variation of the accuracy performance of the tropospheric delay models. Our study focuses on the theoretical background of the assessment of tropospheric delay model performances under worst-case scenarios. The developed computational strategy is capable to estimate the magnitude of extremely rare tropospheric delay error and takes into consideration not only the geographical but also the seasonal variation of model performance. The results show that the proposed methodology provides a conservative model for assessing the maximal tropospheric delay error in worst case scenarios. However, the derived model is significantly less conservative than the RTCA recommendation based on radiosonde observations obtained in Budapest.

1 Introduction

The global navigation satellite systems (GNSS) use ranging between the satellites and the receivers to determine the position of the user with respect to a geocentric reference system. Ranging is realized by the time-of-flight observation of the satellite signals. Since these electromagnetic signals travel through the atmosphere, they suffer considerable delay in the troposphere. Tropospheric delays are usually taken into consideration by empirical models in absolute positioning. To assess the integrity of the satellite signal, the performance of these models must be evaluated to ensure that the safety-of-life users (e.g. aviators) can absolutely rely on the coordinates provided by onboard GNSS receivers. Error models used in current 'standard' (RTCA, 2006) for safety-of-life GNSS applications are considered very conservative in case of residual error modelling. Although it is advantageous for the safety, it has a negative effect on the availability and continuity of the positioning service. The tropospheric delay model recommended in the RTCA MOPS (RTCA, 2016) has an associated maximum ver-

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tical error of 0.12 m in terms of standard deviation globally. Although the RTCA MOPS does not specify how this value was obtained, it agrees well with the value found by Collins and Langley (1998).

van Leeuwen et al. (2004) studied the validity of this model in the Neatherlands and concluded that the model seems to be too conservative. Thus, there seems to be some room for developing a less conservative model without loosing the safety. In the near future, more demanding applications are expected to arise and as most of these will be based on multi-frequency and multi-constellation use of GNSS, they suffer from ionospheric delays less than today. Since the tropospheric effects cannot be eliminated by satellite signals using different frequencies, they need to be taken into consideration with empirical models in the future, too. This creates a demand for more accurate tropospheric error modelling and ensures its importance in approximating integrity while maintaining sufficient system availability. This paper proposes an improved method using the generalized extreme value theory to estimate the maximal tropospheric delay error in the vertical and in the satellite direction. To optimize the model for not only safety but also availability, the model incorporates geographical and seasonal dependent variables. Thus, a less conservative but still safe model can be developed for the existing and future tropospheric delay models.

2 Assessment of the integrity of GNSS service

The integrity of the satellite signal is assessed by the concept of the protection levels. Protection level provides an overbounding model of positioning error. The user must be very confident about his position, thus the protection level is usually calculated with approximately 4σ - 6σ confidence intervals. According to RTCA (2006) the following formula is used to calculate the residual error for GPS pseudorange measurements for the satellites used for the positioning:

$$\sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,air}^2 + \sigma_{i,tropo}^2 \,, \tag{2.1}$$

where:

σ_i is the standard deviation of the pseudorange measurement of satellite i [m],

$\sigma_{i,flt}^2$ is the model variance of the residual errors for fast and long-term corrections [m],

$\sigma_{i,UIRE}^2$ is the model variance of the slant range ionospheric delay estimation error [m],

$\sigma_{i,air}^2$ is the variance of the airborne receiver errors [m],

$\sigma_{i,tropo}^2$ is the variance of tropospheric delay estimation error [m].

