FOIL AIR BEARING ROTOR INTERACTION - BIFURCATION ANALYSIS OF A LAVALROTOR

C. Baum, T. Leister and W. Seemann

Karlsruhe Institute of Technology (KIT), Institute for Engineering Mechanics (ITM) , Kaiserstr. 10, 76131 Karlsruhe, Germany
E-mail: {*christoph.baum, tim.leister, wolfgang.seemann*}*@kit.edu*

Abstract: A simple elastic rotor model is coupled with a new physical and mathematical model of foil air bearings. The novelty is the nonlinear stiffness effects of the compliant bearing structure. Self-excited vibrations are investigated.

Keywords: *foil air bearing, elastic rotor, bifurcation analysis, self-excited vibrations*

1 Introduction

Air bearings offer a high potential for improving high speed rotating machinery. The major advantage is the low friction loss, due to the low viscosity of the lubricating fluid. However, these bearings require either an external pressure supply or high rotating speeds to build up a load carrying fluid film. Furthermore, aerodynamic bearings, which refer to the latter mentioned kind, are prone to instabilities, Gross et al. (1980). A compliant bearing structure is supposed to suppress these vibrations, or at least minimize the amplitudes, Howard et al. (2001). Within this contribution the effects of the bearing compliance, neglecting friction and damping, on the onset and resulting rotor vibration are investigated.

A vast variety of foil air bearing models exist. In most cases the bearing models are investigated for a fixed rotor state. Surveys on the dynamics of the rotor, coupled to air bearing models are rather rare. Complex FE-models used to be expensive in terms of computational costs. Consequently, these models are not appropriate for rotordynamic investigations. Bonello and Phan use a classical linear elastic Winklerfoundation model for the compliant bearing structure to investigate the dynamics of a rigid rotor, Bonello and Pham (2014). Bhore and Darpe use the same structure model, but a linear elastic Lavalrotor, Bhore and Darpe (2013). Here, a new nonlinear physical and mathematical model for the bearing structure is used and coupled with a Lavalrotor.

2 Model Description

Rotor Model An elastic Lavalrotor of mass M with external damping d_e is considered. In addition, the mass of the shaft is modelled by discrete particles of mass $\frac{m}{2}$ located at each bearing, see Figure 1 (a). The shaft has a stiffness k and the configuration is symmetric so that the equations of motion are given by

$$[M\ddot{x}_W + d_e\dot{x}_W + k(x_W - x_P)]\,\boldsymbol{e}_x + [M\ddot{y}_W + d_e\dot{y}_W + k(y_W - y_P)]\,\boldsymbol{e}_y \quad = \quad Mg\,\boldsymbol{e}_x \tag{1}$$

$$[m\ddot{x}_P + 2F_x - k(x_W - x_P)]\,\boldsymbol{e}_x + [m\ddot{y}_P + 2F_y - k(y_W - y_P)]\,\boldsymbol{e}_y \quad = \quad mg\,\boldsymbol{e}_x. \tag{2}$$

$\boldsymbol{e}_x, \boldsymbol{e}_y$ are the base vectors, x_P, y_P the Cartesian coordinates of the journal center P within the bearing, x_W, y_W the coordinates of the rotor's center W and $F_i = F_i(x_P, y_P, \dot{x}_P, \dot{y}_P, \omega)$, $i = x, y$ the bearing forces resulting from the fluid pressure, ω being the speed of rotation.

Bearing Model Based on the conventional assumptions of lubricating fluid film theory, the unsteady Reynolds equation for ideal gases is chosen to model the fluid behavior, in particular the fluid pressure p. The structure of the foil is modeled such that the displacement does not depend on the axial coordinate (z_f) and the displacement is not coupled in circumferential direction (x_f). This yields the following equation

$$\mathcal{D}\{p\} = \frac{\partial}{\partial x_f}\left[\frac{ph^3}{\mu}\frac{\partial p}{\partial x_f}\right] + \frac{\partial}{\partial z_f}\left[\frac{ph^3}{\mu}\frac{\partial p}{\partial z_f}\right] - 6\omega R\frac{\partial}{\partial x_f}[ph] - 12\frac{\partial}{\partial t}[ph] = 0, \tag{3}$$

on $\Omega_{2D} = \{(x_f, z_f)|\ x_f = 0...2\pi R, z_f = -\frac{L}{2}...\frac{L}{2}\}$. Within (3), $h \approx h_0 - x_P\cos(\frac{x_f}{R}) - y_P\sin(\frac{x_f}{R}) + \frac{1}{b}\ln\left[1 + \alpha(\frac{\overline{p}}{p_0 L} - 1)\right]$ states the fluid boundary determined by the journal position and the deformation of the foil, h_0 is the nominal clearance, p_0 is the ambient pressure, $\overline{p} = \int_{-\frac{L}{2}}^{\frac{L}{2}} p\,\mathrm{d}z_f$, is the axial - integrated pressure, L is the bearing length and R the nominal bearing radius. α and b are parameters of the nonlinear foundation model.

