On the Solution of Crack Identification Problems in Composite Materials

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Abstract. A two-dimensional problem of wave propagation in an infinite multilayered structure containing an internal or interface crack is considered. To solve the problem, two approaches are applied, namely reducing of the problem to a system of boundary integral equations relative to unknown displacements of the crack surfaces or modeling of multilayered infinite media infinite using finite element Software ABAQUS by introducing a nonreflecting absorbing boundary. Alongside with the problem of wave propagation in the media with the given crack configuration, the work deals with the inverse problem of crack identification using surface displacements measured within given points.

Introduction

Today composite materials find ever-widening applications in many industrial sectors. Due to their high strength and lightness, they play a particularly important role in the aerospace industry. Thus, for instance, the share of composite materials in the construction of modern large-size airliners A380 and B787 is 25 and 50 percent [1], respectively. However, the complex manufacturing process of composite materials causes relatively frequent occurrence of damages both in the composite matrix and on the ply interfaces. The defect is called an internal crack in the first case and an interface crack, or delamination, in the second. Since the material is subjected to high loads in the process of operation, the presence of even small damage may lead to the crack growth and total destruction of the construction.

Methods of nondestructive testing are used for monitoring of the structures health. Analysis of Lamb waves propagation in the investigated structure is one of the most widespread methods of nondestructive evaluation [1, 2]. The high practical potential of Lamb waves in the nondestructive testing problems has been known for a long time [1-7]. There exist many other methods, see review in [1, 2], however, the method of Lamb waves is considered to the most time-efficient one and can be more simply implemented for the online monitoring in the automatic operation mode [1].

The problem of crack parameters identification belongs to the kind of geometric inverse problems. Taking into account dispersion characteristics of a wave packet, the arrival time of the signal reflected from the crack to the sensor can be used as additional information required for the solution of the inverse problem. Problems of investigation of dispersion characteristics of multilayered composites are studied in a large number of works [4, 6-8]. Based on the estimation of the propagation velocity of Lamb waves, influence maps are constructed with the use of a large number of actuators and sensors [9], as well as preliminary data filtration. This method makes it possible to locate cracks rather efficiently. The crack size, however, can be identificated only roughly. The advantage of this approach lies in the simplicity of the method and the fact that it is not necessary to model the defect itself, since the method can be used for defects of any type [9].

Another approach is based on the comparison of the measured surface displacements and displacements caused by the same surface load under different crack configurations [5, 10]. To apply this approach, it is necessary to solve the problem of wave propagation from a surface excitation source under the given parameters of the crack rather quickly, which is a very complicated problem by itself.

The present paper considers a two-dimensional problem for a multilayered structure containing a through-thickness crack located parallel to the layers boundaries. A method based on a solution of boundary integral equations [11] as well as a method based on the finite-element method [12] is used for modeling of wave propagation within the structure with damage. Inverse problem of identification of the crack parameters is formulates an optimization problem. A genetic algorithm (see for example [5]) and a pattern search algorithm implemented in Software DIRECT 2.0 [13] are used to solve the related optimization problem. The authors discuss the convergence of the methods considered. In the course of a number of numerical experiments it was proved that both methods can be used to solve the identification problem. Suggested methods proved their availability in the application for structural health monitoring.

1. Mathematical formulation

1.1 Modelling in time and frequency domains

In the nondestructive testing problems, cyclic loading during a finite time period is used most often, while excited surface displacements are measured starting from the moment of signal excitation to the moment of arrival of the waves reflected from the structure boundaries. In this case, the structure can be assumed to be infinite in horizontal directions. Moreover, the surface load distribution does not depend on time and the structure is loaded during a relatively short time interval according to the given law $f(t)$, therefore, to analyze the measured displacements, it is possible to transform time-dependent data to the frequency domain

$$\boldsymbol{u}(\boldsymbol{x},t) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty \boldsymbol{u}(\boldsymbol{x},\omega)F(\omega)e^{i\omega t}d\omega, \qquad (1)$$

where the function $F(\omega)$ is the Fourier transform of the function $f(t)$, and displacements in the frequency domain $\boldsymbol{u}(\boldsymbol{x},\omega)$ do not depend on $f(t)$ and describe harmonic vibrations of the infinite layered structure at a frequency ω excited by the independent of the frequency surface load $\boldsymbol{q}(x,y)$. Due to the presence of the noise in the measured data, use of filtration algorithms before the transformation is required [8]. Hereinafter the problem of wave propagation is considered in the frequency domain.

