Model-independent analysis of semileptonic B decays to D^{**} for arbitrary new physics

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We explore semileptonic B decays to the four lightest excited charm mesons, $D^{**} = \{D_0^*, D_1^*, D_1, D_2^*\}$, for nonzero charged lepton mass and for all $b \to c\ell\bar{\nu}$ four-Fermi interactions, including calculation of the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ and $\mathcal{O}(\alpha_s)$ corrections to the heavy quark limit for all form factors. In the heavy quark limit, some form factors are suppressed at zero recoil; therefore, the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections can be very important. The D^{**} rates exhibit sensitivities to new physics in $b \to c\tau\bar{\nu}$ mediated decays complementary to the D and D^* modes. Since they are also important backgrounds to $B \to D^{(*)}\tau\bar{\nu}$, the correct interpretation of future semitauonic $B \to D^{(*)}$ rate measurements requires consistent treatment of both the D^{**} backgrounds and the signals. Our results allow more precise and more reliable calculations of these $B \to D^{**}\ell\bar{\nu}$ decays and are systematically improvable by better data on the e and μ modes. As an example, we show that the D^{**} rates are more sensitive to a new $\bar{c}\sigma_{\mu\nu}b$ tensor interaction than the $D^{(*)}$ rates.

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I. INTRODUCTION

The measurements of the ratio of semitauonic B decays compared to the light-lepton final states,

$$R(X) = \frac{\Gamma(B \to X\tau\bar{\nu})}{\Gamma(B \to Xl\bar{\nu})}, \qquad l = \mu, e, \tag{1}$$

show a 4σ tension with the standard model (SM) expectations [1], when the $X = D$ and D^* results are combined. Improving our understanding of the $B \to D^{(*)}$ form factors, required for precision calculations of $R(D^{(*)})$, has received renewed attention recently [2–8]. To maximize future sensitivity to new physics (NP) contributions, measuring and understanding contributions for additional semileptonic decay modes mediated by the same parton-level transition is important and necessary, not only as they can give complementary information on the new physics, but also as they constitute backgrounds to the $R(D^{(*)})$ measurements.

In this paper, we study $B \to D^{**}\ell\bar{\nu}$ decays, where

$$D^{**} \in \{D_0^*, D_1^*, D_1, D_2^*\}, \tag{2}$$

denotes the four lightest excited charmed mesons, above the $\{D, D^*\}$ ground-state doublet of heavy quark symmetry (HQS) [9–11]. (The D^{**} notation is common in the experimental literature; these are the $1P$ orbitally excited states in the quark model.) In Ref. [12], SM predictions for $R(D^{**})$ were derived, extending results for massless leptons [13,14], but a comprehensive study of NP effects has not been carried out yet. We include contributions from all possible four-fermion operators (assuming no right-handed neutrinos), and derive the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ and $\mathcal{O}(\alpha_s)$ terms in the expansions of the form factors, going beyond the leading order in the heavy quark expansion. The $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections to the SM matrix elements were calculated a long time ago [13,14], and can be substantial, due to the suppressions of certain leading-order matrix elements near zero recoil, imposed by heavy quark symmetry. We show that the available $B \to D_2^* l\bar{\nu}$ and $B \to D_1 l\bar{\nu}$ data are in severe tension with the heavy quark limit, that is alleviated by including $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections. Similarly, $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ terms can generate numerically dominant contributions to NP matrix elements as well, and must be included.

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Understanding the $B \to D^{**}\ell\bar{\nu}$ decays as precisely as possible, both theoretically and experimentally, is important for several reasons. First, as in $B \to D^{(*)}\ell\bar{\nu}$ decays, certain form factor combinations are suppressed by the light lepton mass, and thus cannot be constrained by $B \to D^{**}l\nu$ measurements, but enter unsuppressed in the semitauonic rates. The use of heavy quark effective theory (HQET) [15,16] allows more precise future measurements of $B \to D^{**}l\bar{\nu}$ to systematically improve the predictions for $B \to D^{**}\tau\bar{\nu}$ [12], which will provide complementary sensitivity to new physics compared to $B \to D^{(*)}\tau\bar{\nu}$. Second, $B \to D^{**}\ell\bar{\nu}$ decays also constitute a significant background to the measurements of $R(D^{(*)})$, contributing significantly to its uncertainty at present. As certain $B \to D^{**}\tau\bar{\nu}$ modes may exhibit high sensitivity to NP, good theoretical control of these backgrounds is required in order to understand which NP operators may best fit the data. Third, better theoretical control of these modes will help to improve the determinations of the CKM elements $|V_{cb}|$ and $|V_{ub}|$, both from exclusive and inclusive B decays. The study of these decay modes [17] and their contributions to the Bjorken sum rule [18] will help understanding the composition of the inclusive $B \to X_c\ell\bar{\nu}$ decay in terms of exclusive modes.

In Sec. II, we establish notations and calculate all $B \to D^{**}$ form factors, including the complete set of order $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{c,b})$ and $\mathcal{O}(\alpha_s)$ effects. Section III contains expressions for the differential decay rates for arbitrary currents and charged lepton mass. In Sec. IV, we study observables that are particularly sensitive to the $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{c,b})$ corrections and plot effects of a NP tensor interaction which could not be evaluated previously with comparable accuracy. Section V concludes.

II. HQET EXPANSION OF THE FORM FACTORS

We are interested in the $\bar{B} \to D^{**}$ matrix elements of operators with all possible Dirac structures, for which we choose the basis

$$\begin{aligned} &O_V = \bar{c}\gamma_\mu b, \qquad O_A = \bar{c}\gamma_\mu\gamma_5 b, \\ &O_S = \bar{c}b, \qquad O_P = \bar{c}\gamma_5 b, \qquad O_T = \bar{c}\sigma_{\mu\nu}b, \end{aligned} \tag{3}$$

with $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$. Throughout this paper, we assume isospin symmetry, and $\bar{B}$ denotes $\bar{B}^0$ or B^-. As in Refs. [12–14], we use the conventions $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho\gamma^5] = -4i\epsilon^{\mu\nu\rho\sigma}$, so that $\sigma^{\mu\nu}\gamma^5 \equiv +(i/2)\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}$. (This is the opposite of the common convention in the $\bar{B} \to D^{(*)}\ell\bar{\nu}$ literature, which typically chooses $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho\gamma^5] = +4i\epsilon^{\mu\nu\rho\sigma}$, so that $\sigma^{\mu\nu}\gamma^5 \equiv -(i/2)\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}$.)

A. Spectroscopy

The spectroscopy of the D^{**} states is important, because in addition to the impact on the kinematics, it also affects the HQET expansion of the form factors [13,14]. The isospin averaged masses and widths for the six lightest charm meson states are shown in Table I. (The level of agreement between the measurements of the masses and widths of the D^{**} mesons, especially those of D_0^* in the top row of Table I, is presently unsatisfactory [12].)

TABLE I. Isospin averaged masses and widths of the six lightest charm mesons, rounded to 1 MeV [19].

Particle	$s_l^{\pi_l}$	J^P	m (MeV)	Γ (MeV)
D_0^*	$\frac{1}{2}^+$	0^+	2349	236
D_1^*	$\frac{1}{2}^+$	1^+	2427	384
D_1	$\frac{3}{2}^+$	1^+	2421	31
D_2^*	$\frac{3}{2}^+$	2^+	2461	47
D^*	$\frac{1}{2}^-$	1^-	2009	0.
D	$\frac{1}{2}^-$	0^-	1866	0.

In the heavy quark limit, the spin-parity of the light degrees of freedom (d.o.f.), $s_l^{\pi_l}$, is a conserved quantum number, yielding doublets of heavy quark symmetry, as the spin s_l is combined with the heavy quark spin [11]. In the quark model, the four D^{**} states correspond to combining the heavy quark and light quark spins with $L = 1$ orbital angular momentum. The masses of each heavy quark spin symmetry doublet of hadrons, $H_\pm$, with total spin $J_\pm = s_l \pm \frac{1}{2}$ can be expressed in HQET as

$$m_{H_\pm} = m_Q + \bar{\Lambda}^H - \frac{\lambda_1^H}{2m_Q} \pm \frac{n_\mp\lambda_2^H}{2m_Q} + \ldots, \tag{4}$$

where $n_\pm = 2J_\pm + 1$ is the number of spin states of each hadron, and the ellipses denote terms suppressed by more powers of $\Lambda_{\mathrm{QCD}}/m_Q$. The parameter $\bar{\Lambda}^H$ is the energy of the light degrees of freedom in the $m_Q \to \infty$ limit and plays an important role, as it is related to the semileptonic form factors [13,14]. The λ_1^H and λ_2^H parameters are related to the heavy quark kinetic energy and chromomagnetic energy in the hadron H. We use the notation $\bar{\Lambda}$, $\bar{\Lambda}'$, and $\bar{\Lambda}^*$ for the $\frac{1}{2}^-$, $\frac{3}{2}^+$, and $\frac{1}{2}^+$ doublets, respectively, and for the states in each doublet:

$$D^{1/2^+} \in \{D_0^*, D_1^*\}, \qquad D^{3/2^+} \in \{D_1, D_2^*\}. \tag{5}$$

The current data suggest that the $m_{D_1^*} - m_{D_0^*}$ mass splitting is substantially larger than $m_{D_2^*} - m_{D_1}$. This possibility was not considered in Refs. [13,14], since at that time both of these mass splittings were about 40 MeV. The smallness of $m_{D_2^*} - m_{D_1}$ and $m_{D_1^*} - m_{D_0^*}$ compared to $m_{D^*} - m_D \simeq 140$ MeV was taken as an indication that the chromomagnetic operator matrix elements are suppressed for the four D^{**} states, in agreement with quark model predictions. We relax this constraint, as in Ref. [12].

While the measured masses of the broad D_0^* and D_1^* states changed substantially over the last 20 years, their

TABLE II. The HQET parameter estimates used [12].

Parameter	$\bar\Lambda$	$\bar\Lambda'$	$\bar\Lambda^*$	$\bar\Lambda_s$	$\bar\Lambda_s'$	$\bar\Lambda_s^*$
Value [GeV]	0.40	0.80	0.76	0.49	0.90	0.77

$2J+1$ weighted average is essentially unchanged compared to Ref. [14]. We use $\bar\Lambda' - \bar\Lambda = 0.40$ GeV and $\bar\Lambda' - \bar\Lambda^* \simeq 0.04$ GeV, and summarize the parameters used in Table II. The uncertainty of $\bar\Lambda$ is substantially greater than that of $\bar\Lambda' - \bar\Lambda$ and $\bar\Lambda' - \bar\Lambda^*$; as we see below, the form factors are less sensitive to $\bar\Lambda$ than to these splittings.

B. Matrix elements to order $\Lambda_{\mathrm{QCD}}/m_{c,b}$ and α_s

It is simplest to calculate the $\bar B \to D^{**}$ matrix elements in HQET using the trace formalism [20–22]. It allows a straightforward evaluation of the matrix elements of the five operators in Eq. (3), as well as those of additional operators generated by perturbative corrections (for a review, see Ref. [23]). The $\mathcal{O}(\alpha_s)$ corrections are given explicitly in Appendix A of Ref. [2], extracted from Refs. [20,24,25]. The $\mathcal{O}(\alpha_s \Lambda_{\mathrm{QCD}}/m_{c,b})$ corrections are known for the SM currents and would be straightforward to calculate for any new physics, but are neglected below.

The three heavy quark spin symmetry doublets relevant for this paper can be represented by the (super)fields, which have the correct transformation properties under Lorentz and heavy quark symmetries [22],

$$\begin{aligned}
H_v &= \frac{1+\slashed{v}}{2}[B_v^* \slashed{\epsilon} - B_v \gamma^5],\\
K_v &= \frac{1+\slashed{v}}{2}[V_v \gamma^5 \slashed{\epsilon} + P_v],\\
F_v^\alpha &= \frac{1+\slashed{v}}{2}\left\{T_v \epsilon^{\alpha\beta}\gamma_\beta - V_v \sqrt{\frac{3}{2}}\gamma^5 \left[\epsilon^\alpha - \frac{1}{3}\slashed{\epsilon}(\gamma^\alpha - v^\alpha)\right]\right\}.
\end{aligned} \tag{6}$$

In this paper, each representation occurs for only one heavy quark flavor, so for simplicity we denote the components of H_v by B_v and B_v^*. The $\epsilon^{\alpha\beta}$ denote a normalized traceless symmetric spin-2 polarization tensor.

