Recovering undifferenced GNSS observations from double differences – use and limits

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Abstract

Double differences (DD) of GNSS phase observations are often used to eliminate or largely reduce systematic effects in the observations, especially the satellite and receiver clock errors. However, during this operation not only errors are eliminated but also information of four different lines-of-sights is combined into one DD observation. Consequently, specific effects of interest like e. g. multipath signatures or tropospheric slant delay variations can be hardly attributed to a specific line-of-sight. To overcome these problems, Alber et al. (2000) proposed a strategy to recover undifferenced observations (ZD) from DD which is now widely used in geodesy and GPS data assimilation in meteorology. In this paper, the explicit analytical solution of the strategy proposed by Alber et al. (2000) will be given. The findings describe directly the repartition of the information contained in the DD on the recovered ZD and the difference between the original ZD and the recovered one. Using simulated and real data the benefits and limitations of the strategy are discussed. It is shown that individual signatures cannot be completely recovered. We found that the success of recovering individual signatures and the degree of contamination of other observations by these signatures depend on the number of stations in the network, the number of satellites in common view, and the uniqueness of the signatures.

1 Motivation

Double differencing GPS phase observations is a standard approach in relative positioning. The impact of distance dependent systematics can be largely reduced as the satellite and receiver clock errors cancel out. In addition, depending on the network size the unmodelled tropospheric and ionospheric propagation effects are also lower. However, if double differences (DD) instead of undifferenced observations (zero differences, ZD) are used, specific effects of interest, like e. g., multipath signatures or tropospheric slant delay variations can be hardly attributed to a specific line-of-sight. As a consequence, it becomes difficult to study these effects in detail, i. e. each DD includes four different paths.

Information from high-resolution line-of-sight is increasingly requested. Prominent applications are the description of temporal and spatial variations of the water vapor in the atmosphere which are mandatory for weather prediction. A direct use of ZD is generally difficult for these applications since the information of interest is masked by or highly correlated with the dominant receiver clock error (Luo et al., 2007).

To overcome this problem, Alber et al. (2000) proposed a method for obtaining single-path delays from DD using the so-called “zero mean” assumption. This method has been rapidly adopted in meteorological modeling and multipath characterization. For example, Braun et al. (2001) and Braun et al. (2003) adopted

Erschienen bei KIT Scientific Publishing
Schriftenreihe des Studiengangs
Geodäsie und Geoinformatik 2018,1

DOI Einzelbeitrag:
10.5445/KSP/1000080246

Festschrift zur Verabschiedung von
Prof. Dr.-Ing. Dr. h.c. Bernhard Heck
(Schw)Ehre, wem (Schw)Ehre gebührt

DOI Festschrift:
10.5445/KSP/1000080324

the zero mean assumption and derived integrated water vapor along the ray path between a ground-based GPS receiver and GPS satellites. MacDonald et al. (2002) or Bender et al. (2011) used the obtained phase delays to determine and analyze 3D water vapor fields. Within the TOUGH-project (Targeting Optimal Use of GPS Humidity Measurements in Metrology) this methodology was also applied to retrieve slant delays for use in numerical weather prediction assimilation, cf. e. g. van der Marel and Gundlich (2006).

A second application is the mitigation or monitoring of GPS multipath effects. Here DDs are transformed to inbetween station single difference, in order to take the individual satellite repeat time and thus the repeatability of multipath patterns into account (Zhong et al., 2010). Huisman et al. (2009) transform DD residuals from a CORS network in Australia into ZD residuals and propose finger print maps for the analysis of multipath.

Besides its attractiveness, reconverting DDs into ZDs is based on different assumptions that can restrict the possible interpretations. In this contribution we present the explicit algebraic formulation of the recovered ZDs from SD as well as from DD. These results enable to evaluate if recovered ZD may answer the specific scientific question of interest.

In Section 2, the different strategies for double differencing are briefly revisited. Two algebraic solutions for the recovered ZD from DD are derived and discussed. Finally, we will use simulated and real data to illustrate the mathematical properties of the recovering procedure. The remainder of the paper will summarize in a comprehensive way the benefits and limits of using recovered ZD.