The residual tropospheric error is modelled as a probabilistic variable with the standard deviation of $\sigma_{i,tropo}$ in the i-th satellite direction and it is calculated as:

$$\sigma_{i,tropo} = \left(\sigma_{TVE} \cdot m(\theta_i)\right) \,, \tag{2.2}$$

$$m(\theta_i) = \frac{1.001}{\sqrt{0.002001 + sin^2(\theta_i)}} \,, \tag{2.3}$$

where σ_{TVE} denotes the vertical residual error of the tropospheric delay estimation and is equal to 0.12 meters and θ_i is the satellite elevation angle. Note that the vertical residual error of the tropospheric delay estimation is a constant value which globally overbounds the standard deviation of the residuals, but as it neglects the effect of latitude on the accuracy of the tropospheric delay estimation, leads to an overly conservative model in many regions. Combining these terms, one ends up with the variance of the total residual error which enables the system to calculate the horizontal and vertical protection levels (HPL and VPL, see figure 2.1) for a given position as follows:

$$HPL = K_H \cdot d_{major} \,, \tag{2.4}$$

$$VPL = K_V \cdot d_U \,, \tag{2.5}$$

where K_H and K_V are constants depending on the different approach type and d_{major} [m] corresponds to the uncertainty along the semimajor axis of the error ellipse:

$$d_{major} \equiv \sqrt{\frac{d_{east}^2 + d_{north}^2}{2} + \sqrt{\left(\frac{d_{east}^2 - d_{north}^2}{2}\right)^2 + d_{EN}^2}} \,, \tag{2.6}$$

The terms in the equation stand for the following:

d_{east}^2 is the variance of model distribution that overbounds the true error distribution in the east axis [m^2],

d_{north}^2 is the variance of model distribution that overbounds the true error distribution in the north axis [m^2],

d_{EN} is the covariance of the model distribution in the east and the north axes [m],

d_U^2 is the variance of model distribution that overbounds the true error distribution in the vertical axis [m^2].

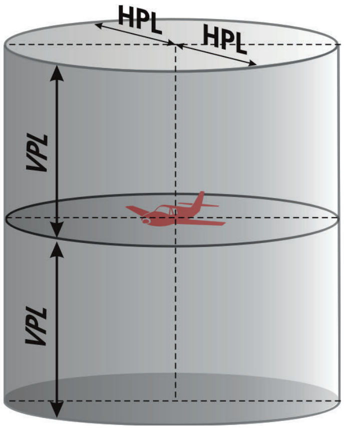

Figure 2.1: The concept of the protection levels.

All the model variances are calculated using the partial derivatives of the position error in the respective direction with respect to the pseudorange error on each satellite. Using the HPL and the VPL values, the instrument can decide whether current accuracy of the position is suitable for navigational purposes during the different approach types. When the calculated protection level is less than the precision requirement of the approach, than the integrity of the GNSS service is good, the service is available, whereas in case of a larger protection level than the needed requirement the service is not available anymore.

3 The proposed approach

The general integrity requirements of radio navigational aids used in civil aviation is formulated in *Aeronautical Telecommunication* (2006). According to this document, the integrity of GNSS positioning service must be evaluated at the extremely rare probability level of $2 \cdot 10^{-7}$ in any approach. Assuming the duration of an average approach of 150 seconds and no concurrent approaches in the same time, the recurrence interval of an integrity event would be 25 years.
Since no continuously available stationary error samples are available for the performance analysis, a probabilistic approach must be used for this purpose. It would be straightforward to fit a normal distribution to the residuals of the estimated tropospheric delays, and extrapolate it to the tails of the distribution. However, the probability plot of the residuals (see figure 3.1) clearly indicates that the tails of the residuals significantly deviate from the normal distribution. Thus, the extreme value theory must be applied for this problem. This mathematical approach is widely used in the prediction of flood levels with 100 years of recurrence time using a records from a much shorter time span.

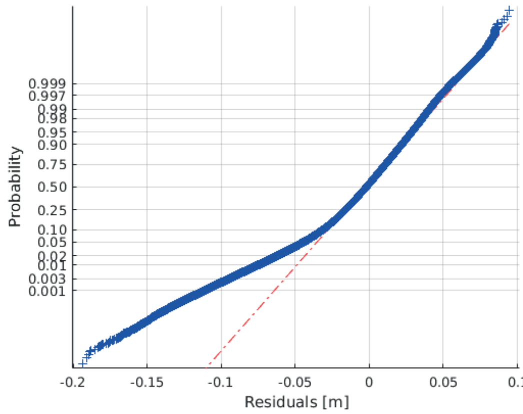

Figure 3.1: Normal probability plot of the hydrostatic tropospheric delay model residuals for the latitude band of N41°-N50° for the years 2000-2016 for the RTCA tropospheric delay model (reference values are calculated from ECMWF ERA-Interim numerical weather models using raytracing).