Similar to Ref. [26], including $\Lambda_{\mathrm{QCD}}/m_{c,b}$ corrections, the $\bar B \to D^{**}$ matrix elements can be written as

$$\frac{\langle D^{1/2^+}|\bar c \Gamma b|\bar B\rangle}{\sqrt{m_{D^{1/2^+}} m_B}} = \zeta(w)\left\{\mathrm{Tr}[\bar K_{v'}\Gamma H_v] + \varepsilon_c \mathrm{Tr}[\bar K^{(1)}_{v',v}\Gamma H_v] - \varepsilon_b \hat G_b \mathrm{Tr}\left[\bar K_{v'}\Gamma \frac{1-\slashed{v}}{2}\gamma^5 B_v\right]\right\}, \tag{7a}$$

$$\frac{\langle D^{3/2^+}|\bar c \Gamma b|\bar B\rangle}{\sqrt{m_{D^{3/2^+}} m_B}} = \tau(w)\left\{\mathrm{Tr}[v_\sigma \bar F^\sigma_{v'}\Gamma H_v] + \varepsilon_c \mathrm{Tr}[\bar F^{(1)}_{v',v}\Gamma H_v] + \varepsilon_b \hat F_b \mathrm{Tr}\left[v_\sigma \bar F^\sigma_{v'}\Gamma \frac{1-\slashed{v}}{2}\gamma^5 B_v\right]\right\}, \tag{7b}$$

where $\varepsilon_{c,b} = 1/(2m_{c,b})$, Γ is an arbitrary Dirac matrix, and

$$\bar K^{(1)}_{v',v} = [V_{v'}\gamma^5(\slashed{\epsilon}\hat M_2 + \epsilon\cdot v \hat M_3) + P_{v'}\hat M_1]\frac{1+\slashed{v}'}{2} + [V_{v'}\gamma^5(\slashed{\epsilon}\hat M_5 + \epsilon\cdot v\hat M_6) + P_{v'}\hat M_4]\frac{1-\slashed{v}'}{2}, \tag{8a}$$

$$\begin{aligned}
\bar F^{(1)}_{v',v} = &\left[T_{v'}(\epsilon^{\mu\nu}\gamma_\mu\gamma_\nu \hat N_1 + \epsilon^{\mu\nu}v_\mu\gamma_\nu\hat N_2 + \epsilon^{\mu\nu}v_\mu v_\nu \hat N_3) + \frac{V_{v'}}{\sqrt6}(\slashed{\epsilon}\hat N_4 + \epsilon\cdot v\hat N_5)\gamma^5\right]\frac{1+\slashed{v}'}{2}\\
&+\left[T_{v'}(\epsilon^{\mu\nu}\gamma_\mu\gamma_\nu \hat N_6 + \epsilon^{\mu\nu}v_\mu\gamma_\nu\hat N_7 + \epsilon^{\mu\nu}v_\mu v_\nu \hat N_8) + \frac{V_{v'}}{\sqrt6}(\slashed{\epsilon}\hat N_9 + \epsilon\cdot v\hat N_{10})\gamma^5\right]\frac{1-\slashed{v}'}{2}.
\end{aligned} \tag{8b}$$

At leading order, in the heavy quark limit, all $\bar B \to D^{1/2^+}$ form factors are determined by one Isgur-Wise function, $\zeta(w)$, while all $\bar B \to D^{3/2^+}$ form factors are determined by another, $\tau(w)$. (In the notation of Ref. [17], $\zeta(w)$ is twice the function $\tau_{1/2}$ and $\tau(w)$ is $\sqrt3$ times the function $\tau_{3/2}$.) All form factors are viewed as functions of the dimensionless kinematic variable w, instead of $q^2 = (p_B - p_{D^{**}})^2$, with

$$w = v\cdot v' = \frac{m_B^2 + m_{D^{**}}^2 - q^2}{2m_B m_{D^{**}}}. \tag{9}$$

Here $v = p_B/m_B$ and $v' = p_{D^{**}}/m_{D^{**}}$ are the four-velocities of the initial and final states.

The coefficients $\hat M_i$ and $\hat N_i$ contain order $\Lambda_{\mathrm{QCD}}/m_{c,b}$ corrections. These are expressed in terms of subleading Isgur-Wise functions, which arise either from corrections to the HQET Lagrangian or from matching the current operators onto HQET [27–29]. Specifically, (i) matrix elements of the $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{c,b})$ current operators give rise to the subleading (dimensionful) Isgur-Wise functions ζ_1 and $\tau_{1,2}$; (ii) matrix elements involving the $\Lambda_{\mathrm{QCD}}/m_{c,b}$ suppressed kinetic energy operator, $\bar h_v (iD)^2 h_v/(2m_Q)$, are

TABLE III. Leading and subleading Isgur-Wise functions that parametrize $\bar{B} \to D^{**}$ form factors at $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$. The arrows in the last column indicate the minimal set of functions needed for the $1/m$ Lagrangian corrections, if the replacements in Eq. (10) are made. Omitted upper indices mean 'c'.

Doublet	Leading order	$1/m$ current	$1/m$ Lagrangian
$D^{1/2^+}$	ζ	ζ_1	$\chi_{\rm ke}^{c,b}, \chi_{1,2}^{c,b} \to \chi_{1,2}$
$D^{3/2^+}$	τ	$\tau_{1,2}$	$\eta_{\rm ke}^{c,b}, \eta_{1,2,3}^{c,b} \to \eta_{1,2,3}$

spin symmetry conserving and generate the functions $\chi_{\rm ke}^{c,b}$ and $\eta_{\rm ke}^{c,b}$; (iii) matrix elements involving the chromomagnetic operator in the HQET Lagrangian, $(g_s/2)\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v/(2m_Q)$, which violates spin symmetry, generate the functions $\chi_{1,2}^{c,b}$ and $\eta_{1,2,3}^{c,b}$. The notations for these subleading Isgur-Wise functions are summarized in Table III.

As we consider only B (and not B^*) decays, the $\Lambda_{\rm QCD}/m_b$ corrections from the chromomagnetic operator in the Lagrangian—the terms involving $\chi_{1,2}^b$ ($\eta_{1,2,3}^b$)—enter in just one linear combination for all $D^{1/2^+}$ ($D^{3/2^+}$) form factors, as do the heavy quark spin symmetry conserving subleading Isgur-Wise functions, $\chi_{\rm ke}^{c,b}$ ($\eta_{\rm ke}^{c,b}$). These can, therefore, be absorbed into the leading-order Isgur-Wise functions via the replacements,

$$\zeta + \varepsilon_c\chi_{\rm ke}^c + \varepsilon_b[\chi_{\rm ke}^b + 6\chi_1^b - 2(w+1)\chi_2^b] \to \zeta,$$
$$\tau + \varepsilon_c\eta_{\rm ke}^c + \varepsilon_b[\eta_{\rm ke}^b + 6\eta_1^b - 2(w-1)\eta_2^b + \eta_3^b] \to \tau. \quad (10)$$

These replacements formally introduce $\mathcal{O}(\Lambda_{\rm QCD}^2/m_{c,b}^2)$ errors. Because the $\Lambda_{\rm QCD}/m_{c,b}$ terms themselves can be dominant near zero recoil, these $\mathcal{O}(\Lambda_{\rm QCD}^2/m_{c,b}^2)$ corrections may in practice be sizable.

Hereafter it is understood that the replacements in Eq. (10) are made, unless explicitly noted otherwise. As in Table III, we omit the c superscript from the remaining (unabsorbed) subleading Isgur-Wise functions. We further define $\hat{\chi}_{1,2} = \chi_{1,2}/\zeta$, $\hat{\zeta}_1 = \zeta_1/\zeta$, $\hat{\eta}_{1,2,3} = \eta_{1,2,3}/\tau$, and $\hat{\tau}_{1,2} = \tau_{1,2}/\tau$. With these conventions, the $\hat{M}_i$ and $\hat{N}_i$ coefficient functions in Eq. (8) are

$$\hat{M}_1 = 6\hat{\chi}_1 - 2\hat{\chi}_2(w+1), \qquad \hat{M}_2 = -2\hat{\chi}_1,$$
$$\hat{M}_3 = 2\hat{\chi}_2, \qquad \hat{M}_4 = 2\hat{\zeta}_1(w+1) - 3\frac{\bar{\Lambda}^* w - \bar{\Lambda}}{w+1},$$
$$\hat{M}_5 = -\frac{\bar{\Lambda}^* w - \bar{\Lambda}}{w+1}, \qquad \hat{M}_6 = -2\hat{\zeta}_1,$$
$$\hat{G}_b = \frac{(1+2w)\bar{\Lambda}^* - (2+w)\bar{\Lambda}}{w+1} - 2(w-1)\hat{\zeta}_1(w), \quad (11)$$

and

$$\hat{N}_1 = -\hat{\eta}_3(w+1), \qquad \hat{N}_2 = \hat{\eta}_3 - 2\hat{\eta}_1,$$
$$\hat{N}_3 = -2\hat{\eta}_2, \qquad \hat{N}_4 = -(2\hat{\eta}_1 + 3\hat{\eta}_3)(w+1),$$
$$\hat{N}_5 = 10\hat{\eta}_1 + 4\hat{\eta}_2(w-1) - 5\hat{\eta}_3,$$
$$\hat{N}_6 = (\hat{\tau}_1 - \hat{\tau}_2)(w-1) - (\bar{\Lambda}' w - \bar{\Lambda}),$$
$$\hat{N}_7 = -(\hat{\tau}_1 - \hat{\tau}_2), \qquad \hat{N}_8 = -2\hat{\tau}_1,$$
$$\hat{N}_9 = 3(\hat{\tau}_1 - \hat{\tau}_2)(w-1) - 4(\bar{\Lambda}' w - \bar{\Lambda}),$$
$$\hat{N}_{10} = \hat{\tau}_1(4w-1) + 5\hat{\tau}_2,$$
$$\hat{F}_b = \bar{\Lambda} + \bar{\Lambda}' - (2w+1)\hat{\tau}_1 - \hat{\tau}_2. \quad (12)$$

The $\Lambda_{\rm QCD}/m_b$ corrections not absorbed into the leading-order Isgur-Wise functions occur exclusively in the $\hat{F}_b$ and $\hat{G}_b$ linear combinations. (The sign difference between these terms in Eqs. (7a) and (7b) is simply due to defining their known $\bar{\Lambda}^{(\prime,*)}$ parts to be positive.)

C. $\bar{B} \to D^{1/2^+}$ form factors

We define form factors in agreement with those in Refs. [13,14] for the SM currents. For $\bar{B} \to D_0^*$,

$$\langle D_0^*|\bar{c}b|\bar{B}\rangle = \langle D_0^*|\bar{c}\gamma_\mu b|\bar{B}\rangle = 0,$$
$$\langle D_0^*|\bar{c}\gamma_5 b|\bar{B}\rangle = \sqrt{m_{D_0^*}m_B}\,g_P,$$
$$\langle D_0^*|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}\rangle = \sqrt{m_{D_0^*}m_B}[g_+(v_\mu + v'_\mu) + g_-(v_\mu - v'_\mu)],$$
$$\langle D_0^*|\bar{c}\sigma_{\mu\nu} b|\bar{B}\rangle = \sqrt{m_{D_0^*}m_B}\,g_T\varepsilon_{\mu\nu\alpha\beta}v^\alpha v'^\beta, \quad (13)$$

and for $\bar{B} \to D_1^*$,

$$\langle D_1^*|\bar{c}b|\bar{B}\rangle = -\sqrt{m_{D_1^*}m_B}\,g_S(\epsilon^*\cdot v),$$
$$\langle D_1^*|\bar{c}\gamma_5 b|\bar{B}\rangle = 0,$$
$$\langle D_1^*|\bar{c}\gamma_\mu b|\bar{B}\rangle = \sqrt{m_{D_1^*}m_B}[g_{V_1}\epsilon^*_\mu + (g_{V_2}v_\mu + g_{V_3}v'_\mu)(\epsilon^*\cdot v)],$$
$$\langle D_1^*|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}\rangle = i\sqrt{m_{D_1^*}m_B}\,g_A\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v^\beta v'^\gamma,$$
$$\langle D_1^*|\bar{c}\sigma_{\mu\nu} b|\bar{B}\rangle = i\sqrt{m_{D_1^*}m_B}[g_{T_1}(\epsilon^*_\mu v_\nu - \epsilon^*_\nu v_\mu) + g_{T_2}(\epsilon^*_\mu v'_\nu - \epsilon^*_\nu v'_\mu) + g_{T_3}(\epsilon^*\cdot v)(v_\mu v'_\nu - v_\nu v'_\mu)]. \quad (14)$$

These form factors are dimensionless functions of w, and hereafter we often suppress displaying the w variable. In the heavy quark limit, each of these form factors either vanishes or is determined by the Isgur-Wise function,

$$g_+ = g_{V_2} = g_{T_3} = 0, \qquad g_P = g_{V_1} = (w-1)\zeta,$$
$$g_- = g_T = g_S = g_A = -g_{V_3} = -g_{T_1} = g_{T_2} = \zeta. \quad (15)$$

Unlike the leading-order Isgur-Wise function in $\bar{B} \to D^{(*)}$ decays, the function $\zeta(w)$ is not subject to any symmetry imposed normalization condition. Near zero recoil, only g_P, g_{V_1}, and the linear combination $g_{T_1} + g_{T_2}$ contribute to the decay rates without a $(w-1)$ suppression [in addition to the $\sqrt{w^2-1}$ phase space factor; see Eqs. (30) and (31) below], so heavy quark symmetry implies that these form factors in the heavy quark limit are suppressed near zero recoil as $(w-1)\zeta$. This is why the order $\Lambda_{\rm QCD}/m_{c,b}$ terms have enhanced significance for these decays [13,14]. At order $\Lambda_{\rm QCD}^2/m_{c,b}^2$ and higher, the expansion is expected to behave as suggested by the power counting.