2 Mathematical concept

We can compute linearly independent DD using different construction sequences, like e. g. the fixed differencing basis selecting one reference satellite and one reference station (Remondi 1984, also named pivoting) or the sequential differencing basis (Beutler et al. 1984, also named cycling). Lindlohr and Wells (1985) showed that the estimated parameters and their associated variancecovariance matrices are independent of the specific strategies used for computing the DD. Following the strategy of first differencing between stations and then between satellites, the DD can be expressed as follows

$$d_{AB}^{12} = s_{AB}^{1} - s_{AB}^{2} = (z_A^1 - z_B^1) - (z_A^2 - z_B^2)\,, \qquad (2.1)$$

where z_A^1 and z_B^1 are the original undifferenced (zerodifferenced) observations of satellite 1 by station A and B, respectively, and z_A^2 and z_B^2 are observations of satellite 2 by station A and B, respectively. The difference between z_A^1 and z_B^1 is the single difference s_{AB}^1. Similarly s_{AB}^2 denotes the SD for the satellite 2. The two single differences can then be combined into the double difference d_{AB}^{12}.

Considering a network with m stations $A, B, ..., M$ observing n satellites $1, 2, ..., n, m-1$ linearly independent baselines can be formed. In matrix vector notation we can write

$$\mathbf{s}^i = \begin{bmatrix} s_{IA}^{\prime} \\ s_{IB}^{\prime} \\ \vdots \\ \\ s_{IM}^{\prime} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & ... & 0 \\ 1 & 0 & -1 & ... & 0 \\ \vdots & & & & \vdots \\ 1 & 0 & 0 & ... & -1 \end{bmatrix} \begin{bmatrix} z_I^{\prime} \\ z_A^{\prime} \\ z_B^{\prime} \\ \vdots \\ z_M^{\prime} \end{bmatrix} = \mathbf{A}_P \mathbf{z}^{\prime}\,, \qquad (2.2)$$

where the $(m-1) \times m$ matrix $\mathbf{A}_P$ presents the functional relationship between ZDs and SDs in case of pivoting and station I was arbitrarily chosen as reference station. Similarly, we form the DD for one baseline IJ as the product of a $(n-1) \times n$ matrix $\mathbf{B}_P$ for pivoting the SDs vector $\mathbf{s}_{IJ}$

$$\mathbf{d}_{IJ} = \begin{bmatrix} d_{IJ}^{12} \\ d_{IJ}^{13} \\ \vdots \\ \\ d_{IJ}^{1n} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & ... & 0 \\ 1 & 0 & -1 & ... & 0 \\ \vdots & & & & \vdots \\ 1 & 0 & 0 & ... & -1 \end{bmatrix} \begin{bmatrix} s_{IJ}^{1} \\ s_{IJ}^{2} \\ s_{IJ}^{3} \\ \vdots \\ s_{IJ}^{n} \end{bmatrix} = \mathbf{B}_P \mathbf{s}_{IJ}\,, \qquad (2.3)$$

Finally, the $(m-1)(n-1) \times 1$ vector of all linear independent DD of the whole network reads

$$\begin{aligned} \mathbf{d} &= (\mathbf{B} \otimes \mathbf{A})\mathbf{z} = \mathbf{M}\mathbf{z} \\ &= (\mathbf{B} \otimes \mathbf{I}_{m-1})(\mathbf{I}_n \otimes \mathbf{A})\mathbf{z} \end{aligned} \qquad (2.4)$$

where $\otimes$ is the ‘Kronecker-Zehfuss product’ (e. g., Koch 1999). The matrix $\mathbf{M}$ relates the ZDs to the

DDs. The $mn \times 1$ vector $\mathbf{z}$ is sorted by satellites (i. e. $z_I^1, z_A^1, \ldots, z_M^1, z_I^2, z_A^2, \ldots, z_M^2, \ldots, z_I^n, z_A^n, \ldots, z_M^n$). The vector $\mathbf{s}_S$ and $\mathbf{s}_B$ contain the SD sorted by satellites and baselines, respectively. These formulations are developed for observed- computed values. However, they are also valid for DD residuals.

Eqs. (2.2),(2.3) and (2.4) are three under-determined equation systems of ZDs as function of linearly independent DDs. Thus, the matrix $\mathbf{M}$ cannot be directly inverted. Consequently, recovering ZDs from the DDs is only possible under some assumptions.

In order to invert the matrix $\mathbf{M}$, Alber et al. (2000) added constraints to Eqs. (2.2) and (2.3), assuming that the residual delay in the direction of one GPS satellite at each epoch, averaged over the entire GPS network, is equal to zero. They stated that this assumption is generally valid for a network distributed over a large area. This 'zero mean' assumption is also used for the transformation between DDs and SDs.

