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Aspects of CP Violation

An E_6 Symmetric Nelson-Barr Model and a Supersymmetric Solution to ϵ'_K/ϵ_K

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"Nenne keinen weise,
ehe er nicht bewiesen hat,
dass er eine Sache von wenigstens
acht Seiten her beurteilen kann."

\- Konfuzius

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Preface

A doctoral thesis is by its definition required to advance science. It should contain an answer to a previously unanswered scientific question. It is, however, common nowadays to work in collaborations and possibly even to work at multiple projects during the timespan of the thesis. The doctoral researcher grinding one question all by herself for three years and then coming up with an answer is nowadays probably the minority. These projects usually culminate in papers and therefore the proof that the doctoral researcher has conducted scientific work and contributed to advance the field has already been given prior to submitting the thesis. The same applies for this thesis. Since the scientific results are already given in the papers, this renders the doctoral thesis as a proof of scientific work obsolete. What the thesis, however, can do, is give insight in the very work the doctoral researcher conducted, the ideas that inspired the work, the thoughts that led to the results. In this sense, it can be more than a mere list of results of what the doctoral researcher has done, but it can be a portrayal of the work that the researcher has done in the timespan of the whole PhD. In my 3 years of PhD, I mainly worked on two different projects with several collaborators. Both projects culminated in scientific papers. With Teppei Kitahara and my advisor Ulrich Nierste, we investigated whether the present discrepancy of the flavor observable ϵ'_K/ϵ_K can be plausibly explained within the Minimal Supersymmetric Standard Model (MSSM) [1]. With Jakob Schwichtenberg and Robert Ziegler, we investigated whether an E_6 Grand Unified Theory with spontaneously broken CP symmetry can explain the absence of CP violation in the strong sector while still producing the Standard Model at low energies [2].

The results have already been stated for everyone to read. I will therefore intend to write this thesis the way I laid down above: as a portrayal not only of the results, but of the work that we did in the collaborations, focusing on my part of the work where it is possible to entangle it. As with discussions that lead to ideas and cross-checking calculations, sometimes there is nothing to entangle but the work is a team effort.

In the end, a doctoral thesis is graded by the supervisor, and - having today's collaborative environment in mind - I do feel that a doctoral student of physics should show signs of understanding in her thesis rather than merely presenting the numbers. A thesis should demonstrate that the student thought about physics at a level deeper than mere application. In the end, it all comes down to what we expect from the PhD title. It is my personal opinion that a person who holds a PhD in theoretical physics should aspire to have broad knowledge of physics theory in general and most importantly be curious about the concepts we can use to describe nature.

An E_6 Symmetric Nelson-Barr Model 1

This chapter is based on our paper "A Grand-Unified Nelson-Barr Model" [2]. The project started with the following considerations.

The strong interaction Lagrangian does allow for a CP violating term of the form $\bar{\theta}G\tilde{G}$, where G is the gluon field strength tensor and $\tilde{G}$ its dual - the contraction of G with the four dimensional Levi Civita symbol. The term is allowed and there is no reason, why its coupling parameter $\bar{\theta}$ should be zero. However, precision measurements of the neutron dipole moment restrict $\bar{\theta}$ to $\leqslant 10^{-10}$ [3] [4]. This result can be interpreted in two ways: either the parameter $\bar{\theta}$ is very small or zero because of some underlying reason. Or, it could mean that there is something structurally misunderstood about certain parts of Yang Mills theory and the term itself does not even arise. Either way, the smallness of $\bar{\theta}$ is known as the 'strong CP problem'. Formulated differently, why is the $\bar{\theta}$ parameter zero or close to zero when it could have any value? What is the underlying reason? While in the past decades the Axion solution enjoyed most dedication by phenomenologists, there are also other solutions.

One particular attractive solution is to say that CP is a symmetry of the Lagrangian. Then, $\bar{\theta}$ is naturally zero and the strong CP problem is solved. This, however, requires one to explain where the well established CP violation in the weak sector comes from. A very straight forward solution to this is to say that CP is a good symmetry at a high scale, then it is spontaneously broken in a way which gives rise to weak CP violation while the strong sector remains unaffected. While the strong sector is pretty much a paragon of a Yang Mills theory, the weak sector is kind of a bad-boy anyway: mediating across generations, having massive gauge bosons due to electroweak symmetry breaking, violating P, violating CP. So why shouldn't the CP violation be due to another symmetry breaking where the strong sector keeps a clean sheet.

Combining this idea with a GUT seems like the reasonable next step. You break a GUT group in a way that also breaks CP and you get all the nice features of GUTs for free. There are some reasons why GUTs might be a good idea [5]: One of the reasons is that you want the CP breaking scale rather high, otherwise you can easily get in trouble with FCNCs thanks to flavor precision constraints. Another reason is that if you break CP at a low scale, you might have to explain why there are no visible domain walls in the universe, whereas when you go to a sufficient high scale, then inflation just blows them away.

The interesting perspective here is that historically people thought about spontaneous CP breaking around the weak scale and had these kinds of problems with it. When they came up with the idea to combine it with a GUT, most of the problems just vanished [5], which is remarkable. Whenever an idea solves multiple problems at once, the idea usually has something good to offer.

The next question is how to break a GUT in a way that CP is violated while the strong sector remains unaffected? First of all, you generically break CP if your Higgs VEV is complex.[1] This way we obtain complex mass matrices and by diagonalizing them, your diagonalization matrices become complex and upon forming the CKM matrix (and also the PMNS matrix!), they give rise to a complex CKM (and PMNS) phase. So how could this affect the strong sector? The key lies within the $\bar{\theta}$ parameter. The actual $\bar{\theta}$ parameter in the Lagrangian is actually a sum of two parameters: $\bar{\theta} = \theta_{QCD} + \theta_F$ and usually denoted with a bar over the symbol. The first contribution, θ_{QCD}, goes by the name 'vacuum angle' and comes from the topological structure of the QCD Yang Mills vacuum. The second contribution, θ_F, comes from the fermion sector of the theory and is the argument of the determinant of the quark mass matrix[2]. How this comes to merge with a parameter of the topology of the Yang Mills vacuum seems like a miracle at first - two sectors, which appear to have nothing to do with each other - but does have a deeper reason. This reason has something to do with the chiral anomaly and the very structure of quantum field theory and is not only vastly interesting, but, as I think, a key progress to our understanding in quantum field theory and how its different aspects work together to create the phenomena we observe. Having seen, that θ_F gives rise to CP violation in the strong sector, the task at hand in constructing a model of spontaneous CP breaking is then to ensure that this determinant is real. This is most conveniently - and one could argue most naturally - ensured by implementing the Barr criteria [6]. These criteria simply use the fact that a determinant is a product of the entries of the matrix and then demand that only certain entries may be complex while other, complementing entries have to be zero. This ensures, that in the calculation of the determinant only real entries survive and the determinant thus by construction has no phase. While this sounds somewhat arbitrary and artificial, it will hopefully become clear in the following sections that this is merely a constraint on the GUT group breaking VEVs: Only few of them can be complex, while some have to be zero to arrive at a low energy theory with vanishing strong CP violation. After all, when constructing a model one has to make certain choices, most obviously: we will only give nonzero VEVs to scalar fields that do not break the Standard Model. Similarly here, we only make VEVs complex that do not violate the strong CP conservation. In this sense, the restriction from the Barr criteria can be interpreted as more of a guideline for which VEVs to pick and which not to, in the very same way as the Standard Model gauge group tells us which VEVs to pick and which not to.

This is a subtle point we would like to emphasize again: In phenomenological GUT model building it is common sense to consider only breaking chains and thus values for VEVs which reproduce the Standard Model. All the numerous other choices are discarded because they contradict what we measure. In the same way, starting with the ambition to construct a GUT model which breaks CP spontaneously, the mere requirement that we have to end up with the Standard Model which includes a CP invariant strong sector

[1] To be specific, a combination of Higgs VEVs needs to differ in their complex phase, otherwise you can just make it real by redefinition of the physical field.

[2] θ_F is the argument of the determinant of the product of the up quark and down quark mass matrices. It is often presented in a way that suggests it is only about the down quark mass matrix, but this presentation assumes working in a basis where the up quark mass matrix is diagonal and thereby the relevant misalignment is entirely transported into the down quark mass matrix.

constrains the breaking chain and the VEVs. The term 'Barr criteria' is just a label for this.

This summarizes the approach we took and gives already an outline of our research question: We take CP to be a symmetry of the Lagrangian to naturally have $\theta_{QCD} = 0$. Then we employ a GUT group and break it through a complex VEV in order to generate weak CP violation. The obvious question is: is this possible? If so, is it reasonable? If it turns out that the model is not reasonable, what can we learn from this? If it turns out to be reasonable, what can we learn from that? The term 'reasonable' differs from person to person. There is, for example, some consensus in the community that overly fine-tuned models are usually considered not very reasonable. On the other hand, a class of something-phobic models emerged recently which seem to be widely accepted, yet I personally consider them not very reasonable. But in the end we have to judge for ourselves what we consider reasonable and what we do not. As history tells us, the next successful theory of nature will certainly be considered 'not reasonable' by a large number of people before it is verified by experiment.

I should mention, that the model turned out to be what we consider quite reasonable, in that it is consistent to the extend we investigated it. It makes predictions and it is falsifiable. It even turned out to be more reasonable than we had anticipated. These are the kind of pleasant surprises one secretly hopes to encounter in model building.

1.1. Motivation: Essentials of the BBP Study & Barr Criteria

Why E_6? This question is probably best answered by going back to the original question we had when starting with the project: which GUT group to choose? In '91, Bento Branco and Parada (BBP) investigated [7] a simplified model that featured spontaneous CP breaking by employing the Barr criteria. Their study was designed to be a minimalistic realization of the Barr criteria and analyze some of the resulting phenomenology. On top of the Standard Model (SM), they added one heavy vectorlike downtype quark[3] ($D_{L,R}$), and a scalar, which obtained the complex VEV above the EW scale. In our model, we tried to construct a predictive model by realizing that the core idea of BBP's phenomenological study can be implemented in a model when starting with an E_6 GUT. The difference we get is that we have three generations of exotic down type quarks, which seems more natural than having just one. The Barr criteria then state that the complex coupling may only appear in SM-$D_{L,R}$ couplings, while SM-SM couplings and $D_{L,R}$-$D_{L,R}$ couplings need to be real. The right handed SM down quarks d_R thus couple via the complex VEV to the exotic left handed down quark D_L, while a d_L-D_R coupling cannot arise via the complex VEV because of $SU(2)_L$ symmetry.

To clarify this point: Before EWSB, the left handed SM down type quarks d_L are merely gauge degrees of freedom in the left handed quark $SU(2)_L$ doublets Q_L and thus cannot couple to the exotic quark D_R, which is a $SU(2)_L$ singlet, via the exotic scalar. Any coupling to the left handed quarks can thus only arise after EWSB, when the $SU(2)_L$ is broken and the gauge degrees of freedom become physical. But since we need the exotic quark to be heavy, the exotic scalar VEV needs to be far above the EW scale to generate this mass term. This leaves only one option then: coupling the exotic quark D_L to the right handed SM down type quarks d_R in a Yukawa coupling with the exotic scalar. Coupling to up type quarks is out of question because the exotic quark has the same electric charge as the SM down type quarks. To form an electrically neutral term with an SM up type quark would require a charged VEV. Note that in the BBP model, the electric charge assignment for the exotic quark is for convenience only. In a GUT model, these charges - like all quantum numbers - are fixed by the GUT group. Also, unless the GUT group unifies generations which up to today has not brought very satisfactory results, there will be three copies of the GUT group field content, leading certainly to not one exotic quark in total but one per generation. We will see this when we discuss our model in section 1.2.

The addition of one vectorlike quark effectively adds a 4th row and column to the down quark mass matrix, where one part of the new off-diagonal entries are complex while another part is zero. Diagonalizing this mass matrix then leads to complex rotation matrices and subsequently to a complex CKM matrix. This gives rise to CP violation in the weak sector. Meanwhile the Barr criteria ensure that ArgDet $(M_u M_d) = 0$ and thus the $\bar{\theta}$ parameter is not generated through the quark mass matrix.

We see that the Barr criteria are two rules for VEVs in a GUT model, which ensure ArgDet $(M_u M_d) = 0$ at tree level. The first one states, that all EW scale VEVs which mix SM and exotic quarks must be zero. The second one states, that only those (GUT scale) VEVs which mix SM and exotic quarks are allowed to be complex, all others must be real.

[3]Note that they only added one exotic vectorlike quark in total, not one per generation.

To explicitly show this, suppose we have a Lagrangian of the form

$$\mathcal{L}_d = d_L m_d d_R + D_L M_C d_R + d_L m_C D_R + D_L M_R D_R \tag{1.1}$$

where the mass parameters m_d, M_C, m_C and M_R are in general a Higgs VEV times a Yukawa matrix. We can think of the $d_{L,R}$ as the SM down quarks and the $D_{L,R}$ as some exotic quarks with the same quantum numbers. By introducing new fields that are superpositions of the old fields, we can find the mass eigenstates. This is conveniently done by writing the Lagrangian as a scalar product of vectors and matrices and then diagonalizing these matrices. Generations have been suppressed for simplicity.

$$\mathcal{L}_d = \begin{pmatrix} d_L & D_L \end{pmatrix} \begin{pmatrix} m_d & m_C \\ M_C & M_R \end{pmatrix} \begin{pmatrix} d_R \\ D_R \end{pmatrix} \tag{1.2}$$

This defines the full down quark mass matrix $\mathcal{M}_d$. Now we want a complex mass matrix in order to get a complex CKM matrix upon diagonalization, while retaining a real determinant, not to spoil $\bar{\theta} = \theta_{QCD} + \text{ArgDet}\ (M_u M_d) = 0$. This is generically satisfied, if we impose the Barr criteria:

$$m_C = 0 \qquad\qquad M_C \in \mathbb{C}^{\mathrm{n}\times\mathrm{n}} \qquad\qquad m_d \wedge M_R \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}} \tag{1.3}$$

This way, the determinant is trivially real: we demand that every complex entry is cancelled by a zero in the calculation of the determinant. This is what the Barr criteria do. We obtain a little more sophisticated and maybe more physical seeming version of that statement when we remember that each of the mass parameters is the product of a Yukawa matrix and a Higgs VEV. Then, the Barr criteria (1.3) translate into prescriptions on the scalar VEVs: some of the VEVs mixing SM and exotic fields may become complex (those contained in M_C) while others have to be zero (those contained in m_C). We will see this explicitly when discussing the symmetry breaking scheme of our model where some VEVs are required to be zero for exactly this reason.

As we already mentioned, this line of arguments also holds for an arbitrary number of families. Bento Branco and Parada used three generations for the SM down quarks $d_{L,R}$ while adding only one exotic quark $D_{L,R}$. Since our model is based on an E_6 gauge symmetry, we naturally get one exotic down quark $D_{L,R}$ for every generation. Therefore, the mass parameters above are all 3×3 matrices, consisting of combinations of Yukawa matrices with Higgs VEVs.

So what do we learn from BBP and their simplified model? If we want to employ a similar realization of the Barr criteria, then we need vectorlike exotic down quarks. And here is where E_6 comes around. E_6 can provide us with a vectorlike down type quark $D_{L,R}$, a vectorlike lepton doublet $L_{L,R}$ and a SM singlet s, if we choose the right breaking chain. And we get this once for each generation, which does seem more natural than adding just one more field. The exotic downtype quarks transport CP violation into the CKM matrix in the same fashion as in the BBP model. The fact that there are three exotic quarks doesn't alter this mechanism. The lepton doublets, interestingly, do the same in the lepton sector, thereby generating CP violation in the PMNS matrix. This has not been measured yet at the time of writing this thesis and therefore promises a solid prediction already at this very conceptual level. Moreover, the quark CP phase and the lepton CP phase are correlated through the GUT group. So apart from the exotic singlet, the new particles predicted by the E_6 GUT group fit just perfectly with the idea of breaking CP spontaneously and employing the Barr criteria in the BBP manner. It turns out that even the singlet s plays a crucial role in our model. Therefore all exotic particles that emerge in our model matter and are actually necessary for the model to work out. There are no superfluous exotic fermions flying around. The appeal of the traditional GUTs $SU(5)$ and $SO(10)$ lies in the fact, that they contain just the SM and essentially nothing more. With just any other GUT, you get additional particle and raise the question 'what are these good for?'. Taking a large GUT like E_6 and realizing that all the new particles are actually useful and even needed to perform mechanisms in order to realize the original idea, is a particularly attractive observation.

We will introduce our model in Section 1.2 by spelling out the breaking chain from E_6 to the SM. In this process we determine the fermion fields and the scalar VEVs. In Section 1.3 and Section 1.4.1 we examine the quark and the lepton sectors in some detail, showing how the complex GUT VEV feeds into the low energy phenomenology. In Section 1.4.2 we give a detailed treatment of the neutrino sector before we come to the results in Section 1.5.

1.1.1. RG survival of the complex phase

BBP showed in their paper [7], that the phase of the complex VEV that feeds into the CKM matrix is in this particular setup suppressed only by a ratio of high scale VEVs. Naively, one would assume a decoupling behaviour like EW scale over GUT scale. Formula (7a) of [7] generalized to an arbitrary number of heavy exotic quarks, is given by

$$K\overline{m}^2K^\dagger = m_d\left[1 - M_C^\dagger(M^2)^{-1}M_C + \mathcal{O}(m_d^4M_C^2/M_R^6)\right]m_d^\dagger \tag{1.4}$$

We give a detailed derivation in Appendix A.2. Here, m_d, M_C and M_R are defined via the Lagrangian (1.1) and the conditions (1.3). $M^2 := M_CM_C^\dagger + M_RM_R^\dagger$ and $\overline{m} = \mathrm{Diag}(m_b, m_s, m_d)$ is the diagonal matrix of SM down type quark masses. K is the CKM matrix, which is unitary up to corrections of $\mathcal{O}(m_d^4M_C^2/M_R^6)$. We see from Equation (1.4) that the complex phase residing in M_C is essentially only suppressed by the scale of M_R. The scales M_C and M_R thus need to be close together, however, they may be arbitrarily high - and are required to be sufficiently high to guarantee sufficient unitarity of the CKM matrix.

In our model, M_C is proportional to a $SU(5)$ breaking VEV, while M_R is proportional to $SO(10)$ and E_6 breaking VEVs. The takeaway message here is that the survival of the complex phase requires our GUT scales to be fairly near to each other, while the overall position of the GUT scale w.r.t. the EW scale may be arbitrarily high. The lower bounds here being flavor precision experiments which we would certainly contradict should the corrections to the approximate unitarity of the CKM matrix become too large. The CKM matrix is essentially $V_{CKM} = V_{CKM}^{SM} + \mathcal{O}(m_{EW}^2/M_{GUT}^2)$. The upper bound is the Planck scale from physical reasonability arguments of the effective theory and the lower bound is proton decay. As we will see later, the neutrino sector expresses some preference here if we want to obtain the correct mass range of the light neutrinos by means of a seesaw mechanism. Interestingly enough, these requirements constrain the choices for the GUT scales so heavily, that their order of magnitude is essentially fixed.

1.2. Field Content of our Model

The symmetry breaking pattern we chose to work with, is

$$E_6 \longrightarrow SO(10) \longrightarrow SU(5) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \tag{1.5}$$

The rank of the group $SO(10)$ is lower than the rank of E_6. This expresses itself in an additional $U(1)$ factor that is broken when E_6 is broken. The same happens when $SO(10)$ breaks to $SU(5)$. In the notation, we keep the $U(1)_{SO(10)}$ and $U(1)_{SU(5)}$ factors as subscript as a consistency check of the decomposition. They are, however, not to be interpreted as unbroken group factors.

This breaking pattern gives us suitable representations for the Barr criteria (the vectorlike down quarks) and at the same time provides us with the possibility of having the GUT scales close enough to ensure the complex phase is not suppressed. The Pati-Salam model, to name an example, requires an amount of RGE running between the Pati-Salam scale and the $SO(10)$ scale to make the unification work, which would suppress the complex phase way too much.

At the E_6 scale, we have one single fermion field (coming in three generations), which we will call 27. This is the fundamental representation of E_6, analogous to a quark with 27 colours. The gauge bosons are in the adjoint representation 78, as dictated by gauge theory. To employ symmetry breaking, we need scalar fields and we take a 27_H and a symmetric 351_H.

The Yukawa Lagrangian above the E_6 scale is

$$\mathcal{L} = 27\ 27\ (\mathcal{Y}_{27}\ 27_H + \mathcal{Y}_{351}\ 351_H) \tag{1.6}$$

where we choose to work in a basis where the Yukawa matrix $\mathcal{Y}_{27}$ is diagonal. $\mathcal{Y}_{351}$ is symmetric as a result of the E_6 symmetry. The model is CP invariant at the E_6 scale, therefore the Yukawa matrices need to be real [8]. We thus have $3 + 6 = 9$ Yukawa parameters.

A word on the scalar fields

It is worthwhile to note, that $\overline{27} \times \overline{27} = 27 + 351_A + 351$, where 351_A is an antisymmetric representation which we did not require to fit our model. This tensor product leaves room for creativity: a bound state of two fermions in the 27 can decompose in exactly the scalar representations required to break E_6 to the SM. A rather appealing composite Higgs GUT scenario. We will not follow this idea in this thesis. We just comment that from the modern geometric perspective of gauge theories, fundamental scalar fields somewhat seem not to fit in very well. To that end we quote from a review by Francois, Lazzarini and Masson:

"In the early 1950s, while Yang and Mills proposed their idea of non-abelian gauge fields (generalization of electromagnetism), Ehresmann developed the notion of connections on

principal fiber bundles, which turns out to be the natural mathematical framework for Yang-Mills field theories [...] Indeed, the $\mathcal{C}^2$-valued scalar field involved in the SSBM is, at the same time, a section of a (suitable) associated vector bundle [...], and a boson, so that it is an "hybrid structure" [...]. Moreover, in this scheme, its scalar potential does not emerge from a natural mathematical construction." (taken from [9], page 2)

Coupled with the immense size of GUT scalar sectors which make it challenging to even write them down, let alone make predictive statements from them, theory could really use some fundamental work on the origin of scalar fields. Looking at the incredible precision we reach in the determination of some Higgs decays, conceptual theory is nowadays threatened to fall behind the impressive progress of phenomenology and experiment. The case of a GUT, which seemingly offers to make the required scalar fields emergent through a bound state of 'the fermion' present in the theory sounds certainly appealing. Such attempts have been followed in the past, e.g. [10], albeit with little success as a realistic theory. Here we just write down the Yukawa Lagrangian with the side note that the scalar fields may or may not originate from a fermionic bound state, and then focus on working out the phenomenological consequences of the setup.

1.2.1. Fermion Fields

We start at the E_6 scale with three copies of a fermion field in the 27.

All our fermion fields come from the 27 of E_6, which is the fundamental representation.

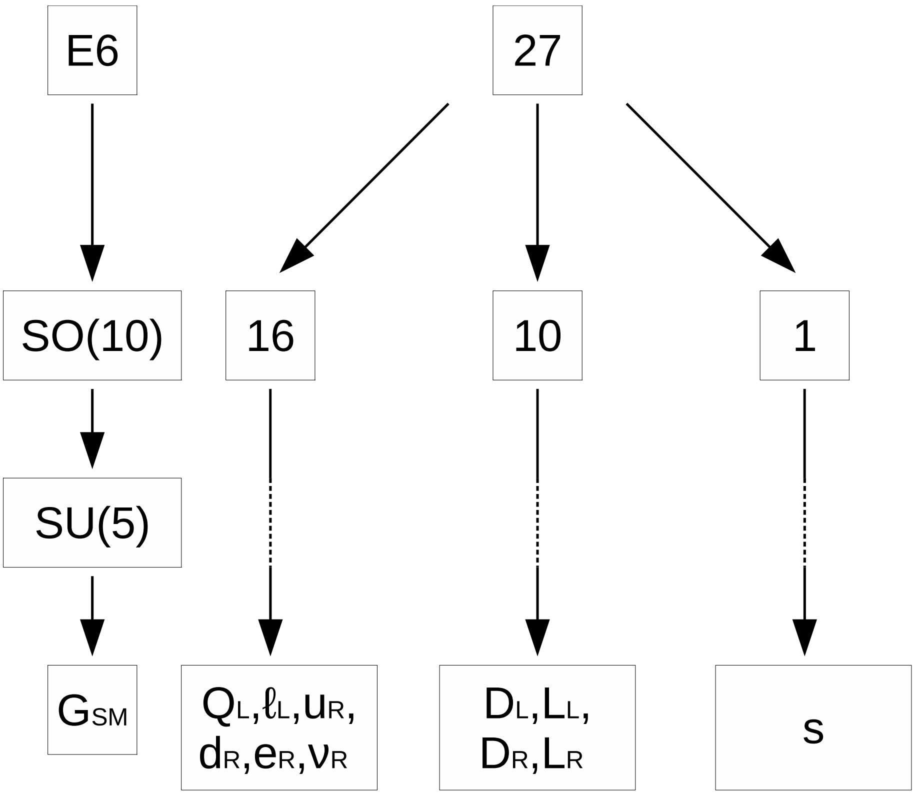

Figure 1.1.: *Sketch of the spectrum of our model.*

The 27 decomposes under $SO(10)$ into a 16, a 10 and a 1 of $SO(10)$ (see sketch in Figure 1.1, which contains the takeaway message of this subsection). The Standard Model fermion fields (plus a sterile neutrino w.r.t. the SM) are entirely contained in the 16 of $SO(10)$, while the 10 of $SO(10)$ gives rise to new exotic fermions. We give a detailed list of the fermion fields and their representations in Table 1.1 (We normalized the SM $U(1)$ hypercharge in a way that it represents the average electric charge of the multiplet)

E_6	$SO(10)\times U(1)_{SO(10)}$	$SU(5)\times U(1)_{SU(5)}$	$SU(3)_C\times SU(2)_L\times U(1)_Y$	Label
27	16_1	10_{-1}	$(3,2)_{1/6}+(\overline{3},1)_{-2/3}+(1,1)_1$	$Q_L+u_R^c+e_R^c$
27	16_1	$\overline{5}_3$	$(1,2)_{-1/2}+(\overline{3},1)_{1/3}$	$\ell_L+d_R^c$
27	16_1	1_{-5}	$(1,1)_0$	ν_R^c
27	10_{-2}	5_2	$(1,2)_{1/2}+(3,1)_{-1/3}$	$L_R^c+D_L$
27	10_{-2}	$\overline{5}_{-2}$	$(1,2)_{-1/2}+(\overline{3},1)_{1/3}$	$L_L+D_R^c$
27	1_4	1_0	$(1,1)_0$	s

Table 1.1.: *The fermion content of our model. $U(1)$ charges are written as indices.*

The superscript c denotes the charge conjugate which accounts for conjugates in the representation and flipping the sign of the $U(1)$ charges. It also flips the chirality of a field, thus, since it only appears on right handed fields, we only deal with left handed fields here. The label ν_R^c is at this point merely a name. It is a SM singlet and has no relation to chirality but we introduce the label here to point at its later use as partner of the SM left handed neutrino ν_L.