To write the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections in a compact form, we define

$$\hat{g}_i(w) = g_i(w)/\zeta(w). \tag{16}$$

Denoting $\hat{\alpha}_s = \alpha_s/\pi$, we obtain

$$\begin{aligned}
\hat{g}_P &= (w-1)(1+\hat{\alpha}_s C_P) + \varepsilon_c\{3(w\bar{\Lambda}^* - \bar{\Lambda}) - 2(w^2-1)\hat{\zeta}_1 + (w-1)[6\hat{\chi}_1 - 2(w+1)\hat{\chi}_2]\} - \varepsilon_b(w+1)\hat{G}_b,\\
\hat{g}_+ &= \hat{\alpha}_s(w-1)\frac{C_{A_2}+C_{A_3}}{2} - \varepsilon_c\left[3\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} - 2(w-1)\hat{\zeta}_1\right] - \varepsilon_b\hat{G}_b,\\
\hat{g}_- &= 1 + \hat{\alpha}_s\left[C_{A_1} + (w-1)\frac{C_{A_2}-C_{A_3}}{2}\right] + \varepsilon_c[6\hat{\chi}_1 - 2(w+1)\hat{\chi}_2],\\
\hat{g}_T &= 1 + \hat{\alpha}_s C_{T_1} + \varepsilon_c\left[3\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} - 2(w-1)\hat{\zeta}_1 + 6\hat{\chi}_1 - 2(w+1)\hat{\chi}_2\right] - \varepsilon_b\hat{G}_b.
\end{aligned}\tag{17}$$

The C_i's encode the α_s corrections (and are given in Appendix A in Ref. [2]) and correspond to integrating out the b and c quarks at a common scale, chosen as $\mu = \sqrt{m_c m_b}$. For the order $\Lambda_{\rm QCD}/m_{c,b}$ and α_s contributions to the $\bar{B} \to D_1^*$ form factors, we obtain

$$\begin{aligned}
\hat{g}_S &= 1 + \hat{\alpha}_s C_S - \varepsilon_c\left[\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} - 2(w-1)\hat{\zeta}_1 + 2\hat{\chi}_1 - 2(w+1)\hat{\chi}_2\right] - \varepsilon_b\hat{G}_b,\\
\hat{g}_{V_1} &= (w-1)(1+\hat{\alpha}_s C_{V_1}) + \varepsilon_c[w\bar{\Lambda}^* - \bar{\Lambda} - 2(w-1)\hat{\chi}_1] - \varepsilon_b(w+1)\hat{G}_b,\\
\hat{g}_{V_2} &= -\hat{\alpha}_s C_{V_2} + \varepsilon_c(2\hat{\zeta}_1 - 2\hat{\chi}_2),\\
\hat{g}_{V_3} &= -1 - \hat{\alpha}_s(C_{V_1}+C_{V_3}) - \varepsilon_c\left(\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} + 2\hat{\zeta}_1 - 2\hat{\chi}_1 + 2\hat{\chi}_2\right) + \varepsilon_b\hat{G}_b,\\
\hat{g}_A &= 1 + \hat{\alpha}_s C_{A_1} + \varepsilon_c\left(\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} - 2\hat{\chi}_1\right) - \varepsilon_b\hat{G}_b,\\
\hat{g}_{T_1} &= -1 - \hat{\alpha}_s[C_{T_1} + (w-1)C_{T_2}] + \varepsilon_c\left(\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} + 2\hat{\chi}_1\right) + \varepsilon_b\hat{G}_b,\\
\hat{g}_{T_2} &= 1 + \hat{\alpha}_s[C_{T_1} - (w-1)C_{T_3}] + \varepsilon_c\left(\frac{w\bar{\Lambda}^*-\bar{\Lambda}}{w+1} - 2\hat{\chi}_1\right) + \varepsilon_b\hat{G}_b,\\
\hat{g}_{T_3} &= -\hat{\alpha}_s C_{T_2} + \varepsilon_c(2\hat{\zeta}_1 + 2\hat{\chi}_2).
\end{aligned}\tag{18}$$

In Eqs. (17) and (18), the expressions for the SM terms (g_+, g_-, g_{V_1}, g_{V_2}, g_{V_3}, and g_A) agree with Refs. [13,14]. Only four functions of w are needed to parametrize all twelve $\bar{B} \to D^{1/2^+}$ form factors in Eqs. (17) and (18) at this order: ζ, $\hat{\zeta}_1$, and $\hat{\chi}_{1,2}$. Only the $6\hat{\chi}_1 - 2(w+1)\hat{\chi}_2 = \hat{M}_1$ linear combination of $\hat{\chi}_1$ and $\hat{\chi}_2$ occurs for $\bar{B} \to D_0^*$ in Eq. (17), as expected from Eq. (8).

D. $\bar{B} \to D^{3/2^+}$ form factors

We define the form factors for $\bar{B} \to D^{3/2^+}$ such that they agree with those in Refs. [13,14] for the SM terms,

$$\begin{aligned}
\langle D_1|\bar{c}b|\bar{B}\rangle &= \sqrt{m_{D_1}m_B}f_S(\epsilon^*\cdot v),\\
\langle D_1|\bar{c}\gamma_5 b|\bar{B}\rangle &= 0,\\
\langle D_1|\bar{c}\gamma_\mu b|\bar{B}\rangle &= \sqrt{m_{D_1}m_B}[f_{V_1}\epsilon^*_\mu\\
&\quad + (f_{V_2}v_\mu + f_{V_3}v'_\mu)(\epsilon^*\cdot v)],\\
\langle D_1|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}\rangle &= i\sqrt{m_{D_1}m_B}f_A\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v^\beta v'^\gamma,\\
\langle D_1|\bar{c}\sigma_{\mu\nu}b|\bar{B}\rangle &= i\sqrt{m_{D_1}m_B}[f_{T_1}(\epsilon^*_\mu v_\nu - \epsilon^*_\nu v_\mu)\\
&\quad + f_{T_2}(\epsilon^*_\mu v'_\nu - \epsilon^*_\nu v'_\mu)\\
&\quad + f_{T_3}(\epsilon^*\cdot v)(v_\mu v'_\nu - v_\nu v'_\mu)],
\end{aligned}\tag{19}$$

while for $\bar{B} \to D_2^*$,

$$\begin{aligned}
\langle D_2^*|\bar{c}b|\bar{B}\rangle &= 0,\\
\langle D_2^*|\bar{c}\gamma_5 b|\bar{B}\rangle &= \sqrt{m_{D_2^*}m_B}\,k_P\epsilon^*_{\alpha\beta}v^\alpha v^\beta,\\
\langle D_2^*|\bar{c}\gamma_\mu b|\bar{B}\rangle &= i\sqrt{m_{D_2^*}m_B}\,k_V\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha\sigma}v_\sigma v^\beta v'^\gamma,\\
\langle D_2^*|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}\rangle &= \sqrt{m_{D_2^*}m_B}[k_{A_1}\epsilon^*_{\mu\alpha}v^\alpha\\
&\quad + (k_{A_2}v_\mu + k_{A_3}v'_\mu)\epsilon^*_{\alpha\beta}v^\alpha v^\beta],\\
\langle D_2^*|\bar{c}\sigma_{\mu\nu} b|\bar{B}\rangle &= \sqrt{m_{D_2^*}m_B}\,\varepsilon_{\mu\nu\alpha\beta}\{[k_{T_1}(v+v')^\alpha\\
&\quad + k_{T_2}(v-v')^\alpha)]\epsilon^{*\gamma\beta}v_\gamma\\
&\quad + k_{T_3}v^\alpha v'^\beta\epsilon^{*\rho\sigma}v_\rho v_\sigma\}. \qquad (20)
\end{aligned}$$

The form factors f_i and k_i are again dimensionless functions of w. In the heavy quark limit, each of these form factors either vanishes or is determined by the Isgur-Wise function, $\tau(w)$. (This Isgur-Wise function is different from $\zeta(w)$, although model calculations can relate the two). The simple parametrizations in Eqs. (19) and (20) yield the slightly complicated relations

$$\begin{aligned}
&k_{A_2} = k_{T_2} = k_{T_3} = 0, \qquad f_{V_1} = (1-w^2)\tau/\sqrt{6},\\
&f_{V_3} = (w-2)\tau/\sqrt{6}, \qquad -f_{V_2} = f_{T_3} = 3\tau/\sqrt{6},\\
&f_A = f_S/2 = -f_{T_1} = f_{T_2} = k_{A_1}/\sqrt{6} = -(w+1)\tau/\sqrt{6},\\
&k_P = -k_V = k_{A_3} = k_{T_1} = \tau. \qquad (21)
\end{aligned}$$

At zero recoil where $w = 1$ and $v = v'$, only the f_{V_1} form factor can contribute (as well as the linear combination $f_{T_1} + f_{T_2}$), since $\epsilon^\mu v'_\mu$ and $\epsilon^{\mu\nu} v'_\nu$ vanish. Heavy quark symmetry implies that $f_{V_1}(1)$ is either of order $\Lambda_{\rm QCD}/m_{c,b}$, or its dependence on the leading Isgur-Wise function, $\tau(w)$, is suppressed by $(w-1)$ [17]. This is why, as explained above, the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ terms are so significant for semileptonic decays to excited charmed mesons.

We define in analogy with Eq. (16),

$$\hat{f}_i(w) = f_i(w)/\tau(w), \qquad \hat{k}_i(w) = k_i(w)/\tau(w). \qquad (22)$$

For the order $\Lambda_{\rm QCD}/m_{c,b}$ and α_s contributions to the $\bar{B} \to D_1$ form factors, we obtain

$$\begin{aligned}
\sqrt{6}\hat{f}_S &= -2(w+1)(1+\hat{\alpha}_s C_S) - \varepsilon_b 2(w-1)\hat{F}_b\\
&\quad - \varepsilon_c\{4(w\bar{\Lambda}'-\bar{\Lambda}) - 2(w-1)[(2w+1)\hat{\tau}_1+\hat{\tau}_2] + 2(w+1)[6\hat{\eta}_1+2(w-1)\hat{\eta}_2-\hat{\eta}_3]\},\\
\sqrt{6}\hat{f}_{V_1} &= (1-w^2)(1+\hat{\alpha}_s C_{V_1}) - \varepsilon_b(w^2-1)\hat{F}_b - \varepsilon_c[4(w+1)(w\bar{\Lambda}'-\bar{\Lambda}) - (w^2-1)(3\hat{\tau}_1-3\hat{\tau}_2+2\hat{\eta}_1+3\hat{\eta}_3)],\\
\sqrt{6}\hat{f}_{V_2} &= -3 - \hat{\alpha}_s[3C_{V_1}+2(1+w)C_{V_2}] - \varepsilon_b 3\hat{F}_b - \varepsilon_c[(4w-1)\hat{\tau}_1+5\hat{\tau}_2+10\hat{\eta}_1+4(w-1)\hat{\eta}_2-5\hat{\eta}_3],\\
\sqrt{6}\hat{f}_{V_3} &= w-2-\hat{\alpha}_s[(2-w)C_{V_1}+2(1+w)C_{V_3}] + \varepsilon_b(2+w)\hat{F}_b\\
&\quad + \varepsilon_c[4(w\bar{\Lambda}'-\bar{\Lambda}) + (2+w)\hat{\tau}_1 + (2+3w)\hat{\tau}_2 - 2(6+w)\hat{\eta}_1 - 4(w-1)\hat{\eta}_2 - (3w-2)\hat{\eta}_3],\\
\sqrt{6}\hat{f}_A &= -(w+1)(1+\hat{\alpha}_s C_{A_1}) - \varepsilon_b(w-1)\hat{F}_b - \varepsilon_c[4(w\bar{\Lambda}'-\bar{\Lambda}) - 3(w-1)(\hat{\tau}_1-\hat{\tau}_2) - (w+1)(2\hat{\eta}_1+3\hat{\eta}_3)],\\
\sqrt{6}\hat{f}_{T_1} &= (w+1)[1+\hat{\alpha}_s[C_{T_1}+(w-1)C_{T_2}]] + \varepsilon_b(w-1)\hat{F}_b - \varepsilon_c[4(w\bar{\Lambda}'-\bar{\Lambda}) - 3(w-1)(\hat{\tau}_1-\hat{\tau}_2) + (w+1)(2\hat{\eta}_1+3\hat{\eta}_3)],\\
\sqrt{6}\hat{f}_{T_2} &= -(w+1)[1+\hat{\alpha}_s[C_{T_1}-(w-1)C_{T_3}]] + \varepsilon_b(w-1)\hat{F}_b - \varepsilon_c[4(w\bar{\Lambda}'-\bar{\Lambda}) - 3(w-1)(\hat{\tau}_1-\hat{\tau}_2) - (w+1)(2\hat{\eta}_1+3\hat{\eta}_3)],\\
\sqrt{6}\hat{f}_{T_3} &= 3+\hat{\alpha}_s[3C_{T_1}-(2-w)C_{T_2}+3C_{T_3}] + \varepsilon_b 3\hat{F}_b - \varepsilon_c[(4w-1)\hat{\tau}_1+5\hat{\tau}_2-10\hat{\eta}_1-4(w-1)\hat{\eta}_2+5\hat{\eta}_3]. \qquad (23)
\end{aligned}$$

For the order $\Lambda_{\rm QCD}/m_{c,b}$ and α_s contributions to the $\bar{B} \to D_2^*$ form factors, we obtain

$$\begin{aligned}
\hat{k}_P &= 1+\hat{\alpha}_s C_P + \varepsilon_b\hat{F}_b + \varepsilon_c[(2w+1)\hat{\tau}_1+\hat{\tau}_2-2\hat{\eta}_1-2(w-1)\hat{\eta}_2+\hat{\eta}_3],\\
\hat{k}_V &= -1-\hat{\alpha}_s C_{V_1} - \varepsilon_b\hat{F}_b - \varepsilon_c(\hat{\tau}_1-\hat{\tau}_2-2\hat{\eta}_1+\hat{\eta}_3),\\
\hat{k}_{A_1} &= -(w+1)(1+\hat{\alpha}_s C_{A_1}) - \varepsilon_b(w-1)\hat{F}_b - \varepsilon_c[(w-1)(\hat{\tau}_1-\hat{\tau}_2) - (w+1)(2\hat{\eta}_1-\hat{\eta}_3)],\\
\hat{k}_{A_2} &= \hat{\alpha}_s C_{A_2} - \varepsilon_c 2(\hat{\tau}_1+\hat{\eta}_2),\\
\hat{k}_{A_3} &= 1+\hat{\alpha}_s(C_{A_1}+C_{A_3}) + \varepsilon_b\hat{F}_b - \varepsilon_c(\hat{\tau}_1+\hat{\tau}_2+2\hat{\eta}_1-2\hat{\eta}_2-\hat{\eta}_3),\\
\hat{k}_{T_1} &= 1+\hat{\alpha}_s\left[C_{T_1}+\frac{w-1}{2}(C_{T_2}-C_{T_3})\right] - \varepsilon_c(2\hat{\eta}_1-\hat{\eta}_3),\\
\hat{k}_{T_2} &= \hat{\alpha}_s\frac{w+1}{2}(C_{T_2}+C_{T_3}) + \varepsilon_b\hat{F}_b - \varepsilon_c(\hat{\tau}_1-\hat{\tau}_2),\\
\hat{k}_{T_3} &= -\hat{\alpha}_s C_{T_2} + \varepsilon_c 2(\hat{\tau}_1-\hat{\eta}_2). \qquad (24)
\end{aligned}$$

In Eqs. (23) and (24), the relations for the SM terms (f_{V_1}, $f_{V_2}, f_{V_3}, f_A, k_{V_1}, k_{V_2}, k_{V_3}$, and k_A) agree with Refs. [13,14]. In this case, six functions of w are needed to parametrize all sixteen $\bar{B} \to D^{3/2^+}$ form factors in Eqs. (23) and (24), including all $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections: τ, $\hat{\tau}_{1,2}$, and $\hat{\eta}_{1,2,3}$.