1.2 Forward problem for a structure with given crack parameters

The present work considers the through-thickness crack in the layered structure. The form of the crack and its location do not depend on the horizontal coordinate y. The structure is subjected to a line load that is independent of the y-axis. Hence, the problem is two-dimensional and the field variables are independent of the y-axis. In this paper, it is offered to model the crack as an infinitely thin cut, the surfaces of which are stress-free and non-interacting. A possible account of the contact between crack surfaces leads to the nonlinearity of the problem and thus substantially complicates analysis.

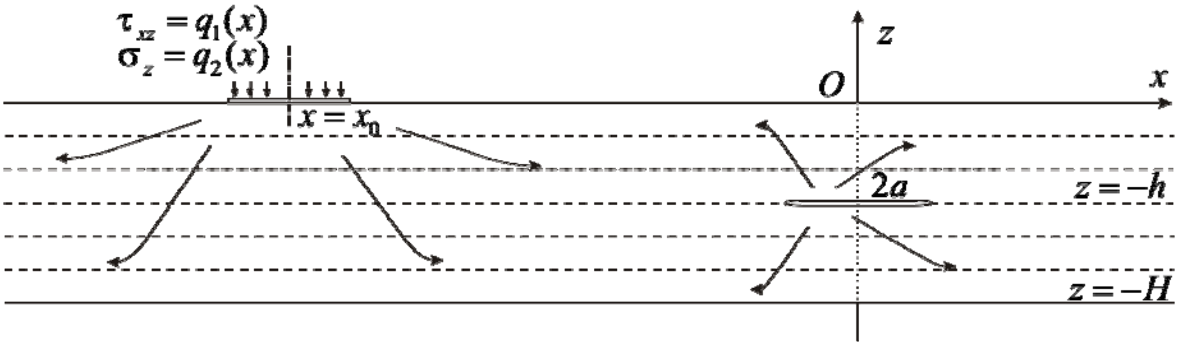

Fig. 1. Multilayered structure with a crack

We consider harmonic vibrations at a frequency ω of the multilayered structure. The layered structure is assumed to be infinite over the horizontal coordinate x. The thickness of a medium is H (Fig. 1). The load $\boldsymbol{q}(x)$ is given on the surface. At a depth h, the structure contains a crack with the length $2a$. The distance along the x-axis between the middle of the crack and the center of the domain with a given load is assumed to be x_0. The displacement field on the crack has a discontinuity with an unknown jump $\boldsymbol{v}(x) = \boldsymbol{u}_2(x,-h,\omega) - \boldsymbol{u}_1(x,-h,\omega)$ for $|x| \le a$, where the indices 2 and 1 correspond to the displacements of the structure above and below the crack respectively. The condition of the stress-free crack surfaces yields the system of boundary integral equations (BIEs) for unknown $\boldsymbol{v}(x)$ [11]

$$\int_{-a}^{a} \boldsymbol{l}(x-\xi,-h)\,\mathbf{v}(\xi)\,d\xi = -\boldsymbol{\tau}_0(x,-h), \quad |x| \le a, \tag{2}$$

where $\boldsymbol{\tau}_0(x,-h)$ describes the crack depth stresses within the structure without a damage, while the integral on the left side describes stresses caused by the displacement jump on the crack. The kernel of the integral equation $\boldsymbol{l}(x-\xi,-h)$ is a hypersingular operator and its representation $\boldsymbol{L}(\alpha,-h)$ in the Fourier domain can be calculated numerically, and in some cases analytically as well [14]

$$\boldsymbol{l}(x,-h) = \int_{\Gamma} \boldsymbol{L}(\alpha,-h) e^{-i\alpha x} d\alpha, \tag{3}$$

where Γ is the integration contour chosen according to the principle of a limiting absorption [14]. The right side of integral equation (2) can be calculated using the methods of Green's functions developed for layered structures [8, 14]. Let us consider the methods used to solve the problem of elastic waves propagation from a source within the layered infinite structure with a crack of the given configuration.

