# On the Influence of Contact Compliance and Stiction on Vibrational Smoothing of Dry Friction

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*Summary*. High-frequency vibrations have been shown to reduce average friction forces and smooth the effective characteristics of dry friction. This effect of vibrational smoothing has been subject of numerous publications and is used in several industrial applications. Within this contribution, the effect of longitudinal and transverse vibrations on a 1-DoF friction oscillator is investigated. Focussing on modelling of stiction and break-away effects, the impact of these phenomena on vibrational smoothing is investigated. The results obtained for an elasto-plastic friction model show the expected behaviour of the system and are consistent with the results obtained for simpler friction models. However, additional effects can be observed when accounting for break-away effects. While the models show mostly quantitative discrepancies over a wide range of parameters, even the qualitative behaviour may change in case of the more evolved friction model.

## Introduction

The effect of high-frequency vibrations on systems with dry friction has been studied intensively in the past. Vibrational excitation has been shown to reduce average friction forces and smooth the effective friction-velocity characteristics [1]. Depending on the system under investigation, the use of simple Coulomb friction models has turned out to yield insufficient agreement with related experimental results [2]. Consequently, several attempts have been made accounting for additional effects, such as velocity dependencies or contact compliance. Based on Dahl's friction model, the effect of high-frequency vibrations on a 1-DoF friction oscillator has been investigated, also showing a reduction of friction forces and smoothed friction-velocity characteristics [3]. However, the results still appear to be insufficient, as Dahl's model does not account for stiction.

Within this contribution, the elasto-plastic friction model suggested by Dupont et al. is considered as an extension of Dahl's model accounting for stiction and break-away effects [4]. Using this extended friction model, the effect of longitudinal and transverse vibrations on a 1-DoF friction oscillator is investigated by means of an averaging procedure.

## Longitudinal Vibrations

The dimensionless equation of motion for the 1-DoF friction oscillator under high-frequency excitation can be stated as

$$\ddot{x} + x = f_R + a\omega^2 \sin \omega t, \tag{1}$$

where $f_R$ is the dimensionless friction force. The excitation is assumed to be of inertia type with dimensionless amplitude $a \ll 1$ and frequency $\omega \gg 1$, such that $a\omega = \mathcal{O}(1)$, which is reasonable in case of high-frequency excitation. For the elasto-plastic friction model, the friction force is given by $f_R = \sigma_0 z$, where $\sigma_0$ represents the tangential contact stiffness, and the evolution of the friction state variable is described by

$$\dot{z} = v_{rel} - \alpha |v_{rel}| \frac{\sigma_0 z}{\mu f_N}. \tag{2}$$

Herein, velocity dependence of steady-state sliding and viscous effects have been neglected. The function $\alpha = \alpha(v_{rel}, z) \in [0, 1]$ controls the transition between stiction and sliding. For small deflections $|z| < z_{ba}$ and at velocity reversal $v_{rel} z < 0$, $\alpha = 0$ implies stiction, whereas $\alpha = 1$ corresponds to pure sliding [4].

Introducing the fast time scale $\tau = \omega t$, $(\cdot)' = \partial/\partial\tau$ and using the friction force as a variable, the system can be written as

$$x' = \varepsilon v, \tag{3}$$

$$v' = -\varepsilon(x + f_R) + a\omega \sin \tau, \tag{4}$$

$$f_R' = \beta \left( v_{rel} - \alpha |v_{rel}| \frac{f_R}{\mu f_N} \right), \tag{5}$$

where $\varepsilon = \omega^{-1} \ll 1$ is a small parameter and $\beta = \sigma_0 \omega^{-1} = \mathcal{O}(1)$. Using the solution of the corresponding unperturbed problem as a transformation of variables, the system can be rewritten in terms of slow motions $X, V$ of the system:

$$X' = \varepsilon(V - a\omega \cos \tau), \tag{6}$$

$$V' = -\varepsilon(X + f_R^*(\tau) + \phi), \tag{7}$$

$$\phi' = -\beta\alpha |V_{rel} - a\omega \cos \tau| \frac{\phi}{\mu f_N}. \tag{8}$$