The content of the $SU(2)$ doublets is $Q_L=(u_L,d_L)$, $\ell_L=(\nu_L,e_L)$, $L_L=(N_L,E_L)$ and $L_R^c=(E_R^c,N_R^c)$. Note the flip in the L_R^c multiplet compared to L_L, which is due them being charge conjugates. SM fields follow the usual notation, the capital letters (except for the Q_L which is the SM left handed quark doublet) denote the exotic vectorlike fermions, where the names are given with respect to their resemblance to the quantum numbers of the respective SM fields.

1.2.2. GUT Breaking Scalar Fields

To find the VEVs which we can use to break the GUT groups to the SM, we work out the scalar fields, which are singlets under the Standard Model gauge group. The relevant scalar fields are (see Appendix A.1.1 for details) given in Table 1.2.

E_6	$SO(10)\times U(1)_{SO(10)}$	$SU(5)\times U(1)_{SU(5)}$	$SU(3)_C\times SU(2)_L\times U(1)_Y$	Label
27	1_4	1_0	$(1,1)_0$	$\phi_{27;1;1}$
351	1_{-8}	1_0	$(1,1)_0$	$\phi_{351;1;1}$
27	16_1	1_{-5}	$(1,1)_0$	$\phi_{27;16;1}$
351	$\overline{16}_{-5}$	1_5	$(1,1)_0$	$\phi_{351;\overline{16};1}$
351	$\overline{126}_{-2}$	1_{10}	$(1,1)_0$	$\phi_{351;\overline{126};1}$
351	54_4	24_0	$(1,1)_0$	$\phi_{351;54;24}$
351	144_1	24_{-5}	$(1,1)_0$	$\phi_{351;144;24}$

Table 1.2.: *The SM singlet scalar fields and the progression of their representations through our breaking chain. These fields can be used to break the GUT groups while leaving the SM intact.*

The labels show the representations these fields live in in order to show which symmetry groups are broken upon giving them a nonzero VEV. We have omitted the SM gauge group in the label since they all are singlets and leave the SM gauge group intact. For the SM breaking scalar fields, we will use a different notation.

Using the above defined labels for fermions and GUT breaking scalars, the Yukawa Lagrangian which includes their interactions is given by

$$\begin{aligned}\mathcal{L}_{\mathcal{G}_{SM}} =&\mathcal{Y}_{27}\Big(\ \ell_L L^c_R \ \phi_{27;16;1} + D_L d^c_R \ \phi_{27;16;1} + L_L L^c_R \ \phi_{27;1;1} + D_L D^c_R \ \phi_{27;1;1}\Big)\\ &\mathcal{Y}_{351'}\Big(\ell_L L^c_R \ \phi_{351;144;24} + D_L d^c_R \ \phi_{351;144;24} + \nu^c_R \nu^c_R \ \phi_{351;\overline{126};1}\\ &+ s\nu^c_R \ \phi_{351;\overline{16};1} + L_L L^c_R \ \phi_{351;54;24} + D_L D^c_R \ \phi_{351;54;24} + ss \ \phi_{351;1;1}\Big)\end{aligned} \tag{1.7}$$

We show the explicit decomposition of the Yukawa terms in Appendix A.1.1. Mind that this is the SM language before EWSB, thus the left handed SM fields are still $SU(2)$ doublets ℓ_L (and Q_L although not appearing here), as are the (left and right) exotic leptons L_L and L^c_R.

We denote the VEVs of these fields $\langle\phi_{(E_6;SO(10);SU(5))}\rangle$ by

$$\begin{aligned}\langle\phi_{(27;1;1)}\rangle &\equiv v_{6,1}\\ \langle\phi_{(351;1;1)}\rangle &\equiv v_{6,2}\\ \langle\phi_{(27;16;1)}\rangle &\equiv v_{10,1}\\ \langle\phi_{(351;\overline{16};1)}\rangle &\equiv v_{10,2}\\ \langle\phi_{(351;\overline{126};1)}\rangle &\equiv v_{10,3}\\ \langle\phi_{(351;54;24)}\rangle &\equiv v_{5,1}\\ \langle\phi_{(351;144;24)}\rangle &\equiv v_{5,2}\end{aligned} \tag{1.8}$$

This encodes the important information, that is down to which group the VEV will break the symmetry.

We so far ignored the Clebsch-Gordan coefficients since we were interested only in the general structure. When we want to consider the mass matrices, we need to include them however and do so in the next step. We express the scalar fields through the respective VEVs and we will also expand the SM $SU(2)$ doublets to explicitly show the form of the Lagrangian after EWSB

$$\begin{aligned}\mathcal{L}_{\mathcal{G}_{SM}} =& \mathcal{Y}_{27}\Big[\ \Big(D_L D^c_R + E_L E^c_R + N_L N^c_R\Big) \ v_{6,1} + \Big(D_L d^c_R + e_L E^c_R + \nu_L N^c_R\Big) \ v_{10,1}\Big]\\ &\mathcal{Y}_{351'}\Big[ss \ v_{6,2} + s\nu^c_R \ v_{10,2} + \nu^c_R\nu^c_R \ v_{10,3}\\ &+ \Big(D_L D^c_R - \frac{3}{2}E_L E^c_R - \frac{3}{2}N_L N^c_R\Big) \ v_{5,1} + \Big(D_L d^c_R - \frac{3}{2}e_L E^c_R - \frac{3}{2}\nu_L N^c_R\Big) \ v_{5,2}\Big]\end{aligned} \tag{1.9}$$

1.2.3. Higgs Fields

By Higgs Fields we denote the scalar fields, which can do the Standard Model symmetry breaking, that is break the electroweak symmetry in the standard way to the electromagnetic $U(1)$. From the representations 144, $\overline{126}$ and 10 of $SO(10)$ we get via $\overline{45}$ and $\overline{5}$ of $SU(5)$ scalar fields that transform under the Standard Model symmetry group as $(1,2)_{-1/2}$ (we normalized the charge to the average electric charge of the multiplet, meaning we divided the Slansky convention [11] by 6).

E_6	$SO(10)\times U(1)_{SO(10)}$	$SU(5)\times U(1)_{SU(5)}$	$SU(3)_C\times SU(2)_L\times U(1)_Y$	Label
27	10_{-2}	5_2	$(1,2)_{1/2}$	$H^u_{27;10;5}$
351	10_{-2}	5_2	$(1,2)_{1/2}$	$H^u_{351;10;5}$
351	$\overline{16}_{-5}$	5_{-3}	$(1,2)_{1/2}$	$H^u_{351;\overline{16};5}$
351	$\overline{126}_{-2}$	5_2	$(1,2)_{1/2}$	$H^u_{351;\overline{126};5}$
351	144_1	5_7	$(1,2)_{1/2}$	$H^u_{351;144;5}$

Table 1.3.: *SM Higgs fields with hypercharge +1/2. These can be used to generate EW mass terms e.g. for up type quarks and neutrinos.*

The up type Yukawa Lagrangian is thus

$$\begin{aligned}\mathcal{L}_{Yuk,SM,u} = &\,\mathcal{Y}_{27}\left(Q_L u_R^c + \ell_L \nu_R^c + L_L s\right) H^u_{27;10;5} \\ &+ \mathcal{Y}_{351}\Big[\left(Q_L u_R^c + \ell_L \nu_R^c + L_L s\right)\left(H^u_{351;10;5} + H^u_{351;\overline{126};5}\right) \\ &\quad + L_L \nu_R^c H^u_{351;144;5} + \ell_L s H^u_{351;\overline{16};5}\Big]\end{aligned} \tag{1.10}$$

Where we colored the Higgs fields that can give masses to the SM up quarks.
We denote the VEVs of these Higgs fields in the following way

$$\begin{aligned}\langle H^u_{27;10;5}\rangle &\equiv v_{u,1} \\ \langle H^u_{351;10;5}\rangle &\equiv v_{u,2} \\ \langle H^u_{351;\overline{16};5}\rangle &\equiv v_{u,3} \\ \langle H^u_{351;\overline{126};5}\rangle &\equiv v_{u,4} \\ \langle H^u_{351;144;5}\rangle &\equiv v_{u,5}\end{aligned} \tag{1.11}$$

Turning to the down type Higgs fields, we have

The down type Yukawa Lagrangian is thus

$$\begin{aligned}\mathcal{L}_{Yuk,SM,d} = &\,\mathcal{Y}_{27}\Big[\left(L_R^c \nu_R^c + L_L e_R^c + Q_L D_R^c\right) H^d_{27;16;\overline{5}} + \left(Q_L d_R^c + \ell_L e_R^c + s L_R^c\right) H^d_{27;10;\overline{5}}\Big] \\ &+ \mathcal{Y}_{351}\Big[\left(Q_L d_R^c + \ell_L e_R^c + s L_R^c\right)\left(H^d_{351;10;\overline{5}} + H^d_{351;\overline{126};\overline{45}}\right) \\ &\quad + \left(L_R^c \nu_R^c + L_L e_R^c + Q_L D_R^c\right)\left(H^d_{351;144;\overline{45}} + H^d_{351;144;\overline{5}}\right)\Big]\end{aligned} \tag{1.12}$$

where we again colored the Higgs fields responsible for down quark mass generation.

E_6	$SO(10)\times U(1)_{SO(10)}$	$SU(5)\times U(1)_{SU(5)}$	$SU(3)_C\times SU(2)_L\times U(1)_Y$	Label
27	10_{-2}	$\overline{5}_{-2}$	$(1,2)_{-1/2}$	$H^d_{27;10;\overline{5}}$
27	16_1	$\overline{5}_3$	$(1,2)_{-1/2}$	$H^d_{27;16;\overline{5}}$
351	10_{-2}	$\overline{5}_{-2}$	$(1,2)_{-1/2}$	$H^d_{351;10;\overline{5}}$
351	$\overline{126}_{-2}$	$\overline{45}_{-2}$	$(1,2)_{-1/2}$	$H^d_{351;\overline{126};\overline{45}}$
351	144_1	$\overline{45}_3$	$(1,2)_{-1/2}$	$H^d_{351;144;\overline{45}}$
351	144_1	$\overline{5}_3$	$(1,2)_{-1/2}$	$H^d_{351;144;\overline{5}}$

Table 1.4.: *SM Higgs fields with hypercharge -1/2. These can be used to generate EW mass terms e.g. for down type quarks and electrons.*

Again we denote the VEVs of these fields by

$$\begin{aligned}
\langle H^d_{27;10;\overline{5}}\rangle &\equiv v_{d,1}\\
\langle H^d_{27;16;\overline{5}}\rangle &\equiv v_{d,2}\\
\langle H^d_{351;10;\overline{5}}\rangle &\equiv v_{d,3}\\
\langle H^d_{351;\overline{126};\overline{45}}\rangle &\equiv v_{d,4}\\
\langle H^d_{351;144;\overline{5}}\rangle &\equiv v_{d,5}\\
\langle H^d_{351;144;\overline{45}}\rangle &\equiv v_{d,6}
\end{aligned} \tag{1.13}$$

1.2.4. First Barr criterion: Complex VEVs

The full Yukawa Lagrangian responsible for the mass generation looks like

$$\begin{aligned}
\mathcal{L}_{Yuk,mass} = \mathcal{Y}_{27}\Big[&\left(D_L D^c_R + E_L E^c_R + N_L N^c_R\right)\, v_{6,1} + \left(D_L d^c_R + e_L E^c_R + \nu_L N^c_R\right)\, v_{10,1}\Big]\\
+ \mathcal{Y}_{351'}\Big[&ss\, v_{6,2} + s\nu^c_R\, v_{10,2} + \nu^c_R \nu^c_R\, v_{10,3}\\
&+ \left(D_L D^c_R - \frac{3}{2}E_L E^c_R - \frac{3}{2}N_L N^c_R\right)\, v_{5,1} + \left(D_L d^c_R - \frac{3}{2}e_L E^c_R - \frac{3}{2}\nu_L N^c_R\right)\, v_{5,2}\Big]\\
+ \mathcal{Y}_{27}\Big[&u_L u^c_R + \nu_L \nu^c_R + N_L s\Big] v_{u,1}\\
+ \mathcal{Y}_{351}\Big[&\left(u_L u^c_R - 3\nu_L\nu^c_R - 3N_L s\right)\left(v_{u,2} + v_{u,4}\right) + N_L \nu^c_R v_{u,5} + \nu_L s v_{u,3}\Big]\\
+ \mathcal{Y}_{27}\Big[&\left(N^c_R \nu^c_R + E_L e^c_R + d_L D^c_R\right) v_{d,2} + \left(d_L d^c_R + e_L e^c_R + s N^c_R\right) v_{d,1}\Big]\\
+ \mathcal{Y}_{351}\Big[&\left(d_L d^c_R - 3e_L e^c_R - 3sN^c_R\right)\left(v_{d,3} + v_{d,4}\right) + \left(N^c_R \nu^c_R + E_L e^c_R + d_L D^c_R\right)\left(v_{d,5} + v_{d,6}\right)\Big]
\end{aligned} \tag{1.14}$$

We recognize the GUT breaking part in the first three lines and the SM breaking part in the remaining four lines. So far, we have not specified which scalar fields acquire a complex VEV. To this end, we need to employ the first Barr criterion, which tells us that only VEVs which mix light and heavy fields may become complex, all others must be real.

We look at the Yukawa Lagrangian before electroweak symmetry breaking, which consists of the first three lines of Equation (1.14) with the $SU(2)_L$ doublets not disassembled. It is given by

$$\begin{aligned}\mathcal{L}_{Yuk,\text{noEWSB}} = &\left(\mathcal{Y}_{27}v_{10,1} + \mathcal{Y}_{351}v_{5,2}\right) D_L d_R^c + \left(\mathcal{Y}_{27}v_{6,1} + \mathcal{Y}_{351}v_{5,1}\right) D_L D_R^c \\ &+ \left(\mathcal{Y}_{27}v_{10,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,2}\right) \ell_L L_R^c + \left(\mathcal{Y}_{27}v_{6,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,1}\right) L_L L_R^c \\ &+ \mathcal{Y}_{351}v_{6,2}\, ss + \mathcal{Y}_{351}v_{10,2}\, s\nu_R^c + \mathcal{Y}_{351}v_{10,3}\nu_R^c\nu_R^c \end{aligned} \tag{1.15}$$

The first line is the down quark sector, the second line the lepton sector and the third line a bunch of neutrino exclusive terms. The first Barr criterion tells us, that only $v_{10,1}$ and $v_{5,2}$ may be complex. Since actually only their difference is required to be complex, it is sufficient to have $v_{5,2}$ complex while $v_{10,1}$ remains real.

1.2.5. Second Barr Criterion: Electroweak Symmetry Breaking

The second Barr criterion says, that in order to get ArgDet $(M_u M_d) = 0$ at tree level, we must not allow EWSB scale VEVs for scalar fields, which mix SM and exotic quarks. These VEVs are colored red in the full Yukawa Lagrangian (1.14). This means

$$v_{d,2} = 0 \qquad v_{d,5} = 0 \qquad v_{d,6} = 0 \tag{1.16}$$

Furthermore, since $v_{d,3} + v_{d,4}$ and $v_{u,2} + v_{u,4}$ only appear in their respective sum, we can just set $v_{u,2} = v_{d,3} = 0$ without loss of generality. The Yukawa Lagrangian is then

$$\mathcal{L}_{Yuk,\text{full}} = \mathcal{L}_{Yuk,\text{u}} + \mathcal{L}_{Yuk,\text{d}} + \mathcal{L}_{Yuk,\text{e}} + \mathcal{L}_{Yuk,\nu} \tag{1.17}$$

where

$$\mathcal{L}_{Yuk,\text{u}} = \left(\mathcal{Y}_{27}v_{u,1} + \mathcal{Y}_{351}v_{u,4}\right) u_L u_R^c \tag{1.18}$$

gives mass to the up quarks. There are no exotic fields affecting this sector. The term

$$\begin{aligned}\mathcal{L}_{Yuk,\text{d}} = &\left(\mathcal{Y}_{27}v_{d,1} + \mathcal{Y}_{351}v_{d,4}\right) d_L d_R^c + \left(\mathcal{Y}_{27}v_{10,1} + \mathcal{Y}_{351}v_{5,2}\right) D_L d_R^c \\ &+ \left(\mathcal{Y}_{27}v_{6,1} + \mathcal{Y}_{351}v_{5,1}\right) D_L D_R^c\end{aligned} \tag{1.19}$$

gives masses to the down quarks and exotic down quarks and will be the part of the Lagrangian responsible for CKM CP violation. We will cover this in Section 1.3 in great detail. The term

$$\mathcal{L}_{Yuk,\mathrm{e}} = \left(\mathcal{Y}_{27}v_{d,1} - 3\mathcal{Y}_{351}v_{d,4}\right) e_L e_R^c + \left(\mathcal{Y}_{27}v_{10,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,2}\right) e_L E_R^c$$
$$+ \left(\mathcal{Y}_{27}v_{6,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,1}\right) E_L E_R^c \tag{1.20}$$

is essentially the leptonic counterpart of $\mathcal{L}_{Yuk,\mathrm{d}}$, yielding SM and exotic electron masses. Together with the respective $SU(2)_L$ counterparts from the neutrino sector, these terms will give rise to CP violation in the PMNS matrix. We will cover this in Section 1.4.

The last term

$$\begin{aligned}\mathcal{L}_{Yuk,\nu} = {} & \mathcal{Y}_{351}v_{6,2}\ ss + \mathcal{Y}_{351}v_{10,2}\ s\nu_R^c + \mathcal{Y}_{351}v_{10,3}\nu_R^c\nu_R^c \\ & + \left(\mathcal{Y}_{27}v_{10,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,2}\right) \nu_L N_R^c + \left(\mathcal{Y}_{27}v_{6,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,1}\right) N_L N_R^c \\ & + \left(\mathcal{Y}_{27}v_{u,1} - 3\mathcal{Y}_{351}v_{u,4}\right)\left(\nu_L\nu_R^c + N_L s\right) + \mathcal{Y}_{351}v_{u,3}\nu_L s + \mathcal{Y}_{351}v_{u,5}N_L\nu_R^c \\ & + \left(\mathcal{Y}_{27}v_{d,1} - 3\mathcal{Y}_{351}v_{d,4}\right) sN_R^c\end{aligned} \tag{1.21}$$

contains the Neutrino sector of our model, which we will also cover in Section 1.4, especially in Section 1.4.2, where we discuss the emergence of the low energy neutrino masses.

1.2.6. Mass Matrices

We define new labels for the mass terms

$$\begin{aligned}M_C &= \left(\mathcal{Y}_{27}v_{10,1} + \mathcal{Y}_{351}v_{5,2}\right) \\ M_R &= \left(\mathcal{Y}_{27}v_{6,1} + \mathcal{Y}_{351}v_{5,1}\right) \\ M_{C2} &= \left(\mathcal{Y}_{27}v_{10,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,2}\right) \\ M_{R2} &= \left(\mathcal{Y}_{27}v_{6,1} - \frac{3}{2}\mathcal{Y}_{351}v_{5,1}\right) \\ M_s &= \mathcal{Y}_{351}v_{6,2} \\ M_{s\nu_R^c} &= \mathcal{Y}_{351}v_{10,2} \\ M_{\nu_R^c} &= \mathcal{Y}_{351}v_{10,3} \to 0\end{aligned} \tag{1.22}$$

where the indices C and R reflect that the matrix is complex or real, respectively. This notation will turn out to be useful when tracking the complex phase in the low energy regime. The neutrino sector mass matrices M_s, $M_{s\nu_R^c}$ and $M_{\nu_R^c}$ are real.

We set $v_{10,3} = 0$ to reproduce acceptable neutrino masses for the light SM neutrinos. The reason is the following: the s and ν_R^c enter a seesaw mechanism where a Majorana term for the ν_R^c would lead to the lighter mass state becoming heavier. This lighter mass state enters another seesaw with the SM neutrino. To obtain SM neutrino masses within the experimentally acceptable region, its seesaw partner needs to have a Majorana mass of order $\mathcal{O}(10^{16}$ GeV). The presence of the $M_{\nu_R^c}$ Majorana mass, set by a $SO(10)$ scale VEV, will cause the resulting state to become too heavy to fill this role. Switching off $v_{10,3}$ causes this 'effective right handed neutrino' to emerge just at the appropriate scale to produce viable and experimentally testable neutrino masses.

The Yukawa Lagrangian before EWSB becomes pretty compact with this notation

$$\begin{aligned}\mathcal{L}_{Yuk,\text{noEWSB}} &= M_C D_L d_R^c + M_R D_L D_R^c \\ &+ M_{C2} \ell_L L_R^c + M_{R2} L_L L_R^c \\ &+ M_s\, ss + M_{s\nu_R^c}\, s\nu_R^c \end{aligned} \tag{1.23}$$

We also define labels for the EWSB mass terms

$$m_u = \left(\mathcal{Y}_{27} v_{u,1} + \mathcal{Y}_{351} v_{u,4}\right) \tag{1.24}$$
$$m_d = \left(\mathcal{Y}_{27} v_{d,1} + \mathcal{Y}_{351} v_{d,4}\right) \tag{1.25}$$
$$m_e = \left(\mathcal{Y}_{27} v_{d,1} - 3\mathcal{Y}_{351} v_{d,4}\right) \tag{1.26}$$
$$m_\nu = \left(\mathcal{Y}_{27} v_{u,1} - 3\mathcal{Y}_{351} v_{u,4}\right) \tag{1.27}$$

and write down the electroweak sector of the mass Yukawa Lagrangian so that $\mathcal{L}_{Yuk,\text{full}} = \mathcal{L}_{Yuk,\text{noEWSB}} + \mathcal{L}_{Yuk,\text{EW}}$

$$\begin{aligned}\mathcal{L}_{Yuk,\text{EW}} &= m_u u_L u_R^c + m_d d_L d_R^c + m_e e_L e_R^c \\ &+ m_\nu \left(\nu_L \nu_R^c + N_L s\right) + \mathcal{Y}_{351} v_{u,3} \nu_L s + \mathcal{Y}_{351} v_{u,5} N_L \nu_R^c + m_e s N_R^c \end{aligned} \tag{1.28}$$

which shows in the second line, that we are about to get some interesting neutrino phenomenology in Section 1.4.2.

Next we take a look at the down quark sector and show how the complex VEV translates into a complex CKM matrix.

1.3. Quark Sector

We work in Weyl spinor notation (two components). Note that the charge conjugate of a right chiral spinor is a left chiral spinor: $\psi_R^c \equiv \epsilon\psi_R^*$ is a left chiral field, however not to be confused with ψ_L! These are independent fields. We also suppress the spinor metric ϵ in all inner products.
The up Quark sector looks exactly like the normal Standard Model up Quark sector

$$\mathcal{L}_{\mathrm{u}} = u_L^T m_u u_R^c \tag{1.29}$$

The down Quark sector contains an additional heavy vectorlike quark $D_{L,R}$

$$\mathcal{L}_{\mathrm{d}} = d_L^T m_d d_R^c + D_L^T M_C d_R^c + D_L^T M_R D_R^c \tag{1.30}$$

where the mass matrices were given in Section 1.2.6. Recall, that M_C is complex, while M_R and m_d are real. Note that we get exactly the structure for the full mass matrix which we used to demonstrate the Barr criteria in Section 1.1.

Bob's method

Before electroweak symmetry breaking (setting the light masses to zero), in the down quark sector of the Yukawa Lagrangian we have

$$\mathcal{L}_{\mathrm{d}} = D_L^T M_C d_R^c + D_L^T M_R D_R^c \tag{1.31}$$

We make an ansatz with undetermined coefficient matrices a, b, A, B to find the mass eigenstates

$$d_R^c = a \cdot \tilde{d}_R^c + b \cdot \tilde{D}_R^c \qquad\qquad D_R^c = A \cdot \tilde{d}_R^c + B \cdot \tilde{D}_R^c \tag{1.32}$$

Inserting this into the kinetic terms

$$\mathcal{L}_{\mathrm{kin}} \supset d_R^{c\dagger} \sigma^\mu \partial_\mu d_R^c + D_R^{c\dagger} \sigma^\mu \partial_\mu D_R^c \tag{1.33}$$

and demanding normalized kinetic terms implies

$$\begin{aligned} \tilde{d}_R^{c\dagger} \left(a^\dagger a + A^\dagger A\right) \sigma^\mu \partial_\mu d_R^c &\stackrel{!}{=} \tilde{d}_R^{c\dagger} \sigma^\mu \partial_\mu d_R^c \\ \tilde{D}_R^{c\dagger} \left(b^\dagger b + B^\dagger B\right) \sigma^\mu \partial_\mu D_R^c &\stackrel{!}{=} \tilde{D}_R^{c\dagger} \sigma^\mu \partial_\mu D_R^c \end{aligned} \tag{1.34}$$

This is true and only true, if the ”kinetic normalization conditions”

$$a^\dagger a + A^\dagger A = 1 \qquad\qquad b^\dagger b + B^\dagger B = 1 \tag{1.35}$$

are fulfilled. We also get two ”diagonalization conditions” from demanding that we do not want any leftover mixed terms between the new eigenstates. After all, that was the whole point of introducing them.
One comes from the kinetic terms

$$\tilde{d}_R^{c\dagger} \left(a^\dagger b + A^\dagger B\right) \sigma^\mu \partial_\mu D_R^c \stackrel{!}{=} 0 \tag{1.36}$$

giving

$$a^\dagger b + A^\dagger B = 0 \tag{1.37}$$

and the other one from the Yukawa terms

$$D_L^T \left(M_C \cdot a + M_R \cdot A\right) \tilde{d}_R^c \stackrel{!}{=} 0 \tag{1.38}$$

giving

$$M_C \cdot a + M_R \cdot A = 0 \tag{1.39}$$

We obtain the latter one once we insert the ansatz into the Yukawa Lagrangian (1.31) and demand that all terms mixing D_L and d_R^c vanish. Now we solve Equation (1.39) for A and obtain

$$A = -Z_q \cdot a \qquad\qquad Z_q := M_R^{-1} M_C \tag{1.40}$$

We insert A into Equation (1.35) from the kinetic normalization and obtain

$$aa^\dagger = \left(1 + Z_q^\dagger Z_q\right)^{-1} \tag{1.41}$$

where the root

$$a_q := \left[1 + Z_q^\dagger Z_q\right]^{-1/2} \tag{1.42}$$

is well defined. We added the label q to distinguish it from its counterpart in the lepton sector, which we will turn to in the next section. Note that $a_q = a_q^\dagger$ is hermitian.