3.1 Principles of extreme value theory

The Fisher-Tippett theorem states that the maximum of a sample of independent and identically distributed probability variables after proper renormalization can converge to one of the three possible distributions, the Gumbel, the Fréchet or the Weibull distribution.
The three distribution functions are the following:

$$H(x) = \begin{cases} 0, & \text{if } x \leqslant 0 \\ exp(-x^{-\alpha}), & \text{if } x > 0 \text{ and } \alpha > 0 \end{cases}$$

for the Fréchet,

$$H(x) = \begin{cases} exp(-(-x^{-\alpha})), & \text{if } x \leqslant 0 \\ 1, & \text{if } x > 0 \text{ and } \alpha > 0 \end{cases}$$

for the Weibull and

$$H(x) = exp(-exp(-x)) \text{ for } x \in R$$

for the Gumbel distribution. The general extreme value (GEV) theory (Jenkinson, 1955) combines the previous three distributions to the general extreme value distribution. The distribution function is given by:

$$H(x) = \begin{cases} exp(-(1-k(x-\xi)/\alpha)^{1/k}), & \text{if } k \neq 0 \\ exp(-exp(-(x-\xi)/\alpha)), & \text{if } k = 0 \end{cases}$$

with x bounded by $\xi + \alpha/k$ from above if $k > 0$ and from below if $k < 0$. Here ξ and α are the location and scale parameters, respectively, while k is the shape parameter. The shape parameter determines which original extreme value is represented by the GEV distribution:
for $k > 0$, the Fréchet distribution (heavy tailed),
for $k = 0$ the Gumbel distribution (light tailed),
for $k < 0$ the short tailed negative Weibull distribution.

3.2 Estimation of extreme tropospheric error using the GEV theory

To study the performance of tropospheric delay models under extreme conditions, firstly, the tropospheric model error must be calculated. In order to achieve this, the tropospheric delay estimates provided by the studied model need to be compared to the 'true' value of the tropospheric delays. Since the 'true' value is not known, some kind of meteorological data sets must be used to calculate the 'true' tropospheric delay. One could use radiosonde datasets or numerical weather models for this purpose. In order to illustrate the proposed model, 16 years of ECMWF ERA-Interim reanalysis numerical weather models were acquired and processed in this paper with the ray-tracing technique. Afterwards the tropospheric delays obtained from the empirical models can be subtracted from the reference values to obtain the tropospheric delay model error. Due to the climatic and seasonal dependency of the tropospheric delay model performances, the error values will show strong seasonal variations all over the globe. Figure 3.2 shows the time series of the hydrostatic delay residuals for the latitude band between N41° to N50° latitudes on the globe. The figure shows, that the spread of the daily residuals has a significant seasonal variation. Since our aim is to derive the maximal tropospheric delay error taking into account seasonal dependency, the residual dataset needs to be normalized by an appropriate time-dependent model describing the seasonal variation of the standard deviation of the daily residuals. When this function is available, then the extreme value analysis can be carried out on the normalized residuals. Later the obtained maximal tropospheric delay error with the recurrence time of 25 years can be scaled using the same function to any day of the year. It must also be mentioned that figure 3.2 shows a significant bias as well, that has a seasonal variation, too. Although theoretically this bias must be taken into account, in our approach the zero-mean assumption was used for the normalization step. The reason of this simplification is that when the bias is removed from the residual error, then it needs to be restored in the protection level calculation during the application of the developed model. However, the RTCA MOPS recommends that the protection level should be calculated with the zero-mean assumption using the propagation of the uncertainties of the various observation and error model components. In order to maintain the consistency with the RTCA recommendation, we decided to adopt the zero-mean assumption in the normalization step.