Herein, $f_R^*(\tau)$ represents the periodic solution of (5) and the perturbation $\phi$ can be identified as a strongly damped variable. Applying an averaging procedure for systems with strong damping, the equation of slow motion is finally given by

$$\ddot{X} + X = \langle f_R^*(\tau) \rangle, \tag{9}$$

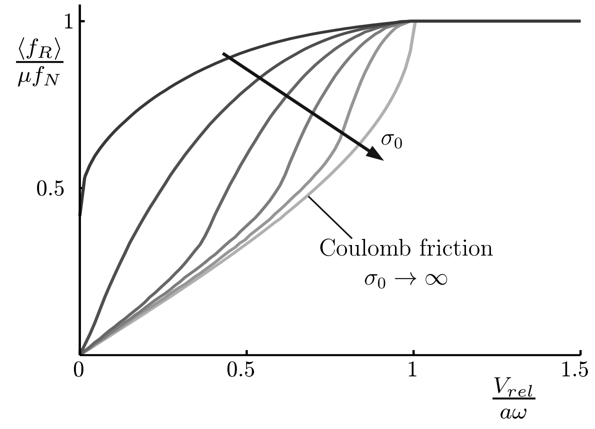


Figure 1: Effective friction-velocity characteristics for longitudinal excitation

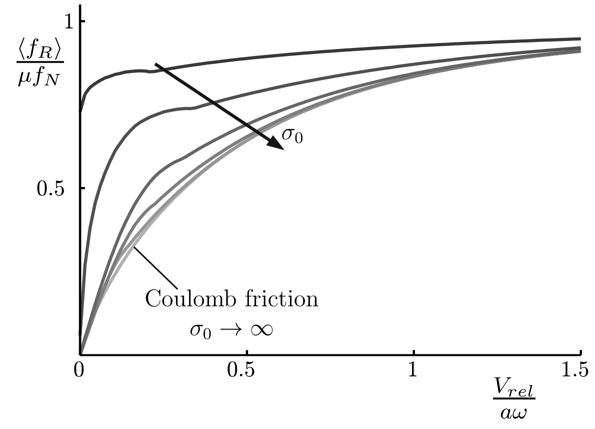


Figure 2: Effective friction-velocity characteristics for transverse excitation

where $\langle \cdot \rangle$ denotes the fast time average.
The results obtained for the average friction force are depicted in fig. 1. As expected, the effective friction-velocity characteristics are smoothed in case of moderate and high contact stiffness and approach the classical results for Coulomb friction. At high relative velocities, the excitation has no effect on the friction characteristics. However, the results change qualitatively in case of small contact stiffness. When accounting for break-away effects, the effect of friction reduction is limited, yielding finite values at zero relative velocity and, consequently, non-smooth friction-velocity characteristics.

## Transverse Vibrations

In order to investigate the effect of transverse vibrations, the model equations have to be extended to two dimensions. Using the quantities $\mathbf{x} = [x; y]$, $\mathbf{v} = [\dot{x}; \dot{y}]$, $\mathbf{x}_r = [x; 0]$, $\mathbf{f}_R = [f_{R,x}; f_{R,y}]$ and $\mathbf{h}(\tau) = [0; a\omega \sin \tau]$, the mechanical model can be described by the system of first order equations

$$\mathbf{x}' = \varepsilon \mathbf{v}, \tag{10}$$

$$\mathbf{v}' = \varepsilon(-\mathbf{x}_r + \mathbf{f}_R) + \mathbf{h}(\tau), \tag{11}$$

$$\mathbf{f}_R' = \beta \left( \mathbf{v}_{rel} - \alpha ||\mathbf{v}_{rel}|| \frac{\mathbf{f}_R}{\mu f_N} \right). \tag{12}$$

In contrast to the one-dimensional case, the switching function has to be adapted. The break-away condition changes to $||\mathbf{z}|| < z_{ba}$ and the kinematic stiction condition can be formulated as $\mathbf{v}_{rel} \cdot \mathbf{z} < 0$.
Performing a similar analysis as before, the equation of slow motion in direction of macroscopic sliding is given by

$$\ddot{X} + X = \langle f_{R,x}^*(\tau) \rangle, \tag{13}$$

where $f_{R,x}^*(\tau)$ is the $x$-component of the periodic solution of (12).
The resulting friction-velocity characteristics are depicted in fig. 2. Like in the one-dimensional case, the characteristics are smoothed for high contact stiffness and approach the classical results using Coulomb friction. For small contact stiffness, the average friction force again tends towards finite values at zero relative velocity.

## Conclusions

Within the present discussion, the effect of stiction and break-away effects on vibrational smoothing of dry friction has been investigated. Using the elasto-plastic friction model proposed by Dupont et al., a reduction of the smoothing effect has been observed compared to the classical results using Coulomb friction. Especially for small contact stiffness, the effect of friction reduction has been shown to reduce dramatically. When accounting for break-away effects, vibrational excitation may yield a finite friction force at zero relative velocity and, consequently, result in non-smooth friction-velocity characteristics. This observation may serve as an explanation approach for the experimental results given in the literature, where the corresponding modelling has lead to an overestimation of the smoothing effect.

## References

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