When we proceed to electroweak symmetry breaking, we can show that no additional mixing between the left handed fields occurs. Using the same approach as above for the d_L and D_L fields, we obtain trivial mixing coefficients. From a physical point of view, this can be understood as follows: At the GUT scale, the states d_R^c and D_R^c mix to produce mass eigenstates in the Lagrangian. Electroweak symmetry breaking introduces mixing between d_L and D_L, however, the large hierarchy between the GUT scale and the EWSB scale dictates a suppression of the EWSB effects of this mixing. To see this, introduce the EWSB mixing term with a mass of zero. In this limit, nothing can happen to observables, we just added a term that is zero. Now we can continuously increase the value of the mass, the resulting change in observables needs to be continuously aswell, up to the limit where the EWSB scale and the GUT scale are equal and in this case our approach of rotating first the fields d_R^c and D_R^c will be completely altered by the introduction of the d_L and D_L mixing terms since they are now at the same scale and thus of equal footing. This quick argument makes clear, that the effect of the EWSB induced mixing between d_L and D_L is suppressed by the scale of the d_R^c, D_R^c mixing, the GUT scale. The result that we obtain exactly trivial mixing coefficients when using the same approach in the d_L, D_L case as we did in the d_R^c, D_R^c case is naively suprising. We would expect corrections which are suppressed by the high scale. But recall, that for this approach, we implied unitarity of the CKM matrix, which however is only true up to corrections which in turn are suppressed by the high scale. It is the absence of these corrections, that leads to trivial mixing in the d_L, D_L case. The fortunate fact, that the d_L, D_L mixing is introduced only at the EWSB scale is of course a result of the SM being a chiral theory: left handed SM quark fields are $SU(2)_L$ doublets and thus cannot mix with the D_L fields which are $SU(2)_L$ singlets.

Electroweak symmetry breaking generates EW mass terms with the mass matrices m_u and m_d for the SM quark fields. Inserting the mass eigenstates into the Lagrangian, we get for the SM quark masses

$$\mathcal{L}_{\text{quarks,SM}} = \tilde{u}_L^T m_u \tilde{u}_R^c + \tilde{d}_L^T \underbrace{m_d \cdot a_q}_{m_d^{\text{eff}}} \tilde{d}_R^c \tag{1.43}$$

where the effective down quark mass matrix is given by

$$m_d^{\text{eff}} = m_d \cdot a_q \tag{1.44}$$

Our original motivation was to construct a model where CP is broken spontaneously to produce the observed CP violation in the weak sector of the SM. We see that a_q involves the quantities M_R and M_C, both being of comparable scale and the latter being complex, thus a_q in general has a sizable complex phase. At this point, we can explicitly check that the first Barr criterion did its job:

$$\text{ArgDet}(m_u m_d^{\text{eff}}) = \text{Arg}(\text{Det}(m_u)(\text{Det}(m_d)\text{Det}(a_q)) = 0 \tag{1.45}$$

The last equality follows from m_u and m_d being real and a_q being hermitian, so all three have real determinants. This shows that there is indeed no contribution from the quark mass matrices to $\overline{\theta}$.

1.4. Lepton Sector

In the lepton sector, we follow essentially the same approach as in the quark sector. Here, however, it is not the conjugated right handed singlets which superpose, but the left handed doublets.

We recall the leptonic part of the Yukawa Lagrangian before EWSB from Equation (1.23). Since we want to obtain the correct form of the low energy mass matrices, it is important that we write down the Lagrangian properly. To this end, we choose the LR convention (left handed fields to the left of the mass matrix, right handed conjugate fields to the right). We still suppress the spinor metric in inner products.

$$\begin{aligned}\mathcal{L}_{Y,\ell} &= \ell_L^T M_{C2} L_R^c + L_L^T M_{R2} L_R^c \\ &+ s^T M_s\, s + s^T M_{s\nu_R^c} \nu_R^c\end{aligned} \tag{1.46}$$

We see that the lepton sector consists of two distinct parts: One part is the mixing between the exotic neutrino fields s and ν_R^c, while the other part contains the exotic and SM lepton doublets. By decoupling arguments analogous to the quark sector, we can treat these parts separately.

1.4.1. Lepton Doublets

We first turn towards finding the mass eigenstates of the lepton doublet part, the first line of Equation (1.46).

$$\mathcal{L}_{\text{lep.dbl.}} = \ell_L^T M_{C2} L_R^c + L_L^T M_{R2} L_R^c \tag{1.47}$$

Essentially, we just repeat the same procedure which we used in the quark sector. We make an ansatz for the mass eigenstates

$$\ell_L = a \cdot \tilde{\ell}_L + b \cdot \tilde{L}_L \qquad\qquad L_L = A \cdot \tilde{\ell}_L + B \cdot \tilde{L}_L \tag{1.48}$$

Inserting this into the kinetic terms and demanding proper normalization implies the "kinetic normalization conditions"

$$a^\dagger a + A^\dagger A = 1 \qquad\qquad b^\dagger b + B^\dagger B = 1 \tag{1.49}$$

and the "kinetic diagonalization condition".

$$a^\dagger b + A^\dagger B = 0 \tag{1.50}$$

Inserting the ansatz into the Yukawa Lagrangian (1.47) gives the "Yukawa diagonalization condition"

$$a^T \cdot M_{C2} + A^T \cdot M_{R2} = 0 \tag{1.51}$$

We solve this equation for A and obtain

$$A = -Z_\ell \cdot a \qquad\qquad Z_\ell := (M_{R2}^{-1})^T M_{C2}^T \tag{1.52}$$

Note that M_{R2} and M_{C2} are symmetric matrices, so we can just drop the transpositions on them. We insert A this into Equation (1.49) from the kinetic normalization and obtain

$$aa^\dagger = \left(1 + Z_\ell^\dagger Z_\ell\right)^{-1} \tag{1.53}$$

and subsequently the root, which is hermitian

$$a_\ell := \left[1 + Z_\ell^\dagger Z_\ell\right]^{-1/2} \tag{1.54}$$

where we added the subscript ℓ to distinguish it from its counterpart in the quark sector. The state $\tilde{\ell}$, as defined above, is due to (1.50) a massless state, while $\tilde{L}$ is massive at the GUT scale. When we switch on EWSB, the SM Higgs mass generation takes place: The $SU(2)_L$ doublets fall apart and its components form Dirac type mass terms with their $SU(2)_L$ singlet counterparts. For these light lepton masses we get

$$\mathcal{L}_{\text{leptons,SM}} = e_L^T m_e^{\text{eff}} e_R^c + \nu_L^T m_\nu^{\text{eff}} \nu_R^c \tag{1.55}$$

where

$$\begin{aligned} m_\nu^{\text{eff}} &= a_\ell^T \cdot m_\nu \\ m_e^{\text{eff}} &= a_\ell^T \cdot m_e \end{aligned} \tag{1.56}$$

The masses m_ν and m_e are generated through EWSB in the Lagrangian. The matrix a_ℓ originated in the mixing of the $SU(2)_L$ doublets before EWSB, therefore it is the same e_L and ν_L after EWSB. Note that in the lepton sector, we get a_ℓ^T from the left because it is the $SU(2)_L$ doublets ℓ_L and L_L which mix. In the quark sector we had a_q from the right because there, the $SU(2)_L$ singlets d_R^c and D_R^c were mixing. It all traces back to the representations of the exotic quarks that emerge through the breakdown of the GUT scales: vectorlike down quark singlets and vectorlike lepton doublets.

1.4.2. Neutrino Sector

The neutrino sector is by far the most involved part of our model. By 'neutrino' we define every field, which is a singlet under $SU(3)_C \times U(1)_{EM}$. There are five fields which qualify and each comes in three generations. We will suppress the generations here, but keep in mind that the mass parameters actually are combinations of VEVs and 3×3 Yukawa matrices. We recall these fields

- s, the $SO(10)$ singlet when the 27 of E_6 breaks to $SO(10)$
- N_R^c, the neutral part of the exotic conjugated right chiral lepton doublet L_R^c and
- N_L, the neutral part of the exotic left chiral lepton doublet L_L, both coming from the 10 of $SO(10)$
- ν_R^c, the SM singlet coming from the 16 of $SO(10)$. The label stems from it forming a Dirac mass term with ν_L.
- ν_L, the observed SM neutrino

Before EWSB, the fields residing in $SU(2)_L$ doublets cannot mix with the $SU(2)_L$ singlets. There is just no way to form a singlet from combining a doublet and a singlet. Recall the Yukawa Lagrangian (1.46) from the previous section

$$\begin{aligned}\mathcal{L}_{Y,\ell} &= \ell_L^T M_{C2} L_R^c + L_L^T M_{R2} L_R^c \\ &+ s^T M_s \, s + s^T M_{s\nu_R^c} \nu_R^c\end{aligned} \tag{1.57}$$

We dealt with the first line in the last section, deriving its influence on the low energy mass terms of the SM neutrino and the charged lepton. Now we will investigate the second line before $SU(2)_L$ is broken. EWSB will introduce terms mixing the fields of both lines, but the large hierarchy between the GUT scale and the EW scale will again allows us to treat these sectors separately.

Seesaw 1

We look at the second line of $\mathcal{L}_{Y,\ell}$, cf. Equation (1.57). What we find is a seesaw scenario

$$\begin{aligned}\mathcal{L}_{\text{seesaw 1}} &= s^T M_s \, s + s^T M_{s\nu_R^c} \nu_R^c \\ &= \begin{pmatrix} \nu_R^{cT} & s^T \end{pmatrix} \begin{pmatrix} 0 & M_{s\nu_R^c}^T \\ M_{s\nu_R^c} & M_s \end{pmatrix} \begin{pmatrix} \nu_R^c \\ s \end{pmatrix}\end{aligned} \tag{1.58}$$

Due to the Majorana nature of these states, the construction of the mass matrix involves considering all combination of states. Also the transposed Dirac mass term exists, since no notion of chirality exists here, despite the misleading label ν_R. We diagonalize the mass matrix using a Takagi decomposition [12] to achieve the form

$$\mathcal{L}_{\text{seesaw 1}} = (\tilde{\nu}_R^c)^T M_{light} \tilde{\nu}_R^c + \tilde{s}^T M_{heavy} \tilde{s} \tag{1.59}$$

and find the new eigenstates parametrized by $\delta \simeq M_{s\nu_R^c}/M_s \ll 1$ [13].

Thus

$$\begin{aligned} \tilde{\nu}_R^c &\simeq \nu_R^c + \delta \cdot s \\ \tilde{s} &\simeq s + \delta \cdot \nu_R^c \end{aligned} \tag{1.60}$$

and we can simply ignore the mixing of states. The new masses are given by [14] [15]

$$\begin{aligned} M_{light} &\simeq -M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \\ M_{heavy} &\simeq M_s \end{aligned} \tag{1.61}$$

where again corrections of $\mathcal{O}(\delta)$ are involved. Neglecting $\delta \ll 1$, we find to a very good approximation

$$\mathcal{L}_{\text{seesaw 1}} = \nu_R^{cT} \left(-M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \right) \nu_R^c + s^T M_s \, s \tag{1.62}$$

After the seesaw, we have a heavy state s and a light state ν_R^c. Funny as it may sound, we here refer to $\mathcal{O}(10^{13}$ GeV) as 'light'.

Seesaw 2

After EWSB, a Dirac type mass term is generated for ν_L and ν_R^c. Recall from Section 1.4.1, that we constructed a massless state $\tilde{\nu}_L$ (the neutral part of the doublet $\tilde{\ell}$). Once we express ν_L through $\tilde{\nu}_L$ and $\tilde{N}_L$, we obtain Dirac mass terms between $\tilde{\nu}_L$ and ν_R^c and between $\tilde{N}_L$ and ν_R^c. We can ignore the latter since the involved states both come with GUT scale masses and this mixing term is of the electroweak order and subsequently will lead to highly suppressed corrections to the states and their masses. The former is the interesting term which we shall consider now. We recall the masses for the states $\tilde{\nu}_L$ and ν_R^c by writing down the relevant terms of the Lagrangian.

$$\mathcal{L}_{\text{seesaw 2}} = \tilde{\nu}_L^T a_\ell^T m_\nu \nu_R^c + (\nu_R^c)^T \left(-M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \right) \nu_R^c \tag{1.63}$$

Here, a_ℓ was the mixing coefficient between ν_L and $\tilde{\nu}_L$, derived in Section 1.4.1, m_ν is the Dirac mass matrix containing electroweak VEVs, and the mass matrix between the ν_R^c states resulted from the $s\nu_R^c$ seesaw which was described above.

From the section above we know, that for largely separate scales, the seesaw mechanism changes the states only in a negligible fashion. We can thus safely apply the seesaw mechanism between $\tilde{\nu}_L$ and ν_R^c without having to worry about the possibility of large mixing induced elsewhere by changing the states in this part of the Lagrangian. Note that we could also apply the seesaw between ν_L and ν_R first and afterwards calculate the mixing between ν_L and N_L with the effective approach of Section 1.4.1. The result is the same.

Applying the seesaw formula to Equation (1.63), we obtain the following Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{seesaw 2}} &= \tilde{\nu}_L^T \left(-(a_\ell^T m_\nu) \left(-M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \right)^{-1} (a_\ell^T m_\nu)^T \right) \tilde{\nu}_L + (\nu_R^c)^T \left(-M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \right) \nu_R^c \\ &= \tilde{\nu}_L^T m_\nu^{\text{SM}} \tilde{\nu}_L + (\nu_R^c)^T \left(-M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \right) \nu_R^c \qquad (1.64)\end{aligned}$$

where

$$m_\nu^{\text{SM}} = a_\ell^T m_\nu \left(M_{s\nu_R^c} M_s^{-1} M_{s\nu_R^c}^T \right)^{-1} m_\nu^T a_\ell \qquad (1.65)$$

is the Majorana mass matrix for the left handed Standard Model neutrino. Here we can clearly see how the GUT scales affect the low energy neutrino mass: Recall from Section 1.2.6, that $M_{s\nu_R^c}$ contains an $SO(10)$ VEV, M_s an E_6 VEV and m_ν an electroweak VEV. Moreover, a_ℓ is $\mathcal{O}(1)$ and complex, thus causing the SM neutrino mass matrix to become complex. Diagonalizing this effective SM mass matrix via a singular value decomposition gives predictions for SM neutrino masses, as well as a generally CP violating PMNS matrix.

To obtain realistic neutrino masses, the seesaw scale for ν_L and ν_R^c needs to be of $\mathcal{O}(10^{12} - 10^{14})$ [16]. Since the ν_R^c mass term is generated through another seesaw mechanism with the heavy $SO(10)$ singlet s, we can determine the scale of the mass parameter of s to be $\mathcal{O}(10^{17} - 10^{19})$ when taking into account that the GUT scale is required to be $\gtrsim 4 \cdot 10^{15}$ by current proton decay bounds [17].

We will turn to the predictions in the next section.

1.5. Results

The model we presented here differs from the final version in our paper [2] in a few aspects. The E_6 VEV which enters in the mass term of the heavy $SO(10)$ singlet s has to be just below the Planck scale, as argued in Section 1.4.2. To avoid this, we broke $SO(10)$ in the paper directly to the Standard Model through an adjoint scalar representation, which allows us to break the accompanying $U(1)_5$ factor at a lower scale than the $SO(10)$ scale. This serves to change the part in the Lagrangian that contains the high scale seesaw between s and ν_R^c from a seesaw scenario to Majorana mass terms by neglecting mixing terms proportional to the $U(1)_5$ VEV. In this way, the E_6 Majorana mass VEV for s can be lowered comfortably below the Planck scale without impacting the low energy neutrino phenomenology through seesaw dependence. Personally, I prefer the version of the model presented in this thesis. A VEV just below the Planck scale is merely an input parameter of the effective theory. We do not speculate about Transplanckian dynamics and therefore a VEV just below the Planck scale is no conceptual problem. It is only a matter of personal philosophy.

The fitting results and thus the low energy predictions are unaffected by this. In fact, we performed the entire fit in the model we presented in this thesis before we altered the model to the version presented in the paper.

1.5.1. Fitting the Standard Model Observables

We take a moment to count the degrees of freedom we have in our model. In the beginning of this section we noted that we have $3+6$ parameters coming from the diagonal $\mathcal{Y}_{27}$ and the symmetric $\mathcal{Y}_{351}$, respectively. We started with 18 VEVs, where 7 were SM singlet GUT breaking VEVs, 5 were 'up-type' Higgs and 6 were 'down-type' Higgs VEVs. The first Barr criterion told us that only $v_{5,2}$ and $v_{10,1}$ may become complex and since we only needed a relative complex phase between them, we could leave $v_{10,1}$ to be real so that $v_{5,2}$ is the only complex VEV of our model, adding only 1 complex phase to the number of parameters. The second Barr criterion dictated that three of the Higgs VEVs need to be zero not to generate a possibly large $\bar{\theta}$ through a complex determinant of the quark mass matrix. Additionally, we could discard $v_{d,3}$ and $v_{u,2}$ because they appeared exclusively in combination with $v_{d,4}$ and $v_{u,4}$, respectively, and therefore do not add anything new. Finally we set the VEV $v_{10,3}$ generating a Majorana mass for ν_R^c to zero, which we require to make the seesaw structure of the neutrino sector work.

Counting the VEVs we retained, we find 11 real numbers: $v_{5,1}$, $|v_{5,2}|$, $v_{10,1}$, $v_{10,2}$, $v_{6,1}$, $v_{6,2}$, $v_{u,1}$, $v_{u,4}$, $v_{d,1}$, $v_{d,4}$ and Arg $v_{5,2}$.

This gives us in total $9+11=20$ real parameters. We can eliminate two more parameters by realizing that the Yukawa matrices and the VEVs always come together. We can thus absorb two VEVs into the definition of the Yukawa matrices. As a consequence of that, we find that only VEV ratios are important for the fit. Upon close inspection, we find that two more VEVs turn out to be unimportant in the fit. We carried out this reparametrization explicitly in our paper [2] and give explicit numbers for our best fit point. We will forbear from doing so in this thesis because it is not very illuminating. The structure of our model is

Fit result at the electroweak scale $\mu = M_Z$					
	fit	pull		fit	pull
m_d(MeV)	3.44	−2.4	$\Delta_{12}(\mathrm{eV}^2)$	7.39×10^{-5}	0.63
m_s(MeV)	50.4	1.4	$\Delta_{13}(\mathrm{eV}^2)$	-0.76×10^{-3}	−0.19
m_b(GeV)	2.85	0.27	$\sin\theta^q_{12}$	0.225	0.56
m_u(MeV)	1.32	−0.08	$\sin\theta^q_{23}$	0.0414	0.1
m_c(GeV)	0.63	−0.07	$\sin\theta^q_{13}$	0.0035	1.1
m_t(GeV)	171.58	0.08	$\sin^2\theta^l_{12}$	0.302	0.37
m_e(MeV)	0.486	0.15	$\sin^2\theta^l_{23}$	0.405	1.5
m_μ(MeV)	102.76	−0.61	$\sin^2\theta^l_{13}$	0.022	−0.26
m_τ(GeV)	1.746	−0.04	δ_{CKM}	1.13	1.5

Table 1.5.: *Result of the fitting procedure, as described in the text, as appeared in our paper [2].*

much clearer in the way we presented it in the previous sections and the reparametrization is only a technical detail required for the numerical fitting procedure. The fit therefore contains 16 parameters which have to match 18 Standard Model observables.

As a measure of error, we gathered statistics on repeated fits where we allowed a total χ^2 of $\lesssim 160$, corresponding to $\chi^2/\mathrm{dof} \lesssim 10$. This gives us a measure of sensitivity of the best fit point on the input variables. This range then expresses itself as error range for our predictions in the Neutrino observables. The statement is: Should one of the predicted observables be measured outside of the quoted range in Table 1.7, the χ^2 per degree of freedom of our fit will rise above $\chi^2/\mathrm{dof} > 10$.

We performed the fit by choosing random input variables at the GUT scale and then evolved them to the electroweak scale by solving the Yukawa RGE numerically using REAP [18]. As measure of quality for our fit is given by

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{\mathcal{O}_i^{\mathrm{exp}} - \mathcal{O}_i^{\mathrm{fit}}}{\sigma_i^{\mathrm{exp}}} \right)^2 \tag{1.66}$$

where $\mathcal{O}_i^{\mathrm{exp}}$ denotes the experimental value of observable $\mathcal{O}_i$ with experimental error σ_i^{exp}. $\mathcal{O}_i^{\mathrm{fit}}$ is the obtained fitting value. The pull of a fit value $\mathcal{O}_i^{\mathrm{fit}}$ is defined as $\mathrm{pull}(\mathcal{O}_i^{\mathrm{fit}}) = \left(\mathcal{O}_i^{\mathrm{exp}} - \mathcal{O}_i^{\mathrm{fit}}\right)/\sigma_i^{\mathrm{exp}}$ and encodes a weighted measure of distance and direction from the experimental value. We give the fit results in Table 1.5 and collected the experimental values in Table 1.6.

The final result gave a best fit point with total $\chi^2 \approx 15$. Taking into account, that we have 16 degrees of freedom (dof) which are relevant for the fit, we obtain $\chi^2/\mathrm{dof} \approx 0.9$. The quality of the fit is quite surprising, considering that we fit 16 parameters on 18 targets (see Table 1.6). The success of the fit implies that there are two relations among SM observables hidden within our model. Unfortunately, we were not able to find them.

Fermion observables at the electroweak scale $\mu = M_Z$			
m_d(MeV)	2.75 ± 0.29	$\Delta_{12}(\mathrm{eV}^2)$	$(7.50 \pm 0.18) \times 10^{-5}$
m_s(MeV)	54.3 ± 2.9	$\Delta_{31}(\mathrm{eV}^2)$	$(2.52 \pm 0.04) \times 10^{-3}$
m_b(GeV)	2.85 ± 0.03	$\sin\theta^q_{12}$	0.2254 ± 0.0007
m_u(MeV)	1.3 ± 0.4	$\sin\theta^q_{23}$	0.0421 ± 0.0006
m_c(GeV)	0.627 ± 0.019	$\sin\theta^q_{13}$	0.0036 ± 0.0001
m_t(GeV)	171.7 ± 1.5	$\sin^2\theta^l_{12}$	0.306 ± 0.012
m_e(MeV)	0.4866 ± 0.0005	$\sin^2\theta^l_{23}$	0.441 ± 0.024
m_μ(MeV)	102.7 ± 0.1	$\sin^2\theta^l_{13}$	0.0217 ± 0.0008
m_τ(GeV)	1.746 ± 0.002	δ_{CKM}	1.21 ± 0.05

Table 1.6.: *The SM input parameters at the electroweak scale we used for the fit, as appeared in our paper [2]. We took quark and lepton masses as well as the quark mixing parameters from Ref. [19], and and the neutrino mixing parameters from Ref. [20] for Normal Ordering (NO). We use a* 0.1% *uncertainty for the charged lepton masses to make sure the fit does not give undue preference to these observables. To simplify the fitting procedure, we used for all observables the arithmetic average of the errors when not symmetric.*

1.5.2. Predictions

The configuration of parameters we obtained from the best fit gives us predictions for the neutrino masses and PMNS Dirac phase δ and Majorana phases φ_1 and φ_1. The different mass observables are defined as follows:

Neutrinoless double beta decay experiments like GERDA [21], EXO-200 [22] or KamLAND-Zen [23] measure the effective Majorana mass

$$m_{0\nu\beta\beta} = \left|\sum U_{ei}^2\, m_i\right| . \tag{1.67}$$

Experiments like KATRIN [24], MARE [25], Project 8 [26], or ECHo [27] measure

$$m_\beta = \sqrt{\sum |U_{ei}|^2\, m_i^2}\, . \tag{1.68}$$

Cosmology neutrino experiments like PLANCK [28] probe the sum of the neutrino masses

$$\Sigma = \sum m_i \tag{1.69}$$

We collected our predictions for these observables and their respective current bounds in Table 1.7.

	m_β [meV]	Σ [meV]	$m_{0\nu\beta\beta}$ [meV]	δ [°]	φ_1 [°]	φ_2 [°]
Prediction	8.8 ± 0.5	59 ± 3	1.8 ± 0.1	157 ± 3	187 ± 4	159 ± 5
Current Bound	$\lesssim 2000$ [29]	$\lesssim 230$ [29, 28]	200 [30, 31]	-	-	-

Table 1.7.: *Predicted values and current bounds for the neutrino observables, as appeared in our paper [2]. The current bounds were taken from Ref. [32]. As explained in the text, the ranges shown here correspond to perturbations of the best fit point with* $\chi^2/\mathrm{dof} \lesssim 10$.

Discussion of θ_F

The Barr criteria ensure $\theta_F = \text{ArgDet}\,(M_u M_d)$ is zero at tree level and arises only at the loop level. The suppression of one-loop effects has been discussed at length in the literature [33] [7] [34] and was found to be small enough in this type of models. Possible two-loop effects are sufficiently suppressed by loop factors and the large mass of the exotic gauge bosons [2] [33]. The leading order value for θ_F is in principle calculable and therefore can be a prediction in these models. In our model, however, the leading order loop calculation of θ_F depends on the unknown specifics of the scalar spectrum. The low energy sector is insensitive to these parameters and therefore they are not fixed by our fit.

1.5.3. Summary and Conclusion

We investigated a Nelson-Barr type model based on the GUT group E_6. The symmetry breaking chain $E_6 \to SO(10) \to SU(5) \to G_{SM}$ turned out to produce exactly the required representations to realize the Barr criteria and obtain the observed weak CP violation analogous to the effective model of [7]. We break the CP symmetry through a complex $SU(5)$ VEV, which gets transported into the quark and lepton sectors via mixing between SM and exotic fermions to produce complex CKM and PMNS matrices. The phase of the complex VEV is not suppressed by the GUT scale. This has been shown by BBP [7] for a single vectorlike quark and we generalized this formula to matrix structures (cf. Equation (1.4)) to incorporate an arbitrary number of vectorlike quarks.

We performed a fit with a best fit point of in total $\chi^2 \approx 15$ while fitting 16 input variables on 18 Standard Model targets. The surprising quality of the fit with less parameters than the Standard Model implies two hidden relations among Standard Model observables within our model. We predict a PMNS Dirac phase, which is correlated with the CKM phase via the complex $SU(5)$ VEV, as well as PMNS Majorana phases and Neutrino masses. These predictions are collected in Table 1.7 and can be tested in the near future.

A Supersymmetric Solution to ϵ'_K/ϵ_K 2

2.1. Motivation: Recent Lattice Results

This chapter is based on our paper "Supersymmetric Explanation of CP Violation in $K \to \pi\pi$ Decays" [1].