1.3 Galerkin method for solving BIEs

The technique for the solution of the system of BIEs (2), offered in [11], is used for the computation of the displacements on the surface of the structure with given damage parameters. It is assumed that the unknown jump (vector-function) of displacements on the crack can be approximately represented as a sum of a finite number of basis functions with coefficients-vectors

$$\boldsymbol{v}(x) \approx \sum_{k=0}^{N_p} \boldsymbol{c}_k p_k(x). \tag{4}$$

Due to the known properties of behavior of a displacement jump function on the crack edges [11], we choose Jacobi polynomials with weight functions as basis functions in the form

$$p_k(x) = P_k^{(\delta_1,\delta_2)}\left(\frac{x}{a}\right)\left(1-\frac{x}{a}\right)^{\delta_1}\left(1+\frac{x}{a}\right)^{\delta_2}. \tag{5}$$

In the general case, the indices δ_1, δ_2 must be calculated numerically, but in the case of the internal crack in the isotropic homogeneous material they are equal to $\delta_1 = \delta_2 = 1/2$. According to the standard Galerkin's scheme, the left and right sides of system (2) are subsequently multiplied scalarwise in L_2 by the basis functions $p_l(x)$ $N_p + 1$ times, and the problem is then reduced to the solution of a system of linear equations

$$\sum_{k=0}^{N_p} a_{lk} c_k = f_l, \tag{6}$$

where the system coefficients can be calculated, using the Parseval's identity, as scalar products in the Fourier domain after the transformation over the coordinate x is performed

$$a_{lk} = \left(L\,P_k, P_l\right)_{L_2},\quad f_l = \left(F, P_l\right)_{L_2}, \tag{7}$$

where the functions, written in capital letters, correspond to the respective functions from equations (2, 5) in the Fourier domain. The scalar products in (7) are calculated numerically in Fourier domain. When solution of (6) is found, the displacements at each point can be computed using the known methods of Green's functions [8, 11, 14].

The number of basis functions N_p is chosen according to the crack length and wave propagation frequency - the larger the values, the larger the number of basis functions that are needed to achieve the required accuracy of the Galerkin method [11].

1.4 FEM for a structure with an absorbing non-reflecting boundary

For calculating surface displacements in the medium with a crack of the known configuration is also used an algorithm, offered in [12]. This algorithm is based on the finite-element method, where an absorbing non-reflecting boundary for modeling of the boundary conditions at infinity is introduced. This algorithm is an alternative to the above method and does not require solution of integral equations (2).

When the problem of wave propagation in the medium infinite along the horizontal coordinate is considered using finite element method (FEM,) boundary conditions must be chosen specially, in order to prevent wave reflection. Note, that infinite elements, which are suggested in the commercial software ABAQUS are not suitable for the modeling of Lamb wave propagation in infinite layered media [12]. According to the algorithm from [12], it is necessary to supplement the area required for modeling by a damping area with internal friction increasing while moving away from the coordinate center. The friction must increase slowly to minimize wave reflection into the undamped area, but should be enough to avoid the reflections from the boundaries of the damping area. The damping parameters are selected numerically depending on the modeled medium properties, crack parameters, as well as frequency and the amplitudes of displacements. It is important to mention that the solution calculated in this way is approximate, firstly, due to the application of FEM and, secondly, due to imperfectness of modeling of a nonreflecting boundary.

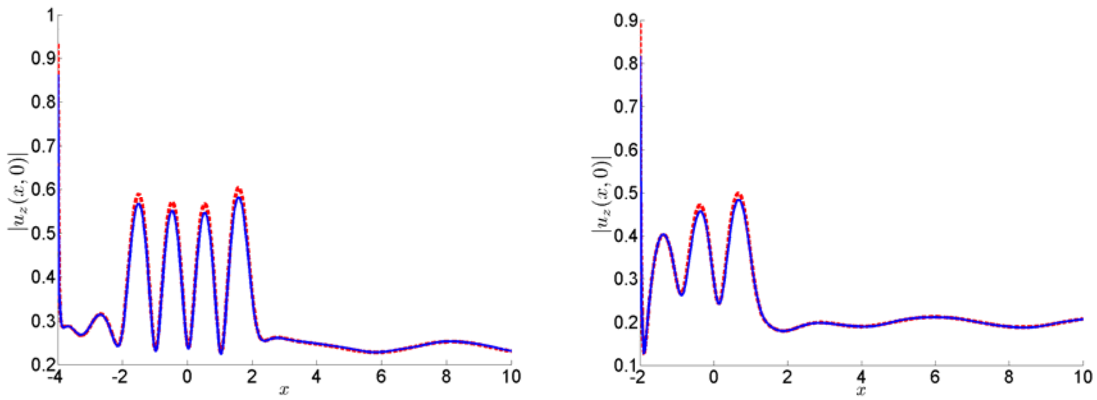

Fig. 2. Amplitudes of vertical displacements on the surface of a steel isotropic layer (the Poisson ratio $\nu = 1/3$) containing a crack under the excitation by a vertical point source: (a) $\overline{\omega} = 0.97$, $a = 1.49$, $h = 0.4$, $x_0 = 4$; (b) $\overline{\omega} = 1.57$, $a = 1$, $h = 0.4$, $x_0 = 2$.