3 Explicit algebraic solution for recovered zerodifferences

In this section, we derive an analytical and explicit solution of the recovered ZD based on DD. Without loss of generality we concentrate on star-like networks. We generalize the allowed constraints so that a weighted sum of the ZDs is fixed to an arbitrary value d . Again, we assume a network of m stations $A, B, \ldots, M$; the station I is the reference station for all $m-1$ baselines and the same $i = 1, \ldots, n$ satellites are observed at each station.

3.1 Two-step solution

The $m \times 1$ vector $\mathbf{z}^{i**}$ is given by:

$$\mathbf{z}^{i**} = \begin{bmatrix} w_I^i & w_A^i & \cdots & & w_M^i \\ 1 & -1 & & & \\ \vdots & & \cdots & & \\ & & & 1 & \\ 1 & & & & -1 \end{bmatrix}^{-1} \begin{bmatrix} \delta^i \\ \mathbf{s}_P^{i*} \end{bmatrix}, \quad (3.1)$$

where $\mathbf{z}^{i**}$ is the vector of ZDs referring to the i-th satellite. The $(m-1) \times 1$ vector $\mathbf{s}_P^{i*} = [s_{IA}^{i*}, s_{IB}^{i*}, \ldots, s_{IM}^{i*}]^T$ contains the corresponding recovered SDs, which we get from DDs using a similar strategy (see Fig. 2.1, red selection path). The required condition is $\sum_K (w_K^i z_K^{i**}) = \delta^i$. After some rearrangements, the solution of Eq. (3.1) is completely described by

$$z_J^{i**} = z_J^i + \frac{\delta^i - \sum_K w_K^i z_K^i}{w^i} + \frac{\sum_k w_{IJ}^k s_{IJ}^k - \delta_{IJ}}{\sum_k w_{IJ}^k} + \sum_K \frac{w_K^i}{w^i} \frac{\delta_{IK} - \sum_k w_{IK}^k s_{IK}^k}{\sum_k w_{IK}^k} \quad (3.2)$$

where $\mathbf{z}_J^{i**}$ is the recovered ZD for the i-th satellite measured at the J-th station.

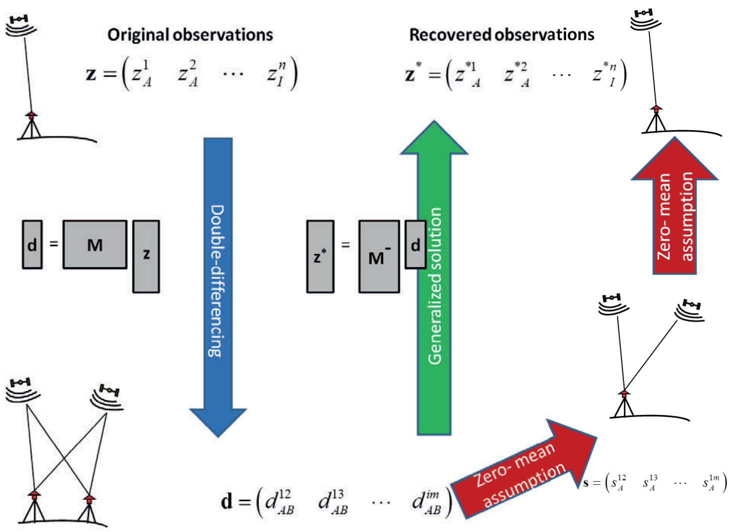

Figure 2.1: Recovery scheme of undifferenced observations (z) from Double Differences d. Blue: Process of double-differencing, green: direct solution by generalized inverse, red: two-step solution by zero mean assumption.

The terms $\delta^i - \sum_K w_K^i z_K^i, \sum_k w_{IJ}^k s_{IJ}^k - \delta_{IJ}$ and $\delta_{IK} - \sum_k w_{IK}^k s_{IK}^k$ represent the constraints applied to the satellite i, the baseline IJ , and all baselines referenced with station I, respectively. Note that for the reference station I, the second and the third term vanish. In the second line of Eq.(3.2) the recovered observation is expressed in terms of ZD and SD.