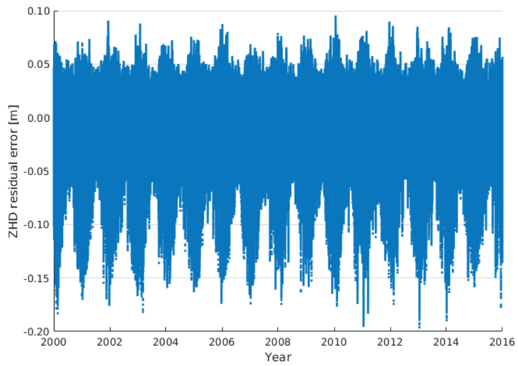

Figure 3.2: Time series of the residuals of the hydrostatic delays wrt the reference values obtained from raytracing numerical weather models.

In order to carry out this normalization, the daily standard deviation of the residual error was calculated and a periodic function was fit to these mean and standard deviation values considering both the annual and the semi-annual components of the seasonal variations (figure 3.3).
The model function for the daily standard deviation values:

$$\sigma(DOY) = \overline{\sigma} + A_1 cos\left(\frac{DOY - DOY_0}{365.25} 2\pi\right) + A_2 cos\left(\frac{DOY - DOY_0}{365.25} 4\pi\right) \quad (3.1)$$

where the unknown parameters are:
$\overline{\sigma}$ is the mean value of the daily mean residuals for the total time series,
DOY_0 is the day of the annual extreme value of the standard deviation of the daily residuals (phase),
A_1 is the amplitude of the annual terms of the seasonal variations of the daily standard deviations,
A_2 is the amplitude of the semi-annual terms of the seasonal variations of the daily standard deviations.
The time series of the daily standard deviation and the fitted model can be seen on figure 3.3.

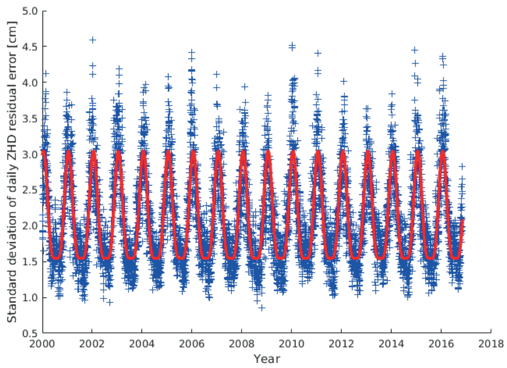

Figure 3.3: The seasonal variation of the daily standard deviation of the residuals and the fitted model containing annual and semi-annual terms.

Afterwards, the residuals (δ) were normalized using a zero-mean assumption with the following equation:

$$\delta_n = \frac{\delta}{\sigma(DOY)} . \quad (3.2)$$

In the next step, the normalized residual error values were used for the extreme value analysis. Since the samples covered 17 years of data, 17 annual extremes (maximum and minimum values) were identified and selected for the analysis. The GEV distribution was fit to these extremes using the MATLAB software, and finally the extreme value representing the recurrence time of 25 years was estimated using the fitted distribution for both the maximal (positive) and minimal (negative) extremes. From these two values, the one with the larger absolute value was chosen as the maximal expected error of the normalized residuals ($\Delta_{n,max}$).
Since the RTCA-MOPS proposes a calculation of the protection levels based on the standard deviation of parameters defined as normally distributed probabilistic variables, the previously estimated extreme values had to be converted to the uncertainty domain by calculating the standard deviation of normally distributed probabilistic variables providing the same maximum error at the given confidence level. Thus:

$$\sigma_{n,max} = \frac{\Delta_{n,max}}{K} , \quad (3.3)$$

where K is the value of the probability density function of the standard normal distribution at the probability level of $1-10^{-7}$.
To estimate the seasonal variations of the troposphere model errors the following overbounding model is formulated for the latitude band of N41° to N50°:

$$\sigma_{max}(DOY, band) = \frac{\Delta_0}{K} + \sigma_{n,max}(DOY) , \quad (3.4)$$

where Δ_0 is an offset parameter, that is necessary for achieving the overbounding of model error. This offset parameter takes into consideration the effect of the zero-mean assumption during the normalization phase. To achieve overbounding, the maximal daily mean value (maximal bias) is estimated by fitting another extreme value distribution to the annual extremes of the daily mean values of the residual error (see figure 3.4). To maintain consistency with the previous steps, the maximal value of the daily bias is calculated with the recurrence time of 25 years, too.