E. Equations of motion

As in Ref. [2], we can verify the relations stemming from the QCD equations of motion between the (pseudo)scalar and the (axial)vector matrix elements,

$$\begin{aligned}
-[\bar{m}_b(\mu) + \bar{m}_c(\mu)]\langle D_0^*|\bar{c}\gamma_5 b|\bar{B}\rangle &= \langle D_0^*|\bar{c}\not{q}\gamma_5 b|\bar{B}\rangle, \\
[\bar{m}_b(\mu) - \bar{m}_c(\mu)]\langle D_1^*|\bar{c} b|\bar{B}\rangle &= \langle D_1^*|\bar{c}\not{q} b|\bar{B}\rangle, \\
-[\bar{m}_b(\mu) + \bar{m}_c(\mu)]\langle D_2^*|\bar{c}\gamma_5 b|\bar{B}\rangle &= \langle D_2^*|\bar{c}\not{q}\gamma_5 b|\bar{B}\rangle, \\
[\bar{m}_b(\mu) - \bar{m}_c(\mu)]\langle D_1|\bar{c} b|\bar{B}\rangle &= \langle D_1|\bar{c}\not{q} b|\bar{B}\rangle. \qquad (25)
\end{aligned}$$

Using $m_b = m_B - \bar{\Lambda} + \mathcal{O}(\Lambda_{\rm QCD}^2/m_b)$ and $m_c = m_{D^{**}} - \bar{\Lambda}^{\prime,*} + \mathcal{O}(\Lambda_{\rm QCD}^2/m_c)$ imply that all first-order $\Lambda_{\rm QCD}/m_{c,b}$ and α_s corrections agree in Eq. (25) as they must. Note that the left-hand sides of Eq. (25) contain the running quark masses at the common scale μ. At order α_s, the results are sensitive to this, and the $\mathcal{O}(\alpha_s)$ terms from the expansion of

$$m_Q = \bar{m}_Q(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\frac{4}{3} - \ln\frac{m_Q^2}{\mu^2}\right) + \cdots\right], \qquad (26)$$

are required to be present for Eq. (25) to be satisfied. As emphasized in Ref. [2], it is probably better to evaluate the scalar and pseudoscalar matrix elements using Eqs. (17), (18), (23), and (24) instead of Eq. (25), because the natural choice for μ is below m_b. In the $\overline{\rm MS}$ scheme, fermions do not decouple for $\mu < m$, introducing artificially large corrections in the running, compensated by corresponding spurious terms in the β function computed without integrating out heavy quarks [30].

III. $B \to D^{**}\ell\bar{\nu}$ RATES FOR GENERIC NP

It is straightforward to calculate the $B \to D^{**}\ell\bar{\nu}$ rates including lepton mass effects and all possible four-fermion operators. The double differential distributions in the SM were written down in Ref. [12], the integrals of which agree with the expressions below. Here we give the single differential distributions as the SM plus generic NP. These expressions can be used with any form factor input to study the SM predictions and possible patterns of NP in $R(D^{**})$. In Appendix A, we provide explicit results for the $\bar{B} \to D^{**}\ell\bar{\nu}$ amplitudes, which may be used in combination with D^{**} and τ decay amplitudes to derive fully differential distributions of the visible decay products in $\bar{B} \to D^{**}\ell\bar{\nu}$, including all interference effects [31].

We consider the following complete basis for the four-Fermi operators mediating $b \to c\ell\bar{\nu}$ decay:

$$\text{SM}: \; i2\sqrt{2}V_{cb}G_F[\bar{c}\gamma^\mu P_L b][\bar{\ell}\gamma_\mu P_L \nu], \qquad (27a)$$

$$\begin{aligned}\text{Vector}: \; & i2\sqrt{2}V_{cb}G_F[\bar{c}(\tilde{\alpha}_L^V\gamma^\mu P_L + \tilde{\alpha}_R^V\gamma^\mu P_R)b] \\ & \times [\bar{\ell}(\tilde{\beta}_L^V\gamma_\mu P_L + \tilde{\beta}_R^V\gamma_\mu P_R)\nu], \qquad (27b)\end{aligned}$$

$$\begin{aligned}\text{Scalar}: \; & i2\sqrt{2}V_{cb}G_F[\bar{c}(\tilde{\alpha}_L^S P_L + \tilde{\alpha}_R^S P_R)b] \\ & \times [\bar{\ell}(\tilde{\beta}_L^S P_L + \tilde{\beta}_R^S P_R)\nu], \qquad (27c)\end{aligned}$$

$$\begin{aligned}\text{Tensor}: \; & i2\sqrt{2}V_{cb}G_F[(\bar{c}\tilde{\alpha}_L^T\sigma^{\mu\nu}P_L b)(\bar{\ell}\tilde{\beta}_L^T\sigma_{\mu\nu}P_L\nu) \\ & + (\bar{c}\tilde{\alpha}_R^T\sigma^{\mu\nu}P_R b)(\bar{\ell}\tilde{\beta}_R^T\sigma_{\mu\nu}P_R\nu)]. \qquad (27d)\end{aligned}$$

The NP couplings to the quark and lepton currents are denoted by $\tilde{\alpha}_j^i$ and $\tilde{\beta}_j^i$, respectively. The lower index of $\tilde{\beta}$ denotes the ν helicity and the lower index of $\tilde{\alpha}$ is that of the b quark. (This notation is a variation of the conventions chosen in Ref. [31], whence the seemingly superfluous tildes. See Appendix A for details.) The NP couplings $\tilde{\alpha}_j^i$ and $\tilde{\beta}_l^k$ may be complex, and $\tilde{\alpha}_j^i\tilde{\beta}_l^i$ products are normalized with respect to the SM couplings, such that setting $\tilde{\alpha}_L^V\tilde{\beta}_L^V = 1$ would amount to doubling the coefficient of the $(\bar{c}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L\nu)$ operator compared to its SM value. The CP conjugate operators for $\bar{b} \to \bar{c}\,\bar{\ell}\,\nu$ are obtained by Hermitian conjugation. The operators involving right-handed neutrinos are included for completeness, but do not interfere with the SM (neglecting m_ν suppressed terms).

We define the dimensionless ratios,

$$r = m_{D^{**}}/m_B, \qquad \rho_\ell = m_\ell^2/m_B^2, \qquad (28)$$

as well as

$$\hat{q}^2 = \frac{q^2}{m_B^2} = 1 + r^2 - 2rw, \qquad \Gamma_0 = \frac{G_F^2|V_{cb}|^2 m_B^5}{192\pi^3}. \qquad (29)$$

In the rest of this section, for brevity, we suppress indication of absolute value squared for the NP^2 terms, so that all $(\tilde{\alpha}_j^i\tilde{\beta}_l^i)^2$ terms mean $|\tilde{\alpha}_j^i\tilde{\beta}_l^i|^2$. Similarly, all interference terms are understood as the appropriate real parts: terms linear in NP couplings (coming from SM–NP interference) of the form $\tilde{\alpha}_j^i\tilde{\beta}_k^i$ mean $\mathrm{Re}(\tilde{\alpha}_j^i\tilde{\beta}_l^i)$, while bilinear terms in NP couplings (from NP–NP interference) of the form $\tilde{\alpha}_j^i\tilde{\beta}_m^i\tilde{\alpha}_l^k\tilde{\beta}_n^k$ mean $\mathrm{Re}(\tilde{\alpha}_j^i\tilde{\beta}_m^i\tilde{\alpha}_l^{k\,*}\tilde{\beta}_n^{k\,*})$.

Considering only left-handed neutrinos, we obtain for the $B \to D_0^*\ell\bar{\nu}$ rate,

$$\frac{\mathrm{d}\Gamma^{(\mathrm{SM})}_{D_0^*}}{\mathrm{d}w} = 4\Gamma_0 r^3\sqrt{w^2-1}\,\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{g_-^2(w-1)[\rho_\ell[(1+r^2)(2w-1)+2r(w-2)]+(1-r)^2(w+1)\hat{q}^2] + g_+^2(w+1)[\rho_\ell[(1+r^2)(2w+1)-2r(w+2)]+(1+r)^2(w-1)\hat{q}^2]-2g_-g_+(1-r^2)(w^2-1)(\hat{q}^2+2\rho_\ell)\}. \tag{30a}$$

$$\frac{\mathrm{d}\Gamma_{D_0^*}}{\mathrm{d}w} = \frac{\mathrm{d}\Gamma^{(\mathrm{SM})}_{D_0^*}}{\mathrm{d}w}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)^2 + 2\Gamma_0 r^3\sqrt{w^2-1}\,\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^4}\Big\{3[(\tilde{\alpha}_R^S-\tilde{\alpha}_L^S)\tilde{\beta}_L^S]^2 g_P^2\hat{q}^2 + 6(\tilde{\alpha}_R^S-\tilde{\alpha}_L^S)\tilde{\beta}_L^S g_P\sqrt{\rho_\ell}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)[g_-(1+r)(w-1)-g_+(1-r)(w+1)] + 8\tilde{\alpha}_L^T\tilde{\beta}_L^T g_T(w^2-1)[2\tilde{\alpha}_L^T\tilde{\beta}_L^T g_T(\hat{q}^2+2\rho_\ell)+3\sqrt{\rho_\ell}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)[g_+(1+r)-g_-(1-r)]]\Big\}. \tag{30b}$$

For the narrow D_1 state ($s_\ell^{\pi_\ell}=\frac{3}{2}^+$), we find

$$\frac{\mathrm{d}\Gamma^{(\mathrm{SM})}_{D_1}}{\mathrm{d}w} = 2\Gamma_0 r^3\sqrt{w^2-1}\,\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{f_{V_1}^2[2\hat{q}^2[(w-r)^2+2\hat{q}^2]+\rho_\ell[4(w-r)^2-\hat{q}^2]] + (w^2-1)(f_{V_2}^2[2r^2\hat{q}^2(w^2-1)+\rho_\ell[3\hat{q}^2+4r^2(w^2-1)]]+f_{V_3}^2[2\hat{q}^2(w^2-1)+\rho_\ell[4(w-r)^2-\hat{q}^2]] + 2f_A^2\hat{q}^2(2\hat{q}^2+\rho_\ell)+2f_{V_1}f_{V_2}[2r\hat{q}^2(w-r)+\rho_\ell(3-r^2-2rw)]+4f_{V_1}f_{V_3}(w-r)(\hat{q}^2+2\rho_\ell) + 2f_{V_2}f_{V_3}[2r\hat{q}^2(w^2-1)+\rho_\ell[3w\hat{q}^2+4r(w^2-1)]])\}, \tag{31a}$$

$$\frac{\mathrm{d}\Gamma_{D_1}}{\mathrm{d}w} = \frac{\mathrm{d}\Gamma^{(\mathrm{SM})}_{D_1}}{\mathrm{d}w}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)^2 + 2\Gamma_0 r^3\sqrt{w^2-1}\,\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{3[(\tilde{\alpha}_L^S+\tilde{\alpha}_R^S)\tilde{\beta}_L^S]^2 f_S^2(w^2-1)\hat{q}^4 + 6(\tilde{\alpha}_L^S+\tilde{\alpha}_R^S)\tilde{\beta}_L^S(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V+\tilde{\alpha}_R^V\tilde{\beta}_L^V)f_S(w^2-1)\hat{q}^2\sqrt{\rho_\ell}[f_{V_1}+f_{V_2}(1-rw)+f_{V_3}(w-r)] + 16(\tilde{\alpha}_L^T\tilde{\beta}_L^T)^2(\hat{q}^2+2\rho_\ell)(f_{T_1}^2[\hat{q}^2(2+w^2)+4r^2(w^2-1)]+f_{T_2}^2[4(w-r)^2-\hat{q}^2]+f_{T_3}^2\hat{q}^2(w^2-1)^2 + 2f_{T_1}f_{T_2}[3w\hat{q}^2+4r(w^2-1)]-2f_{T_3}(f_{T_1}w+f_{T_2})\hat{q}^2(w^2-1)) - 24\tilde{\alpha}_L^T\tilde{\beta}_L^T\sqrt{\rho_\ell}\hat{q}^2((1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)2f_A(f_{T_1}r+f_{T_2})(w^2-1) - (1+\tilde{\alpha}_L^V\tilde{\beta}_L^V+\tilde{\alpha}_R^V\tilde{\beta}_L^V)[2f_{T_1}f_{V_1}(1-rw)+[wf_{T_1}+3f_{T_2}-f_{T_3}(w^2-1)]f_{V_1}(w-r) + [wf_{T_1}+f_{T_2}-f_{T_3}(w^2-1)](f_{V_2}r+f_{V_3})(w^2-1)]) + 4\tilde{\alpha}_R^V\tilde{\beta}_L^V(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V)(3f_{V_1}^2\hat{q}^2(2\hat{q}^2+\rho_\ell)+2f_{V_1}[f_{V_1}+2f_{V_3}(w-r)](w^2-1)(\hat{q}^2+2\rho_\ell) + (w^2-1)(f_{V_2}^2[2r^2\hat{q}^2(w^2-1)+\rho_\ell[3\hat{q}^2+4r^2(w^2-1)]]+f_{V_3}^2[2\hat{q}^2(w^2-1)+\rho_\ell[4(w-r)^2-\hat{q}^2]] + 2f_{V_1}f_{V_2}[2r\hat{q}^2(w-r)+\rho_\ell(3-r^2-2rw)]+2f_{V_2}f_{V_3}[2r\hat{q}^2(w^2-1)+\rho_\ell[3w\hat{q}^2+4r(w^2-1)]]))\}. \tag{31b}$$