A recent lattice QCD analysis [35] revealed a correction in the prediction for direct CP violation in Kaon decays (ϵ'_K/ϵ_K), making it deviate largely from the measured value.

$$\frac{\epsilon'_K}{\epsilon_K} = \begin{cases} (16.6 \pm 2.3) \times 10^{-4} & \text{(PDG [29])} \\ (1.0 \pm 4.7 \pm 1.5 \pm 0.6) \times 10^{-4} & \text{(SM-NLO)} \end{cases} \tag{2.1}$$

The first uncertainty in the theory prediction stems from the non-perturbative lattice computation. The second one stems from higher order corrections and the third one stems from isospin violating terms [36] [37]. The experimental values essentially stem from measurements from 1999 by KTeV [38] and NA48 [39] collaborations and have not changed since then.

The Standard Model theory prediction consists of two separate parts: One part is the perturbative calculation of the Wilson Coefficients, which encode the high energy physics. This has been done at next-to-leading order (NLO) [40] [41] [42] [43]. The other part is the calculation of the hadronic matrix elements, which encode the low energy physics and therefore cannot be treated with perturbation theory. These non-perturbative calculations have been done in various methods like $1/N_C$ [44], chiral perturbation theory (χPT) [45] and lattice QCD [35] [46].

The theory prediction has always been plagued by uncertainties in the non-perturbative quantities, so that more or less only an order of magnitude prediction was possible. In retrospective, with $(\epsilon'_K/\epsilon_K)_{1/N_c} = (8.6 \pm 3.2) \times 10^{-4}$ [42], the $1/N_c$ approach has actually been closer to the now established lattice value than to the experimental value, but people were cautious to believe their own results. After all, these non-perturbative estimation methods rely on some kind of confidence in the applicability of the method in the case at hand, and thus it was always a possibility, that the $1/N_c$ limit would receive large corrections and thus be a poor estimate. Additionally, another non-perturbative method - chiral perturbation theory - happened to predict $(\epsilon'_K/\epsilon_K)_{\chi PT} = (19 \pm 11) \times 10^{-4}$ [45], a value apparently in perfect agreement with experiment. This made it hard to believe that there is much room for new physics in ϵ'_K/ϵ_K. In the end, even the $1/N_c$ value was still substantially far away from today's lattice value, requiring to cut the relevant parameter $B_6^{(1/2)}$ nearly by half[1]. This naively suggests that $1/N_c$ got lucky landing between experiment and the lattice value, rather than being overly precise in the prediction. Doubling the same parameter would increase the prediction so far that it would even overshoot the experimental result. Ironically, χPT had the bad luck of landing right on the experimental value, which seemed to give a lot of credibility to the method. To be fair, without sufficient accuracy, ϵ'_K/ϵ_K is an incredibly ungrateful observable to predict because of the large cancellation between the amplitudes A_0 and A_2[2]. Every expert knew fully well that as long as the full range between zero and the experimental value is comfortably within the theoretical possibility, the actual number is more like an experts opinion than a real, reliable prediction.

2.2. New Physics in ϵ'_K

Talking about ϵ'_K/ϵ_K rather than ϵ'_K is just a historical convention that somehow still made it up to today. We thus talk about ϵ'_K/ϵ_K whenever we refer to actual numbers, calculations or measurements, and ϵ'_K, when it comes to talking about the physics behind the numbers and calculations. ϵ'_K labels direct CP violation in Kaon decay. Dividing it by ϵ_K is just convenient for calculating numbers since the phases of both quantities accidentally coincide, making the ratio a real number.

If we take the discrepancy between the experimental value and the lattice prediction for ϵ'_K/ϵ_K serious and want to think about how to satisfy it with new physics, we actually need a contribution that is larger than the Standard Model value. This naively seems impossible without fine tuning because of the following argument:

The parameters which govern ϵ'_K also govern ϵ_K. The latter, however, is measured to great accuracy and the theory predictions comply with these measurements, leaving hardly any room for new physics contributions.

The important quantities are $\tau \sim V_{td}V_{ts}^*$, which is a combination of CKM elements involving the CKM phase[3], and the mass of the W-Boson. These give a measure of likelihood of CP violation and a weak interaction process taking place, respectively. Thus, together, these are the essential quantities when talking about CP violation in Kaon processes (ϵ'_K and ϵ_K).

[1] The hadronic matrix element $\langle Q_6\rangle_0$ can be parametrized in the following way: $\langle Q_6\rangle_0 = -4\sqrt{\frac{3}{2}}\left[\frac{m_K^2}{m_s+m_d}\right]^2 \frac{F_\pi}{\kappa} B_6^{(1/2)}$, where $B_6^{(1/2)} = 1$ corresponds to the vacuum insertion approximation. m_K, m_s and m_d are the Kaon, strange quark and down quark masses, respectively. $\kappa = F_\pi/(F_K - F_\pi)$, where F_K and F_π are the Kaon and Pion decay constants, respectively [40]. The lattice results of Reference [35] imply $B_6^{(1/2)} = 0.57 \pm 0.19$ [47].

[2] For a definition of the amplitudes A_0 and A_2 see Section 3.2.6. For their relation to ϵ'_K/ϵ_K see Sections 3.2.9 and 3.2.10.

[3] Which CKM elements involve the complex phase depends on the convention used in the CKM matrix. Should one choose another convention, rendering the combination $V_{td}V_{ts}^*$ real, another combination of CKM elements inevitably becomes complex. The point is, that ϵ'_K and ϵ_K contain several combinations of CKM elements in such a way that they are invariant under the CKM convention change.

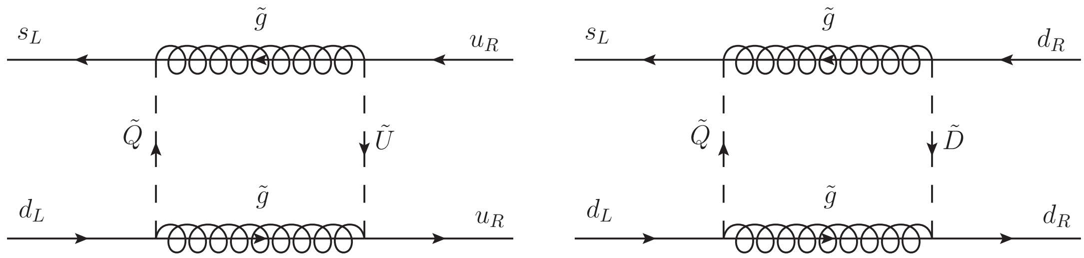

Figure 2.1.: *Trojan penguins [48] [49]: Gluino boxes contributing to ϵ'_K/ϵ_K*

The Standard Model (SM) prediction $\epsilon_K'^{\mathrm{SM}}$ is proportional to Im τ/M_W^2, involving one strange-top vertex and one W-Boson propagator. ϵ_K^{SM} is proportional to Im τ^2/M_W^2, involving two strange-top vertices and two W-Boson propagators, whereas one of the latter is cancelled in the process of the loop integration.

If we want new physics (NP) to contribute to ϵ'_K, we will likely have the same Feynman diagram structure, but with a new parameter δ encoding the CP violation at the vertices, and a new mediator with mass M taking on the role of the W-Boson. We thus get $\epsilon_K'^{\mathrm{NP}} \propto \mathrm{Im}\ \delta/M^2$ and $\epsilon_K^{\mathrm{NP}} \propto \mathrm{Im}\ \delta^2/M^2$.

Now we expect $M \gg M_W$ and thus require Im $\delta \gg$ Im τ to obtain Im $\tau/M_W^2 \approx$ Im δ/M^2 to have $\epsilon_K'^{\mathrm{SM}}$ and $\epsilon_K'^{\mathrm{NP}}$ in the same order of magnitude.

Taking into account, that we need $\epsilon_K'^{\mathrm{NP}}$ at least as large as $\epsilon_K'^{\mathrm{SM}}$ (implying $\epsilon_K'^{\mathrm{NP}}/\epsilon_K'^{\mathrm{SM}} \geq 1$) and that ϵ_K^{SM} has barely any room for new physics, implying $\epsilon_K^{\mathrm{NP}}/\epsilon_K^{\mathrm{SM}} < 1$, we can estimate

$$1 \leq \frac{\epsilon_K'^{\mathrm{NP}}}{\epsilon_K'^{\mathrm{SM}}} < \frac{\epsilon_K'^{\mathrm{NP}}}{\epsilon_K'^{\mathrm{SM}}}\frac{\epsilon_K^{\mathrm{SM}}}{\epsilon_K^{\mathrm{NP}}} = \mathcal{O}\left(\frac{\mathrm{Im}\ \delta/M^2}{\mathrm{Im}\ \tau/M_W^2} \cdot \frac{\mathrm{Im}\ \tau^2/M_W^2}{\mathrm{Im}\ \delta^2/M^2}\right) = \mathcal{O}\left(\frac{\mathrm{Re}\ \tau}{\mathrm{Re}\ \delta}\right) \tag{2.2}$$

Thus we need Re $\delta <$ Re τ if we want a large $\epsilon_K'^{\mathrm{NP}}$ while keeping a small ϵ_K^{NP} compared to the respective SM part. At the same time, we need Im $\delta \gg$ Im τ to obtain a large enough $\epsilon_K'^{\mathrm{NP}}$ to resolve the discrepancy. This implies we need a largely imaginary δ. In other words, Arg δ would need to be finetuned very close to $\frac{\pi}{2}$. Therefore, if we do not want to finetune our model, it naively seems impossible to noticeably enhance ϵ'_K without overshooting the bound for ϵ_K. In the following, we will show that this assumption turns out to be too naive and that it is possible to construct a supersymmetric model that can satisfy ϵ'_K while staying within the ϵ_K bounds without any finetuning [1].

2.3. ϵ'_K in the MSSM

Any NP contribution to ϵ'_K can simply be added to the SM value since ϵ'_K, as an amplitude level quantity, is linear in the effective operators. The contributions we are interested in come from gluino boxes like Figure 2.1.

These boxes contribute to both, the A_0 and the A_2 amplitude at about equal magnitude. While A_0 is dominated by QCD penguins, the gluino box contribution is numerically unimportant there. In A_2, however, which is dominated by electroweak penguins, the gluino boxes are numerically comparable because of the larger coupling of the strong interaction. Hence the witty name *Trojan Penguins* [48]: they are strong box processes which kind of infiltrate the domain of electroweak penguins. It was shown [48] [49], that these contributions can become large. In the calculation of ϵ'_K, the CP violating part of the amplitude A_2 gets enhanced by the ratio of the CP conserving parts of A_0 and A_2. This ratio is known as $\Delta I = 1/2$ rule and experimentally turns out to be roughly 22 [50]. Thus, it numerically boosts all the processes contributing to A_2, including the gluino boxes, by a significant amount.

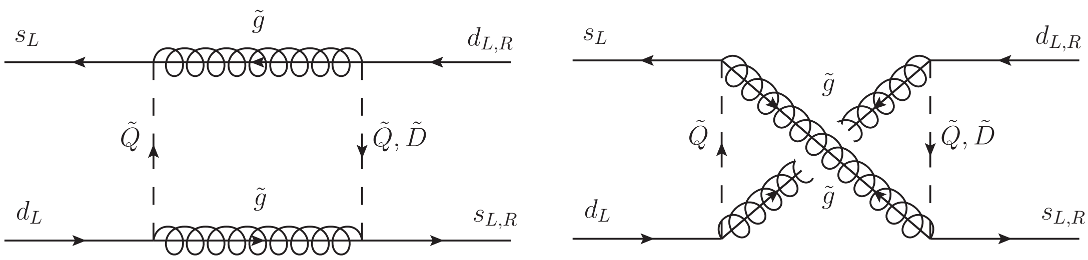

Figure 2.2.: *Suppression of diagrams contributing to ϵ_K: For $m_{\tilde{g}} = 1.5 M_S$, the diagrams with outgoing left chiral quarks cancel exactly. The diagrams with outgoing right chiral quarks vanish in the limit of negligible mixing among right chiral squarks.*

The parameters, which have the potential to make Trojan Penguin contribution to ϵ'_K large in supersymmetric models, also feed into ϵ_K. To see this, all we need to do is to change the outgoing states in any ϵ'_K gluino box (either *dd* or *uu*) into *sd*. The resulting diagrams contribute to ϵ_K and contain the flavor-changing parameter which mediates the $\tilde{s}_L$-$\tilde{d}_L$ mixing twice. The very same parameter, which governs the ϵ'_K contribution. This was the argument around Equation (2.2), naively forbidding significant NP contributions to ϵ'_K. But here a remarkable property of the gluinos comes in. They are Majorana fermions and thereby allow the constructions of diagrams with crossed boxes like Figure 2.2. These diagrams come with a minus sign with respect to the uncrossed boxes. The suppression of the Trojan penguin contribution to ϵ_K differs depending on chirality, therefore it is instructive to present the cases $s_L \equiv P_L s$ and $s_R \equiv P_R s$ separately. The authors of [51] showed, that for the diagram with outgoing left chiral quarks (LL), the two diagrams in Figure 2.2 cancel exactly for $m_{\tilde{g}} = 1.5 M_S$. The cancellation becomes imperfect thereafter, behaving like $[m_{\tilde{g}}^2 - (1.5 M_S)^2]/m_{\tilde{g}}^4$ and is depicted in Figure 2.3. At around $m_{\tilde{g}} \simeq 2.5 M_S$, the largeness of the gluino mass starts to dominate and numerically suppresses the whole process. In the case of outgoing right chiral quarks (LR), the diagrams vanish on their own in the limit of negligible mixing among right chiral squarks. In the case of ϵ'_K, the LR diagrams do not vanish since no mixing on the squark line is needed (see Figure 2.1). For these LR diagrams, there is no cancellation with the crossed boxes. The reason lies with the QCD color factors, which are different when coupled to 'left chiral' or 'right chiral' squarks, resulting in different numerical factors that do not cancel. The dominant diagrams are shown in Figure 2.1. These diagrams come with opposite signs, we therefore also require mass splitting between the right handed up and down squarks for these diagrams not to cancel.

We therefore can have large LL mixing allowing a potentially large $\epsilon_K'^{\mathrm{NP}}$ while ϵ_K^{NP} remains sufficiently small without having to fine-tune our model.

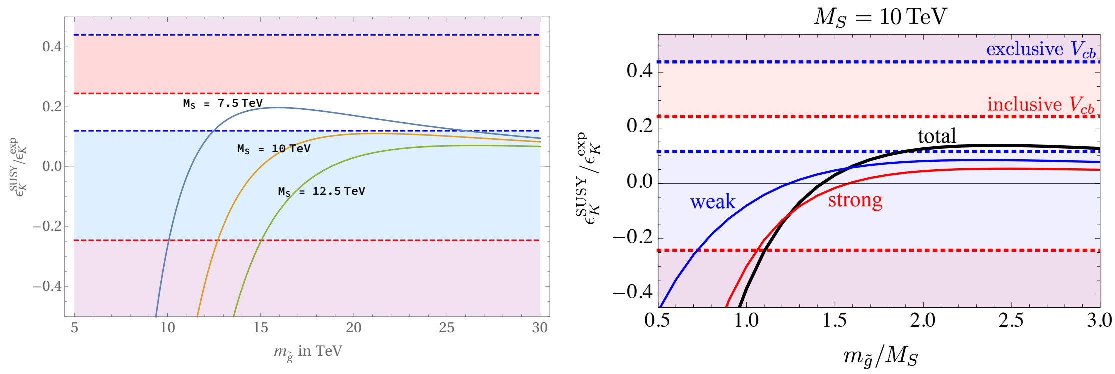

Figure 2.3.: *The behaviour of the gluino box contribution to ϵ_K^{NP} normalized to the SM value. The blue (red) shading represents the regions which are excluded at 95 % confidence level by ϵ_K measurements, if the exclusive (inclusive) value for $|V_{cb}|$ is taken for the SM prediction. See section 2.5 for a discussion of the exclusive and inclusive values of $|V_{cb}|$. The left plot shows the behaviour according to the approximate formula given in the text, while the right plot is taken from our paper [1] and shows the exact numerical behaviour. There, the red line corresponds to gluino box contribution while the blue line corresponds to the sum of the box contributions with one or two winos.*

2.3.1. Explicit Calculation of the Gluino Box Diagram

In this section, we show an exemplary calculation of a gluino box diagram, namely the left diagram of Figure 2.1. Applying Feynman rules [52] [53] to this diagram, we obtain the matrix element

$$\begin{aligned}
i\mathcal{M} =& \bar{s}_\alpha \left[ig_3\sqrt{2}T^a_{\alpha\beta}\left(-\Gamma^{I2*}_{DL}P_R + \Gamma^{I2*}_{DR}P_L\right)\right]\left(i\frac{(-1)\gamma^\mu(p_s+k)_\mu + m_{\tilde{g}}}{(p_s+k)^2 - m_{\tilde{g}}^2}\right)\left[ig_3\sqrt{2}T^a_{\delta\sigma}\left(-\Gamma^{J1}_{UL}P_L + \Gamma^{J1}_{UR}P_R\right)\right]u_\sigma \\
&\cdot \bar{u}_\rho\left[ig_3\sqrt{2}T^b_{\rho\delta}\left(-\Gamma^{J1*}_{UL}P_R + \Gamma^{J1*}_{UR}P_L\right)\right]\left(i\frac{(-1)\gamma^\nu(p_d-k)_\nu + m_{\tilde{g}}^2}{(p_d-k)^2 - m_{\tilde{g}}}\right)\left[ig_3\sqrt{2}T^b_{\beta\gamma}\left(-\Gamma^{I1}_{DL}P_L + \Gamma^{I1}_{DR}P_R\right)\right]d_\gamma \\
&\cdot\left(\frac{i}{k^2 - m^2_{\tilde{d}_I}}\right)\left(\frac{i}{(p_s - p_u + k)^2 - m^2_{\tilde{u}_J}}\right) \qquad (2.3)
\end{aligned}$$

where integration over the loop momentum k is implied. Γ^{Ii}_{DL} denotes the mixing matrix at the vertex of a down type quark-squark mixing (first subscript), left handed current (second subscript), of a squark generation I with quark of generation i. The translation to the convention used in [52] is given by [54]

$$\Gamma^{iI}_{DL} = Z^{Ii}_D \qquad \Gamma^{iI}_{DR} = Z^{(I+3)i}_D \qquad \Gamma^{iI}_{UL} = Z^{Ii*}_U \qquad \Gamma^{iI}_{UR} = Z^{(I+3)i*}_U \qquad (2.4)$$

We focus on the part of the calculation with incoming left chiral quark fields and outgoing right chiral quark fields. The other combinations of chirality proceed in the same way and lead to subleading operators. We will give the results at the bottom of this section. Note that for conjugate spinors taking a right chiral projector leads to a left chiral conjugate field: $\bar{s}_L = \bar{s}P_R$. Due to the properties of the projectors P_L and P_R, the choice of the $L \to R$ transition eliminates the tensor and vector integral structures (after we sent external momenta to zero) and leaves us with only the scalar integral to be evaluated.

We rearrange the expression we obtained from evaluating the Feynman rules

$$
\begin{aligned}
i\mathcal{M} =&(-1)^2 i^8 4 g_3^4 T^a_{\alpha\beta} T^a_{\delta\sigma} T^b_{\rho\delta} T^b_{\beta\gamma} \left(\Gamma^{I2}_{DL}\right)^* \left(\Gamma^{I1}_{DL}\right) \left(\Gamma^{J1}_{UR}\right)^* \left(\Gamma^{J1}_{UR}\right) (\bar{s}_{L,\alpha} u_{R,\sigma})(\bar{u}_{R,\rho} d_{L,\gamma}) \\
&\cdot m_{\tilde{g}}^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{(p_s+k)^2 - m_{\tilde{g}}^2}\right) \left(\frac{1}{(p_d-k)^2 - m_{\tilde{g}}^2}\right) \left(\frac{1}{k^2 - m_{\tilde{d}_I}^2}\right) \left(\frac{1}{(p_s - p_u + k)^2 - m_{\tilde{u}_J}^2}\right)
\end{aligned}
\tag{2.5}
$$

The color structure evaluates to

$$
\sum_a T^a_{\alpha\beta} T^a_{\delta\sigma} \sum_b T^b_{\rho\delta} T^b_{\beta\gamma} = \frac{1}{36}\delta_{\alpha\gamma}\delta_{\sigma\rho} + \frac{7}{12}\delta_{\alpha\sigma}\delta_{\gamma\rho} \tag{2.6}
$$

For the loop integral, all masses are of the order of the SUSY scale, while the external momenta correspond to the Kaon scale, thus $p_i \ll m_j$ for all external momenta p_i and all masses m_j involved. The diagram remains finite when we take all external momenta to zero and with $\tau_I^{d,u} := m^2_{\tilde{d}_I,\tilde{u}_I}/m^2_{\tilde{g}}$ we obtain for the scalar integral I_S:

$$
\begin{aligned}
I_S &= m_{\tilde{g}}^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{(p_s+k)^2 - m_{\tilde{g}}^2}\right) \left(\frac{1}{(p_d-k)^2 - m_{\tilde{g}}^2}\right) \left(\frac{1}{k^2 - m_{\tilde{d}_I}^2}\right) \left(\frac{1}{(p_s - p_u + k)^2 - m_{\tilde{u}_J}^2}\right) \\
&\overset{p_i \to 0}{=} \frac{1}{m_{\tilde{g}}^2} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2-1}\right) \left(\frac{1}{k^2-1}\right) \left(\frac{1}{k^2 - \tau_I^d}\right) \left(\frac{1}{k^2 - \tau_J^u}\right) \\
&= \frac{1}{m_{\tilde{g}}^2} \frac{i}{16\pi^2} F(\tau_I^d, \tau_J^u)
\end{aligned}
\tag{2.7}
$$

The loop function $F(\tau_I^d, \tau_J^u)$ is given by [54]

$$
F[x,y] = -\frac{x\ln x}{(x-y)(x-1)^2} - \frac{y\ln y}{(y-x)(y-1)^2} - \frac{1}{(x-1)(y-1)} \tag{2.8}
$$

For convenience, we also give the loop function $G(\tau_I^d, \tau_J^u)$, which will appear later

$$
G[x,y] = \frac{x^2\ln x}{(x-y)(x-1)^2} + \frac{y^2\ln y}{(y-x)(y-1)^2} + \frac{1}{(x-1)(y-1)} \tag{2.9}
$$

Inserting this into the amplitude, we get

$$
\begin{aligned}
i\mathcal{M} =& \, i4\alpha_s^2 \frac{1}{m_{\tilde{g}}^2} \left(\Gamma^{I2}_{DL}\right)^* \left(\Gamma^{I1}_{DL}\right) \left(\Gamma^{J1}_{UR}\right)^* \left(\Gamma^{J1}_{UR}\right) F(\tau_I^d, \tau_J^u) \\
&\cdot \left[\frac{1}{36}(\bar{s}_{L,\alpha} u_{R,\beta})(\bar{u}_{R,\beta} d_{L,\alpha}) + \frac{7}{12}(\bar{s}_{L,\alpha} u_{R,\alpha})(\bar{u}_{R,\beta} d_{L,\beta})\right]
\end{aligned}
\tag{2.10}
$$

To get to the traditional operator basis, we use the following Fierz transformation [55]

$$
(\bar{s}_i P_R u_k)(\bar{u}_l P_L d_j) = -\frac{1}{2}(\bar{s}_i \gamma^\mu P_L d_j)(\bar{u}_l \gamma_\mu P_R u_k) \tag{2.11}
$$

so the two types of fermion chains we have turn into

$$
\begin{aligned}
(\bar{s}_{L,\alpha} u_{R,\beta})(\bar{u}_{R,\beta} d_{L,\alpha}) &= (\bar{s}_\alpha P_R u_\beta)(\bar{u}_\beta P_L d_\alpha) \\
&= -\frac{1}{2}(\bar{s}_\alpha \gamma^\mu P_L d_\alpha)(\bar{u}_\beta \gamma_\mu P_R u_\beta) \\
&:= -\frac{1}{8}(sd)_{V-A}(uu)_{V+A} := -\frac{1}{8}Q_1'^{,u}
\end{aligned}
\tag{2.12}
$$

and

$$\begin{aligned}(\bar{s}_{L,\alpha}u_{R,\alpha})(\bar{u}_{R,\beta}d_{L,\beta}) &= (\bar{s}_\alpha P_R u_\alpha)(\bar{u}_\beta P_L d_\beta)\\ &:= -\frac{1}{8}(s_\alpha d_\beta)_{V-A}(u_\beta u_\alpha)_{V+A} := -\frac{1}{8}Q_2^{\prime,u} \end{aligned}\tag{2.13}$$

We insert this into the amplitude along with $1 = \frac{G_F}{\sqrt{2}}\frac{\sqrt{2}}{G_F}$, then read off $-\mathcal{M} = \mathcal{H}$

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\left[\frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DL}^{I2}\right)^*\left(\Gamma_{DL}^{I1}\right)\left(\Gamma_{UR}^{J1}\right)^*\left(\Gamma_{UR}^{J1}\right)F(\tau_I^d,\tau_J^u)\left[\frac{1}{18}Q_1^{\prime,u}+\frac{7}{6}Q_2^{\prime,u}\right]\right]\tag{2.14}$$

and with the definition $\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\cdot\sum_i c_i Q_i$, we can read up the contributions of this diagram to the Wilson Coefficients $c_1^{\prime,u}$ and $c_2^{\prime,u}$.