3.2 Equivalence conditions and final solution

Since in Eq. (3.2) the correction for the reference station differs from that of the other stations, the solution is obviously dependent on the strategy used to compute the DD. However, this is in contradiction with the fundamental differencing theorem (Lindlohr and Wells, 1985). Therefore, we propose the following equivalence conditions to be fulfilled in order to guarantee independency on the DD processing strategy:

EC1. The weights w_{ST}^k must be identical for a given satellite k in all baselines ST with arbitrary endpoints S and T.

EC2. The constraints for the baselines cannot be chosen in an arbitrary way. They must fulfill the following equation:

$$\sum_K \frac{w_K^i}{w^i} \delta_{IK} - \delta_{IJ} = \sum_K \frac{w_K^i}{w^i} \delta_{JK}\,, \tag{3.3}$$

note that $\delta_{II} = 0$.

With these two conditions we can rewrite our solution, which is now independent on the selection of the DD. For the given station J and satellite i the solution reads:

$$\begin{aligned} z_J^{i**} = j_J^i &+ \frac{\delta^i}{\sum\limits_K w_K^i} - \frac{\sum\limits_K w_K^i z_K^i}{\sum\limits_K w_K^i} - \frac{\sum\limits_k w_{ST}^k z_J^k}{\sum\limits_K w_{ST}^k} \\ &+ \sum_K \frac{w_K^i}{\sum\limits_K w_K^i} \frac{\sum\limits_k w_{ST}^k z_K^k}{\sum\limits_k w_{ST}^k} + \sum_K \frac{w_K^i}{\sum\limits_K w_K^i} \frac{\delta_{JK}}{\sum\limits_k w_{ST}^k} \end{aligned} \tag{3.4}$$

with w_{ST}^k according to the EC1, and an arbitrary starting point K of the baseline instead of the firm reference station. The corrections are now independent of the choice of reference station or baseline selection in the network. If we use the assumption that δ^i and δ_{JK} equal zero and $w_I^i = w_{ST}^k = w$ are identical for all stations and satellites, the solution can be expressed in its most simplified case

$$z_J^{i**} = z_J^i - \frac{\sum\limits_K z_K^i}{m} - \frac{\sum\limits_k z_J^k}{n} + \frac{\sum\limits_K \sum\limits_k z_K^k}{mn}\,. \tag{3.5}$$

3.3 Direct inversion

The general solution for the consistent equation $\mathbf{Mz} = \mathbf{d}$ can be expressed according to Koch (1999) as follows:

$$\begin{aligned} \mathbf{z} &= \mathbf{M}^- \mathbf{d} + (\mathbf{I} - \mathbf{M}^- \mathbf{M})\beta \\ &= \mathbf{M}^- \mathbf{Mz} + (\mathbf{I} - \mathbf{M}^- \mathbf{M})\beta \\ &= \mathbf{z} + (\mathbf{I} - \mathbf{M}^- \mathbf{M})(\beta - \mathbf{z}) \end{aligned} \tag{3.6}$$

where $\mathbf{M}^-$ denotes the generalized inverse of the matrix $\mathbf{M}$, and β is an arbitrary $mn \times 1$ vector. Using the pseudo inverse, a special solution of Eq. (3.6) reads

$$\mathbf{z}^\sharp = \mathbf{M}^+ \mathbf{d}\,, \text{where } \mathbf{M}^+ = \mathbf{M^T}(\mathbf{MM^T})^{-1} \tag{3.7}$$

for the matrix $\mathbf{M}$ with full row rank and refers to the minimal norm of $||\mathbf{z}||$ In order to compare the pseudo-inverse solution and Eq. (3.4) we can describe the inversion problem as

$$\begin{bmatrix} \mathbf{W} \\ \mathbf{M} \end{bmatrix} \mathbf{z}^{**} = \begin{bmatrix} \delta \\ \mathbf{d} \end{bmatrix}, \tag{3.8}$$

where $\begin{bmatrix} \mathbf{W} \\ \mathbf{M} \end{bmatrix}$ has a suitable inversion and combines the two-step strategy in one step. The two-step solution and the MINOS solution provide both a unique solution because of the uniqueness of the inverse and pseudo inverse. Through $\mathbf{d} = \mathbf{Mz}$ and the matrix identity $\mathbf{MM^+M} = \mathbf{M}$ (see Koch, 1999) we can obtain $\mathbf{Mz}^\sharp = \mathbf{MM^+Mz} = \mathbf{Mz} = \mathbf{Mz}^{**}$. If the upper part of equation system (3.8) holds, i. e. $Wz^\sharp = \delta$, both solutions give the identical result. In addition, they are identical under the condition $\delta = 0$ and $\mathbf{w_i} \in null(\mathbf{M})$ with $\mathbf{W^T} = [\mathbf{w_1}, \ldots, \mathbf{w_i}, \ldots, \mathbf{w_{n+m-1}}]$, and $null$ denoting the nullspace of the matrix M. This special scenario is given under the zero mean assumption designed by Alber et al. (2000) when the weights that appear in the same row of the matrix W are identical.