The result for the broad D_1^* state ($s_\ell^{\pi_\ell}=\frac{1}{2}^+$) can be obtained from Eqs. (31a) and (31b) via the replacements $f_S\to -g_S, f_A\to g_A, f_{V_i}\to g_{V_i}, f_{T_i}\to g_{T_i}$, where $i=1, 2, 3$. For $\mathrm{d}\Gamma(B\to D_2^*\ell\bar{\nu})/\mathrm{d}w$, the result is

$$\frac{\mathrm{d}\Gamma^{(\mathrm{SM})}_{D_2^*}}{\mathrm{d}w} = \frac{2\Gamma_0}{3}r^3(w^2-1)^{3/2}\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{k_{A_1}^2[2\hat{q}^2[2(w-r)^2+3\hat{q}^2]+\rho_\ell[8(w-r)^2-3\hat{q}^2]] + 2(w^2-1)(k_{A_2}^2[2r^2\hat{q}^2(w^2-1)+\rho_\ell[3\hat{q}^2+4r^2(w^2-1)]]+k_{A_3}^2[2\hat{q}^2(w^2-1)+\rho_\ell[4(w-r)^2-\hat{q}^2]] + 3k_V^2\hat{q}^2(\hat{q}^2+\rho_\ell/2)+2k_{A_1}k_{A_2}[2r\hat{q}^2(w-r)+\rho_\ell(3-r^2-2rw)]+4k_{A_1}k_{A_3}(w-r)(\hat{q}^2+2\rho_\ell) + 2k_{A_2}k_{A_3}[2r\hat{q}^2(w^2-1)+\rho_\ell[3w\hat{q}^2+4r(w^2-1)]])\}, \tag{32a}$$

$$\frac{\mathrm{d}\Gamma_{D_2^*}}{\mathrm{d}w} = \frac{\mathrm{d}\Gamma_{D_2^*}^{(\mathrm{SM})}}{\mathrm{d}w}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V - \tilde{\alpha}_R^V\tilde{\beta}_L^V)^2 + \frac{4\Gamma_0}{3} r^3 (w^2-1)^{3/2}\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}$$
$$\times \{6\tilde{\alpha}_R^V\tilde{\beta}_L^V(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V)k_V^2(w^2-1)\hat{q}^2(2\hat{q}^2+\rho_\ell) + 3[(\tilde{\alpha}_R^S-\tilde{\alpha}_L^S)\tilde{\beta}_L^S]^2 k_P^2(w^2-1)\hat{q}^4$$
$$+ 6(\tilde{\alpha}_L^S-\tilde{\alpha}_R^S)\tilde{\beta}_L^S k_P(w^2-1)\hat{q}^2\sqrt{\rho_\ell}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)[k_{A_1}+k_{A_2}(1-rw)+k_{A_3}(w-r)]$$
$$+ 16(\tilde{\alpha}_L^T\tilde{\beta}_L^T)^2(\hat{q}^2+2\rho_\ell)(k_{T_1}^2(w+1)[\hat{q}^2(4w+1)+6r(w^2-1)] + k_{T_2}^2(w-1)[\hat{q}^2(4w-1)+6r(w^2-1)]$$
$$+ k_{T_3}(w^2-1)\hat{q}^2[k_{T_3}(w^2-1)+2k_{T_1}(w+1)+2k_{T_2}(w-1)] - 4k_{T_1}k_{T_2}(w^2-1)(1+rw-2r^2))$$
$$+ 12\tilde{\alpha}_L^T\tilde{\beta}_L^T\sqrt{\rho_\ell}\hat{q}^2((w^2-1)(2(k_{A_2}r+k_{A_3})(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)[k_{T_1}(w+1)+(w-1)(k_{T_2}+k_{T_3}(1+w))]$$
$$- 3k_V(1+\tilde{\alpha}_R^V\tilde{\beta}_L^V+\tilde{\alpha}_L^V\tilde{\beta}_L^V)[k_{T_1}(1+r)-k_{T_2}(1-r)]) + k_{A_1}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)$$
$$\times [k_{T_1}(w+1)(3+2w-5r) - k_{T_2}(w-1)(3-2w+5r) + 2k_{T_3}(w^2-1)(w-r)])\}. \tag{32b}$$

In the heavy quark limit, for the SM and the tensor coupling contributions, using Eqs. (15) and (21), our results in Eqs. (30)–(32) agree with the results in Eqs. (B9)—(B12) in Ref. [32] (see also Ref. [33]), which studied the $R(D^{**})$ predictions for a NP tensor interaction using QCD sum rule predictions for the leading-order Isgur-Wise functions. In Appendix B, for completeness, we include the analogous expressions for the $B \to D^{(*)}$ rates.

In the SM, the only form factors that enter these rates without additional $(w-1)$ suppressions are g_+ for D_0^*, g_{V_1} for D_1^*, and f_{V_1} for D_1. In the infinite mass limit, these form factors must vanish at $w=1$ due to heavy quark symmetry. The model-independent result derived in the SM is that the $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{c,b})$ leading terms for these form factors at $w=1$ are determined by hadron mass splittings and the leading-order Isgur-Wise functions [14],

$$\hat{g}_+(1) = -\frac{3}{2}(\varepsilon_c+\varepsilon_b)(\bar{\Lambda}^*-\bar{\Lambda}) + \cdots,$$
$$\hat{g}_{V_1}(1) = (\varepsilon_c-3\varepsilon_b)(\bar{\Lambda}^*-\bar{\Lambda}) + \cdots,$$
$$\sqrt{6}\hat{f}_{V_1}(1) = -8\varepsilon_c(\bar{\Lambda}'-\bar{\Lambda}) + \cdots, \tag{33}$$

where the ellipses denote $\mathcal{O}[\alpha_s(w-1), \varepsilon_{c,b}(w-1), \varepsilon_{c,b}^2]$ and higher-order terms. All terms in $\mathrm{d}\Gamma_{D_2^*}/\mathrm{d}w$ have an overall $(w^2-1)^{3/2}$ suppression, so there is no similar constraint for that channel.

In the presence of new physics, the g_P (for D_0^*), $g_{T_{1,2}}$ (for D_1^*), and $f_{T_{1,2}}$ (for D_1) form factors also have unsuppressed contributions at $w=1$. Of these, $g_P(1)$ vanishes in the heavy quark limit, and is determined at order $\Lambda_{\mathrm{QCD}}/m_{c,b}$ by hadron mass splittings and the leading-order Isgur-Wise function similar to Eq. (33) above

$$\hat{g}_P(1) = 3(\varepsilon_c-\varepsilon_b)(\bar{\Lambda}^*-\bar{\Lambda}) + \cdots. \tag{34}$$

The $f_{T_{1,2}}$ and $g_{T_{1,2}}$ form factors are proportional to the respective Isgur-Wise functions, ζ and τ, in the heavy quark limit, without any $(w-1)$ factors, and their contributions to $\mathrm{d}\Gamma/\mathrm{d}w$ neither include a $(w-1)$ suppression. However, in the heavy quark limit, $g_{T_1} = -g_{T_2}$ and $f_{T_1} = -f_{T_2}$, so only $f_{T_1}+f_{T_2}$ and $g_{T_1}+g_{T_2}$ terms appear in the rates. One sees in Eq. (31b) that the sum of the terms proportional to $f_{T_1}^2$, $f_{T_2}^2$, and $f_{T_1}f_{T_2}$ vanishes at $w=1$ in the heavy quark limit, as it must (and similarly for the $g_{T_1}^2$, $g_{T_2}^2$, and $g_{T_1}g_{T_2}$ terms). The contributions to the rate proportional to the linear combinations $(f_{T_1}-f_{T_2})$ and $(g_{T_1}-g_{T_2})$ are suppressed by $(w-1)$, while

$$\sqrt{6}[\hat{f}_{T_1}(1)+\hat{f}_{T_2}(1)] = -8\varepsilon_c(\bar{\Lambda}'-\bar{\Lambda}) + \cdots,$$
$$\hat{g}_{T_1}(1)+\hat{g}_{T_2}(1) = (\varepsilon_c+3\varepsilon_b)(\bar{\Lambda}^*-\bar{\Lambda}) + \cdots, \tag{35}$$

are again determined at order $\Lambda_{\mathrm{QCD}}/m_{c,b}$ by hadron mass splittings and the leading-order Isgur-Wise function, similar to Eqs. (33) and (34).

IV. SOME PREDICTIONS

The data which can be used to constrain the leading and subleading Isgur-Wise functions are the four $\bar{B} \to D^{**}l\bar{\nu}$ branching ratios, the $\bar{B} \to D_2^*l\bar{\nu}$ and $\bar{B} \to D_0^*l\bar{\nu}$ spectra measured [34] in four and five bins of q^2, respectively, and the $B \to D^{3/2^+}\pi$ rates which are related to the semileptonic rates at $q^2 = m_\pi^2$ using factorization. The available data, used in our fits, is identical to that collected in Tables V–VII in Ref [12] and are not repeated here. In all fits we perform, we expand the leading-order Isgur-Wise functions in $(w-1)$ to linear order,

$$\tau(w) \simeq \tau(1)[1+\tau'(w-1)],$$
$$\zeta(w) \simeq \zeta(1)[1+\zeta'(w-1)]. \tag{36}$$

In our predictions for the $\bar{B} \to D^{**}\tau\nu$ rates and $R(D^{**})$, for simplicity we assume that the $l = e, \mu$ rates are given by the SM, and that NP may only enter the τ mode.

A. Fits in the heavy quark limit

To understand the importance of the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections, it is instructive to attempt fitting the data using the form factor parametrizations in the heavy quark limit. Fitting the $\mathrm{d}\Gamma/\mathrm{d}w$ data for D_2^* and D_0^* and the branching fractions of all four states in the heavy quark limit, using Eqs. (15) and (21), yields an unacceptably poor fit ($\chi^2/\mathrm{d.o.f.} = 80./4$). The fit is not improved with the addition of quadratic terms to Eqs. (36). Given the branching ratios and some gain in efficiencies in Belle II over Belle, one may expect sensitivity to such quadratic terms with about (5–10)/ab of Belle II data.

To better quantify this tension, we instead fit $\tau(1)$ and τ' ($\zeta(1)$ and ζ') using the branching ratio and $\mathrm{d}\Gamma/\mathrm{d}w$ for the D_2^* (D_0^*) alone, and the $\bar{B}\to D_2^*\pi$ rate. This yields good fits, with the results shown in Table IV. From these fits, one can predict the ratios,

$$R_{3/2} = \frac{\Gamma[\bar{B}\to D_2^* l\bar{\nu}]}{\Gamma[\bar{B}\to D_1 l\bar{\nu}]} \simeq 1.67\pm 0.09,$$
$$R_{1/2} = \frac{\Gamma[\bar{B}\to D_0^* l\bar{\nu}]}{\Gamma[\bar{B}\to D_1^* l\bar{\nu}]} \simeq 0.88\pm 0.07, \qquad (37)$$

to be compared to the current experimental values [12,34], $R_{3/2}\simeq 0.45\pm 0.07$ and $R_{1/2}\simeq 2.2\pm 0.7$. (The $R_{3/2}=1.67$ central value is very close to the 'B_∞' result, 1.65, in Table II in Ref. [14].) The severe tension for $R_{3/2}$, about 10σ, is evidence for the presence of large deviations from the heavy quark limit. Adding quadratic terms to Eqs. (36) does not resolve this tension. However, including the $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ corrections yields good fits [12], in alignment with the expectation that these corrections can be large because of the zero recoil suppression of the heavy quark limit terms. Therefore, hereafter, we consider only fits including both $\Lambda_{\rm QCD}/m_{c,b}$ and α_s terms.