The crossed diagram contributes with the loop function $G(\tau_I^d,\tau_J^u)$ and a different coefficient due to the color structure. The calculation, however, proceeds in the same manner as above. Summing up all contributing diagrams, we end up with the full contributions of the gluino boxes to the coefficients $c_1^{\prime,u}$ and $c_2^{\prime,u}$:

$$\begin{aligned}
c_1^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DL}^{I2}\right)^*\left(\Gamma_{DL}^{I1}\right)\left(\Gamma_{UR}^{J1}\right)^*\left(\Gamma_{UR}^{J1}\right)\left[\frac{1}{18}F(\tau_I^d,\tau_J^u)-\frac{5}{18}G(\tau_I^d,\tau_J^u)\right]\\
c_2^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DL}^{I2}\right)^*\left(\Gamma_{DL}^{I1}\right)\left(\Gamma_{UR}^{J1}\right)^*\left(\Gamma_{UR}^{J1}\right)\left[\frac{7}{6}F(\tau_I^d,\tau_J^u)+\frac{1}{6}G(\tau_I^d,\tau_J^u)\right]
\end{aligned}\tag{2.15}$$

The coefficients $c_1^{\prime,d}$ and $c_2^{\prime,d}$ can easily be obtained by replacing the up type squark with a down type squark by simply switching $\tau_J^u \to \tau_J^d$ and $\Gamma_U \to \Gamma_D$ in the above expressions. These coefficients belong to the numerically largest matrix elements within the A_2 amplitude - Q_7 and especially Q_8 in the notation of the traditional SM basis [40] - and thus constitute the largest contribution to ϵ_K'/ϵ_K in our model. Calculating the remaining gluino box diagrams with $L \to L$, $R \to L$ and $R \to R$ chiral transitions, we obtain the contributions to the remaining operators. The operator basis and the effective Hamiltonian are given in our paper [1]. The result for the Wilson coefficients complies with [49].

$$\begin{aligned}
c_3^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DL}^{I2}\right)^*\left(\Gamma_{DL}^{I1}\right)\left(\Gamma_{UL}^{J1}\right)^*\left(\Gamma_{UL}^{J1}\right)\left[-\frac{5}{9}F(\tau_I^d,\tau_J^u)+\frac{1}{36}G(\tau_I^d,\tau_J^u)\right]\\
c_4^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DL}^{I2}\right)^*\left(\Gamma_{DL}^{I1}\right)\left(\Gamma_{UL}^{J1}\right)^*\left(\Gamma_{UL}^{J1}\right)\left[\frac{1}{3}F(\tau_I^d,\tau_J^u)+\frac{7}{12}G(\tau_I^d,\tau_J^u)\right]\\
\tilde{c}_1^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DR}^{I2}\right)^*\left(\Gamma_{DR}^{I1}\right)\left(\Gamma_{UL}^{J1}\right)^*\left(\Gamma_{UL}^{J1}\right)\left[\frac{1}{18}F(\tau_I^d,\tau_J^u)-\frac{5}{18}G(\tau_I^d,\tau_J^u)\right]\\
\tilde{c}_2^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DR}^{I2}\right)^*\left(\Gamma_{DR}^{I1}\right)\left(\Gamma_{UL}^{J1}\right)^*\left(\Gamma_{UL}^{J1}\right)\left[\frac{7}{6}F(\tau_I^d,\tau_J^u)+\frac{1}{6}G(\tau_I^d,\tau_J^u)\right]\\
\tilde{c}_3^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DR}^{I2}\right)^*\left(\Gamma_{DR}^{I1}\right)\left(\Gamma_{UR}^{J1}\right)^*\left(\Gamma_{UR}^{J1}\right)\left[-\frac{5}{9}F(\tau_I^d,\tau_J^u)+\frac{1}{36}G(\tau_I^d,\tau_J^u)\right]\\
\tilde{c}_4^{\prime,u} &= \frac{\alpha_s^2}{2\sqrt{2}G_F}\frac{1}{m_{\tilde{g}}^2}\left(\Gamma_{DR}^{I2}\right)^*\left(\Gamma_{DR}^{I1}\right)\left(\Gamma_{UR}^{J1}\right)^*\left(\Gamma_{UR}^{J1}\right)\left[\frac{1}{3}F(\tau_I^d,\tau_J^u)+\frac{7}{12}G(\tau_I^d,\tau_J^u)\right]
\end{aligned}\tag{2.16}$$

2.4. Results

For definiteness, we use the following set of input parameters. These mainly serve to constrain the large parameter space of the MSSM and thereby to show that our results do not depend on specifically tuned choices of MSSM parameters.

The input parameters we used are

- SUSY scale: Universal mass of the left handed squark doublet, the right handed down squark and the higgsino mass parameter: $m_Q = m_{\overline{D}} = \mu_{SUSY} = M_S$
- Universal gaugino masses at the GUT scale $M_G \sim 10^{16}$
- $m_{\tilde{g}}/M_S = 1.5$
- No trilinear SUSY terms: $A_q = 0$
- $\tan\beta = 10$
- Squark mass matrix offdiagonal elements for LL mixing $\Delta_{Q,12,13,23} = 0.1 \exp(-i\pi/4)$, all others zero

As mentioned above, this choice of parameters is mainly for simplification as to only include the sources of the mechanism we wished to show. To this end, we need the mixing in the left handed squark mass matrix, which we chose as a 10 % effect of the diagonal elements, together with a phase choice which maximizes the CP phase in ϵ_K, the hardest constraint on our model. In this way, we showed that our solution is not finetuned in the CP phase but even persists for the maximal choice. The right handed squark mixing needs to be absent (or at least $\mathcal{O}(\lesssim 10^{-5})$ for the ϵ_K suppression to work. The $m_{\tilde{g}}/M_S = 1.5$ relation is the prime spot for the ϵ_K suppression, although a ratio > 1.5 also works, as discussed above.

The essential result of our investigation is that under reasonable assumptions it is possible to resolve the ϵ'_K/ϵ_K tension within the MSSM. Of course, the term 'reasonable assumptions' is our own interpretation of things and since we were the people conducting the analysis, this could be viewed as biased by others. So what did we view as 'reasonable assumptions' here, under which conditions can the MSSM resolve the ϵ'_K/ϵ_K tension?

The main result is collected in Figure 2.4. The plot shows in which region of $m_{\overline{U}}$ and M_S the ϵ'_K/ϵ_K discrepancy can be resolved together with satisfying the ϵ_K bounds. $m_{\overline{U}}$ is the right handed up squark mass while M_S is the universal SUSY mass, including the right handed down squark mass. The dark green (light green) region resolves the ϵ'_K/ϵ_K discrepancy at 1σ (2σ). The red shaded region between the dashed red lines is excluded through ϵ_K using the inclusive V_{cb} measurement. In the case of the exclusive V_{cb} measurement, ϵ_K even asks a small contribution from new physics (see Figure 2.3). The region between the blue dashed lines depicts the area, where the contribution to ϵ_K resolves its discrepancy.

We deliberately chose the phase of $\Delta_{Q,12}$ (the squark mass matrix element which mediates

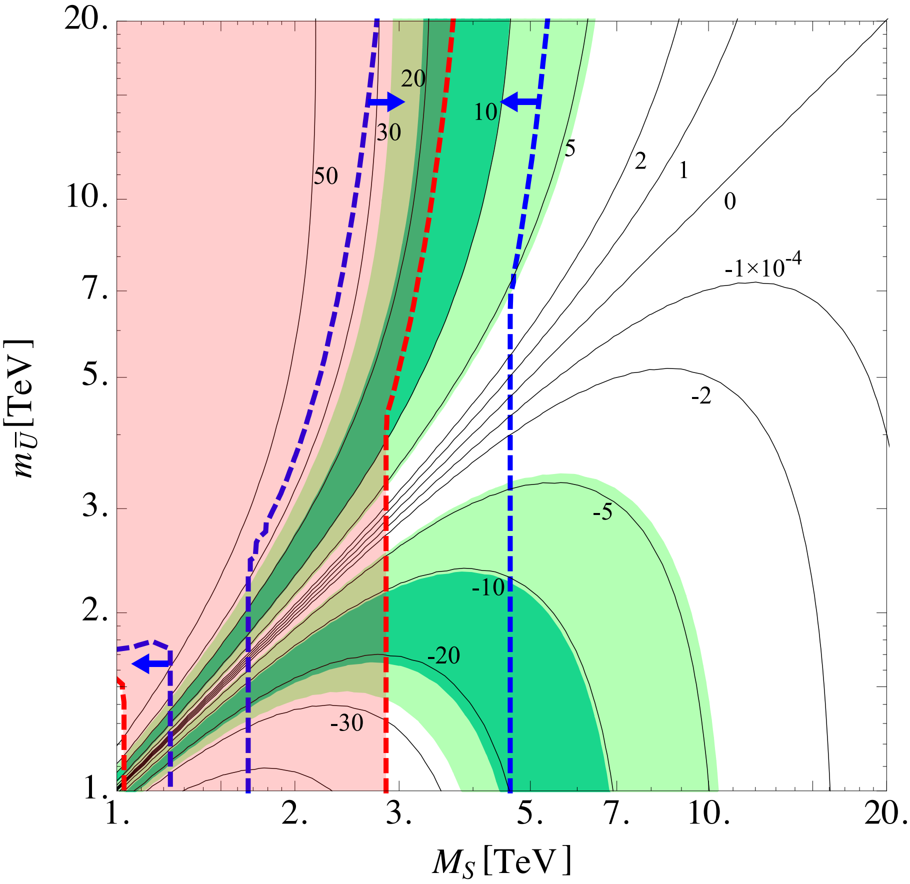

Figure 2.4.: *Contour plot of the supersymmetric contributions to ϵ'_K/ϵ_K in 10^{-4} as appeared in our paper [1]. The dark (light) green bands resolve the ϵ'_K/ϵ_K discrepancy at 1σ (2σ). The red shaded region is the 95% exclusion region of ϵ_K in case of an inclusive V_{cb}; the region between the two blue dashed lines is the favored region of ϵ_K in case of an exclusive V_{cb}.*

the $s-d$ transition) to be $-i\pi/4$ to maximize the CP phase in the $K^0 - \overline{K}^0$ mixing amplitude. This way we show that our suppression of ϵ_K is not finetuned at all. Flipping the sign of this phase, ϵ_K does not change, while ϵ'_K/ϵ_K flips its sign, indicated by the two green branches in Figure 2.4.

The dominant contribution comes from gluino-gluino box diagrams like the one we calculated in Section 2.3.1. The next largest contribution likely comes from gluon gluino chromomagnetic penguins [56] [57] (an explicit calculation of the Wilson coefficients is attached in Appendix B.1). However, the hadronic matrix element is poorly known with a B_G parameter of $B_G = 1 \pm 3$, which makes a precision calculation of the perturbative parts practically useless until non-perturbative methods improve. This amounts to an uncertainty in the contribution of the chromomagnetic penguin of about an order of magnitude. For that reason, we excluded the chromomagnetic penguin contribution from Figure 2.4. Note that it is easy to reproduce this plot for any given value of the matrix element of the chromomagnetic penguin. The statement stays the same, just the allowed areas could shift a bit to even higher squark masses in case the hadronic matrix element turns out to be large.

Very recently, a new calculation of the relevant matrix element for the chromomagnetic

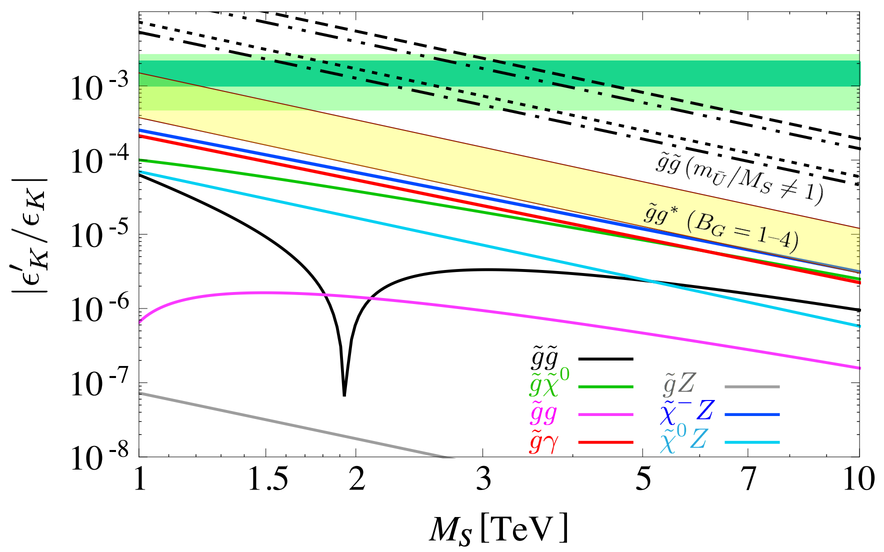

Figure 2.5.: *All contributions to ϵ'_K/ϵ_K as appeared in our paper [1]. The dark (light) green bands resolve the ϵ'_K/ϵ_K discrepancy at 1σ (2σ).*

penguin in the dual-QCD approach has been performed [58]. The authors come to the conclusion, that the parameter is small and hence the chromomagnetic penguin contribution to ϵ'_K in our analysis is subleading.

Additional subleading contributions

Apart from the gluino-gluino boxes and the gluino chromomagnetic penguin, we also calculated gluino-neutralino boxes, gluon, photon and Z penguin diagrams involving a gluino in the loop and Z penguins with charginos and neutralinos in the loop. All these various diagrams turned out to be subleading and have little effect as shown in Figure 2.5.

The black dashed lines show the gluino-gluino box contributions for $m_{\tilde{U}}/m_{\tilde{D}} = 0.5, 2.0, 0.8, 1.2$ from top to bottom. The yellow band shows the contribution from the chromomagnetic penguin for a B_G parameter of $B_G = 1-4$. The solid black line shows the gluino-gluino box contribution in case of degenerate masses $m_{\tilde{U}} = m_{\tilde{D}}$, in which case the various diagrams mostly cancel (see Figure 2.1. Only in this degeneracy limit, together with a $B_G \lesssim 1$, the various subleading contributions become meaningful for our choice of parameters. Note that gluino-photon (red line) and chargino-Z penguin (blue line) have opposite signs and almost cancel. We neglected gluino-W penguin and chargino box contributions which contribute at most $\mathcal{O}(10^{-5})$ to ϵ'_K/ϵ_K.

2.5. A word on the status of V_{cb}

The calculation of ϵ'_K/ϵ_K and ϵ_K involves many different CKM elements. ϵ_K is measured so precisely [29] that it is beneficial to express the CKM matrix elements with larger uncertainties through the use of CKM unitarity by others with smaller uncertainties.

Especially V_{td} and V_{ts} are hard to measure precisely [59]. We can express the imaginary part of $\lambda_t = V_{td}V_{ts}^*$, which is essential for the determination of ϵ_K and ϵ_K'/ϵ_K (see Section 3.2.10), through Im $\lambda_t = |V_{ub}||V_{cb}|\sin\gamma$ [50]. In the case of ϵ_K'/ϵ_K, the errors of the hadronic matrix elements are presently so large, that such a tradeoff would avail to nothing. In this subsection, we would like to give a brief overview on the status of the matrix element V_{cb}.

The current status of V_{cb} is [60] [29]

$$\begin{aligned} |V_{cb}^{\text{excl.}}| &= (39.2 \pm 0.7) \times 10^{-3} \\ |V_{cb}^{\text{incl.}}| &= (42.2 \pm 0.8) \times 10^{-3} \end{aligned} \tag{2.17}$$

The exclusive value is extracted from $\overline{B} \to D^{(*)}\ell\overline{\nu}$ decays, while the inclusive value sums over final states in $\overline{B} \to X_c\ell\overline{\nu}$.

While the inclusive V_{cb} measurement is in perfect agreement with the SM value for ϵ_K, it does allow for a NP contribution of about 20% the size of the SM value. The exclusive measurement on the other hand actually disfavors the SM value at 95% confidence level and asks for a small, positive NP contribution of about 15% to 40% of the SM value. In a general analysis from 2014, Crivellin and Pokorski [61] come to the conclusion that the discrepancy between the inclusive and exclusive measurements cannot be explained by new physics and thus must stem from underestimated uncertainties. Of course, one can construct specifically tailored models to account for this discrepancy, but they usually require absurd assumptions (like most something-phobic models) in order not to violate one of the numerous other experimental bounds that include the same elementary processes but do not show discrepancies.

Recently [62] [63], a reparametrization of the form factor using new Belle data pushed the exclusive V_{cb} value to perfect accordance with the inclusive value, possibly putting an end to the conundrum by giving

$$|V_{cb}^{\text{excl.}}| = (41.9 \pm 2) \times 10^{-3} \qquad [29]$$

This should be taken with a grain - or rather a truckload - of salt since the current situation is very biased. It remains to be seen whether this development is further solidified and subsequently reveals a flaw in the use of the old parametrization or goes away with further data and leaves us stuck with the former conundrum that the results of two established and theoretically well-founded methods exclude each other.

Theoretical Background 3

In this chapter we review the concepts and the formalism of weak CP violation in Kaon decays with a focus on direct CP violation. The content of this chapter is entirely common knowledge in the field and subject to a vast amount of reviews, lectures and books. Most formulae appear in most reviews and thus the citations are often meant exemplary and in no way exhausting.

3.1. Theory of Weak CP Violation

Weak CP violation in the Standard Model manifests itself in a single complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [64]. The CKM matrix describes the misalignment between up-type and down-type quarks. Experimental evidence of neutrino mixing angles imply that neutrinos have mass [29]. The addition of a mass term to the Standard Model Lagrangian leads to a misalignment matrix between charged and neutral leptons in the same way as in the quark sector. The misalignment matrix is denoted Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [65] and can in general contain complex phases. In the same way as in the quark sector, this can lead to CP violating phenomenology. In this chapter we will review how the CKM matrix and the PMNS matrix appear as misalignment matrices between the upper and lower component of their respective $SU(2)_L$ doublet after the electroweak symmetry has been broken.

3.1.1. The CKM matrix

After electroweak symmetry breaking (EWSB), mass terms for the matter fields are generated via the Higgs VEV $\langle\phi\rangle$. The quark mass matrices are then combinations of the 3×3 Yukawa matrices and the Higgs VEV $M_u := \langle\phi\rangle\mathcal{Y}_u$ and $M_d := \langle\phi\rangle\mathcal{Y}_d$.

$$\mathcal{L}_{q,\text{Mass}} = \bar{u}_L M_u u_R + \bar{d}_L M_d d_R \tag{3.1}$$

u and d are triplets of the up-type and down-type quarks in generation space. We diagonalize the mass matrices by singular value decomposition [66].

$$M_u^{\text{diag}} := \text{diag}\left(m_u, m_c, m_t\right) = U_u M_u V_u^\dagger \qquad M_d^{\text{diag}} := \text{diag}\left(m_d, m_s, m_b\right) = U_d M_u V_d^\dagger \tag{3.2}$$

With U_q and V_q unitary, we can rewrite the Yukawa Lagrangian in the following way

$$\mathcal{L}_{\mathcal{Y}} = \underbrace{\bar{u}_L U_u^\dagger}_{\tilde{\bar{u}}_L} \underbrace{U_u M_u V_u^\dagger}_{M_u^{\text{diag}}} \underbrace{V_u u_R}_{\tilde{u}_R} + \underbrace{\bar{d}_L U_d^\dagger}_{\tilde{\bar{d}}_L} \underbrace{U_d M_d V_d^\dagger}_{M_d^{\text{diag}}} \underbrace{V_d d_R}_{\tilde{d}_R} \tag{3.3}$$

Where we defined the mass eigenstates of the quark fields.

$$\begin{aligned} \tilde{u}_L &= U_u u_L \\ \tilde{u}_R &= V_u u_R \\ \tilde{d}_L &= U_d d_L \\ \tilde{d}_R &= V_d d_R \end{aligned} \tag{3.4}$$

The charged current interaction term for the left handed quarks is not invariant under this transformation. Using (3.4), we can express it in terms of mass eigenstates $\bar{q}_L = \tilde{\bar{q}}_L U_q$

$$\begin{aligned} \mathcal{L}_{W,q} &= \frac{g}{\sqrt{2}} \left(W_\mu^+ \bar{u}_L \gamma^\mu d_L + W_\mu^- \bar{d}_L \gamma^\mu u_L \right) \\ \mathcal{L}_{W,q} &= \frac{g}{\sqrt{2}} \left(W_\mu^+ \tilde{\bar{u}}_L \gamma^\mu \underbrace{U_u U_d^\dagger}_{=:V_{\text{CKM}}} \tilde{d}_L + W_\mu^- \tilde{\bar{d}}_L \underbrace{U_d U_u^\dagger}_{=:V_{\text{CKM}}^\dagger} \gamma^\mu \tilde{u}_L \right) \end{aligned} \tag{3.5}$$

There is no equivalent for the right handed quarks because they do not take part in the $SU(2)_L$ interaction, hence this term is absent. The fact that the charged current interaction is not invariant is not suprising. The weak interaction mediates between the components of the $SU(2)_L$ doublet Q_L. After EWSB this interaction gets suppressed by the energy of the breaking scale, which we express through the mass of the W boson. In this way, the CKM matrix becoming physical is a remnant of the left handed quark fields being correlated by originating from a $SU(2)_L$ doublet.

We interprete the CKM matrix as the misalignment matrix between down quark mass eigenstates and flavor eigenstates.

$$\boxed{d_L = V_{\text{CKM}} \tilde{d}_L} \tag{3.6}$$

We note that we could equally well have attributed the CKM matrix to the up type quarks. Attributing it to the down type quarks is just a convention. In the neutrino sector, the corresponding matrix is attributed to the upper part of the $SU(2)_L$ doublet, the neutrinos.

3.1.2. The PMNS matrix

In the lepton sector, we have to distinguish two cases. If there is a Majorana term present, we need to treat the mass matrix in a little more general and complicated way. If no Majorana term is present, we can proceed in exactly the same way as we did in the quark sector. The Lepton mass Lagrangian then is

$$\mathcal{L}_{\ell,\mathrm{Mass}} = \bar{\nu}_L M_{\nu,D} \nu_R + \bar{l}_L M_l l_R \tag{3.7}$$

ν_L and l are triplets in generation space, containing the neutral leptons (in the following just called neutrinos) and charged leptons, respectively. This is the same form as the quark mass Lagrangian (3.1) and the procedure is exactly the same as in the quark sector. We diagonalize the mass matrices,

$$\begin{aligned} \bar{\nu}_L M_\nu \nu_R &= \bar{\nu}_L U_\nu^\dagger U_\nu M_\nu V_\nu^\dagger V_\nu \nu_R \\ &= \bar{\tilde{\nu}}_L M_\nu^{\mathrm{diag}} \tilde{\nu}_R \\ \bar{l}_L M_l l_R &= \bar{l}_L U_l^\dagger U_l M_l V_l^\dagger V_l l_R \\ &= \bar{\tilde{l}}_L M_l^{\mathrm{diag}} \tilde{l}_R \end{aligned} \tag{3.8}$$

thus the mass eigenstates are

$$\begin{aligned} \bar{\tilde{l}}_L = \bar{l}_L U_l^\dagger \quad &\longleftrightarrow \quad l_L = U_l^\dagger \tilde{l}_L \\ \bar{\tilde{\nu}}_L = \bar{\nu}_L U_\nu^\dagger \quad &\longleftrightarrow \quad \nu_L = U_\nu^\dagger \tilde{\nu}_L \end{aligned} \tag{3.9}$$

and the charged currents look like

$$\begin{aligned} \mathcal{L}_{W,\ell} &= \frac{g}{\sqrt{2}} \left(W_\mu^+ \bar{\nu}_L \gamma^\mu l_L + W_\mu^- \bar{l}_L \gamma^\mu \nu_L \right) \\ \mathcal{L}_{W,\ell} &= \frac{g}{\sqrt{2}} \left(W_\mu^+ \bar{\tilde{\nu}}_L \gamma^\mu \underbrace{U_\nu U_l^\dagger}_{=:U_{\mathrm{PMNS}}} \tilde{l}_L + W_\mu^- \bar{\tilde{l}}_L \underbrace{U_l U_\nu^\dagger}_{=:U_{\mathrm{PMNS}}^\dagger} \gamma^\mu \tilde{\nu}_L \right) \end{aligned} \tag{3.10}$$

This allows us to define the PMNS matrix as the misalignment matrix between the neutrino mass eigenstates and flavor states

$$\boxed{\nu_L = U_{\mathrm{PMNS}}^\dagger \tilde{\nu}_L} \tag{3.11}$$

Note the Hermitian conjugate on the PMNS matrix, which is purely conventional. In this convention, the PMNS matrix is defined in line with the CKM matrix, such that they are without Hermitian conjugate when interpreted as mixing matrices of the lower component of the $SU(2)_L$ doublet (d_L and l_L, respectively) or with Hermitian conjugate when they are interpreted as mixing matrices of the upper component of the $SU(2)_L$ doublet (u_L and ν_L).

Presence of a Majorana mass term - Takagi diagonalization

In presence of a Majorana mass term, the left and right handed fields obtain separate masses. To treat this, we switch to two component spinor notation in LR convention. The mass Lagrangian for the leptons then looks like

$$\mathcal{L}_{\ell,\text{Mass}} = \nu_L^T M_{\nu,D} \nu_R^c + l_L^T M_l l_R + \nu_L^T M_{\nu,M} \nu_L \tag{3.12}$$

We obtain the full neutrino mass matrix M_ν in the following way [12]

$$M_\nu = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}^T \begin{pmatrix} M_{\nu,M} & M_{\nu,D} \\ M_{\nu,D}^T & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \tag{3.13}$$

Here, ν_L and ν_R^c are still triplets and the matrix (3.13) is really a 6×6 matrix. When trying to diagonalize this matrix via a SVD, a subtle problem arises: The matrix M_ν is symmetric and this leaves an arbitrariness in the determination of the phases of the diagonalization matrices U_ν and V_ν:

$$U_\nu M_\nu V_\nu^\dagger = M_\nu^{\text{diag}} = M_\nu^{\text{diag},T} = V_\nu^* M_\nu^T U_\nu^T = V_\nu^* M_\nu U_\nu^T \tag{3.14}$$

where we explicitly used the symmetry of M_ν in the last step. The symmetry of M_ν thus allows us to identify

$$U_\nu = V_\nu^* \tag{3.15}$$

which corresponds to a choice of the phases, making the diagonalization matrices unique. We find the Takagi diagonalization [12]

$$U_\nu M U_\nu^T = M_\nu^{\text{diag}} \tag{3.16}$$

or equivalently

$$V_\nu^* M V_\nu^\dagger = M_\nu^{\text{diag}} \tag{3.17}$$

Note that this only applies to complex symmetric matrices. Real symmetric matrices are considered hermitian and an eigenvalue decomposition gives the correct results.

3.2. Phenomenology of Weak CP Violation in the Kaon System

The *raison d'être* of phenomenology is to make the connection between theory and experiment. This means to extract and calculate observables from a given theory and compare these theory predictions with existing measurements. This connection is where the scientific statement is made, whether a theory coincides with measurements and thus establishes more and more confidence in its power to model a part of nature, or whether it is falsified, contradicting measurements.

Phenomenology does thus play a crucial role in modern day physics, where theories are most of the time so complex that it requires a lot of dedicated work by well-trained specialists to make the connection to experiment. Without phenomenology, statements like 'our model contains a complex phase in the quark mixing matrix' and 'neutral long living Kaons decay occasionally to two Pions instead of three' would stand each for themselves without the crucial connection that this part of the theory actually manifests itself in exactly this measurement. Each statement for itself does not hold scientific value, it is only in connecting them that there is scientific value, that there is science, that there actually is theory and experiment. Without the connection to experiment, theory are just random ideas. Without the connection to theory, experiment is just playing games. Throwing an apple becomes an experiment when you try to investigate Newtonian gravity, otherwise you just throw an apple.