4 Discussion and examples

4.1 General discussion

In Section 3 we derived explicit formulas for the recovered undifferenced observations in terms of DD and ZD. In all cases three corrections are applied to the original observation, namely:

i) For common mode errors in all observations of the satellite i. This correction includes mainly the satellite clock error, but also satellite hardware delays, and satellite specific parts of ionospheric and tropospheric refraction that are common over the whole network.

ii) For common mode errors in all observations of the station J. This correction includes primary the receiver clock error and receiver hardware delays as well as station specific common parts of ionospheric and tropospheric refraction.

iii) For common mode errors in all observations of the whole network. The specific combination of these corrections yields that the signatures that are common to all observations of one satellite or common to all observations at one station cannot be recovered. Consequently, the recovered observations are free of the receiver clock error in J and the satellite clock error of the satellite i. This property is one of the basic benefit of the strategy, since the clock errors mask many line of sight effects, like slant trospospheric delays or multipath effects.

4.2 Discussion of special properties

The recovering by the simplest model was evaluated. From Eq. (3.5), the individual signature can only be recovered to a fraction of $f_R = \frac{mn-n-m+1}{mn}$ of the original signature. Assuming n = 10 satellites observed at each station, then f_R equals 45% in a two-station-network ($m = 2$) and 85%(89%) for 20 (100) stations. For even larger networks, f_R is limited by the number of satellites n and tends to $f_R = 1 - \frac{1}{n}$, e. g. for $n = 10, f_R \rightarrow 90\%$. Consequently, with Eq. (3.5) the individual signature cannot be completely recovered. In addition, we can see that the smaller the number of stations or satellites, the smaller the percentage of recovery, like e. g., for $n = 5$ satellites the maximum recovery tends to 80%. Finally, all observations of the network will be contaminated by the signature. The degree of contamination depends on the relationship between each observation and the observation with signature. Four groups of observations can be distinguished:

1) The observation to satellite i at station J with individual signature.
2) The observations to satellite i at any arbitrary station L. It is affected by $f_{C1} = -\frac{n-1}{mn}$ times the signature of the observation to satellite i at station J.
3) The observations to any arbitrary satellite k at any station L. It is affected by $f_{C2} = \frac{1}{mn}$ times the signature of the observation to satellite i at station J.
4) The observation of any satellite k at station J. It is affected by $f_{C3} = -\frac{m-1}{mn}$ times the signature of the observation to satellite i at station J.

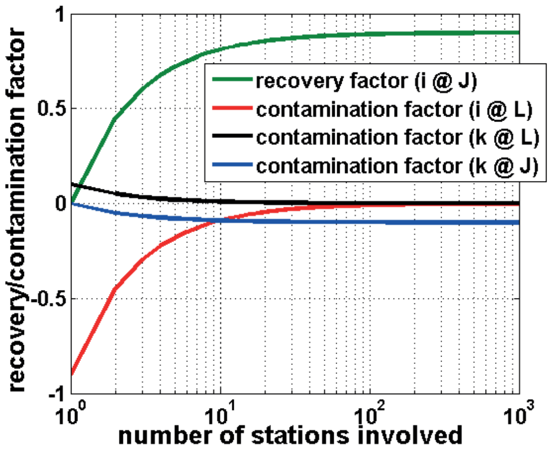

Figure 4.1: Dependency of the recovery and contamination factors on the network size (number of stations involved) for a scenario of $n = 10$ common satellites visible at all stations.

Figure 4.1 shows the variation of the contamination depending on the number of stations m involved in the network and the relation of the observation k to the one with the signature i. For large networks only observations at station J are affected, since f_{C1} and f_{C2} tend to zero. If less than 10 satellites are visible, the recovery is less successful and the contamination is increased.