B. Fits including the subleading terms

To deal with the several unknown subleading Isgur-Wise functions and the limited amount of experimental data, some approximations need to be made. In what is called "Approximation A" in Ref. [14], $(w-1)$ is treated as a small parameter of order $\Lambda_{\rm QCD}/m_{c,b}$, up to second-order terms are kept, and chromomagnetic terms are neglected. This reduces the number of subleading Isgur-Wise functions that enter and allows parametrization of the rates with a few numbers (rather than functions). In "Approximation B" one does not expand in $(w-1)$, but still neglects chromomagnetic terms. Finally, Ref. [12] introduced an "Approximation C", which treats $\hat{\tau}_{1,2}$ and $\hat{\zeta}_1$ as constant fit parameters. It also does not neglect the subleading Isgur-Wise functions parametrizing matrix elements of the chromomagnetic term in the Lagrangian, motivated by the fact that the $m_{D_1^*}-m_{D_0^*}$ mass splitting no longer appears much smaller than $m_{D^*}-m_D$.

We have updated the Approximation C fit to include the α_s corrections neglected in Ref. [12]. This changes the fit parameters shown in Table V only slightly compared to Ref. [12]. For $R(D^{**})$ defined in Eq. (1), we obtain

$$\begin{aligned} R(D_0^*) &= 0.08\pm 0.03, & \tilde{R}(D_0^*) &= 0.24\pm 0.05,\\ R(D_1^*) &= 0.05\pm 0.02, & \tilde{R}(D_1^*) &= 0.18\pm 0.02,\\ R(D_1) &= 0.10\pm 0.02, & \tilde{R}(D_1) &= 0.20\pm 0.02,\\ R(D_2^*) &= 0.07\pm 0.01, & \tilde{R}(D_2^*) &= 0.17\pm 0.01, \end{aligned} \qquad (38)$$

and the phase-space constrained ratio is defined as

$$\tilde{R}(X) = \frac{\int_{m_\tau^2}^{(m_B-m_X)^2} \frac{\mathrm{d}\Gamma(\bar{B}\to X\tau\bar{\nu})}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{m_\tau^2}^{(m_B-m_X)^2} \frac{\mathrm{d}\Gamma(\bar{B}\to Xl\bar{\nu})}{\mathrm{d}q^2}\mathrm{d}q^2}. \qquad (39)$$

Our results in Eq. (38) are nearly identical with Eq. (38) in Ref. [12]. The ratio for the sum of the four D^{**} states is

$$\bar{R}(D^{**}) = \frac{\sum_{X\in D^{**}}\Gamma[\bar{B}\to X\tau\bar{\nu}]}{\sum_{X\in D^{**}}\Gamma[\bar{B}\to Xl\bar{\nu}]} = 0.08\pm 0.01. \qquad (40)$$

TABLE IV. Fit results and correlations, fitting at leading order in HQET the $\bar{B}\to D_2^* l\bar{\nu}$ data (above) and $D_0^* l\bar{\nu}$ (below).

$\chi^2/\mathrm{d.o.f.}$	$\tau(1)$	τ'
2.5/3	0.65 ± 0.08	-1.3 ± 0.4
$\tau(1)$	1	−0.90
τ'	−0.90	1

$\chi^2/\mathrm{d.o.f.}$	$\zeta(1)$	ζ'
9.1/4	1.14 ± 0.32	-0.20 ± 1.0
$\zeta(1)$	1	−0.95
ζ'	−0.95	1

TABLE V. Fit results and correlations for Approximation C, for the narrow $D^{3/2^+}$ (above) and broad $D^{1/2^+}$ (below) states.

$\chi^2/\mathrm{d.o.f.}$	$\tau(1)$	τ'	$\hat{\tau}_1$	$\hat{\tau}_2$
2.4/4	0.70 ± 0.07	-1.6 ± 0.2	-0.5 ± 0.3	2.9 ± 1.4
$\tau(1)$	1	−0.85	0.53	−0.49
τ'	−0.85	1	−0.17	0.086
$\hat{\tau}_1$	0.53	−0.17	1	−0.89
$\hat{\tau}_2$	−0.49	0.086	−0.89	1

$\chi^2/\mathrm{d.o.f.}$	$\zeta(1)$	ζ'	$\hat{\zeta}_1$
9.1/4	0.70 ± 0.21	0.2 ± 1.4	0.6 ± 0.3
$\zeta(1)$	1	−0.95	−0.44
ζ'	−0.95	1	0.61
$\hat{\zeta}_1$	−0.44	0.61	1

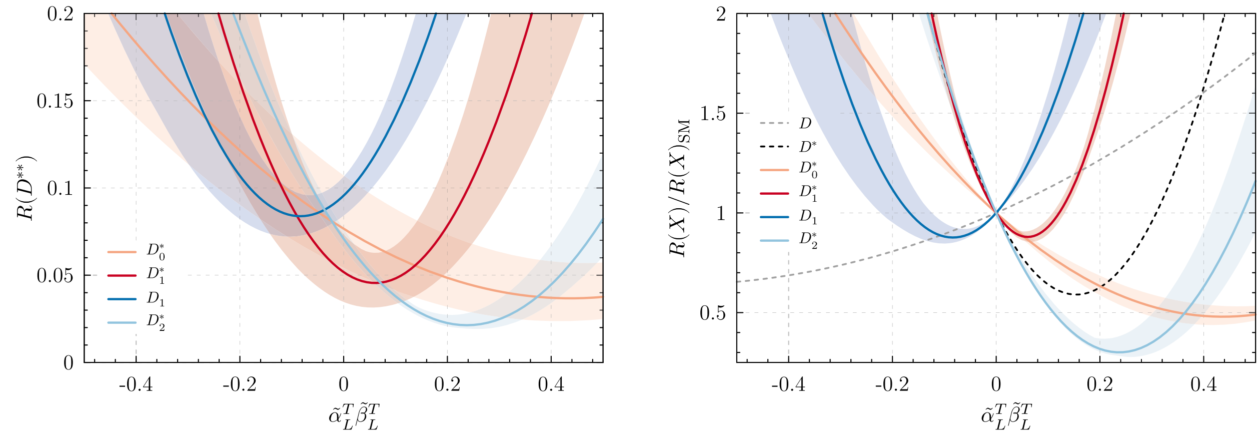

FIG. 1. Predictions for $R(D^{**})$ as functions of a NP tensor coupling (left), and for $R(X)/R(X)_{\rm SM}$ for all six $D^{(*,**)}$ states (right).

We next consider new physics generating one of the interactions in the basis defined by Eq. (3). The scalar (O_S) and pseudoscalar (O_P) matrix elements could be estimated in the past using Eq. (25) [12]. A right-handed vector current, $O_V + O_A$, does not help to fit the current data, and $O_V - O_A$ is the SM operator. Hence, we study here in some detail NP generating the tensor operator, O_T, which has best fit for the present data at $\tilde{\alpha}_L^T \tilde{\beta}_L^T \simeq 0.35$, assuming it is real (corresponding to $C_T \simeq 0.48$ in the conventions of Ref. [35]). This is primarily for illustration: While exclusively O_T cannot be generated by a dimension-six new physics operator, it can arise from Fierzing interactions generated in viable scenarios.

Figure 1 (left) shows our predictions for $R(D^{**})$ as functions of $\tilde{\alpha}_L^T \tilde{\beta}_L^T$, and Fig. 1 (right) shows the predictions for $R(X)/R(X)_{\rm SM}$ for all six $D^{(*,**)}$ states. In the vicinity of $\tilde{\alpha}_L^T \tilde{\beta}_L^T \simeq 0.35$, where the current $R(D)$ and $R(D^*)$ data can be fit well, measurements of $R(D^{**})$ will have a lot of discriminating power. We obtain for $\tilde{\alpha}_L^T \tilde{\beta}_L^T = 0.35$ the central values for $R(X)/R(X)_{\rm SM} = \{0.46, 4.3, 4.3, 0.47\}$ for $\{D_0^*, D_1^*, D_1, D_2^*\}$, respectively, whereas the corresponding values for $\{D, D^*\}$ are $\{1.51, 1.25\}$. The uncertainty bands are dominated by the first principal components of the fit covariance matrices, added in quadrature with variations in ζ_1 and τ_1 (see Ref. [12] for details). Figure 2 shows the predicted $d\Gamma/dw$ spectra both for the SM and for $\tilde{\alpha}_L^T \tilde{\beta}_L^T \simeq 0.35$.

Understanding $\bar{B} \to D^{**} l \bar{\nu}$ is also important because they give some of the largest experimental backgrounds to the measurements of $R(D^{(*)})$. For example, for Belle, Table IV in Ref. [36] showed that the $\bar{B} \to D^{**} \ell \bar{\nu}$ shapes and composition are significant backgrounds to the $R(D^{(*)})$

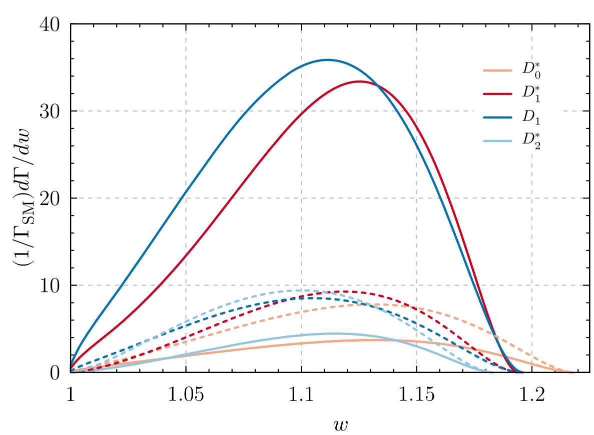

FIG. 2. Predicted $d\Gamma/dw$ distributions normalized to the SM rates for each of the four $D^{(**)}$ states, in the SM (dashed curves) and for $\tilde{\alpha}_L^T \tilde{\beta}_L^T = 0.35$ (solid curves), as determined from the Approximation C best-fit result, including $\Lambda_{\rm QCD}/m_{c,b}$ and α_s corrections.

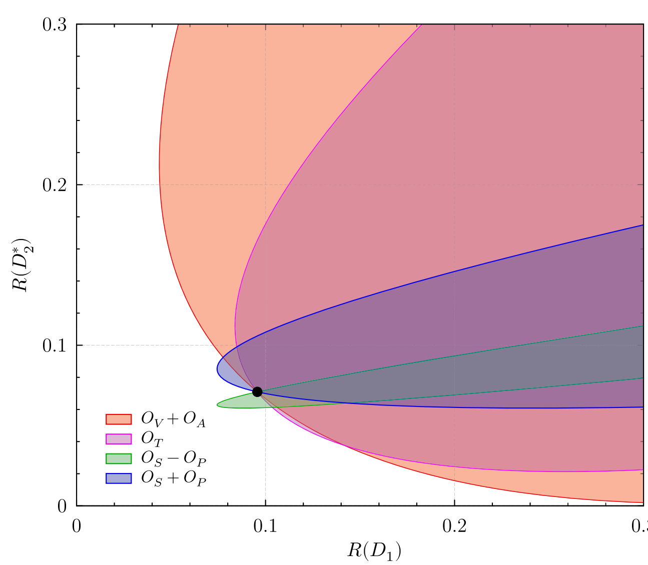

FIG. 3. Allowed ranges of $R(D_1)$ and $R(D_2^*)$ for the two narrow states, in the presence of any one of the four non-SM currents with arbitrary weak phases, as determined from the Approximation C best-fit result, including $\Lambda_{\rm QCD}/m_{c,b}$ and α_s corrections. The SM prediction is shown by the black dot.

measurements. For *BABAR*, Table V in Ref. [37] lists separately the uncertainties due to $\bar{B} \to D^{**} l\bar{\nu}$ and $\bar{B} \to D^{**}\tau\bar{\nu}$, which are both significant. The sensitive dependence of the $\bar{B} \to D^{**}\ell\bar{\nu}$ rates on $\tilde{\alpha}_L^T \tilde{\beta}_L^T$ shown in Figs. 1 and 2 illustrate the importance of treating these D^{**} backgrounds to $\bar{B} \to D^{(*)}\ell\bar{\nu}$ consistently, when fitting NP to more precise future $R(D^{(*)})$ data.

Figure 3 shows the possible ranges of $R(D_1)$ and $R(D_2^*)$, for the two narrow states, allowing any one of the four non-SM interactions, with arbitrary relative phases compared to the SM. The Approximation C best-fit result is used, including $\Lambda_{\rm QCD}/m_{c,b}$ and α_s corrections.

Predictions for $\bar{B}_s \to D_s^{**}\ell\bar{\nu}$ can be made using the same formalism with the appropriate hadron masses, the $\bar{\Lambda}^{(\prime,*)}$ parameters in Table II, and flavor $SU(3)$ symmetry for the form factors. Three of the D_s^{**} states have widths below a few MeV, and may become the first $\bar{B}_{(s)} \to D_{(s)}^{**}\tau\bar{\nu}$ decay modes measured by LHCb. The $R(D_s^{**})$ predictions are numerically close to those for $\bar{B} \to D^{**}\ell\bar{\nu}$, but larger uncertainties arise from $SU(3)$ violation and questions remain regarding the interpretation of D_s^{**} as simply orbitally excited $\bar{s}c$ states [12].

V. CONCLUSIONS

We derived the $\bar{B} \to D^{**}\ell\bar{\nu}$ decay rates for arbitrary beyond SM $b \to c$ currents and finite charged lepton mass, including all order $\Lambda_{\rm QCD}/m_{c,b}$ and α_s terms in the heavy quark effective theory expansion of the form factors.