Nondimensional Reduced Bearing-, Rotor - and Coupled Overall Model At first, to reduce the number of model parameters, the overall model is transformed into nondimensional form, where a star $(.)^*$ denotes the nondimensional variable. Furthermore, with the objective of a computational efficient bearing model a single term axial shape function $p_a^* = \overline{p}_a^*(z_f^*)\hat{p}_a^*(\tau,\varphi) + 1$ is proposed. Based on the theory of weighted residuals, in particular applying Kantorovich's method, the dependency on the axial coordinate z_f^* is eliminated by evaluating $\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathcal{D}\{p_a^*\}\ \overline{p}_a^*(z_f^*)\mathrm{d}z_f^* = 0$, with the nonlinear differential operator $\mathcal{D}$ defined in (3). The boundary condition is chosen as $\hat{p}^*(\varphi = -\pi) = \hat{p}^*(\varphi = \pi) = 0$. Furthermore, substituting finite differences (FD) for the remaining spatial derivatives w.r.t. φ in the fluid pressure and rearranging gives a nonlinear system of coupled ODEs $\hat{p}_j^{*'} = f(\hat{p}_{j-1}^*, \hat{p}_j^*, \hat{p}_{j+1}^*),\ j = 1\ldots n$, for n collocation points.

Introducing a state space vector $\boldsymbol{x}$ containg both, the rotor states and the centerline pressure at each collocation point $\hat{p}_j^*$, enables an overall formulation of the coupled fluid-bearing-rotor problem as an autonomous nonlinear system of first order ODEs.

$$\boldsymbol{x}' = \boldsymbol{X}(\boldsymbol{x}(\tau)), \quad \boldsymbol{x} = [x_P^*, y_P^*, x_P^{*'}, y_P^{*'}, x_W^*, y_W^*, x_W^{*'}, y_W^{*'}, \hat{p}_1^*, \ldots, \hat{p}_j^*, \ldots, \hat{p}_n^*]^T. \tag{4}$$

3 Results

To analyze the system's dynamical behavior a bifurcation analysis is conducted. The angular velocity ω^* is used as a bifurcation parameter. Stationary solutions $\boldsymbol{x}_0$ of (4) are determined by solving a system of algebraic nonlinear equations $\boldsymbol{X}(\boldsymbol{x}_0) = 0$. Their stability is determined by the eigenvalues of the Jacobian. Periodic solutions are investigated within MatCont.

Rigid Rotor For a rigid rotor ($k \to \infty$), without external damping ($d_e = 0$) the stationary solutions for low rotor speeds are stable. With increasing rotor speed a threshold $\omega_{s,rgd}^*$ can be identified, at which two conjugate complex eigenvalues with vanishing real part, an Andronov-Hopf-Bifurcation, occurs. For $\omega^* > \omega_{s,rgd}^*$ the stationary solutions are unstable. Moreover, at $\omega^* = \omega_{s,rgd}^*$ an unstable limit cycle is born. Following the unstable branch a fold bifurcation is detected. Unstable limit cycles switch to stable limit cycles. Following the stable branch again, a transition to unstable limit cycles can be observed. On the stable limit cycles the rotor whirls with approximately the half angular velocity, period $T \approx \frac{4\pi}{\omega^*}$.

Lavalrotor When considering the elasticity ($k < \infty$) for a rotor of the same mass, the journal loci coincide with the journal loci of the rigid rotor. But the stability threshold of the Lavalrotor $\omega_{s,lvl}^*$ does not coincide with the stability threshold of the rigid rotor $\omega_{s,rgd}^*$. Both rotor stiffness k and mass distribution $(m, M,\ m + M = const.)$ influence the critical rotor speed $\omega_{s,lvl}^*$ and the amplitudes of the whirl motion for $\omega^* > \omega_{s,lvl}^*$. Figure 1 (b) shows exemplarily a trajectory for a constant angular velocity. Further results will be shown within the presentation.

References

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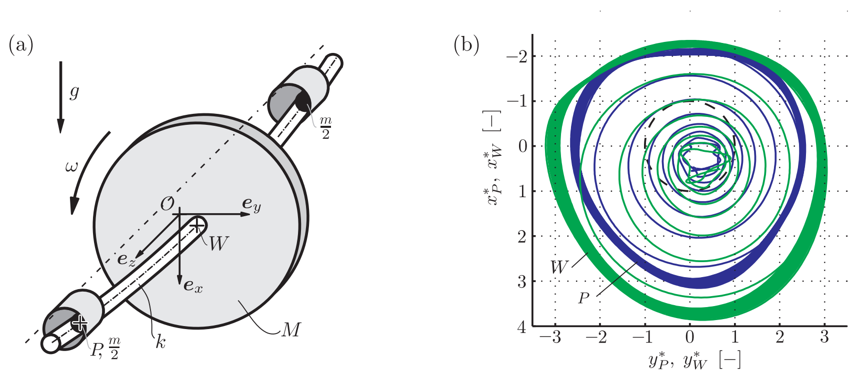

Figure 1: Model of the symmetric Lavalrotor (a), example trajectory of P and W for $\omega^* > \omega_{s,lvl}^*$ (b)