Fig. 2 shows calculation results of surface displacements based on the Galerkin method (dashed line) and using ABAQUS (solid line). Crack parameters and frequency are given in dimensionless form, where the spatial variables and frequency variable are normalized with respect to the thickness of the layer H and to the shear waves velocity in a steel $\overline{\omega} = \omega H / c_S$. As is seen in the figures, the results are in good agreement. For the ABAQUS calculations, 16000 elements are used, while for the calculations using the Galerkin method, 20 basis functions are used. Increasing of the number of basis functions and elements did not lead to considerable changes in the solution. The calculation time in case of the Galerkin method (less than 5 seconds on a standard PC) is from two to four times less than that of the ABAQUS method.

2. Inverse problem of the identification of crack parameters

2.1 Optimization problem

It is known [10] that the inverse problem of the identification of the damage with arbitrary configuration using the measurement data of surface displacements has a unique solution when measurements are performed on the stress-free part of the surface of the medium and measurement data are known for the frequency range $[\omega_1, \omega_2]$ that does not contain resonance frequencies. If the measuring area does not intersect an area located directly above the crack, the problem is ill-posed and, therefore, is unstable relative to small perturbations of input data. In this case, to find the stable approximation of the solution of the inverse problem, it is required to use regularization algorithms. At the first stage, the method aimed at finding the crack configuration is based on the procedure of regularization over compact sets, specifically on preliminary parameterization of the crack through the values of its half-width a, location of the load application zone relative to the crack center x_0 and its occurrence depth h.

The displacements $\boldsymbol{u}(x_j)=\boldsymbol{g}_j$ specified in the measurement points x_j on the medium's surface are used in the paper as known inputs for the inverse problem. The goal of the identification is to find such crack parameters, for which the displacements at the same points x_j are nearly the same as measured values $\boldsymbol{g}_j$. This inverse problem is formulated using non-linear least squares method as the optimization problem [5, 10]

$$\min \Phi\left(a, x_0\right)=\sum_j\left\|\boldsymbol{u}\left(x_j, 0, a, x_0\right)-\boldsymbol{g}_j\right\| \rightarrow 0, \tag{4}$$

$$a_0 \leq a \leq a_M, \quad x_{0,L} \leq x_0 \leq x_{0,R}, \tag{5}$$

where the objective function describes the differences between the displacement values calculated for some "trial" parameters of the cracks a, x_0 and the known measurement data $\boldsymbol{u}(x_j)=\boldsymbol{g}_j$ in the points x_j. It is assumed that the depth of the crack h is known (this algorithm can be also extended for the case of the unknown crack depth), while properties of the material under study and frequency are the desired parameters.

The statement of the problem (4-5) does not satisfy the uniqueness theorem [10]. Using of the finite set of points for measuring displacements, as well as using only one frequency ω instead of the frequency range $\left[\omega_1, \omega_2\right]$ can lead to non-uniqueness of the solution of the inverse problem. On the other hand, the presence of a priori information about the solution, namely the crack depth h, as well as limitations on the unknown parameters (5) restricts the searching area and can make it possible to uniquely identify the unknown parameters of the crack. The question of the uniqueness of the inverse problem solution in the statement (4-5) requires additional investigations. In case of the non-uniqueness of problem solution (4-5), it is offered to use additional information about the solution, i.e. the larger number of displacement measurement points [5, 10] or the data about the displacement field during frequency variation [10] to get the over-posed problem.

2.2 Some aspects of numerical solution of the inverse problem

To solve the optimization problem, it is required to solve the forward problem of computing surface displacements many times for different crack parameters. Both algorithms described above are quick and stable and can be used as a forward solver. Local optimization methods are not effective in solving the optimization problem (4-5), since the objective function can have a large number of local maxima and minima and initial approximations of crack parameters are unknown. Genetic algorithms (GAs) and pattern search algorithms have proved their efficiency in problems of global optimization. Pattern search methods, also known as direct-search methods, do not require the gradient of the problem to be optimized and can hence be used on functions that are not continuous or differentiable. The global minimization software DIRECT 2.0 [13], based on the pattern search, is used to the optimization problem (4-5). The DIRECT 2.0 switches to the local search algorithm after a good approximation to the solution is obtained. Genetic Algorithms are derived on the principles of natural evolution and are widely used in many optimization problems, computational search algorithms and machine learning. The Matlab function "ga" with standard settings is also applied for searching the minimum.