3.2.1. Qualitative discussion

Kaon CP Violation History

In 1964, Christenson, Cronin, Fitch and Turlay measured [67] a K_L decay to 2π and thereby found experimental evidence for CP violation in the neutral Kaon system. How does that show CP violation? To see this, we first look at how the Kaon flavor states behave under CP transformation. With $|K^0\rangle = |\bar{s}d\rangle$ and $|\bar{K}^0\rangle = |s\bar{d}\rangle$ we get

$$\begin{aligned} \mathrm{CP}|K^0\rangle &= |\bar{K}^0\rangle \\ \mathrm{CP}|\bar{K}^0\rangle &= |K^0\rangle \end{aligned} \tag{3.18}$$

where the CP transformation introduces an unobservable phase factor to the Kaon state which we just set to 1. We know that the weak force can mediate flavor transitions and thus there is a probability for the transition shown in figure 3.1. The two quarks exchange two W bosons and what initially was a K^0 ($\bar{K}^0$) is now a $\bar{K}^0$ (K^0). At the level of the theory of mesons, we talk about it in a language like 'particles which are identical in all conserved quantum numbers can mix'. Once we go to the levels of the theory of quarks, we can actually model the dynamics which underlie this mixing via the diagram shown in figure 3.1.

The Kaon oscillation period is determined by the mass splitting of the mass eigenstates and can be measured to be $\Delta m/2\pi \approx 1.2 \times 10^{-9}s$ [29]. From this experimental result we

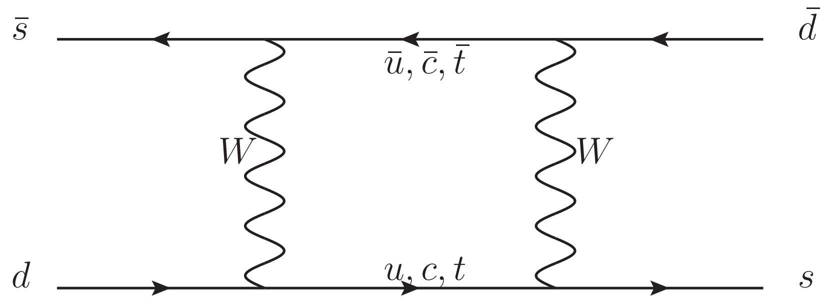

Figure 3.1.: *Feynman diagram representing the transition of a K^0 into a $\bar{K}^0$ by the exchange of two W bosons.*

can qualitatively conclude that we should expect on average around 40-50 oscillations in the lifetime of the long lived Kaon mass eigenstate K_L with $\tau_L \approx 0.5 \times 10^{-7} s$. Although a classical picture, it is similar to flipping a coin where we have no possibility of tracking the number of flips but expecting several flips midair. We have to go with a 50/50 heads/tails expectation value. Hence we take a superposition of the Kaon flavor states. In the end, we want the expectation value for finding a K^0 or a $\bar{K}^0$ in the final state. Should the weak interaction, which is responsible for the oscillation diagrams, couple equally to the constituent quarks of K^0 and $\bar{K}^0$ (and as such not differentiate between particle and antiparticle), we take superpositions with even weights.

$$
\begin{aligned}
|K_{CP+}\rangle &= \frac{1}{\sqrt{2}} \left[|K^0\rangle + |\bar{K}^0\rangle \right] \\
|K_{CP-}\rangle &= \frac{1}{\sqrt{2}} \left[|K^0\rangle - |\bar{K}^0\rangle \right]
\end{aligned}
\tag{3.19}
$$

These happen to be CP eigenstates with eigenvalues $+1$ and -1 as we can easily verify.

Kaons decay via the weak interaction into Pions. The only possible Pion final states are $(\pi\pi)$ and $(\pi\pi\pi)$ since four Pions would exceed the Kaon rest mass. A single Pion is a CP eigenstate with eigenvalue -1, thus the $(\pi\pi)$ state has CP eigenvalue $(-1)(-1) = +1$ while the $(\pi\pi\pi)$ state has CP eigenvalue $(-1)(-1)(-1) = -1$. This dictates $K_{CP+} \rightarrow (\pi\pi)$ and $K_{CP-} \rightarrow (\pi\pi\pi)$ as the only possibilities.

The decay into a three Pion final state has a very small phase space because the production of three Pions take nearly all the energy of the Kaon rest mass. This results in a large difference in lifetime between the two physical Kaon states of ~ 600. Hence the experimentally observed Kaon states were labelled K_S for the short lived state, to be identified with the K_{CP+} state, and K_L for the long lived state, to be identified with K_{CP-}.

This large difference in lifetime is unique to the Kaon system (as opposed to the B and B_s meson systems for example). It presents a great opportunity to test the assumption we made about whether the weak interaction actually treats the constituents of the K^0 equally to the constituents of the $\bar{K}^0$ - whether it treats particles and antiparticles in the same way. If we were to prepare a Kaon beam and waited a sufficient amount of time of the order of the K_L lifetime, we would expect all K_S to have decayed long ago. Hence only K_L particles remain, which by CP conservation can only decay into three Pions. Should we observe a two Pion final state, we found that CP is not conserved.

This is what Christenson, Cronin, Fitch and Turlay found in 1964. Thus the weak interaction does not conserve CP, albeit the violation is very tiny as can be seen by the smallness of the fraction of K_L which decay to two Pions, which is only about 0.3 % of all decaying K_L. Nevertheless, the CP violation is there. In our line of arguments, there are two spots where we assumed CP invariance of the weak interaction.

CP violation in mixing

First we assumed that the mixing via the box diagrams would be equally weighed because the exchange of the weak bosons would not differentiate between particle and antiparticle. This turns out to be not quite right, one seems to be slightly favored over the other, leading to slightly uneven weights in the superposition [68].

$$
\begin{aligned}
|K_S\rangle &\sim (1+\tilde{\epsilon})|K^0\rangle + (1-\tilde{\epsilon})|\bar{K}^0\rangle \\
|K_L\rangle &\sim (1+\tilde{\epsilon})|K^0\rangle - (1-\tilde{\epsilon})|\bar{K}^0\rangle
\end{aligned} \tag{3.20}
$$

both having a slightly larger portion of K^0 than $\bar{K}^0$. The proportionality is just a normalization factor. Therefore $\tilde{\epsilon}$ is a measure of CP violation in mixing. The states K_L and K_S are not orthogonal anymore as a result of this [69]

$$
\langle K_S|K_L\rangle = \frac{2\ \mathrm{Re}\ \tilde{\epsilon}}{1+|\tilde{\epsilon}|^2} \approx 2\ \mathrm{Re}\ \tilde{\epsilon} = 2\ \mathrm{Re}\ \epsilon_K \tag{3.21}
$$

$\tilde{\epsilon}$ is not convention independent and the above picture is valid only in a certain class of conventions which we will assume from now on. These are the so called 'physical phase conventions' [70] which assume Arg $A_0 \ll 1$ [71] which implies $\tilde{\epsilon}$ is a small parameter. We will define the convention independent observable called ϵ_K in Section 3.2.4. The relation between these quantities is given by $\epsilon_K = \tilde{\epsilon} + i\ \mathrm{Im}A_0/\mathrm{Re}A_0$ [72] [29], where A_0 is a Kaon to Pion isospin amplitude to be defined in Section 3.2.6. The quantities A_0 and A_2 depend on the phase convention chosen for the strange quark state and it is possible to make either of them real [29]. For example, the Wu Yang convention is defined by $\mathrm{Im}A_0 = 0$ [71], which is the choice that gives $\tilde{\epsilon} = \epsilon_K$. Neverthless, the real parts of $\tilde{\epsilon}$ and ϵ_K are identical and measure the non-orthogonality of the physical Kaon states, as indicated by equation (3.21). The normalization factor is negligible since $|\tilde{\epsilon}| \sim \mathcal{O}(10^{-3})$.

We note, that the non-orthogonality of the Kaon mass eigenstates can be measured via the semileptonic charge asymmetry [29]

$$
A_L = \frac{\Gamma\left(K_L \to \pi^- \ell^+ \nu\right) - \Gamma\left(K_L \to \pi^+ \ell^- \nu\right)}{\Gamma\left(K_L \to \pi^- \ell^+ \nu\right) + \Gamma\left(K_L \to \pi^+ \ell^- \nu\right)} = \frac{2\mathrm{Re}\ \tilde{\epsilon}}{1+|\tilde{\epsilon}|^2} \tag{3.22}
$$

and was found to be [29]

$$A_L = (3.32 \pm 0.06) \times 10^{-3} \tag{3.23}$$

therefore Re $\epsilon_K \approx 1.6 \times 10^{-3}$. Compared with the measurements of $|\epsilon_K| = 2.228 \pm 0.011 \times 10^{-3}$, this implies an imaginary part of roughly Im $\epsilon_K \approx 1.5 \times 10^{-3}$ or a phase of about Arg $\epsilon_K \approx 44°$.

CP violation in decay

Second we assumed that the decay was only possible from states of a certain CP parity to states of the same CP parity, meaning CP were conserved in the decay. So even if we retained a negative CP eigenstate, that is were to project the K_L state on its K_{CP-} part, the CP (-1) Kaon state could only decay into the CP (-1) $(\pi\pi\pi)$ state. However, also being mediated through the weak interaction, this does not hold. There is a slight chance to go from a definite CP initial state to a different CP final state through a weak decay. This is more clearly shown in charged Kaon decays, where mixing is absent and hence the only source of CP violation is CP violation in the decay.

3.2.2. Constructing Observables - η_{00} and η_{+-}

We now know that we have CP violation when we observe a K_L decay into two Pions. As phenomenologists, we want to construct observables that can be calculated and compared with the experimental numbers. The central observables in the neutral Kaon system are the amplitude ratios [68]

$$\begin{aligned} \eta_{00} &= \frac{\langle \pi^0\pi^0 | \mathcal{H}_{eff} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_{eff} | K_S \rangle} \\ \eta_{+-} &= \frac{\langle \pi^+\pi^- | \mathcal{H}_{eff} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_{eff} | K_S \rangle} \end{aligned} \tag{3.24}$$

We want to take some time to address some central questions about these quantities which are usually glossed over in technical reviews. Like: 'why do we take amplitude ratios? I can't measure amplitudes, can I?', 'why is K_L normalized to K_S? How much sense does that make?' or to rephrase that a little 'what do the magnitudes of these η's mean, how much CP violation do I have if they are 3, 10^{-3} or 10.000?'. Clarifying questions like that is essential to understand the results.

The η FAQ

On the theory side, we take ratios of amplitudes to discriminate 'direct CP violation' and 'indirect CP violation'.

We can calculate the amount of the probability amplitude that comes from decay type diagrams (direct CP violation) and the amount that comes from mixing type diagrams (indirect CP violation). If we can combine the experimental observables in such a way that we can compare the measured numbers with our Standard Model calculations, we can obtain values for these amplitude level quantities. As it turns out, this is indeed possible as we will see later.

Since 'direct CP violation' and 'indirect CP violation' are concepts defined at amplitude level, it is most convenient to work with amplitudes which we can formally just separate into a sum of these parts.

On the experimental side, we can access these combinations of amplitudes because it is a ratio which has a common final state. We can take a ratio of decay rates and see that the phase space dependence cancels and all that remains is the ratio of amplitudes.

$$\frac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} = \frac{|\langle\pi^0\pi^0|\mathcal{H}|K_L\rangle|^2}{|\langle\pi^0\pi^0|\mathcal{H}|K_S\rangle|^2} = |\eta_{00}|^2 \tag{3.25}$$

The parameters ϵ_K and ϵ_K' correspond to 'indirect CP violation' and direct 'CP violation', respectively. Barring coefficients, they are

$$\begin{aligned} \epsilon_K &\sim \eta_{+-} + \eta_{00} \\ \epsilon_K' &\sim \eta_{+-} - \eta_{00} \end{aligned} \tag{3.26}$$

We will discuss these relations in Section 3.2.4 in more detail and derive the coefficients explicitly in Appendix C.3. Here we can understand them qualitatively as follows: The amplitudes η_{+-} and η_{00} only differ in their final states, which means that the mixing parts of the amplitudes, which have no business with those final states, are the same and cancel in the difference. Hence, if the difference of the η's is nonzero it is a measure of direct CP violation.

We should mention that according to [72], while *"a nonzero ϵ_K' is an unambiguous indication of direct CP violation [...] However, even if the theory has direct CP violation, ϵ_K' may still be zero. This occurs if the two CP-violating phases are equal." ([72], p.1115)* Experiments did measure a nonzero ϵ_K' and thereby established direct CP violation in the SM.

Why exactly the sum of the ηs is proportional *only* to indirect CP violation is not that easy to answer. We hope to clarify that by the end of Section 3.2.4. Let's consent ourselves for the moment with the limit in which the ηs are equal and hence identical to ϵ_K. This may make sense because this is the limit in which the final states contribute not at all (or equally for that matter). We see from experiment, that $|\eta_{+-}| \approx |\eta_{00}|$ is sufficiently fulfilled, therefore we can say $|\epsilon_K| \approx |\eta_{+-}| \approx |\eta_{00}|$. In this limit, we neglected direct CP violation $|\epsilon_K'| \approx 0$. But $|\epsilon_K'| \ll |\epsilon_K|$ anyway and in this limit, we get a very clear picture and a more accessible and even not that inaccurate measure of $|\epsilon_K|$.

"How much CP violation do we have if $|\eta_{00}| = 2 \times 10^{-3}$?"

Phrases like 'CP violation is small. It's 10^{-3}' are very common. Yet, that statement in itself is without meaning. We need a standard against which we can define something as 'small' or 'large'. Would $5 \cdot 10^{-3}$ be a lot of CP violation? Would 10^4 be? Or would these numbers also be considered 'small'? So what is the standard against which 'small' and 'large' are defined? And to give this standard meaning, we need most of all a context that allows us to grasp these concepts.

We give this context by starting with branching ratios. We laid out in section 3.2.1 in detail why observing a K_L decay into two Pions means that CP is violated in the process. Branching ratios give us a very accessible idea on what we can define as 'small' and 'large'. They tell us how many percent of all decays of a given initial state decay to this specific final state. For the K_L, there are two possible final states for the pair of Pions, namely $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$. We can thus define the question how much CP violation we talk about through the sum of the CP violating decays

$$\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \neq 0 \tag{3.27}$$

It is actually possible to relate the sum of the branching ratios to the ηs if we take some minor approximations of the order of neglecting the difference of the ηs. Hence, we have $|\eta_{00}| \approx |\eta_{+-}| \approx |\epsilon_K|$ as argued above. We end up with

$$\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \approx \tau_L/\tau_S \cdot |\epsilon_K|^2 \tag{3.28}$$

we give an explicit derivation of this formula in Appendix C.1. This formula is actually also used in the original paper by Christenson et al [67], although they seem to have accidentally multiplied the lifetimes instead of taking the ratio. Also, compare with equation (8.26) of [73], where the ratio of total widths has not been replaced by the inverse ratio of the lifetimes. Otherwise, its this equation.

The nonlinear relation and the lifetime factor suggests that the branching ratios and the ηs are only by accident numerically similar. However, there are often deeper connections in such accidents and it may be that such a connection here merely escaped us. In any case, we find that if the sum of the branching ratios was 10% or even 50% - so by all means large - the ηs would be $\eta \approx 1.32 \times 10^{-2}$ and $\eta \approx 2.96 \times 10^{-2}$ where the latter had to be interpreted as a large number!

3.2.3. Mixing Formalism - ϵ_K

Working in the well known two state formalism for mixing and propagation for neutral mesons, we start with the Schroedinger equation [74]

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}|K^0\rangle\\|\bar{K}^0\rangle\end{pmatrix} = H\begin{pmatrix}|K^0\rangle\\|\bar{K}^0\rangle\end{pmatrix} \tag{3.29}$$

which governs the time evolution of the two state system. The two state Hamiltonian is given by

$$H = M - \frac{i}{2}\Gamma = \begin{pmatrix}M_{11} & M_{12}\\M_{12}^* & M_{11}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\\Gamma_{12}^* & \Gamma_{11}\end{pmatrix} \tag{3.30}$$

with hermitian matrices M and Γ and $M_{22} = M_{11}$ and $\Gamma_{22} = \Gamma_{11}$ from assuming CPT invariance.

The Kaon mass states can be expressed via the weak interaction states in the following way

$$\begin{aligned}|K_S\rangle &= p|K^0\rangle + q|\bar{K}^0\rangle\\|K_L\rangle &= p|K^0\rangle - q|\bar{K}^0\rangle\end{aligned} \tag{3.31}$$

where the connection to the formulation via $\tilde{\epsilon}$ is given by $\tilde{\epsilon} = \frac{1-q/p}{1+q/p}$ or $\frac{q}{p} = \frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}}$.

The Bell-Steinberger relation [75] links the loss of probability caused by the anti-hermitian part Γ to the Kaon decay amplitudes. Based on physical intuition, this does make a lot of sense: Whatever amount of probability we loose in the oscillating Kaon system must correspond to Kaons which have decayed. This relation ties the mass and decay matrix of the quantum mechanical mixing formalism to the field theoretic amplitudes for mixing and decay. Its success gives credit to the Bell-Steinberger relation, although it is not derived on strict mathematical grounds. The physical idea behind it is solid enough that it should hold to a good approximation. We should, however, be cautious to conclude that it holds to arbitrary precision. This has been discussed in the literature, see e.g. [76] for a recent discussion and references therein. Especially CPT tests based on formalisms involving the Bell-Steinberger relation are criticized as to possibly be problematic.

We find [77]

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \tag{3.32}$$

Note that this is not an absolute square. We can then express the element of the two state Hamiltonian through the mixing matrix element computable by quantum field theory [78]

$$H_{12} = M_{12} - \frac{i}{2}\Gamma_{12} = \langle K^0|\mathcal{H}_{eff}|\bar{K}^0\rangle \tag{3.33}$$

Putting all this together, we can express the deviation from orthogonality of the mass eigenstates K_L and K_S from equation (3.21) in the following way [74]

$$2\ \mathrm{Re}\ \epsilon_K \approx \langle K_L|K_S\rangle = \frac{1-|q/p|^2}{1+|q/p|^2} \tag{3.34}$$

where the approximation made is that $1+|\tilde{\epsilon}|^2 \approx 1$ holds, as already mentioned in section 3.2.1.

3.2.4. Decay Formalism - ϵ'_K

The formalism for ϵ'_K has been developed in more than 50 years of science. Having been improved at many stages depending on the necessities of that time, it carries some nowadays impractical and outdated formalism. We try to give a comprehensive review here, although trying to keep a slightly more modern perspective than most standard literature. The following is largely based on the reviews [78] and [69].

The focal point for our construction of an observable is the decay from a Kaon CP eigenstate with eigenvalue -1 to a $(\pi\pi)$ final state. We take this as the **definition** of direct CP violation. [79]

$$\boxed{\tilde{\epsilon}'_f := \frac{\langle f|\mathcal{H}_{eff}|K_{CP-}\rangle}{\langle f|\mathcal{H}_{eff}|K_{CP+}\rangle}} \tag{3.35}$$

where $f = \{(\pi^+\pi^-), (\pi^0\pi^0)\}$. We normalized the expression to the amplitude of the CP $+1$ eigenstate for convenience. How can we relate this to experimental observables?

First, we express the CP eigenstates through flavor eigenstates (cf. Equation (3.19)) and choose the CP phase to be trival

$$\tilde{\epsilon}'_f = \frac{\langle f|\mathcal{H}_{eff}|K^0\rangle - \langle f|\mathcal{H}_{eff}|\bar{K}^0\rangle}{\langle f|\mathcal{H}_{eff}|K^0\rangle + \langle f|\mathcal{H}_{eff}|\bar{K}^0\rangle} =: \frac{1-g_f/h_f}{1+g_f/h_f} \tag{3.36}$$

We defined $g_f := \langle f|\mathcal{H}_{eff}|K^0\rangle$ and $h_f := \langle f|\mathcal{H}_{eff}|\bar{K}^0\rangle$. In the standard literature, these quantities are often denoted A_f and $\bar{A}_f$. Note that $\tilde{\epsilon}'_f$ depends on the final state, whereas for mixing, there was trivially just one such quantity $\tilde{\epsilon}$. In this CP phase convention, the form (3.36) again shows clearly why this quantity is related to CP violation: it is zero if there is no difference between K^0 and its antiparticle $\bar{K}^0$ in the decay to a certain final state.

We now define the amplitude ratio η_f for the physical Kaon states [78]

$$\eta_f := \frac{\langle f|\mathcal{H}_{eff}|K_L\rangle}{\langle f|\mathcal{H}_{eff}|K_S\rangle} = \frac{1 - qg_f/ph_f}{1 + qg_f/ph_f} \tag{3.37}$$

which can be determined experimentally. This is the definition for the aforementioned η_{+-} and η_{00}, now explicitly using the definitions for the mixing and decay coefficients q, p, h_f and g_f, given in Equations (3.31) and (3.36). The task at hand is to entangle g_f and h_f from q and p and at the same time construct phase convention independent quantities from then. Through a gruesome calculation we get (see Appendix C.2 for details. The authors of [78] sarcastically comment on this *"it is not difficult to show, that η_f can be rewritten as: ([78], p.2)"*)

$$\eta_f = \frac{a_{\tilde{\epsilon}} + a_{\tilde{\epsilon}'_f} + i a_{\tilde{\epsilon}+\tilde{\epsilon}'_f}}{2 + a_{\tilde{\epsilon}} a_{\tilde{\epsilon}'_f} + a_{\tilde{\epsilon}\tilde{\epsilon}'_f}} \tag{3.38}$$

where the as are constructed to be phase convention independent. They are given by [78]

$$\begin{aligned}
a_{\tilde{\epsilon}} &= \frac{1 - |q/p|^2}{1 + |q/p|^2} = \frac{2 \text{ Re } \tilde{\epsilon}}{1 + |\tilde{\epsilon}|^2} \\
a_{\tilde{\epsilon}'_f} &= \frac{1 - |g_f/h_f|^2}{1 + |g_f/h_f|^2} = \frac{2 \text{ Re } \tilde{\epsilon}'_f}{1 + |\tilde{\epsilon}'_f|^2} \\
a_{\tilde{\epsilon}+\tilde{\epsilon}'_f} &= \frac{-4 \text{ Im}\left(qg_f/ph_f\right)}{\left(1 + |g_f/h_f|^2\right)\left(1 + |q/p|^2\right)} = \frac{2 \text{ Im}\tilde{\epsilon}\left(1 - |\tilde{\epsilon}'_f|^2\right) - 2 \text{ Im}\tilde{\epsilon}'_f\left(1 - |\tilde{\epsilon}|^2\right)}{\left(1 + |\tilde{\epsilon}'_f|^2\right)\left(1 + |\tilde{\epsilon}|^2\right)} \\
a_{\tilde{\epsilon}\tilde{\epsilon}'_f} &= \frac{4 \text{ Re}\left(qg_f/ph_f\right)}{\left(1 + |g_f/h_f|^2\right)\left(1 + |q/p|^2\right)} - 1 = \frac{4 \text{ Im } \tilde{\epsilon} \text{ Im } \tilde{\epsilon}'_f - 2\left(|\tilde{\epsilon}'_f|^2 + |\tilde{\epsilon}|^2\right)}{\left(1 + |\tilde{\epsilon}'_f|^2\right)\left(1 + |\tilde{\epsilon}|^2\right)}
\end{aligned} \tag{3.39}$$

We know experimentally that CP violation is small and we work in a physical phase convention which reflects this ($|\tilde{\epsilon}'_f|, |\tilde{\epsilon}| \ll 1$). Therefore we can safely neglect terms quadratic in $|\tilde{\epsilon}'_f|$ and $|\tilde{\epsilon}|$. We then get

$$\begin{aligned}
a_{\tilde{\epsilon}} &\approx 2 \text{ Re } \tilde{\epsilon} =: 2 \text{ Re } \epsilon_K \\
a_{\tilde{\epsilon}'_f} &\approx 2 \text{ Re } \tilde{\epsilon}'_f =: 2 \text{ Re } \epsilon'_f \\
a_{\tilde{\epsilon}+\tilde{\epsilon}'_f} &\approx 2 \text{ Im } \tilde{\epsilon} + 2 \text{ Im } \tilde{\epsilon}'_f \equiv 2 \text{ Im } \epsilon_K + 2 \text{ Im } \epsilon'_f \\
a_{\tilde{\epsilon}\tilde{\epsilon}'_f} &\approx 0
\end{aligned} \tag{3.40}$$

As already shown in the last section, $a_{\tilde{\epsilon}}$ purely measures indirect CP violation. In the case that the final states are CP eigenstates (which is the case for $K \to \pi\pi$ decays we are interested in), $a_{\tilde{\epsilon}'_f}$ purely measures direct CP violation. $a_{\tilde{\epsilon}+\tilde{\epsilon}'_f}$ purely measures interference CP violation [78]. With the approximation that CP violation is small, we recover here that the imaginary parts of $\tilde{\epsilon}$ and $\tilde{\epsilon}'_f$ indicate interference type CP violation, while the real parts are measures of mixing and decay type CP violation, respectively. Note that while $a_{\tilde{\epsilon}+\tilde{\epsilon}'_f}$ is convention independent, Im $\tilde{\epsilon}$ and Im $\tilde{\epsilon}'_f$ themselves are not, as already mentioned for $\tilde{\epsilon}$ in Section 3.2.1. Hence, the convention dependence of Im $\tilde{\epsilon}$ and Im $\tilde{\epsilon}'_f$ must cancel in the sum. Re $\tilde{\epsilon}$ and Re ϵ'_f are both convention independent, thus we symbolically removed the tildes in equation (3.40) to indicate that we ended up with convention independent quantities.

We write η_f with this approximation [78]

$$\eta_f = \frac{a_{\tilde{\epsilon}} + a_{\tilde{\epsilon}'_f} + i a_{\tilde{\epsilon}+\tilde{\epsilon}'_f}}{2 + a_{\tilde{\epsilon}} a_{\tilde{\epsilon}'_f} + a_{\tilde{\epsilon}\tilde{\epsilon}'_f}} \approx \text{ Re } \epsilon_K + \text{ Re } \epsilon'_f + i \text{ Im } \epsilon_K + i \text{ Im } \epsilon'_f = \epsilon_K + \epsilon'_f \tag{3.41}$$

As we already mentioned, the convention dependence must cancel in the sum of the imaginary parts, hence we can identify it with the sum of the imaginary parts of the observables. Here we see very clearly that the experimentally accessible amplitude ratio measuring CP violation gets split into two parts: One part, ϵ_K, is independent of the final state and thus contains mixing type CP violation while the other part, ϵ'_f, depends on the final state and thus contains decay type CP violation.

So for the two Pion final states in neutral Kaon decays, we have

$$\eta_{+-} = \epsilon_K + \epsilon'_{+-} \qquad\qquad \eta_{00} = \epsilon_K + \epsilon'_{00} \tag{3.42}$$

In the next section we will relate ϵ'_{00} to ϵ'_{+-}. This leaves us with two unknowns in two equations (3.42) and allows us to express ϵ_K and ϵ'_K through the experimental quantities η_{+-} and η_{00}.

