Investigation of optimal bipedal walking gaits subject to different energy-based objective functions

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Optimal bipedal walking gaits subject to different energy-based objective functions are investigated using a simple planar rigid body model of a bipedal robot with upper body, thighs and shanks. The robot's segments are connected by revolute joints actuated by electric motors. The actuators' torques are generated by a trajectory tracking controller to produce periodic walking gaits. A numerical optimization routine is used to find optimal reference trajectories for average speeds in the range of 0.3 – 2.3 m/s to investigate the influence of different objective functions.

1 Introduction

There is a variety of optimality criteria for the analysis and generation of bipedal as well as quadrupedal gaits in literature [1–3]. While exceptions exist (e.g. maximum walking speed [3]), most of those criteria optimize either energy efficiency or stability. A combination of different criteria – based on energy and/or stability – is employed in inverse optimal control approaches [4]. The aim here was to identify the human objective criterion by matching generated gaits with measurements. Furthermore, energy efficiency is frequently used as criterion for changes of locomotion patterns like transition from walking to running in bipedal locomotion [5] or transition between walking, trotting and galloping in quadrupedal locomotion [6].

Frequently encountered energy-based criteria are torques squared ($\Phi_{torques}$), the mechanical work or the absolute value of the mechanical work ($\Phi_{mech,abs}$) [2]. Those works are often normalized by weight and step length ($\Psi_{torques}$, $\Psi_{mech,abs}$). This results in specific energies which allow for better comparability of different models. While some of those criteria are motivated by physical properties of the actuating motors or muscles, others – especially torques squared – are often used because of mathematical properties like convexity and differentiability. This contribution investigates the objective criteria's influence on generated walking gaits.

2 Method

A five-link robot with rigid links for trunk, thighs and shanks connected by actuated revolute joints (Fig. 1) is introduced to model bipedal walking. The lengths, masses and inertias of the robot's links are set to match an average human with a height of $H = 1.8$ m and a mass of $m = 80$ kg [7]. The model has point feet, which means no torque can be transmitted between the legs and the ground directly. DC servo motors are assumed in the hip and knee joints. The controller for the actuated joints can be derived employing input-output linearization to track desired joint angle trajectories [8]. The hybrid zero dynamics of the controlled system, which corresponds to a model with a perfect controller, is used to compute periodic walking gaits.

The controlled model is simulated in *Matlab*. Walking gaits are generated using *Matlab*'s implementation of the SQP algorithm to optimize a criterion of energy efficiency. The gait generation incorporates constraints for feasible solutions (inequality constraints: positive normal force in ground contact of stance leg, stiction ($\mu = 0.6$), no ground penetration of swing leg; equality constraint: average speed $\bar{v}$). The following criteria are considered in this study:

$$\begin{aligned}
\Phi_{torques} &= (mg)^{-1}\int_0^{T_{Step}} \sum_{j=1}^{4} c_{stat} u_j^2 \,\mathrm{d}t\,, & \Psi_{torques} &= (L_{Step})^{-1}\Phi_{torques}\,,\\
\Phi_{mech,abs} &= (mg)^{-1}\int_0^{T_{Step}} \sum_{j=1}^{4} |u_j \dot{q}_j| \,\mathrm{d}t\,, & \Psi_{mech,abs} &= (L_{Step})^{-1}\Phi_{mech,abs}\,,\\
\Phi_{mech,pos} &= (mg)^{-1}\int_0^{T_{Step}} \sum_{j=1}^{4} \max(u_j \dot{q}_j, 0) \,\mathrm{d}t\,, & \Psi_{mech,pos} &= (L_{Step})^{-1}\Phi_{mech,pos}\,,\\
\Phi_{electric} &= (mg)^{-1}\int_0^{T_{Step}} \sum_{j=1}^{4} \max(c_{stat} u_j^2 + u_j \dot{q}_j, 0) \,\mathrm{d}t\,, & \Psi_{electric} &= (L_{Step})^{-1}\Phi_{electric}\,,
\end{aligned}$$

where g is the gravitational acceleration, T_{Step} the duration of one step, L_{Step} the step length, u_j the motor torque and $\dot{q}_j$ the angular velocity in the j-th joint. The actuators are characterized by the coefficient of static power $c_{stat} = 1.81 \cdot 10^{-3}$ W/(Nm)^2 [9] which depends on the electric resistance and the gear ratio. The heat losses in the j-th motor winding are $c_{stat} u_j^2$.

The objective function $\Phi_{torques}$ is the torques squared criterion which accounts only for the heat losses in the electric motors. In contrast, $\Phi_{mech,abs}$ and $\Phi_{mech,pos}$ consider the mechanical work which the motors supply to the robots motion. In reality, most of the electric energy produced when the motors are operated as generators (braking) is lost and cannot be

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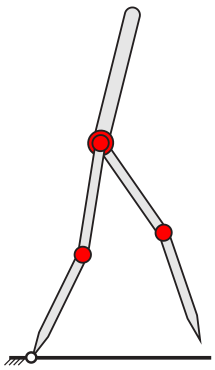

Fig. 1: Robot model (actuated joints in red).