In this work the measured data is simulated using calculation of displacements based on the Galerkin method or based on the application of the above described algorithm using FEM-Software ABAQUS for the given crack parameters. Random errors that are unavoidable during experimental measurement of surface displacements are modeled by adding the uniformly distributed random values to the displacements computed at the measuring points. The maximal relative error between the true and identified crack parameters is used in order to analyze the accuracy of the solution of inverse problem.

The regularization effect is achieved in this paper due to parameterization, i.e. the selection of the finite elements dimension in the FEM-based solution and the number of basis functions in the Galerkin method. The choice of these parameters can have a significant effect on the accuracy of the inverse problem solution and requires additional investigation. Presence of a large relative error in the measured data requires increasing the size of finite elements or decreasing the number of basis functions in the Galerkin method. More detailed information about the regularization effect due to the choice of parameters of a forward solver can be found in [5].

3. Numerical examples and discussion

Numerical calculations are carried out using the data about surface displacements in 16 points distributed on the surface of the structure. Furthermore, reaching the value of the specified objective function ($\Phi\left(a, x_0\right) < 10^{-4}$) or reaching the specified number of evaluations of the forward solver is used as a stopping criteria for the global search algorithms. The numerical experiments carried out in this paper are aimed to investigate how the number of basis functions in the Galerkin method and finite elements size influence the solution accuracy of the forward problem, to work out the methods for solving the inverse problem of crack identification based on the concept of the crack configuration parameterization, and to study the effect of the solution accuracy of the forward problem on the accuracy of the solution of the inverse problem.

Numerical calculations carried out for different locations of cracks in an isotropic layer and different locations of delaminations in a composite showed a high accuracy of the identification of the desired parameters of the damage. In the majority of simulations performed so far, the identification error, even in case of noisy measurement data with a relative error of 10 %, did not exceed 7.5 % when ABAQUS is used as a forward solver (Table 1) and 6% when Galerkin method is used (Table 2). Genetic algorithm applied to the optimization problem can quickly give a good approximation of the solution (10-15%), but in the vicinity of the minimum converges slowly. After 1000 evaluations of the forward solver the accuracy from 0.5% to 7.5% is reached. On the other side, the DIRECT 2.0 Software due to a switch between global and local optimization algorithms allows to find more accurate (from 0.1% to 5%) solution as the GA. The computational time of DIRECT 2.0 Software is about half of time, needed for the GA. It should be noted that, the accuracy of the crack position identification comparing to the accuracy of the crack length identification was in all simulations significantly higher (about 2-3 times). The results of numerical experiments for identification of delaminations in anisotropic composites also revealed that the error in the detection of damage parameters does not exceed the noise level introduced into the displacements at measuring points.

Table 1. Results of crack identification for isotropic layer of steel using ABAQUS as a forward solver, $\bar{\omega} = 1.88$, $h = 0.3$, $x_0 = 4.0$, $a = 1.5$

Relative error in measured data	**Relative error of identification (GA)**	**Relative error of identification (DIRECT 2.0)**	**Number of forward solver evaluations (GA)**	**Number of forward solver evaluations (DIRECT 2.0)**
0.5%	1.5%	1.3%	452	180
2.5%	3.9%	1.7%	567	245
10%	7.4%	4.9%	1000	789

Table 2. Results of crack identification for isotropic layer of steel using Galerkin method as a forward solver, $\bar{\omega} = 0.97$, $h = 0.5$, $x_0 = 2.0$, $a = 0.82$

Relative error in measured data	**Relative error of identification (GA)**	**Relative error of identification (DIRECT 2.0)**	**Number of forward solver evaluations (GA)**	**Number of forward solver evaluations (DIRECT 2.0)**
0.05%	1.0%	0.1%	327	133
5%	2.3%	0.3%	658	292
10%	6%	0.7%	1000	456

Based on the performed numerical experiments, it is possible to conclude that cracks and delaminations in the elastic medium can be successfully identified using the known measurement data of surface displacements and methods described in the present paper. The algorithms used to solve the forward problem have proved their effectiveness when applied to the real problems of elastic wave propagation in the media with defects [5, 11, 12], and the methods offered to solve damage identification problems make it possible to identify defects in case when the noise level in input data exceeds the noise level of real measurements. Therefore, the results obtained in the paper indicate the possibility of application of the above methods in the nondestructive testing.

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