In order to highlight the special properties of the recovering strategy, simulations were carried out. A forward strategy is applied to assess how well signatures can be recovered. We consider a network of 4 stations (A, B, C, and D) observing 6 satellites $(1, 2, ..., 6)$. For each of the 24 observations, time series of undifferenced observations are generated as a series of normal distributed random variables. Since common mode signatures cancel during double differencing, we transform the simulated time series in such a way, that the data fulfill exactly the zero mean assumption epoch by epoch. In a next step, we add an individual signature to one observation time series, like e. g., variations due to multipath in the observation of satellite 2 at station *B*. Then, the DD are computed assuming a star-like network with station *A* as reference station and satellite 1 as reference satellite.

Figure 4.2 shows that for our example, the sine-oscillation of the original time series from satellite 2 at station *B* can only partially be recovered. An estimation of the amplitude gives a recovery factor of 62.3% which is close to the theoretical value of 62.5%. All observations at station B and all observations of satellite 2 are distorted after recovering with a contamination factor of 12.5% and 20%, respectively. Please note the phase shift of 180° in the sineoscillation. All other observations are contaminated by 5% which is not visible from the data depicted in Fig. 4.1.

Corresponding results are obtained if the satellite and/or station with individual signature are changed. If the weight w_J^i of the individual signature is small, the factor of recovery can be improved. In this case even for small networks (i. e. few stations) the factor of recovery tends to $f_R \rightarrow 1-\frac{1}{n}$, i. e. in our example to 83%. The contamination of the satellite 2 at all other station tends to 0. However, the contamination factor of all other satellites is unchanged, especially the contamination factor f_{C3} of the observations at station B does not vanish, i. e. in our example it is 12%. Alber et al. (2000) proposed the selection of the weights according to the satellites' elevations. For small networks (i. e. station separation smaller than 10 km), the individual weight for a particular satellite differs only little, so that this selection equals an assignment of an identical weight for the satellite.

5 Recovering real data series

Up to now we have considered only one individual signature. In real data however, different signatures may occur that may have common mode parts. Starting with ZD residuals from PPP processing, we compute DD and reconstruct the ZD. Test data sets from two networks are investigated. The first data set is from