To describe all $b \to c$ current matrix elements, including $\Lambda_{\rm QCD}/m_{c,b}$ and α_s corrections, only four functions of w (ζ, $\hat{\zeta}_1$, and $\hat{\chi}_{1,2}$) are needed to determine all twelve $B \to D^{1/2^+}$ form factors in Eqs. (17) and (18) to this order, as well as the mass parameters $\bar{\Lambda}^*$ and $\bar{\Lambda}$. For $B \to D^{3/2^+}$ decays, six functions of w (τ, $\hat{\tau}_{1,2}$, and $\hat{\eta}_{1,2,3}$) describe the sixteen form factors in Eqs. (23) and (24) at this order, plus the mass parameters $\bar{\Lambda}'$ and $\bar{\Lambda}$.

With the above results, we have now all ingredients in place to consistently study semileptonic B decays to the six lightest charm mesons, $\bar{B} \to D^{(*,**)}\ell\bar{\nu}$, for arbitrary new physics and for arbitrary charged lepton masses. These results are being implemented in the `Hammer` [38] analysis software, which will allow reweighing fully simulated data for fully differential decays to arbitrary new physics, including arbitrary NP contributions for each of the three lepton flavors. This will lead to better control of both theoretical and experimental uncertainties in $R(D^{(*,**)})$ measurements, as well as in the determinations of $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays.

Unlike calculations using model-dependent inputs on the form factors, our predictions are systematically improvable with more data on the $\bar{B} \to D^{**}l\bar{\nu}$ decays to light lepton final states and/or input from lattice QCD. The upcoming much larger data sets at LHCb and Belle II will answer many important questions.

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APPENDIX A: NP AMPLITUDES

In this appendix, we provide explicit results for the $B \to \bar{D}^{**}\bar{\ell}\nu$ amplitudes themselves. These $B \to \bar{D}^{**}$ amplitudes correspond to those used in the HAMMER code [38].

As in Ref. [31], we consider the $\bar{b} \to \bar{c}$ amplitudes, defining the basis of NP operators to be

$$\text{SM:}\quad i2\sqrt{2}V_{cb}^* G_F[\bar{b}\gamma^\mu P_L c][\bar{\nu}\gamma_\mu P_L \ell], \tag{A1a}$$

$$\begin{aligned}\text{Vector:}\quad & i2\sqrt{2}V_{cb}^* G_F[\bar{b}(\alpha_L^V\gamma^\mu P_L + \alpha_R^V\gamma^\mu P_R)c] \\ & \times[\bar{\nu}(\beta_L^V\gamma_\mu P_L + \beta_R^V\gamma_\mu P_R)\ell],\end{aligned} \tag{A1b}$$

$$\begin{aligned}\text{Scalar:}\quad & -i2\sqrt{2}V_{cb}^* G_F[\bar{b}(\alpha_L^S P_L + \alpha_R^S P_R)c] \\ & \times[\bar{\nu}(\beta_L^S P_R + \beta_R^S P_L)\ell],\end{aligned} \tag{A1c}$$

$$\begin{aligned}\text{Tensor:}\quad & -i2\sqrt{2}V_{cb}^* G_F[(\bar{b}\alpha_R^T\sigma^{\mu\nu}P_R c)(\bar{\nu}\beta_L^T\sigma_{\mu\nu}P_R\ell) \\ & + (\bar{b}\alpha_L^T\sigma^{\mu\nu}P_L c)(\bar{\nu}\beta_R^T\sigma_{\mu\nu}P_L\ell)].\end{aligned} \tag{A1d}$$

The lower index of β denotes the ν helicity and the lower index of α is that of the c quark. The correspondence between these coefficients and those defined in Eq. (27) is (equivalent to)

$$\begin{aligned}&\alpha_L^V = \tilde{\alpha}_L^{V\,*}, \qquad \alpha_R^V = \tilde{\alpha}_R^{V\,*}, \\ &\alpha_L^S = -\tilde{\alpha}_R^{S\,*}, \qquad \alpha_R^S = -\tilde{\alpha}_L^{S\,*}, \\ &\alpha_L^T = -\tilde{\alpha}_R^{T\,*}, \qquad \alpha_R^T = -\tilde{\alpha}_L^{T\,*}, \\ &\beta_j^i = \tilde{\beta}_j^{i\,*},\end{aligned} \tag{A2}$$

Operators for the CP conjugate $b \to c$ processes are obtained by Hermitian conjugation.

The $B \to \bar{D}^{**}\bar{\ell}\nu$ process features only a single physical polar helicity angle, θ_ℓ, defined in Fig. 4 below. (Helicity angles and momenta are defined with respect to the $\bar{b} \to \bar{c}$ process. Definitions for the conjugate process follow by replacing all particles with their antiparticles.) With respect to the $D^{**} \to DY$ decay

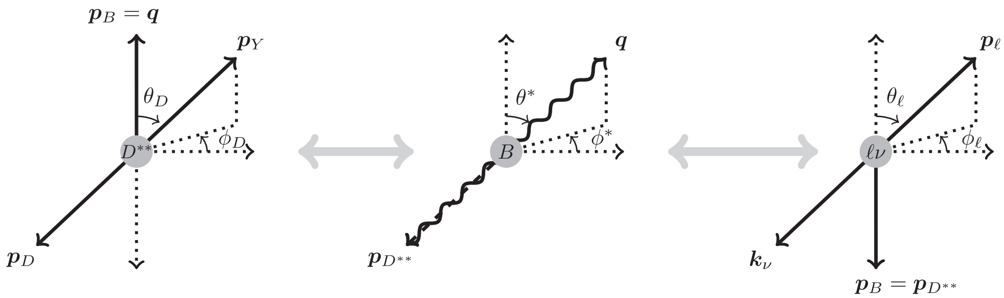

FIG. 4. Helicity angle definitions.

products, one may define ϕ_ℓ and ϕ_D as twist angles of the ℓ–ν and D–Y decay planes, in accordance with Fig. 4: The combination $\phi_\ell - \phi_D$ becomes a physical phase in the presence of D^{**} decays. Anticipating the need to account for interference effects once D^{**} and τ decays are included, we therefore write results for the helicity amplitudes including this physical phase combination. We adopt conventions that match the spinor conventions of Ref. [31] for the τ decay amplitudes. For left-handed neutrino amplitudes, this is achieved by including an extra D^{**} phase factor $e^{i\lambda_{D^{**}}\phi_D}$ for D^{**} spin λ_D, and an additional spinor phase function, $h_{s_\ell} = 1,\ e^{-i\phi_\ell}$ for $s_\ell = 1,\ 2$, respectively. (We label the leptonic spin with 1 and 2, to distinguish it from the D^{**} spins as well as to match the conventions of Ref. [31] for massive spinors.)

Under these conventions, the fully differential $B \to \bar{D}^{**}\bar{\ell}\nu$ rates may be written as

$$\frac{d^2\Gamma}{dw d\cos\theta_\ell} = \frac{G_F^2 m_B^5}{64\pi^3}\frac{2r^3\sqrt{w^2-1}(\hat{q}^2 - r_\ell^2)^2}{\hat{q}^2} \times \sum_{\lambda_{D^{**}}, s_\ell} |A_{s_\ell}^{\lambda_{D^{**}}}|^2, \tag{A3}$$

in which $r_\ell = m_\ell/m_B = \sqrt{\rho_\ell}$.

In the following subsections, we write the $A_{s_\ell}^{\lambda_{D^{**}}}$ helicity amplitudes for the $\bar{b} \to \bar{c}$ processes, for left-handed neutrinos only. As in Ref. [31], the amplitudes for right-handed neutrinos are straightforward to include. The CP conjugate $b \to c$ processes are obtained via

$$\mathcal{A}^s_{b\to c}(\theta, \phi; \alpha, \beta) = \mathcal{A}^{\bar{s}}_{\bar{b}\to\bar{c}}(\theta, -\phi; \alpha^*, \beta^*), \tag{A4}$$

where s is the set of external state quantum numbers, and $\bar{s}$ denotes their corresponding CP conjugates. For the D_2^* processes, the $\lambda_{D_2^*} = \pm 2$ amplitudes are all zero and are not included. We label the remaining amplitudes via $\lambda_{D_2^*} = \pm, 0$.

1. $B \to \bar{D}_0^* \ell\nu$

$$\begin{aligned}
A_1/\sqrt{2} = \bigg\{&\frac{1}{2} g_P(\alpha_L^S - \alpha_R^S)\beta_L^S + 2g_T\sqrt{w^2-1}\alpha_R^T\beta_L^T\cos\theta_\ell \\
&- \frac{g_+ r_\ell(1 + (\alpha_L^V - \alpha_R^V)\beta_L^V)((r-1)(1+w) + (1+r)\sqrt{w^2-1}\cos\theta_\ell)}{2\hat{q}^2} \\
&- \frac{g_- r_\ell(1 + (\alpha_L^V - \alpha_R^V)\beta_L^V)((1+r)(w-1) + (r-1)\sqrt{w^2-1}\cos\theta_\ell)}{2\hat{q}^2}\bigg\} \\
A_2/\sqrt{2} = \sin\theta_\ell\bigg\{&-2g_T r_\ell\sqrt{\frac{w^2-1}{\hat{q}^2}}\alpha_R^T\beta_L^T + \frac{1}{2}g_+(1+r)\sqrt{\frac{w^2-1}{\hat{q}^2}}(1 + (\alpha_L^V - \alpha_R^V)\beta_L^V) \\
&+ \frac{1}{2}g_-(r-1)\sqrt{\frac{w^2-1}{\hat{q}^2}}(1 + (\alpha_L^V - \alpha_R^V)\beta_L^V)\bigg\}.
\end{aligned}$$

2. $B \to \bar{D}_1 \ell \nu$

$$A_1^- = \sin\theta_\ell e^{-i(\phi_D-\phi_\ell)}\left\{\frac{1}{2}f_A r_\ell\sqrt{\frac{w^2-1}{\hat{q}^2}}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)+\frac{f_{V_1}r_\ell(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{2\sqrt{\hat{q}^2}}\right.$$
$$\left.+\frac{2(f_{T_2}(r-w+\sqrt{w^2-1})+f_{T_1}(-1+rw+r\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}\right\}$$

$$A_2^- = \cos^2\frac{\theta_\ell}{2}e^{-i(\phi_D-\phi_\ell)}\left\{f_A\sqrt{w^2-1}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)+f_{V_1}(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)\right.$$
$$\left.+\frac{4r_\ell(f_{T_2}(r-w+\sqrt{w^2-1})+f_{T_1}(-1+rw+r\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\hat{q}^2}\right\}$$

$$A_1^0 = \left\{-\frac{f_S\sqrt{w^2-1}(\alpha_L^S+\alpha_R^S)\beta_L^S}{\sqrt{2}}+\frac{f_{V_1}r_\ell(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)(\sqrt{w^2-1}+(r-w)\cos\theta_\ell)}{\sqrt{2}\hat{q}^2}\right.$$
$$-\frac{r_\ell\sqrt{w^2-1}(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)(f_{V_3}(r-w)+f_{V_2}(rw-1)+(f_{V_3}+f_{V_2}r)\sqrt{w^2-1}\cos\theta_\ell)}{\sqrt{2}\hat{q}^2}$$
$$\left.+2\sqrt{2}(f_{T_2}+f_{T_3}+f_{T_1}w-f_{T_3}w^2)\alpha_R^T\beta_L^T\cos\theta_\ell\right\}$$

$$A_2^0 = \sin\theta_\ell\left\{-\frac{f_{V_1}(r-w)(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{\sqrt{2}\sqrt{\hat{q}^2}}+\frac{(f_{V_3}+f_{V_2}r)(w^2-1)(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{\sqrt{2}\sqrt{\hat{q}^2}}\right.$$
$$\left.-\frac{2\sqrt{2}r_\ell(f_{T_2}+f_{T_3}+f_{T_1}w-f_{T_3}w^2)\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}\right\}$$

$$A_1^+ = \sin\theta_\ell e^{+i(\phi_D-\phi_\ell)}\left\{\frac{1}{2}f_A r_\ell\sqrt{\frac{w^2-1}{\hat{q}^2}}(-1+(\alpha_R^V-\alpha_L^V)\beta_L^V)+\frac{f_{V_1}r_\ell(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{2\sqrt{\hat{q}^2}}\right.$$
$$\left.-\frac{2(f_{T_2}(w-r+\sqrt{w^2-1})+f_{T_1}(1-rw+r\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}\right\}$$

$$A_2^+ = \sin^2\frac{\theta_\ell}{2}e^{+i(\phi_D-\phi_\ell)}\left\{f_A\sqrt{w^2-1}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)+f_{V_1}(-1-(\alpha_L^V+\alpha_R^V)\beta_L^V)\right.$$
$$\left.+\frac{4r_\ell(f_{T_2}(w-r+\sqrt{w^2-1})+f_{T_1}(1-rw+r\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\hat{q}^2}\right\}.$$