There can be only one ϵ'_K

When you get a chance to quote Christopher Lamberts most famous role (rivaling his appearance as thundergod in 'Mortal Kombat'), you should generally do it. The reason why there is only one ϵ'_K instead of two - which we ended up with in the last section - is, that they are related by CPT invariance [69]. In Appendix C.3 we calculate ϵ'_{+-} and ϵ'_{00} explicitly and obtain

$$\begin{aligned} \epsilon'_{+-} &= \tilde{\epsilon}'_K \frac{1}{1+\omega/\sqrt{2}} \\ \epsilon'_{00} &= -2\tilde{\epsilon}'_K \frac{1}{1-\sqrt{2}\omega} \end{aligned} \tag{3.43}$$

as found in [69] [80]. This gives us the relation

$$\epsilon'_{00} = -\frac{1}{2}\left(\frac{1-\sqrt{2}\omega}{1+\omega/\sqrt{2}}\right)\epsilon'_{+-} \tag{3.44}$$

ω is the parameter of the "*poorly understood ([69], p.2)*" $\Delta I = 1/2$ rule, for a definition see Section 3.2.7.

With Equation (3.42) we find

$$\begin{aligned}\eta_{+-} &= \epsilon_K + \tilde{\epsilon}'_K \frac{1}{1+\omega/\sqrt{2}} \\ \eta_{00} &= \epsilon_K - 2\tilde{\epsilon}'_K \frac{1}{1-\sqrt{2}\omega}\end{aligned} \tag{3.45}$$

For completions sake, we give the widely used standard approximations, which correspond to discarding higher orders in ω (Note that ϵ'_K includes one power of ω). We then have

$$\epsilon'_K \approx \epsilon'_{+-} \approx -1/2\ \epsilon'_{00} \tag{3.46}$$

and subsequently get [29] [68] the famous Wu Yang triangle relations [81]

$$\eta_{+-} = \epsilon_K + \epsilon'_{+-} \approx \epsilon_K + \epsilon'_K \qquad \eta_{00} = \epsilon_K + \epsilon'_{00} \approx \epsilon_K - 2\ \epsilon'_K \tag{3.47}$$

and thereby

$$\epsilon_K \approx \frac{2\eta_{+-} + \eta_{00}}{3} \qquad \epsilon'_K \approx \frac{\eta_{+-} - \eta_{00}}{3} \tag{3.48}$$

which are commonly used in relating the ηs to ϵ_K and ϵ'_K.

3.2.5. The Ratio ϵ'_K/ϵ_K

It is very convenient to normalize ϵ'_K to ϵ_K. On the one hand, the phases of the two quantities coincide [68] and $\left|\epsilon'_K/\epsilon_K\right| \approx \mathrm{Re}(\epsilon'_K/\epsilon_K)$. On the other hand, the ratio is historically particularly accessible to experiment [82] [83]. We work with the Wu Yang triangle relations, see Equation (3.47), which means neglecting higher powers of ω.

Through the relations (3.47) between η_{00}, η_{+-} and ϵ'_K, ϵ_K, we can write the ratio of the ηs as

$$\frac{\eta_{00}}{\eta_{+-}} = \frac{\epsilon_K - 2\,\epsilon'_K}{\epsilon_K + \epsilon'_K} = \frac{1 - 2\,\epsilon'_K/\epsilon_K}{1 + \epsilon'_K/\epsilon_K} \tag{3.49}$$

Then taking the absolute square

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \left|\frac{1 - 2\,\epsilon'_K/\epsilon_K}{1 + \epsilon'_K/\epsilon_K}\right|^2 = \frac{1 - 2\left(\epsilon'_K/\epsilon_K + (\epsilon'_K/\epsilon_K)^*\right) + 4\,|\epsilon'_K/\epsilon_K|^2}{1 + \epsilon'_K/\epsilon_K + (\epsilon'_K/\epsilon_K)^* + |\epsilon'_K/\epsilon_K|^2} \tag{3.50}$$

We now use $\epsilon'_K/\epsilon_K + (\epsilon'_K/\epsilon_K)^* = 2\,\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right)$ and because of $|\epsilon'_K/\epsilon_K| \ll 1$ (which implies $\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right) \ll 1$) we can restrict ourselves to linear terms and get

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \approx \frac{1 - 4\,\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right)}{1 + 2\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right)} \approx \left(1 - 4\,\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right)\right)\left(1 - 2\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right)\right) \tag{3.51}$$

where we used the geometric series $\frac{1}{1-q} = 1 + q + \mathcal{O}(q^2)$ with $q < 1$. We thus find

$$\boxed{\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \approx 1 - 6\,\mathrm{Re}\left(\epsilon'_K/\epsilon_K\right)} \tag{3.52}$$

up to corrections of order $\mathcal{O}(|\epsilon'_K/\epsilon_K|^2)$. This is a form often quoted in the literature [29] [83] [82]

We want to go one step further here, because connecting abstract quantities to quantities which most people have an intuition for is what nurtures understanding. In the ratio of decay rates to the same final state, the phase space dependence cancels and thus we have

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\left|\frac{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_L\rangle}{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_S\rangle}\right|^2}{\left|\frac{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_L\rangle}{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_S\rangle}\right|^2} = \frac{\left(\frac{\Gamma\left(K_L\to\pi^0\pi^0\right)}{\Gamma\left(K_S\to\pi^0\pi^0\right)}\right)}{\left(\frac{\Gamma\left(K_L\to\pi^+\pi^-\right)}{\Gamma\left(K_S\to\pi^+\pi^-\right)}\right)} = \frac{\left(\frac{\mathrm{BR}\left(K_L\to\pi^0\pi^0\right)}{\mathrm{BR}\left(K_S\to\pi^0\pi^0\right)}\right)}{\left(\frac{\mathrm{BR}\left(K_L\to\pi^+\pi^-\right)}{\mathrm{BR}\left(K_S\to\pi^+\pi^-\right)}\right)} \tag{3.53}$$

where we used that the lifetime factor drops out in this expression and we can therefore express it directly via branching ratios.

We thus find

$$\frac{\left(\frac{\mathrm{BR}\left(K_L\to\pi^0\pi^0\right)}{\mathrm{BR}\left(K_S\to\pi^0\pi^0\right)}\right)}{\left(\frac{\mathrm{BR}\left(K_L\to\pi^+\pi^-\right)}{\mathrm{BR}\left(K_S\to\pi^+\pi^-\right)}\right)} \approx 1 - 6\ \mathrm{Re}\left(\epsilon_K'/\epsilon_K\right) \tag{3.54}$$

up to corrections of order $\mathcal{O}(|\epsilon_K'/\epsilon_K|^2)$ and $\mathcal{O}(|\omega|^2)$ due to the ansatz.

Now recall that the phases for ϵ_K' and ϵ_K coincide. Therefore $\mathrm{Im}\left(\epsilon_K'/\epsilon_K\right) \approx 0$ and $\mathrm{Re}\left(\epsilon_K'/\epsilon_K\right) \approx |\epsilon_K'/\epsilon_K| \approx \epsilon_K'/\epsilon_K$. This gives us

$$\frac{\epsilon_K'}{\epsilon_K} \approx \frac{1}{6}\left[1 - \frac{\left(\frac{\mathrm{BR}\left(K_L\to\pi^0\pi^0\right)}{\mathrm{BR}\left(K_S\to\pi^0\pi^0\right)}\right)}{\left(\frac{\mathrm{BR}\left(K_L\to\pi^+\pi^-\right)}{\mathrm{BR}\left(K_S\to\pi^+\pi^-\right)}\right)}\right] \tag{3.55}$$

This way we connect ϵ_K'/ϵ_K directly to Kaon to Pion branching ratios.

3.2.6. Isospin Amplitudes

To investigate Kaon CP violation, it is very useful to switch to the Pion isospin basis. We have yet to find a clear statement about the interpretation of these isospin states. Most authors are very conservative about this and just state that the decomposition is possible and useful. Mathematically, it is just a change of basis, but while we have clear interpretations of the Kaon bases in terms of propagating mass states, interacting flavor states or CP states that we need to define CP violation, we have found no comment about the physical interpretation of the isospin states.

The two Pion final states we find in the detector are $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$. Using standard methods for the addition of two spin-1 states, we can obtain the strong isospin states of the two Pion systems

$$\begin{aligned} |\pi^0\pi^0\rangle &= \sqrt{\frac{1}{3}}|(\pi\pi)_{I=0}\rangle - \sqrt{\frac{2}{3}}|(\pi\pi)_{I=2}\rangle \\ |\pi^+\pi^-\rangle &= \sqrt{\frac{2}{3}}|(\pi\pi)_{I=0}\rangle + \sqrt{\frac{1}{3}}|(\pi\pi)_{I=2}\rangle \end{aligned} \tag{3.56}$$

Solving for the isospin eigenstates $|(\pi\pi)_{I=0}\rangle$ and $|(\pi\pi)_{I=2}\rangle$, we get

$$\begin{aligned} |(\pi\pi)_{I=0}\rangle &= \sqrt{\frac{2}{3}}|\pi^+\pi^-\rangle + \sqrt{\frac{1}{3}}|\pi^0\pi^0\rangle \\ |(\pi\pi)_{I=2}\rangle &= \sqrt{\frac{1}{3}}|\pi^+\pi^-\rangle - \sqrt{\frac{2}{3}}|\pi^0\pi^0\rangle \end{aligned} \tag{3.57}$$

There is no $I = 1$ state in the two Pion system because the Pion wave function is bosonic and does not allow an odd state.

We use the following notation for the amplitudes $A_I := \langle(\pi\pi)_I|\mathcal{H}_{eff}|K^0\rangle$ and the CP conjugates $\bar{A}_I := \langle(\pi\pi)_I|\mathcal{H}_{eff}|\bar{K}^0\rangle$. Through Watson's theorem [84], the strong phases δ_I which correspond to strong elastic rescattering of the Pions can be extracted explicitly. Assuming CPT conservation, the amplitudes can be written with explicit phases [68]

$$\begin{aligned} A_I &= a_I e^{i\delta_I} = |a_I|e^{i\phi_I}e^{i\delta_I} \\ \bar{A}_I &= a_I^* e^{i\delta_I} = |a_I|e^{-i\phi_I}e^{i\delta_I} \end{aligned} \tag{3.58}$$

The 'strong phases' δ_I are defined as not to flip sign when transformed under CP, while the 'weak phases' ϕ_I by definition do change sign under CP.

3.2.7. $\Delta I = 1/2$ rule: Omega

The amplitude ratio ω is constructed from the $K_S \to (\pi\pi)_I$ amplitudes. These decays are CP allowed and with CP violation being as small as it is, it can usually be neglected in this quantity, taking K_L and K_S as CP eigenstates by $q/p = 1$.

$$\begin{aligned}\omega :=& \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle} \\ =& \frac{A_2 + \frac{q}{p}\bar{A}_2}{A_0 + \frac{q}{p}\bar{A}_0} \\ \approx& e^{i(\delta_2-\delta_0)}\frac{a_2 + a_2^*}{a_0 + a_0^*} \\ =& e^{i(\delta_2-\delta_0)}\frac{\text{Re } a_2}{\text{Re } a_0}\end{aligned} \tag{3.59}$$

This form is found e.g. in [85]. Therefore we can take $|\omega| \approx \text{Re } a_2/\text{Re } a_0$ and $\text{Arg } \omega \approx \delta_2 - \delta_0$.

The $K_S \to \{\pi^0\pi^0, \pi^+\pi^-\}$ branching ratios

We can use this notation to show explicitly that the ratio of observed branching ratios in K_S decays to two Pions is an expression of the $\Delta I = 1/2$ rule. We start by taking the ratio of branching ratios

$$\begin{aligned}\frac{\text{BR}\left(K_S \to \pi^0\pi^0\right)}{\text{BR}\left(K_S \to \pi^+\pi^-\right)} &= \frac{\Gamma\left(K_S \to \pi^+\pi^-\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} \\ &= \left|\frac{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_S\rangle}{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_S\rangle}\right|^2\end{aligned} \tag{3.60}$$

where we assumed that the phase space for both two Pion final states is *sufficiently identical*. The normalization of decay rates in the branching ratio drops out in the ratio because we have the same initial states. Next, we decompose the Pion final states into Pion isospin states (see equation (3.57)):

$$
\begin{aligned}
\left|\frac{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_S\rangle}{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_S\rangle}\right|^2 &= \left|\frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle + \sqrt{2}\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\sqrt{2}\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle - \langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}\right|^2 \\
&= \frac{1}{2}\left|\frac{1+\sqrt{2}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}}{1-\frac{1}{\sqrt{2}}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}}\right|^2 \\
&= \frac{1}{2}\left|\frac{1+\sqrt{2}\omega}{1-\frac{1}{\sqrt{2}}\omega}\right|^2
\end{aligned}
\tag{3.61}
$$

where we put in our definition for ω as amplitude ratio of the K_S to the two Pion isospin final states.

At this point, we can insert the experimental numbers for ω and basically replace the question of why the branching ratios to the interaction final states of the two Pion system have the observed value by the question why the ratio of the isospin amplitudes have a certain value. This looks like no gain, and indeed the value of ω is poorly understood and coined $\Delta I = 1/2$ rule (see e.g. [86] for a discussion of technical aspects), where the word 'rule' is the usual quantum mechanics jargon for not knowing the mechanics at work. Nevertheless, although we do not know why the value of ω is what it is, the above form helps a lot to understand about the formalism used in Kaon CP violation. The factor in brackets often appears and makes derivations look complicated. Here, we saw that it consists of Clebsch Gordan coefficients originating in the transition from interaction final states and isospin final states of the two Pion system, as well as ω itself, which expresses the preference of the K_S state to decay into the $(\pi\pi)_0$ state or the $(\pi\pi)_2$ state. The experimentally determined smallness of $|\omega| \approx 1/22 \approx 0.045$ [80] means K_S largely prefers to decay into $(\pi\pi)_0$ and is often exploited by expanding in $|\omega|$.

Separating ω in phase and absolute value, we can use the approximations Arg $\omega \approx -\pi/4$ [80] and $|\omega| \ll 1$ and get

$$
\frac{1}{2}\left|\frac{1+\sqrt{2}\omega}{1-\frac{1}{\sqrt{2}}\omega}\right|^2 \approx \frac{1}{2}\left(1-3|\omega|\right) \approx \frac{3}{7}
\tag{3.62}
$$

We thus find for the branching ratios

$$
\frac{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)}{\mathrm{BR}\left(K_S \to \pi^+\pi^-\right)} \approx \frac{3}{7}
\tag{3.63}
$$

which reflects the experimental numbers [29]

$$\begin{aligned}
\mathrm{BR}\left(K_S \to \pi^0\pi^0\right) &\approx (0.3069 \pm 0.0005) \\
\mathrm{BR}\left(K_S \to \pi^+\pi^-\right) &\approx (0.6920 \pm 0.0005)
\end{aligned} \tag{3.64}$$

to a very good degree.

From the point of symmetry, we can thus view these numbers in the following way. In the limit $|\omega| \to 0$, the K_S state would only decay to the $(\pi\pi)_0$ final state. We can view this as an exact symmetry, which is broken by the small parameter ω. In the limit of vanishing ω we would have

$$\left.\frac{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)}{\mathrm{BR}\left(K_S \to \pi^+\pi^-\right)}\right|_{\omega=0} \approx \frac{1}{2} \tag{3.65}$$

With the input that K_S decays to pretty much nothing else than these two states, we get subsequently

$$\begin{aligned}
\left.\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)\right|_{\omega=0} &\approx 0.\bar{3} \\
\left.\mathrm{BR}\left(K_S \to \pi^+\pi^-\right)\right|_{\omega=0} &\approx 0.\bar{6}
\end{aligned} \tag{3.66}$$

which shows that the effect of ω is up to 10% in the branching ratios and far away from the quoted experimental errors. Nevertheless, the main effect on the split between the two interaction final states comes from Clebsch Gordan coefficients. In all the fuss about ω, this shouldn't be swept under the carpet. The split between the observed branching ratios for K_S to $\pi^+\pi^-$ and $\pi^0\pi^0$ is 90% due to Clebsch Gordan coefficients. The picture is then following: The values of the branching ratios from K_S to $\pi^+\pi^-$ and $\pi^0\pi^0$ are due to the $\Delta I = 1/2$ rule, which says: K_S mostly decays to the $(\pi\pi)_0$ isospin state, while the transition to the $(\pi\pi)_2$ state is suppressed by the small parameter $\omega = \langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle / \langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle \approx 0.045 \cdot \exp(-i\pi/4)$. [86] [87]

3.2.8. Isospin expressions for η_{+-}, η_{00}, ϵ_K and ϵ'_K

To find the final form for ϵ'_K/ϵ_K which we used in our analysis, we first express η_{00} and η_{+-} through isospin amplitudes (3.56) and use the definition (3.59) for ω

$$\begin{aligned}
\eta_{+-} &= \frac{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_L\rangle}{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_S\rangle} \\
&= \frac{\sqrt{2}\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle + \langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\sqrt{2}\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle + \langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle} \\
&= \frac{\sqrt{2}\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle + \langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle\left[\sqrt{2} + \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]} \\
&= \left[\frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle} + \frac{1}{\sqrt{2}}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]\left(\frac{1}{1+\frac{\omega}{\sqrt{2}}}\right)
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
\eta_{00} &= \frac{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_L\rangle}{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_S\rangle} \\
&= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle - \sqrt{2}\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle - \sqrt{2}\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle} \\
&= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle - \sqrt{2}\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle\left[1 - \sqrt{2}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]} \\
&= \left[\frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle} - \sqrt{2}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]\left(\frac{1}{1-\sqrt{2}\omega}\right)
\end{aligned} \tag{3.68}$$

ϵ_K - Recall from equation (3.48) that $\epsilon_K = (2\eta_{+-} + \eta_{00})/3$

$$\begin{aligned}
3\,\epsilon_K &= 2\eta_{+-} + \eta_{00} \\
&= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{2}{1+\omega/\sqrt{2}} + \frac{1}{1-\sqrt{2}\omega}\right] + \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{\sqrt{2}}{1+\omega/\sqrt{2}} - \frac{\sqrt{2}}{1-\sqrt{2}\omega}\right] \\
&= 3\frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{1-\omega/\sqrt{2}}{(1+\omega/\sqrt{2})(1-\sqrt{2}\omega)}\right] - 3\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{\omega}{(1+\omega/\sqrt{2})(1-\sqrt{2}\omega)}\right] \\
&= 3\frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[1+\omega^2+\mathcal{O}(\omega^3)\right] - 3\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\omega + \frac{\omega^2}{\sqrt{2}}\mathcal{O}(\omega^3)\right]
\end{aligned} \tag{3.69}$$

And with $\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle} = \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle} = \omega\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}$ we have

$$\epsilon_K = \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[1+\mathcal{O}(\omega^2)\right] \tag{3.70}$$

$\boldsymbol{\epsilon'_K}$ - Recall from equation (3.48) that $\epsilon'_K = (\eta_{+-} - \eta_{00})/3$

$$\begin{aligned}
3\,\epsilon'_K &= \eta_{+-} - \eta_{00} \\
&= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{1}{1+\omega/\sqrt{2}} - \frac{1}{1-\sqrt{2}\omega}\right] + \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{\sqrt{2}}{1+\omega/\sqrt{2}} + \frac{\sqrt{2}}{1-\sqrt{2}\omega}\right] \\
&= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{-3\omega/\sqrt{2}}{(1+\omega/\sqrt{2})(1-\sqrt{2}\omega)}\right] \\
&\qquad + \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{1}{\sqrt{2}}\frac{3}{(1+\omega/\sqrt{2})(1-\sqrt{2}\omega)}\right] \\
&= \left[\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle} - \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right] 3\frac{1}{\sqrt{2}}\omega\left[\frac{1}{(1+\omega/\sqrt{2})(1-\sqrt{2}\omega)}\right]
\end{aligned} \tag{3.71}$$

Expanding in ω we find

$$\epsilon'_K = \frac{1}{\sqrt{2}}\left[\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle} - \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]\left[\omega + \frac{\omega^2}{\sqrt{2}} + \mathcal{O}(\omega^3)\right] \tag{3.72}$$

To find an often used expression, we push the amplitudes around a little

$$\begin{aligned}
&\left(\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle} - \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right)\omega \\
&= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\left[\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle} - \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]
\end{aligned} \tag{3.73}$$

and insert ϵ_K

$$\epsilon'_K = \frac{1}{\sqrt{2}}\epsilon_K\left[\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle} - \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right]\left[1 + \frac{\omega}{\sqrt{2}} + \mathcal{O}(\omega^2)\right] \tag{3.74}$$

Ignoring subleading contributions in ω, we recover

$$\frac{\epsilon'_K}{\epsilon_K} \approx \frac{1}{\sqrt{2}}\left[\frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle} - \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right] \tag{3.75}$$

see e.g. [88].

3.2.9. Final Formula for ϵ'_K/ϵ_K

We will now cast this into a different form with explicit dependence on the isospin amplitudes

$$
\begin{aligned}
\sqrt{2}\frac{\epsilon'_K}{\epsilon_K} &= \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\cdot\left(\frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle}\right)^{-1} - \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle} \\
&= \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_L\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_L\rangle} - \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_S\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_S\rangle} \\
&= \frac{A_2-(q/p)\bar{A}_2}{A_0-(q/p)\bar{A}_0} - \omega \\
&= e^{i(\delta_2-\delta_0)}\left(\cdot\frac{a_2-(q/p)a_2^*}{a_0-(q/p)a_0^*} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{\text{Re } a_2 + i\text{ Im } a_2 - (q/p)\text{ Re } a_2 + i(q/p)\text{ Im } a_2}{\text{Re } a_0 + i\text{ Im } a_0 - (q/p)\text{ Re } a_0 + i(q/p)\text{ Im } a_0} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{(1-q/p)\text{ Re } a_2 + i(1+q/p)\text{ Im } a_2}{(1-q/p)\text{ Re } a_0 + i(1+q/p)\text{ Im } a_0} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{\tilde{\epsilon}\text{ Re } a_2 + i\text{ Im } a_2}{\tilde{\epsilon}\text{ Re } a_0 + i\text{ Im } a_0} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{\text{Re } a_2}{\text{Re } a_0}\cdot\frac{\tilde{\epsilon}+i\frac{\text{Im } a_2}{\text{Re } a_2}}{\tilde{\epsilon}+i\frac{\text{Im } a_0}{\text{Re } a_0}} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{\text{Re } a_2}{\text{Re } a_0}\cdot\frac{\tilde{\epsilon}+i\frac{\text{Im } a_0}{\text{Re } a_0}-i\frac{\text{Im } a_0}{\text{Re } a_0}+i\frac{\text{Im } a_2}{\text{Re } a_2}}{\tilde{\epsilon}+i\frac{\text{Im } a_0}{\text{Re } a_0}} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{\text{Re } a_2}{\text{Re } a_0}\cdot\frac{\epsilon_K-i\frac{\text{Im } a_0}{\text{Re } a_0}+i\frac{\text{Im } a_2}{\text{Re } a_2}}{\epsilon_K} - \frac{\text{Re } a_2}{\text{Re } a_0}\right) \\
&= e^{i(\delta_2-\delta_0)}\cdot\left(\frac{\text{Re } a_2}{\text{Re } a_0}\cdot\frac{-i\frac{\text{Im } a_0}{\text{Re } a_0}+i\frac{\text{Im } a_2}{\text{Re } a_2}}{\epsilon_K}\right) \\
&= -ie^{i(\delta_2-\delta_0-\phi_{\epsilon_K})}\frac{|\omega|}{|\epsilon_K|}\cdot\left(\frac{\text{Im } a_0}{\text{Re } a_0} - \frac{\text{Im } a_2}{\text{Re } a_2}\right)
\end{aligned}
\tag{3.76}
$$

We find

$$
\boxed{\frac{\epsilon'_K}{\epsilon_K} = -ie^{i(\delta_2-\delta_0-\phi_{\epsilon_K})}\frac{1}{\sqrt{2}}\frac{|\omega|}{|\epsilon_K|}\cdot\left(\frac{\text{Im } a_0}{\text{Re } a_0} - \frac{\text{Im } a_2}{\text{Re } a_2}\right)}
\tag{3.77}
$$

as found in e.g. [85], [29] and [79]. We used $\omega = e^{i(\delta_2-\delta_0)}\cdot(\text{Re } a_2/\text{Re } a_0)$ as defined in equation (3.59), $\phi_{\epsilon_K} := \text{Arg } \epsilon_K$, $\tilde{\epsilon} = \frac{1-q/p}{1+q/p}$ from section 3.2.3 and the relation $\epsilon_K = \tilde{\epsilon} + i\frac{\text{Im } a_0}{\text{Re } a_0}$ from section 3.2.1. Note that the prefactor experimentally evaluates as $-ie^{i(\delta_2-\delta_0-\phi_{\epsilon_K})} \approx 1$ [68]. This is a numerical accident, causing the ratio ϵ'_K/ϵ_K to be approximately real.

In practice, we take the phase factor to be unity, take $|\omega|$ and $|\epsilon_K|$ from experiment, as well as the real parts of the amplitudes Re a_0 and Re a_2.

3.2.10. Calculating Im a_0 and Im a_2

We are left to calculate the imaginary parts Im a_0 and Im a_2. The full determination of ϵ'_K/ϵ_K is quite involved. It requires calculating the hadronic matrix elements by non-perturbative methods like lattice QCD [35], as well as performing a matching calculation of the perturbative Wilson coefficients [40]. The latter capture the high energy effects and as such the appearance of new physics particles in virtual intermediate states. These parts need to be connected through a renormalization proceedure [43] which correctly converts the high energy expectation values to the low energy scale at which the decay or reaction in question actually takes place in experiment.

The formulae for the isospin amplitudes are given by [40]

$$a_I = \langle(\pi\pi)_0|\mathcal{H}_{eff}|K^0\rangle = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_{i=1}^{10} \Big[(z_i(\mu) - \tau y_i(\mu)) \langle(\pi\pi)_0|Q_i^{|1/2-I|}(\mu)|K^0\rangle \Big] \tag{3.78}$$

where $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$ and $y_i(\mu) = v_i(\mu) - z_i(\mu)$. Because we are ultimately interested in Im a_I, the relevant part is Im τ. In the standard parametrization of the CKM matrix [29], V_{ud} and V_{us} are both real and the complex CKM phase, from which all weak CP violation in the Standard Model originates, causes Im $V_{td}V_{ts}^* \equiv$ Im $\lambda_t \neq 0$ and thereby in general $\epsilon'_K/\epsilon_K \neq 0$.