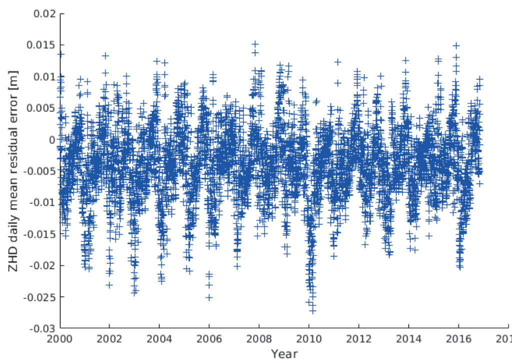

Figure 3.4: The seasonal variation of the daily mean values of the residuals.

3.3 Model formulation

Although the maximal tropospheric delay error in terms of standard deviation could even be given for any specific gridpoint of the input numerical weather data, the derived models must be as simple as possible, since they need to be incorporated in receiver firmware. One of the advantage of the RTCA MOPS recommendation in this respect is the simplicity. It provides a single value for the maximal tropospheric delay error in the vertical direction for the whole globe.

To maintain the simplicity of the model as much as possible and to provide a less conservative but still safe model for the calculation of the tropospheric protection level, the introduced calculations have been done for latitude bands of 10°. To further simplify the models, the following solutions have been formulated:

- Band Seasonal Model (BSM): this model provides the parameters to consider the seasonal variations of σ_{max} in each latitude band of 10°. Thus, this model takes into consideration not only the geographical but also the seasonal variations of the uncertainties.
- Band Constant Model (BCM): this model provides in each 10° latitude band a single value for the estimation of maximal tropospheric vertical error in terms of standard deviation for both the hydrostatic and wet delays.

In the next sections some important aspects of these models are discussed.

Band seasonal model

This model is the most sophisticated one among the derived integrity models. It provides the following parameters for any latitude bands:

- $\overline{\sigma}$: the mean value of the standard deviation of the daily tropospheric delay error (see eq. 3.1)
- DOY_0: the phase of the annual and semiannual variations of the daily standard deviation of the tropospheric delay error (see eq. 3.1)
- A_1: the amplitude of the annual terms of the seasonal variation (see eq. 3.1)
- A_2: the amplitude of the semi-annual terms of the seasonal variation (see eq. 3.1)
- Δ_0: the maximal value of the daily bias (see eq. 3.4)
- $\sigma_{n,max}$: the maximal uncertainty of the normalized tropospheric delay error for each band (see eq. 3.3)

Thus, the maximal uncertainty of the vertical tropospheric delay error can be calculated by combining the equations 3.1 and 3.4 using the following expression:

$$\begin{aligned}\sigma_{max}(DOY,band) &= \frac{\Delta_0}{K} \\ &+ (\overline{\sigma} + A_1 cos(\frac{DOY - DOY_0}{365.25}2\pi) \\ &+ A_2 cos(\frac{DOY - DOY_0}{365.25}4\pi)) \cdot \sigma_{n,max}\,.\end{aligned} \quad (3.5)$$

The BSM models can be derived for the hydrostatic and the wet delays separately, since advanced tropospheric delay models use different mapping functions for the calculation of the slant delays.

Band constant model

Although the Band Seasonal Model (BSM) is capable to take into consideration both the geographical and the seasonal variations of the uncertainties, the complexity of the model may cause some problems in the application in GNSS receivers. Thus, it was decided to provide simpler models as well for positioning applications.