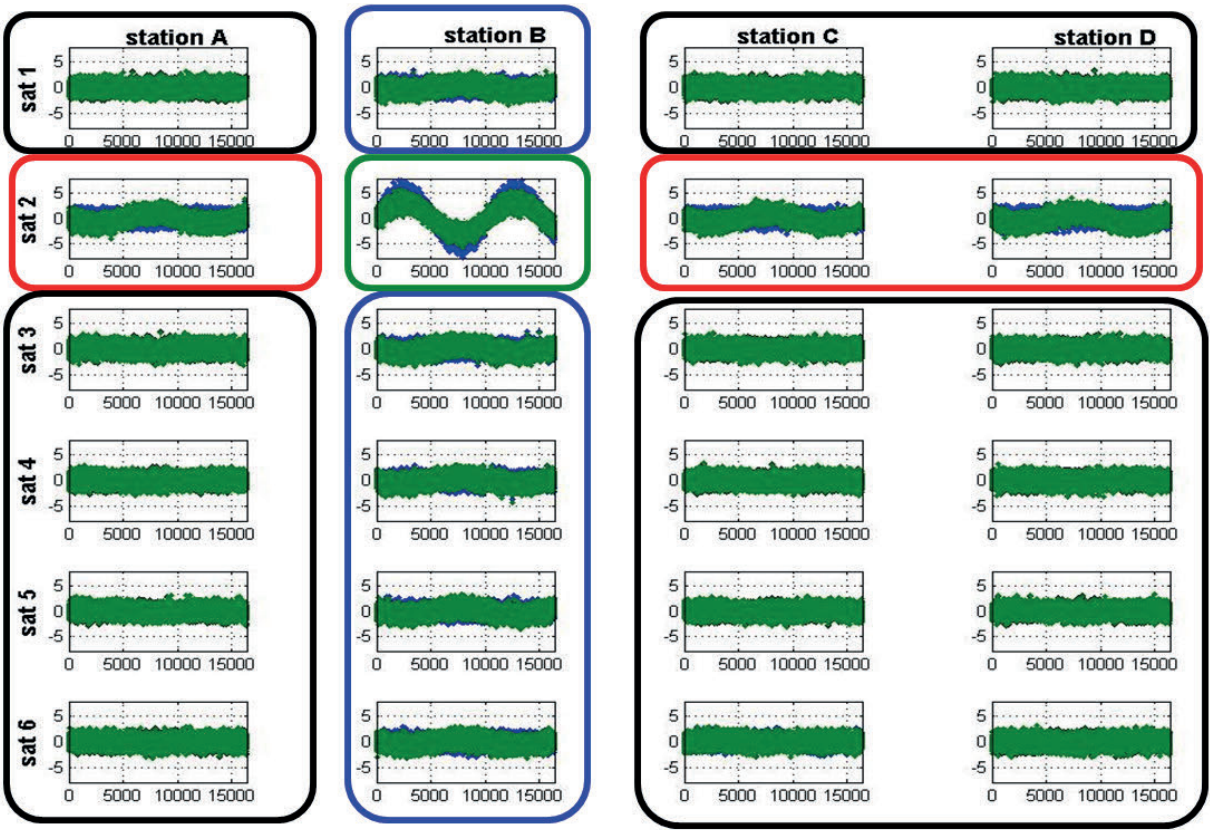

Figure 4.2: Simulated undifferenced data for 4 stations observing the identical 6 satellites. Comparison of original (blue) and recovered (green) simulated time series. The subplots are arranged in a matrix, each column refers to one station and contains the time series of all 6 satellites. Identical weights have been used.

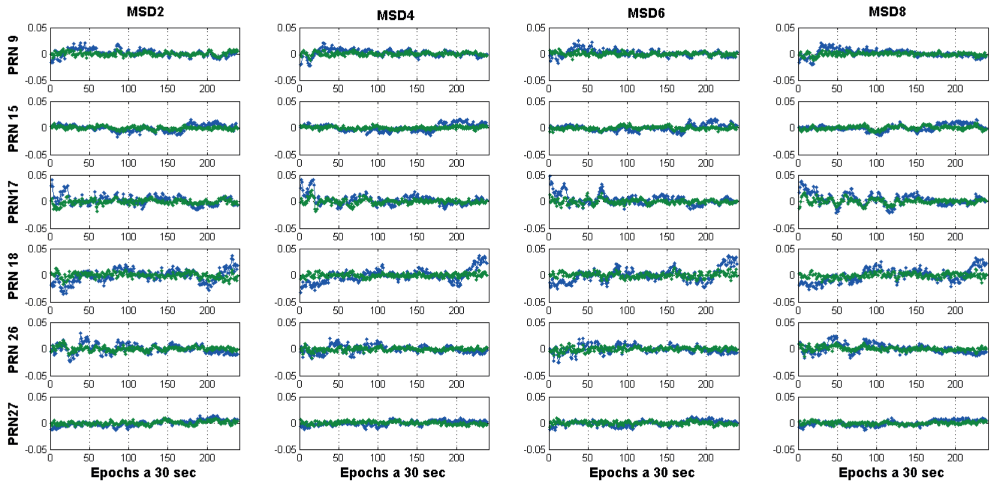

Figure 4.3: Comparison of the original (blue) and recovered (green) time series of undifferenced phase residuals from the LUH network when using Eq. (3.5) for recovering. The subplots are organized in a matrix; each column contains the observations to 6 satellites at the four stations MSD 2, MSD4, MSD6 and MSD8. Data from a 2 h segment (12:00–14:00 GPS time) on 7. July 2009

a very small network on the roof top of the Geodetic Institute of Leibniz Universität Hannover (LUH network). Four equally equipped and aligned stations were used that are separated by 10 m each. The second network is a subset of 6 stations of the EPN network (Bruyninx et al., 2004). Both data sets were processed by the IfE-developed PPP-software, Weinbach and Schön (2011, 2015).

The evaluation of the recovering success is not straight forward and may depend on the specific applications. As a first quality measure, we propose the cumulative histogram of absolute deviations of the recovered time series $\bar{x}_J^i(t)$ with respect to the original one $x_J^i(t)$. This quantity is computed as

$$\begin{aligned} \Delta_J^i &= cumhist\left\{\Delta_J^i(t)\right\} \\ \Delta_J^i(t) &= |\bar{x}_J^i(t) - x_J^i(t)|\,. \end{aligned} \tag{5.1}$$