3. $B \to \bar{D}_2^* \ell \nu$

$$A_1^- = \sin\theta_\ell e^{-i(\phi_D-\phi_\ell)}\left\{\frac{k_V r_\ell(w^2-1)(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{2\sqrt{2}\sqrt{\hat{q}^2}}+\frac{k_{A_1}r_\ell\sqrt{w^2-1}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)}{2\sqrt{2}\sqrt{\hat{q}^2}}\right.$$
$$+\frac{\sqrt{2}k_{T_1}(1+w)((1+r)(w-1)-(1-r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}$$
$$\left.+\frac{\sqrt{2}k_{T_2}(w-1)((r-1)(1+w)+(1+r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}\right\}$$

$$
\begin{aligned}
A_2^- = \cos^2\frac{\theta_\ell}{2}e^{-i(\phi_D-\phi_\ell)}\Bigg\{&\frac{k_V(w^2-1)(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{\sqrt{2}}+\frac{k_{A_1}\sqrt{w^2-1}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)}{\sqrt{2}}\\
&+\frac{2\sqrt{2}k_{T_1}r_\ell(1+w)((1+r)(w-1)-(1-r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\hat{q}^2}\\
&+\frac{2\sqrt{2}k_{T_2}r_\ell(w-1)((r-1)(1+w)+(1+r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\hat{q}^2}\Bigg\}\\
A_1^0 = \Bigg\{&\frac{k_P(w^2-1)(\alpha_L^S-\alpha_R^S)\beta_L^S}{\sqrt{3}}+\frac{k_{A_1}r_\ell(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)(w^2-1+(r-w)\sqrt{w^2-1}\cos\theta_\ell)}{\sqrt{3}\hat{q}^2}\\
&-\frac{r_\ell(w^2-1)(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)(k_{A_3}(r-w)+k_{A_2}(rw-1)+(k_{A_3}+k_{A_2}r)\sqrt{w^2-1}\cos\theta_\ell)}{\sqrt{3}\hat{q}^2}\\
&+\frac{4k_{T_1}(1+w)\sqrt{w^2-1}\alpha_R^T\beta_L^T\cos\theta_\ell}{\sqrt{3}}+\frac{4k_{T_2}(w-1)\sqrt{w^2-1}\alpha_R^T\beta_L^T\cos\theta_\ell}{\sqrt{3}}+\frac{4k_{T_3}(w^2-1)^{3/2}\alpha_R^T\beta_L^T\cos\theta_\ell}{\sqrt{3}}\Bigg\}\\
A_2^0 = \sin\theta_\ell\Bigg\{&\frac{k_{A_1}(w-r)\sqrt{w^2-1}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)}{\sqrt{3}\sqrt{\hat{q}^2}}+\frac{(k_{A_3}+k_{A_2}r)(w^2-1)^{3/2}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)}{\sqrt{3}\sqrt{\hat{q}^2}}\\
&-\frac{4k_{T_1}r_\ell(1+w)\sqrt{\hat{q}^2(w^2-1)}\alpha_R^T\beta_L^T}{\sqrt{3}\hat{q}^2}-\frac{4k_{T_2}r_\ell(w-1)\sqrt{\hat{q}^2(w^2-1)}\alpha_R^T\beta_L^T}{\sqrt{3}\hat{q}^2}\\
&-\frac{4k_{T_3}r_\ell(w^2-1)^{3/2}\alpha_R^T\beta_L^T}{\sqrt{3}\sqrt{\hat{q}^2}}\Bigg\}\\
A_1^+ = \sin\theta_\ell e^{+i(\phi_D-\phi_\ell)}\Bigg\{&-\frac{k_Vr_\ell(w^2-1)(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{2\sqrt{2}\sqrt{\hat{q}^2}}+\frac{k_{A_1}r_\ell\sqrt{w^2-1}(1+(\alpha_L^V-\alpha_R^V)\beta_L^V)}{2\sqrt{2}\sqrt{\hat{q}^2}}\\
&-\frac{\sqrt{2}k_{T_1}(1+w)((1+r)(w-1)+(1-r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}\\
&+\frac{\sqrt{2}k_{T_2}(w-1)((1-r)(1+w)+(1+r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\sqrt{\hat{q}^2}}\Bigg\}\\
A_2^+ = \sin^2\frac{\theta_\ell}{2}e^{+i(\phi_D-\phi_\ell)}\Bigg\{&\frac{k_V(w^2-1)(1+(\alpha_L^V+\alpha_R^V)\beta_L^V)}{\sqrt{2}}+\frac{k_{A_1}\sqrt{w^2-1}(-1+(\alpha_R^V-\alpha_L^V)\beta_L^V)}{\sqrt{2}}\\
&+\frac{2\sqrt{2}k_{T_1}r_\ell(1+w)((1+r)(w-1)+(1-r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\hat{q}^2}\\
&-\frac{2\sqrt{2}k_{T_2}r_\ell(w-1)((1-r)(1+w)+(1+r)\sqrt{w^2-1})\alpha_R^T\beta_L^T}{\hat{q}^2}\Bigg\}.
\end{aligned}
$$

Finally, the $B\to\bar{D}_1^*\ell\nu$ amplitudes are obtained from the $B\to\bar{D}_1\ell\nu$ results, with the form factor mapping

$$f_S\mapsto -g_S,\qquad f_{A,V_i,T_i}\mapsto g_{A,V_i,T_i},\tag{A5}$$

where $i=1$, 2, 3, as follows from the definitions in Eqs. (14) and (19).

APPENDIX B: $B\to D^{(*)}\ell\bar{\nu}$

For completeness, and to have all six $B\to D^{(*,**)}$ rates together in the same notation, we list here $\mathrm{d}\Gamma/\mathrm{d}w$ for arbitrary charged lepton mass and weak current for the $B\to D^{(*)}\ell\bar{\nu}$ modes as well. We use the form factors defined as in Ref. [2]. For $\bar{B}\to D$,

$$\begin{aligned}\langle D|\bar{c}b|\bar{B}\rangle &= \sqrt{m_B m_D} h_S(w+1),\\ \langle D|\bar{c}\gamma_5 b|\bar{B}\rangle &= \langle D|\bar{c}\gamma^\mu\gamma^5 b|\bar{B}\rangle = 0,\\ \langle D|\bar{c}\gamma_\mu b|\bar{B}\rangle &= \sqrt{m_B m_D}[h_+(v+v')_\mu + h_-(v-v')_\mu],\\ \langle D|\bar{c}\sigma_{\mu\nu} b|\bar{B}\rangle &= i\sqrt{m_B m_D}[h_T(v'_\mu v_\nu - v'_\nu v_\mu)], \end{aligned} \tag{B1}$$

while for the $\bar{B}\to D^*$,

$$\begin{aligned}\langle D^*|\bar{c}b|\bar{B}\rangle &= 0,\\ \langle D^*|\bar{c}\gamma_5 b|\bar{B}\rangle &= -\sqrt{m_B m_{D^*}} h_P(\epsilon^*\cdot v),\\ \langle D^*|\bar{c}\gamma_\mu b|\bar{B}\rangle &= i\sqrt{m_B m_{D^*}} h_V \varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} v'^\alpha v^\beta,\\ \langle D^*|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}\rangle &= \sqrt{m_B m_{D^*}}[h_{A_1}(w+1)\epsilon^*_\mu - h_{A_2}(\epsilon^*\cdot v)v_\mu - h_{A_3}(\epsilon^*\cdot v)v'_\mu],\\ \langle D^*|\bar{c}\sigma_{\mu\nu} b|\bar{B}\rangle &= -\sqrt{m_B m_{D^*}}\varepsilon_{\mu\nu\alpha\beta}[h_{T_1}\epsilon^{*\alpha}(v+v')^\beta + h_{T_2}\epsilon^*_\alpha(v-v')^\beta + h_{T_3}(\epsilon^*\cdot v)v^\alpha v'^\beta]. \end{aligned} \tag{B2}$$

The common sign convention in $B\to D^{(*)}\ell\bar{\nu}$ papers is $\sigma^{\mu\nu}\gamma^5 \equiv -(i/2)\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}$ such that $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho\gamma^5] = +4i\epsilon^{\mu\nu\rho\sigma}$. This corresponds to the heavy quark symmetry relations with signs $h_+ = h_V = h_{A_1} = h_{A_3} = h_S = h_P = h_T = h_{T_1} = \xi$ (and $h_- = h_{A_2} = h_{T_2} = h_{T_3} = 0$). This convention is only used in this appendix and is the opposite of that for $B\to D^{(**)}\ell\bar{\nu}$ used in Refs. [12–14] and the rest of this paper.

Then we find

$$\begin{aligned}\frac{\mathrm{d}\Gamma_D^{(\mathrm{SM})}}{\mathrm{d}w} &= 4\Gamma_0 r^3\sqrt{w^2-1}\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{(w^2-1)\hat{q}^2[h_+(1+r)-h_-(1-r)]^2\\ &\quad + [h_+^2(w+1)[2(w-2r+r^2w)+\hat{q}^2] + h_-^2(w-1)[2(w-2r+r^2w)-\hat{q}^2] + 4h_-h_+(r^2-1)(w^2-1)]\}, \end{aligned} \tag{B3a}$$

$$\begin{aligned}\frac{\mathrm{d}\Gamma_D}{\mathrm{d}w} &= \frac{\mathrm{d}\Gamma_D^{(\mathrm{SM})}}{\mathrm{d}w}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V + \tilde{\alpha}_R^V\tilde{\beta}_L^V)^2 + 4\Gamma_0 r^3\sqrt{w^2-1}\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\Big\{\frac{3}{2}[(\tilde{\alpha}_L^S+\tilde{\alpha}_R^S)\tilde{\beta}_L^S]^2 h_S^2(w+1)^2\hat{q}^4\\ &\quad + 3\sqrt{\rho_\ell}(\tilde{\alpha}_L^S+\tilde{\alpha}_R^S)\tilde{\beta}_L^S(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V+\tilde{\alpha}_R^V\tilde{\beta}_L^V)h_S(1+w)\hat{q}^2[h_-(1+r)(1-w)+h_+(1-r)(1+w)]\\ &\quad + 4\tilde{\alpha}_L^T\tilde{\beta}_L^T h_T(w^2-1)\hat{q}^2[2\tilde{\alpha}_L^T\tilde{\beta}_L^T h_T(\hat{q}^2+2\rho_\ell) + 3\sqrt{\rho_\ell}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V+\tilde{\alpha}_R^V\tilde{\beta}_L^V)[h_+(1+r)-h_-(1-r)]]\Big\}. \end{aligned} \tag{B3b}$$

The $B\to D^*$ result is

$$\begin{aligned}\frac{\mathrm{d}\Gamma_{D^*}^{(\mathrm{SM})}}{\mathrm{d}w} &= 2\Gamma_0 r^3\sqrt{w^2-1}\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{2(w+1)\hat{q}^2(2\hat{q}^2[h_{A_1}^2(w+1)+h_V^2(w-1)]\\ &\quad + (w+1)[h_{A_1}(w-r)-(rh_{A_2}+h_{A_3})(w-1)]^2) + \rho_\ell(w+1)([h_{A_1}^2(w+1)+h_{A_3}^2(w-1)][4(w-r)^2-\hat{q}^2]\\ &\quad + 2h_{A_1}(w^2-1)[h_{A_2}(r^2+2rw-3)+4h_{A_3}(r-w)] + (w-1)[(2h_V^2-h_{A_2}^2)\hat{q}^2 + h_{A_2}^2 4(rw-1)^2\\ &\quad + 2h_{A_2}h_{A_3}[3w+3r^2w-2r(w^2+2)]])\}. \end{aligned} \tag{B4a}$$

$$\begin{aligned}\frac{\mathrm{d}\Gamma_{D^*}}{\mathrm{d}w} &= \frac{\mathrm{d}\Gamma_{D^*}^{(\mathrm{SM})}}{\mathrm{d}w}(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)^2 + 2\Gamma_0 r^3\sqrt{w^2-1}\frac{(\hat{q}^2-\rho_\ell)^2}{\hat{q}^6}\{3[(\tilde{\alpha}_L^S-\tilde{\alpha}_R^S)\tilde{\beta}_L^S]^2 h_P^2(w^2-1)\hat{q}^4\\ &\quad - 6(\tilde{\alpha}_L^S-\tilde{\alpha}_R^S)\tilde{\beta}_L^S(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)h_P(w^2-1)\hat{q}^2\sqrt{\rho_\ell}[h_{A_1}(1+w)-h_{A_2}(1-rw)-h_{A_3}(w-r)]\\ &\quad + 16(\tilde{\alpha}_L^T\tilde{\beta}_L^T)^2(\hat{q}^2+2\rho_\ell)(h_{T_1}^2(w+1)[\hat{q}^2(5w+1)+8r(w^2-1)] + h_{T_2}^2(w-1)[\hat{q}^2(5w-1)+8r(w^2-1)]\\ &\quad + h_{T_3}^2\hat{q}^2(w^2-1)^2 - 2h_{T_1}h_{T_2}(w^2-1)(3+2rw-5r^2) - 2h_{T_3}[h_{T_1}(w+1)+h_{T_2}(w-1)]\hat{q}^2(w^2-1))\\ &\quad + 24\tilde{\alpha}_L^T\tilde{\beta}_L^T\sqrt{\rho_\ell}\hat{q}^2(w+1)((1+\tilde{\alpha}_L^V\tilde{\beta}_L^V+\tilde{\alpha}_R^V\tilde{\beta}_L^V)2h_V[h_{T_1}(1+r)-h_{T_2}(1-r)](w-1) + (1+\tilde{\alpha}_L^V\tilde{\beta}_L^V-\tilde{\alpha}_R^V\tilde{\beta}_L^V)\\ &\quad \times[h_{A_1}[h_{T_2}(2-w+3r)(w-1)-h_{T_1}(2+w-3r)(w+1)+h_{T_3}(w-r)(w^2-1)] + (h_{A_2}r+h_{A_3})[h_{T_1}(w+1)\\ &\quad + h_{T_2}(w-1)-h_{T_3}(w^2-1)](w-1)]) + 8\tilde{\alpha}_R^V\tilde{\beta}_L^V(1+\tilde{\alpha}_L^V\tilde{\beta}_L^V)h_V^2\hat{q}^2(2\hat{q}^2+\rho_\ell)(w^2-1)\}. \end{aligned} \tag{B4b}$$

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