The Band Constant Model (BCM) is derived from the BSM by calculating the annual maximum of the uncertainties using eq. 3.5:

$$\sigma_{max}(band) = max(\sigma_{max}(DOY,band))\,. \quad (3.6)$$

These models are also derived for the hydrostatic and wet components, respectively. The advantage of the model is that only a single constant needs to be stored for each latitude band for the hydrostatic and wet delays in the receiver's memory. The disadvantage is that the seasonal variation of the uncertainties is neglected in this model.

3.4 Calculation of the maximal uncertainty of the slant total delays

According to the RTCA MOPS the uncertainties must be calculated in the satellite direction. Thus, the vertical uncertainties calculated from the proposed models must be converted to slant uncertainties. This can be done by the appropriate mapping functions. Thus, the maximal uncertainty of the slant total delay is:

$$\sigma_{max,STD} = \sqrt{\sigma^2_{max,ZHD} \cdot m_h^2 + \sigma^2_{max,ZWD} \cdot m_w^2} \quad (3.7)$$

where m_h and m_w are the hydrostatic and wet mapping function values calculated for the respective satellite.

3.5 Results and conclusions

To prove the feasibility of the approach, the RTCA MOPS model has been tested against 17 years of ECMWF ERA-Interim reanalysis data. The reference values were obtained by ray-tracing the numerical weather models on a 1°×1° geographical grid for the latitude band of N41°-N50°. The proposed BSM and

BCM models were derived using the aforementioned computational strategy. The model parameters can be found in table 3.1, while the seasonal variation of the BSM is depicted on figure 3.5.

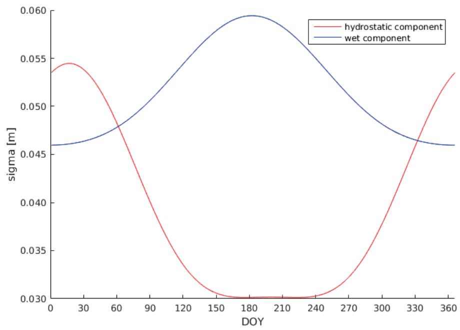

Figure 3.5: Seasonal variations of the BSM.

To check whether the derived models over-bound the tropospheric delay error, a set of radiosonde observations were collected from the radiosonde launching station located in Budapest (WMOID: 12843). The radiosonde profiles were processed and the zenith hydrostatic and wet delays were calculated including the uncertainties of these values according to the process given in Rózsa (2014).

Figure 3.6 and 3.7 show the - so called - Stanford plot of the hydrostatic and wet component of the tropospheric model error for the BSM model. The plot shows the frequency of the tropospheric delay error with respect to the tropospheric protection level for the radiosonde station in Budapest.

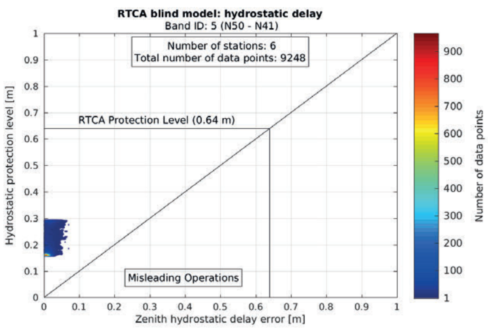

Figure 3.6: The Stanford plot of the tropospheric delay error against the tropospheric protection level for the hydrostatic component in the vertical direction.

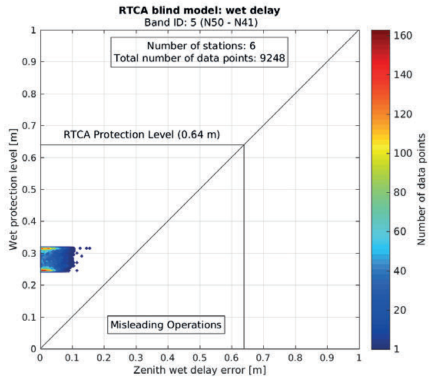

Figure 3.7: The Stanford plot of the tropospheric delay error against the tropospheric protection level for the wet component in the vertical direction.