Due to the very short station separations in the LUH network, many common mode systematic variations exist in the original time series. From the discussion in Sec. 4.1, it is obvious that these common patterns cannot be recovered, they will be lost during doubledifferencing. Consequently, the success of recovering is small. For a network of EPN stations the situation is different since individual signatures exist due to the larger station separations. For the LUH net-

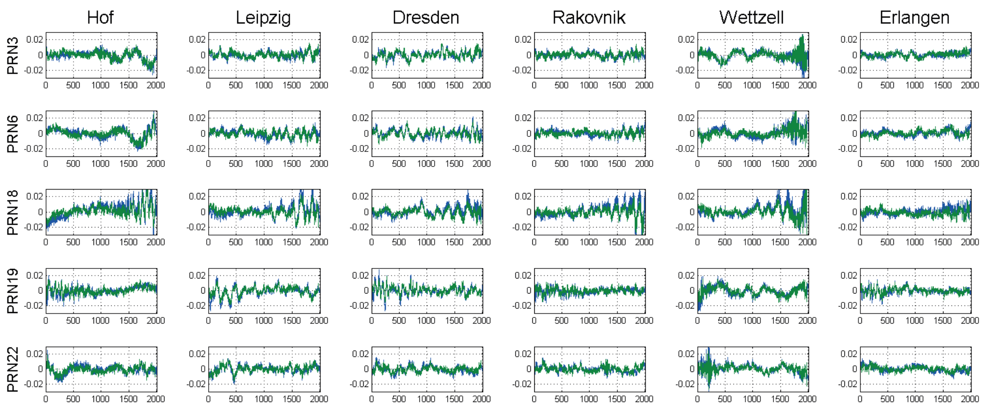

Figure 4.4: Comparison of the original (blue) and recovered (green) time series of undifferenced phase residuals from the EPN subnetwork when using Eq. (3.5) for recovering. The subplots are organized in a matrix; each column contains the observations to 5 satellites at the six stations Hof, Leipzig, Dresden, Rakownik, Wettzell, and Erlangen.

work, the deviation between original and the recovered data is in 55%/85%/95% of all data points smaller 5 mm/10 mm/15 mm, respectively. Maximum deviations of 37 mm occur. In addition, the recovery success depends of the satellite: PRN 27 can be best recovered, i. e. for 70% of the data, deviations smaller 5mm are obtained, and maximum deviations of 13mm occur. However, for PRN 17 in 70% of the data point deviations smaller than 8 mm are obtained. Here maximum deviations of 37 mm can be found.

The change of the weights in the recovering to an elevation dependent weighting yields no significant change of the recovery success.

6 Conclusions

In this paper, the explicit algebraic solution for recovered ZDs from DDs is presented. The strategy initially proposed by Alber et al. (2000) is expanded. It is shown that the recovered ZD of a particular satellite at a distinct station can be expressed as the original ZD plus three correction terms. They represent the common mode errors of all observations of (1) the particular satellite, (2) at the distinct station, and (3) all observations of the whole networks. The main benefits of recovering ZD is that the ZD is consequently free of common mode errors, especially the receiver and satellite clock errors. This enables the analysis of signatures with small amplitudes (i. e. from some mm to a few cm), e. g. like variations in the tropospheric slant delays or multipath patterns. However, it is mathematically shown that the recovery will never be perfect. For an individual signature, we introduce recovery and contamination factors. The recovery success depends on the number of stations involved in the network and the satellites in view at all stations. While the number of stations can be increased and thus the recovering improved, the number of satellites is restricted and currently limits the success of recovery from a theoretical point of view to a maximum of 90% recovery. The operational use of multi GNSS will improve the situation in future. Since the recovered ZD are linear combinations of all original ZD, all recovered ZD will be contaminated by an individual signature. The degree of contamination depends on the relation of each observation to the observation with individual signature and the number of stations and satellites in common view in the network. Considering real PPP residuals as “true” ZD signatures, it could be shown that the success of recovery increases if the signatures of the observation time series of the identical satellite at different stations differ at most and meet the zero mean assumption. Considering original and recovered ZD as identical if their deviations are smaller than the GPS carrier phase noise level of about 2 mm, in our example about 50% of the time series of high elevation satellites could be correctly recovered.

Acknowledgements

Bernhard’s “post” group at GIK is the starting place of my scientific career. I deeply thank Bernhard for his support already during my studies at Universität Karlsruhe, as an enthusiastic teacher who attracted my interests to theoretical questions. His mentorship and friendly interest was a big support during all the last years.

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