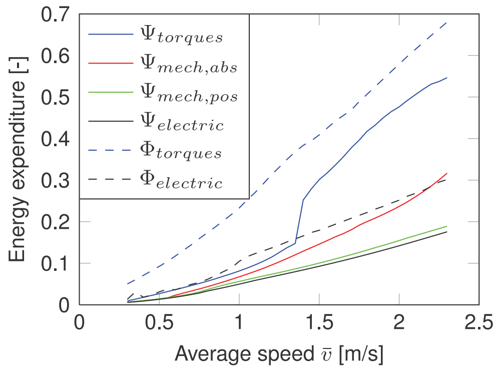

Fig. 2: Energy expenditure of all gaits generated by the optimization.

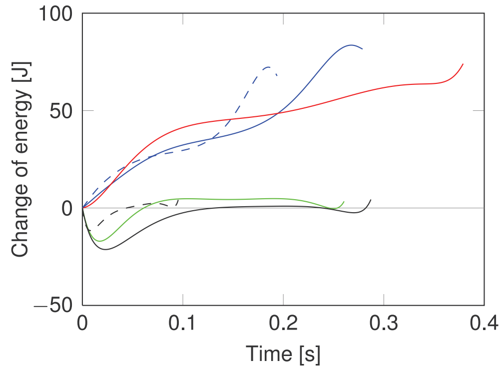

Fig. 3: Change of total energy (kinetic and potential energy) during one step for $\bar{v} = 1.8$ m/s.

recuperated. To account for this, many researchers have optimized the absolute value of the mechanical work $\Phi_{mech,abs}$ in the past [2]. Another way to take this effect into account is to optimize only the positive mechanical work $\Phi_{mech,pos}$. $\Phi_{electric}$ is the electric energy supplied to the motors, combining heat losses and mechanical work. The generator mode is again neglected.

The functions Ψ_i consider the same energy inputs divided by the step length L_{Step}. Optimizing the functions Φ_i minimizes the energy for one step whereas optimizing Ψ_i minimizes the energy per covered distance.

Optimal gaits are generated for all eight objective functions for average speeds from 0.3 to 2.3 m/s.

3 Results

The optimization with $\Phi_{mech,abs}$ and $\Phi_{mech,pos}$ results in gaits with diminishing step length, since the value of the time integral decreases for $T_{Step} \to 0$. These gaits are not meaningful and therefore not considered any further.

When comparing the energy expenditures (electric energy consumption per covered distance: $\Psi_{electric}$) of all gaits in Fig. 2 significant differences between the different objective functions can be observed. The most popular criteria $\Phi_{torques}$, $\Psi_{torques}$ and $\Psi_{mech,abs}$ lead to energy expenditures which are up to eight times as big as the reference ($\Phi_{torques}$, 0.3 m/s). This is also due to the value of c_{stat} which weights the two summands in $\Psi_{electric}$ ($\lim_{c_{stat}\to 0}(\Psi_{electric}) = \Psi_{mech,pos}$ and $\lim_{c_{stat}\to\infty}(\Psi_{electric}) = \Psi_{torques}$). The jump for $\Psi_{torques}$ at $\bar{v} = 1.4$ m/s occurs because the stiction constraint becomes relevant for higher speeds, but is inactive for speeds $\bar{v} \leq 1.4$ m/s. Changing μ shifts this jump to a different average speed.

Comparing the different robot motions for any average speed reveals a strong similarity between $\Psi_{torques}$ and $\Psi_{mech,abs}$ on the one hand as well as $\Psi_{mech,pos}$ and $\Psi_{electric}$ on the other hand. Regarding the change in the robot's total energy (kinetic and potential energy) during one step (exemplary in Fig. 3 for $\bar{v} = 1.8$ m/s) reveals the underlying effect. Optimizing $\Psi_{torques}$, $\Psi_{mech,abs}$ leads to a considerable increase in the total energy over the step. This energy is then dissipated during the impact of the stance leg. $\Psi_{mech,pos}$, $\Psi_{electric}$ on the other hand have almost constant energy during one step. The dissipated energy during the impact is considerably smaller. A very similar behavior has been reported for human locomotion [10].

4 Conclusion

The presented study shows significant differences between the generated optimal gaits corresponding to different objective functions. The objective functions most frequently used in past research ($\Phi_{torques}$, $\Psi_{torques}$ and $\Psi_{mech,abs}$) are considerably less efficient than the optimization of the actual supplied energy per covered distance ($\Psi_{electric}$). Furthermore, using $\Psi_{electric}$ or $\Psi_{mech,pos}$ leads to bipedal gaits with an energetic behavior more similar to human locomotion than the established criteria.

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