Since tropospheric protection level (vertical axis) must always be higher than the tropospheric delay error calculated as the difference between the ground truth obtained from numerically integrating the radiosonde profile and the RTCA tropospheric model estimation (RTCA, 2006), each radiosonde profile should be located above the unit gradient line.

The constant protection level recommended by the RTCA MOPS is also depicted on the figures. The results show that the proposed computational approach successfully provided a model for the protection level calculations, which overbounds the residual error. Moreover, one can notice that all of the calculated protection levels are significantly lower than the recommendation of RTCA MOPS. It means that although our proposed model is conservative, since it overbounds the tropospheric delay error, it is significantly less conservative than the original RTCA model. This effect can result in a higher availability of GNSS positioning services in safety-of-life applications.

It must also be noted, that the proposed computational approach can be applied to any other advanced tropospheric delay models, such as the ESA GALTROPO model (Krueger et al., 2004) or the GPT2W model (Boehm et al., 2014), which is originally only a surface meteorological parameter model, but can be used as a tropospheric model using the approach of Askne and Nordius (1987).

Acknowledgement

The author would like to thank Prof. Bernhard Heck for the warm welcome and continuous support during and after his stay at the University of Karlsruhe be-

Table 3.1: BSM and BCM parameters for the latitude band of N41°-N50°.

	Band seasonal model parameters						**Band constant model parameters**
	Δ_0 [mm]	$\bar{\sigma}$ [mm]	A_1 [mm]	A_2 [mm]	*DOY* [day]	$\sigma_{n,max}$	σ_{max} [mm]
ZHD	27	21	8	2	348	1.63	60
ZWD	60	37	-6	0	0	1.08	60

tween 2001 and 2004. Those years and the memorable experiences significantly contributed to this, and other research conducted by the author in the field of GNSS positioning.

I'd also like to express my gratitude to Prof. Bernhard Heck for the long-lasting cooperation between the Geodetic Institute of the Karlsruhe Institute of Technology and the Department of Geodesy and Surveying of the Budapest University of Technology and Economics.

References

Aeronautical Telecommunication (2006). International Standards and Recommended Practices. Annex 10 to the Convention on International Civil Aviation, Volume 1: 578. International Civil Aviation Organization.

Askne, J. and Nordius, H. (1987): Estimation of tropospheric delay for microwaves from surface weather data. *Radio Science* 22(3):379–386.

Boehm, J., Moeller, G., Schindelegger, M., Pain, G., and Weber, R. (2014): Development of an improved empirical model for slant delays in the troposphere (GPT2w). *GPS Solutions* 19(3):433–441.

Collins, J. P. and Langley, R. B. (1998): The residual tropospheric propagation delay: How bad can it get? In: 11th International Technical Meeting of the Institute of Navigation, Nashville, Tennessee.

Jenkinson, A. F. (1955): The frequency distribution of the annual maximum (or minimum) of meteorological elements. *Quarterly Journal of the Royal Meteorological Society* 81:158–171.

Krueger, E., Schueler, T., Hein, W. G., Martellucci, A., and Blarzino, G. (2004): Galileo Tropospheric Correction Approaches Developed within GSTB-V1. In: Proceedings of the ENC-GNSS, Rotterdam, the Netherlands.

Rózsa, S. (2014): Uncertainty Considerations for the Comparison of Water Vapour Derived from Radiosondes and GNSS. In: IAG Symposia Series, vol. 139.

RTCA (2016): Minimum operational performance standards for global positioning systeme/satellite-based augmentation system airborne equipment. Standard: RTCA DO-229.

Van Leeuwen, S. S., van der Marel, H., Toussaint, M., and Martellucci, A. (2004): Validation of SBAS MOPS troposphere model over the EGNOS service area. In: European Navigation Conference (ENC), Rotterdam, The Netherlands.