The Wilson Coefficients z_i and y_i in Equation (3.78) are given by [40]

$$\begin{aligned} \vec{z}(\mu) &= \hat{U}^3(\mu, m_c)\vec{z}(m_c) \\ \vec{v}(\mu) &= \hat{U}^3(\mu, m_c)M_c\hat{U}^4(m_c, m_b)M_b\hat{U}^5(m_b, m_W)\vec{C}(m_W) \end{aligned} \tag{3.79}$$

Here, $\langle(\pi\pi)_0|Q_i^{|1/2-I|}(\mu)|K^0\rangle \equiv \{\langle Q^{1/2}\rangle, \langle Q^{3/2}\rangle\}$ are the nonperturbative hadronic matrix elements. $\hat{U}^f(\mu_1, \mu_2)$ is the evolution matrix between μ_2 and μ_1 for f active flavor. M_q is the threshold matrix for the quark threshold q. $\vec{C}(m_W)$ and $\vec{z}(m_c)$ are Wilson Coefficients that can be obtained from a matching calculation at the corresponding scale. The next-to-leading order values for all these objects as well as detailed technical discussions can be found in [40] and [42].

ϵ'/ϵ is an amplitude level quantity, hence it is linear in the operators. Therefore, New Physics (NP) contributions can just be added to the Standard Model value. We calculate new Wilson Coefficients w_i, which encode the high energy physics, at the NP scale μ_{NP} and evolve them to the Kaon scale μ by

$$\vec{w}(\mu) = \hat{U}^3(\mu, m_c)M_c \ldots \vec{w}(\mu_{NP}) \tag{3.80}$$

including all necessary thresholds and evolution matrices that lie between the NP scale μ_{NP} and the Kaon scale μ.

Appendix: An E_6 Symmetric Nelson-Barr Model

A

A.1. Symmetry Breaking Pattern

The E_6 Yukawa Lagrangian, as given in Section 1.2, is

$$\mathcal{L} = 27\;27\;\left(\mathcal{Y}_{27}\;27_H + \mathcal{Y}_{351}\;351_H\right) \tag{A.1}$$

The relevant representations decompose to $SO(10)$, from there to $SU(5)$ and finally to $SU(3) \times SU(2) \times U(1)$ according to [11]. $U(1)$ charges are denoted as subscripts.

$E6 \to SO(10) \times U(1)$

$$\begin{aligned} 27 &\to 16_1 + 10_{-2} + 1_4 \\ 351 &\to 144_1 + \overline{126}_{-2} + 54_4 + \overline{16}_{-5} + 10_{-2} + 1_{-8} \end{aligned} \tag{A.2}$$

$SO(10) \to SU(5) \times U(1)$

$$\begin{aligned} 10 &\to 5_2 + \overline{5}_{-2} \\ 16 &\to 10_{-1} + \overline{5}_3 + 1_{-5} \\ 54 &\to 24_0 + \overline{15}_{-4} + 15_4 \\ \overline{126} &\to 50_2 + \overline{45}_{-2} + 15_{-6} + \overline{10}_6 + 5_2 + 1_{10} \\ 144 &\to \overline{45}_3 + 40_{-1} + 24_{-5} + 15_{-1} + 10_{-1} + 5_7 + \overline{5}_3 \end{aligned} \tag{A.3}$$

$SU(5) \to SU(3) \times SU(2) \times U(1)$

$$\begin{aligned}
5 &\to (1,2)_3 + (3,1)_{-2} \\
10 &\to (3,2)_1 + (\overline{3},1)_{-4} + (1,1)_6 \\
15 &\to (6,1)_{-4} + (3,2)_1 + (1,3)_6 \\
24 &\to (8,1)_0 + (\overline{3},2)_5 + (3,2)_{-5} + (1,3)_0 + (1,1)_0 \\
40 &\to (\overline{6},2)_1 + (8,1)_6 + (\overline{3},3)_{-4} + (\overline{3},1)_{-4} + (3,2)_1 + (1,2)_{-9} \\
45 &\to (8,2)_3 + (\overline{6},1)_{-2} + (\overline{3},2)_{-7} + (\overline{3},1)_8 + (3,3)_{-2} + (3,1)_{-2} + (1,2)_3 \\
50 &\to (8,2)_3 + (6,1)_8 + (\overline{6},3)_{-2} + (\overline{3},2)_{-7} + (3,1)_{-2} + (1,1)_{-12}
\end{aligned} \tag{A.4}$$

We spot the SM Higgs field $(1,2)_3$ emerge from a 5 of $SU(5)$ (and the conjugate field $(1,2)_{-3}$ consequently from the conjugate $\overline{5}$). Please note, that this is Slanksy's convention [11] for the $U(1)$ charges, which is very practical for model building since it only involves integer numbers. The frequently used convention for SM purposes, which normalizes hypercharge as the average electric charge, is related to Slanksy's convention by a factor of 6.

A.1.1. SM singlet scalar fields

The SM singlet scalar fields introduced in Section 1.2.2 used in our model. In the $\mathcal{Y}_{27}$ we have the following terms with respective patterns

$$\begin{aligned}
27\,27\,27 &\to 10_{-2}10_{-2}1_4 \to \overline{5}_{-2}5_2 1_0 \to (1,\overline{2})_{-3}(1,2)_3(1,1)_0 + (\overline{3},1)_2(3,1)_{-2}(1,1)_0 \\
&= (L_L L_R^c + D_R^c D_L)\, \phi_{27;1;1} \\
27\,27\,27 &\to 16_1 10_{-2} 16_1 \to \overline{5}_3 5_2 1_{-5} \to (1,\overline{2})_{-3}(1,2)_3(1,1)_0 + (\overline{3},1)_2(3,1)_{-2}(1,1)_0 \\
&= (\ell_L L_R^c + d_R^c D_L)\, \phi_{27;16;1}
\end{aligned} \tag{A.5}$$

and in the $\mathcal{Y}_{351}$ we have

$$\begin{aligned}
27\,27\,351 &\to 16_1 10_{-2} 144_1 \to \overline{5}_3 5_2 24_{-5} \to (1,\overline{2})_{-3}(1,2)_3(1,1)_0 + (\overline{3},1)_2(3,1)_{-2}(1,1)_0 \\
&= (\ell_L L_R^c + d_R^c D_L)\, \phi_{351;144;24} \\
27\,27\,351 &\to 16_1 16_1 \overline{126}_{-2} \to 1_{-5} 1_{-5} 1_{10} \to (1,1)_0(1,1)_0(1,1)_0 \\
&= \nu_R^c \nu_R^c\, \phi_{351;\overline{126};1} \\
27\,27\,351 &\to 10_{-2} 10_{-2} 54_4 \to \overline{5}_{-2} 5_2 24_0 \to (1,\overline{2})_{-3}(1,2)_3(1,1)_0 + (\overline{3},1)_2(3,1)_{-2}(1,1)_0 \\
&= (L_L L_R^c + D_R^c D_L)\, \phi_{351;54;24} \\
27\,27\,351 &\to 16_1 1_4 \overline{16}_{-5} \to 1_{-5} 1_0 1_5 \to (1,1)_0(1,1)_0(1,1)_0 \\
&= \nu_R^c s \phi_{351;\overline{16};1} \\
27\,27\,351 &\to 1_4 1_4 1_{-8} \to 1_0 1_0 1_0 \to (1,1)_0(1,1)_0(1,1)_0 \\
&= s s \phi_{351;1;1}
\end{aligned} \tag{A.6}$$

A.1.2. SM breaking scalars (Higgs fields)

The up-type Higgs fields $(1,2)_{1/2}$ arise as follows

$$
\begin{aligned}
27\,27\,27 &\to 16_1 16_1 10_{-2} \to 10_{-1}10_{-1}5_2 + \overline{5}_3 1_{-5} 5_2 \to (3,2)_1(\overline{3},1)_{-4}(1,2)_3 + (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= \left(Q_L d_R^c H^u_{27;10;5} + \ell_L \nu_R^c H^u_{27;10;5}\right) \\
27\,27\,27 &\to 10_{-2}1_4 10_{-2} \to \overline{5}_{-2}1_0 5_2 \to (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= L_L s H^u_{27;10;5} \\
27\,27\,351 &\to 16_1 16_1 10_{-2} \to 10_{-1}10_{-1}5_2 + \overline{5}_3 1_{-5} 5_2 \to (3,2)_1(\overline{3},1)_{-4}(1,2)_3 + (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= \left(Q_L d_R^c H^u_{351;10;5} + \ell_L \nu_R^c H^u_{351;10;5}\right) \\
27\,27\,351 &\to 10_{-2}1_4 10_{-2} \to \overline{5}_{-2}1_0 5_2 \to (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= L_L s H^u_{351;10;5} \\
27\,27\,351 &\to 16_1 1_4 \overline{16}_{-5} \to \overline{5}_3 1_0 5_{-3} \to (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= \ell_L \nu_R^c H^u_{351;\overline{16};5} \\
27\,27\,351 &\to 16_1 16_1 \overline{126}_{-2} \to 10_{-1}10_{-1}5_2 + \overline{5}_3 1_{-5} 5_2 \to (3,2)_1(\overline{3},1)_{-4}(1,2)_3 + (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= \left(Q_L d_R^c H^u_{351;\overline{126};5} + \ell_L \nu_R^c H^u_{351;\overline{126};5}\right) \\
27\,27\,351 &\to 10_{-2}1_4 \overline{126}_{-2} \to \overline{5}_{-2}1_0 5_2 \to (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= L_L s H^u_{351;\overline{126};5} \\
27\,27\,351 &\to 16_1 10_{-2} 144_1 \to 1_{-5}\overline{5}_{-2}5_7 \to (1,\overline{2})_{-3}(1,1)_0(1,2)_3 \\
&= L_L \nu_R^c H^u_{351;144;5}
\end{aligned}
\tag{A.7}
$$

and the down-type Higgs fields $(1,2)_{-1/2}$ are given by

$$\begin{aligned}
27\,27\,27 &\to 16_1 16_1 10_{-2} \to 10_{-1}\overline{5}_3\overline{5}_{-2} \to (3,2)_1(\overline{3},1)_2(1,2)_{-3} + (1,1)_6(1,\overline{2})_{-3}(1,\overline{2})_{-3}\\
&= \left(Q_L d_R^c H^d_{27;10;\overline{5}} + e_R^c \ell_L H^d_{27;10;\overline{5}}\right)\\
27\,27\,27 &\to 10_{-2}1_4 10_{-2} \to 5_2 1_0 \overline{5}_{-2} \to (1,2)_3(1,1)_0(1,\overline{2})_{-3}\\
&= L_R^c s H^d_{27;10;\overline{5}}\\
27\,27\,27 &\to 16_1 10_{-2} 16_1 \to 10_{-1}\overline{5}_{-2}\overline{5}_3 \to (1,1)_6(1,\overline{2})_{-3}(1,\overline{2})_{-3} + (3,2)_1(\overline{3},1)_2(1,\overline{2})_{-3}\\
&= \left(e_R^c L_L H^d_{27;16;\overline{5}} + Q_L D_R^c H^d_{27;16;\overline{5}}\right)\\
27\,27\,27 &\to 16_1 10_{-2} 16_1 \to 1_{-5} 5_2 \overline{5}_3 \to (1,1)_0(1,2)_3(1,\overline{2})_{-3}\\
&= \nu_R^c L_R^c H^d_{27;16;\overline{5}}\\
27\,27\,351 &\to 16_1 16_1 10_{-2} \to 10_{-1}\overline{5}_3\overline{5}_{-2} \to (3,2)_1(\overline{3},1)_2(1,2)_{-3} + (1,1)_6(1,\overline{2})_{-3}(1,\overline{2})_{-3}\\
&= \left(Q_L d_R^c H^d_{351;10;\overline{5}} + e_R^c \ell_L H^d_{351;10;\overline{5}}\right)\\
27\,27\,351 &\to 10_{-2}1_4 10_{-2} \to 5_2 1_0 \overline{5}_{-2} \to (1,2)_3(1,1)_0(1,\overline{2})_{-3}\\
&= L_R^c s H^d_{351;10;\overline{5}}\\
27\,27\,351 &\to 16_1 16_1 \overline{126}_{-2} \to 10_{-1}\overline{5}_3\overline{45}_{-2} \to (3,2)_1(\overline{3},1)_2(1,2)_{-3} + (1,1)_6(1,\overline{2})_{-3}(1,\overline{2})_{-3}\\
&= \left(Q_L d_R^c H^d_{351;\overline{126};\overline{45}} + e_R^c \ell_L H^d_{351;\overline{126};\overline{45}}\right)\\
27\,27\,351 &\to 10_{-2}1_4 \overline{126}_{-2} \to 5_2 1_0 \overline{45}_{-2} \to (1,2)_3(1,1)_0(1,\overline{2})_{-3}\\
&= \left(L_R^c s H^d_{351;\overline{126};\overline{45}}\right)\\
27\,27\,351 &\to 16_1 10_{-2} 144_1 \to 1_{-5} 5_2 \overline{5}_3 \to (1,1)_0(1,2)_3(1,\overline{2})_{-3}\\
&= \nu_R^c L_R^c H^d_{351;144;\overline{5}}\\
27\,27\,351 &\to 16_1 10_{-2} 144_1 \to 10_{-1}\overline{5}_{-2}\overline{5}_3 \to (1,1)_6(1,\overline{2})_{-3}(1,\overline{2})_{-3} + (3,2)_1(\overline{3},1)_2(1,\overline{2})_{-3}\\
&= \left(e_R^c L_L H^d_{351;144;\overline{5}} + Q_L D_R^c H^d_{351;144;\overline{5}}\right)\\
27\,27\,351 &\to 16_1 10_{-2} 144_1 \to 1_{-5} 5_2 \overline{45}_3 \to (1,1)_0(1,2)_3(1,\overline{2})_{-3}\\
&= \nu_R^c L_R^c H^d_{351;144;\overline{45}}\\
27\,27\,351 &\to 16_1 10_{-2} 144_1 \to 10_{-1}\overline{5}_{-2}\overline{45}_3 \to (1,1)_6(1,\overline{2})_{-3}(1,\overline{2})_{-3} + (3,2)_1(\overline{3},1)_2(1,\overline{2})_{-3}\\
&= \left(e_R^c L_L H^d_{351;144;\overline{45}} + Q_L D_R^c H^d_{351;144;\overline{45}}\right)
\end{aligned} \tag{A.8}$$

A.2. BBP formula

We provide a generalization of formula (7a) from the paper by Bento, Branco and Parada [7] for an arbitrary number of heavy down type quarks.

The down quark mass matrix is

$$\mathcal{M}_d = \begin{pmatrix} m_d & 0 \\ M_C & M_R \end{pmatrix} \tag{A.9}$$

where we assume $M_C \in \mathbb{C}$ and $m_d \wedge M_R \in \mathbb{R}$ to have a real determinant. Furthermore, we put in a hierarchy $m_d \ll M_C \lesssim M_R$, which is reasonable identifying m_d with the SM down quarks, thus of the EW scale, while M_C and M_R contain GUT scale VEVs.

We follow the line of arguments of BBP and diagonalize the mass matrix

$$U_L^\dagger \mathcal{M}_d U_R = \begin{pmatrix} \overline{m} & 0 \\ 0 & \overline{M} \end{pmatrix} \tag{A.10}$$

where $\overline{m} = \mathrm{diag}(m_d, m_s, m_b)$ are the usual SM down quark masses and $\overline{M} = \mathrm{diag}(m_D, m_S, m_B)$ are the heavy quark masses denoted intuitively from lightest to heaviest. We write U_L in block form

$$\begin{pmatrix} K & R \\ S & T \end{pmatrix} \tag{A.11}$$

with K being the usual 3×3 CKM matrix. The unitarity of U_L has the following consequences

$$\begin{aligned} U_L U_L^\dagger &= \begin{pmatrix} KK^\dagger + RR^\dagger & KS^\dagger + RT^\dagger \\ SK^\dagger + TR^\dagger & SS^\dagger + TT^\dagger \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ U_L^\dagger U_L &= \begin{pmatrix} K^\dagger K + S^\dagger S & K^\dagger R + S^\dagger T \\ R^\dagger K + T^\dagger S & R^\dagger R + T^\dagger T \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned} \tag{A.12}$$

Where we will especially need

$$KK^\dagger + RR^\dagger = 1 \tag{A.13}$$

$$KS^\dagger + RT^\dagger = 0 \tag{A.14}$$

$$K^\dagger K + S^\dagger S = 1 \tag{A.15}$$

From the bi-unitary diagonalization we know that

$$U_L^\dagger \mathcal{M}_d \mathcal{M}_d^\dagger U_L = \mathrm{diag}(\bar{m}^2, \bar{M}^2) \tag{A.16}$$

which gives us the following four equations

$$\begin{aligned}
K^\dagger m_d m_d^\dagger K + S^\dagger M_C m_d^\dagger K + K^\dagger m_d M_C^\dagger S + S^\dagger (M_C M_C^\dagger + M_R M_R^\dagger) S &= \bar{m}^2 && \text{(A.17)}\\
K^\dagger m_d m_d^\dagger R + S^\dagger M_C m_d^\dagger R + K^\dagger m_d M_C^\dagger T + S^\dagger (M_C M_C^\dagger + M_R M_R^\dagger) T &= 0 && \text{(A.18)}\\
R^\dagger m_d m_d^\dagger K + T^\dagger M_C m_d^\dagger K + R^\dagger m_d M_C^\dagger S + T^\dagger (M_C M_C^\dagger + M_R M_R^\dagger) S &= 0 && \text{(A.19)}\\
R^\dagger m_d m_d^\dagger R + T^\dagger M_C m_d^\dagger R + R^\dagger m_d M_C^\dagger T + T^\dagger (M_C M_C^\dagger + M_R M_R^\dagger) T &= \bar{M}^2 && \text{(A.20)}
\end{aligned}$$

For convenience, we define $(M_C M_C^\dagger + M_R M_R^\dagger) := M^2$. The square just helps to remind that it counts as two powers of the high scales.

Now we reproduce Eqs. (5a)-(5d) from [7] by multiplying Equations (A.17) - (A.20) with components of U_L:

$$\begin{aligned}
K(\text{A.17}) + R(\text{A.19}) &\quad\longrightarrow\quad K\bar{m}^2 = m_d m_d^\dagger K + m_d M_C^\dagger S && \text{(A.21)}\\
R(\text{A.20}) + K(\text{A.18}) &\quad\longrightarrow\quad R\bar{M}^2 = m_d m_d^\dagger R + m_d M_C^\dagger T && \text{(A.22)}\\
S(\text{A.17}) + T(\text{A.19}) &\quad\longrightarrow\quad S\bar{m}^2 = M_C m_d^\dagger K + M^2 S && \text{(A.23)}\\
T(\text{A.20}) + S(\text{A.18}) &\quad\longrightarrow\quad T\bar{M}^2 = M_C m_d^\dagger R + M^2 T && \text{(A.24)}
\end{aligned}$$

Now we take

$$K^\dagger(\text{A.21}) \quad\longrightarrow\quad K^\dagger K \overline{m}^2 = K^\dagger m_d m_d^\dagger K + K^\dagger m_d M_C^\dagger S \tag{A.25}$$

and argue, that for sufficient high scales M_C and M_R, K is approximately unitary and we can take $K^\dagger K = 1 + \mathcal{O}(m^4/M^4)$, where m/M denotes generic low scale over high scale suppression. The argument goes as follows: We know that by definition U_L is unitary, thus followed Equation (A.15)

$$U_L^\dagger U_L = K^\dagger K + S^\dagger S = 1$$

We add some physical input, namely that if we let the high scales go towards infinity, the exotic particles become infinitely heavy and decouple from the SM. In that limit, there is no way of finding the exotic particles by e.g. flavor physics experiments because their appearance in loops is suppressed by their heavy masses. Therefore, we would measure a unitary CKM matrix in that limit. For the large hierarchy $m_d \ll M_C \lesssim M_R$ which we assumed, we can conclude, that $K^\dagger K$ is close to unity with corrections from $S^\dagger S$, which are suppressed by the high scales. Therefore, we can assume an approximate unitarity of the CKM matrix K with corrections suppressed by m^2/M^2. And indeed, numerical checks provide

$$K^\dagger K = 1 + \mathcal{O}(m_d^4 M_C^2/M_R^6) \tag{A.26}$$

We apply this to (A.25)

$$\overline{m}^2 = K^\dagger m_d m_d^\dagger K + K^\dagger m_d M_C^\dagger S + \mathcal{O}(m_d^4 M_C^2/M_R^6) \tag{A.27}$$

Next we recall the ansatz, which we used earlier to parametrize the mixing with the heavy quarks in Section 1.3, which gave us an effective down quark mass matrix $m_d^{\text{eff}} = m_d \cdot a_q$ with $a_q = \left[1 + Z_q^\dagger Z_q\right]$ and $Z_q = M_R^{-1} M_C$. This effective quark mass matrix is diagonalized by the CKM matrix, all this assuming CKM unitarity. Therefore

$$\overline{m}^2 = K^\dagger m_d (1 + Z^\dagger Z)^{-1} m_d^\dagger K + \mathcal{O}(m_d^4 M_C^2/M_R^6) \tag{A.28}$$

Taking these two equations we get

$$(1 + Z^\dagger Z)^{-1} = 1 + M_C^\dagger S K^\dagger (m_d^\dagger)^{-1} + \mathcal{O}(m_d^4 M_C^2/M_R^6) \tag{A.29}$$

To extract an expression for $SK^\dagger$, we first rewrite $(1 + Z^\dagger Z)^{-1}$. Starting with

$$\begin{aligned}
1 + Z^\dagger Z &= 1 + M_C^\dagger (M_R M_R^\dagger)^{-1} M_C \\
&= M_C^\dagger \left((M_C^\dagger)^{-1} + (M_R M_R^\dagger)^{-1} M_C \right) \\
&= M_C^\dagger (M_R M_R^\dagger)^{-1} \left((M_R M_R^\dagger)(M_C^\dagger)^{-1} + M_C \right) \\
&= M_C^\dagger (M_R M_R^\dagger)^{-1} \left((M_R M_R^\dagger) + M_C M_C^\dagger \right) (M_C^\dagger)^{-1} \\
&:= M_C^\dagger (M_R M_R^\dagger)^{-1} M^2 (M_C^\dagger)^{-1}
\end{aligned} \tag{A.30}$$

We obtain the desired inverse $(1 + Z^\dagger Z)^{-1} = M_C^\dagger M^{-2} (M_R M_R^\dagger)(M_C^\dagger)^{-1}$. We insert this into Equation (A.29) and for readability drop the higher order correction reminder for now

$$M_C^\dagger M^{-2} (M_R M_R^\dagger)(M_C^\dagger)^{-1} = 1 + M_C^\dagger S K^\dagger (m_d^\dagger)^{-1} \tag{A.31}$$

rearranging this expression a little, we get

$$SK^\dagger = -\left(1 - M^{-2}(M_R M_R^\dagger)\right) (M_C^\dagger)^{-1} m_d^\dagger \tag{A.32}$$

Now we can rearrange $\left(1 - M^{-2}(M_R M_R^\dagger)\right)$ a little, noting that $M^2 = (M_R M_R^\dagger) + (M_C M_C^\dagger)$, we just call $(M_R M_R^\dagger) =: a$ and $(M_C M_C^\dagger) := b$, then

$$\begin{aligned} &1 - (a+b)^{-1}a \\ &= (a+b)^{-1}(a+b) - (a+b)^{-1}a \\ &= (a+b)^{-1}b \end{aligned} \tag{A.33}$$

Therefore

$$\left(1 - M^{-2}(M_R M_R^\dagger)\right) = M^{-2} M_C M_C^\dagger \tag{A.34}$$

and putting this into Equation (A.32) we end up with

$$SK^\dagger = -M^{-2} M_C m_d^\dagger \tag{A.35}$$

Now we take $K(\text{A.27})K^\dagger$, use the approximate unitarity of the CKM matrix again and insert Equation (A.35).

$$\boxed{K\overline{m}^2 K^\dagger = m_d \left[1 - M_C^\dagger (M^2)^{-1} M_C + \mathcal{O}(m_d^4 M_C^2 / M_R^6)\right] m_d^\dagger} \tag{A.36}$$

which is the final result of the derivation, as quoted in Section 1.1.1.

Appendix: A Supersymmetric Solution to ϵ'_K/ϵ_K B

In this section, we give the main parts of the explicit calculation of the gluino chromomagnetic penguin diagram (see Figures B.1 and B.2). This calculation has been done it the literature, see e.g. [56] [57].

B.1. The Chromomagnetic Dipole Contribution

B.1.1. Formulae

Gordon Identity

Derivation according to [89]

With incoming momentum $p = i\partial$ and outgoing momentum $p' = -i\partial$, we have the following Dirac equations:

$$
\begin{aligned}
\not{p}\psi(p,m) &= m\psi(p,m) \\
\not{p}'\psi(p',m') &= -m'\psi(p',m') \\
\bar{\psi}(p,m)\not{p} &= -m\bar{\psi}(p,m) \\
\bar{\psi}(p',m')\not{p}' &= m'\bar{\psi}(p',m')
\end{aligned}
\tag{B.1}
$$

The (anti)commutator of gamma matrices is

$$
\begin{aligned}
\frac{1}{2}\{\gamma^\mu,\gamma^\nu\} &= g^{\mu\nu} \\
\frac{i}{2}[\gamma^\mu,\gamma^\nu] &= \sigma^{\mu\nu}
\end{aligned}
\tag{B.2}
$$

which gives

$$\begin{aligned}
\gamma^\mu\gamma^\nu &= \frac{1}{2}\{\gamma^\mu,\gamma^\nu\} + \frac{1}{2}[\gamma^\mu,\gamma^\nu] \\
&= g^{\mu\nu} - i\sigma^{\mu\nu} \\
g^{\mu\nu} &= \gamma^\mu\gamma^\nu + i\sigma^{\mu\nu} \\
g^{\mu\nu} &= \gamma^\nu\gamma^\mu - i\sigma^{\mu\nu}
\end{aligned} \tag{B.3}$$

where in the last line, the symmetry of the metric and the antisymmetry of the sigma tensor has been used. Using the last two lines of equation (B.3), we can choose to write any momenta in the following form

$$\begin{aligned}
p^\mu &= p_\nu g^{\mu\nu} = p_\nu(\gamma^\mu\gamma^\nu + i\sigma^{\mu\nu}) \\
p^{\mu\prime} &= p'_\nu g^{\mu\nu} = p'_\nu(\gamma^\nu\gamma^\mu - i\sigma^{\mu\nu})
\end{aligned} \tag{B.4}$$

adding these two lines, we have

$$(p^\mu + p^{\mu\prime}) = \gamma^\mu \not{p} + \not{p}'\gamma^\mu + i\sigma^{\mu\nu}(p_\nu - p'_\nu) \tag{B.5}$$

Equation (B.5) does not make any assumptions about the momenta, nor has the Dirac equation been used so far, this is just a general Identity.

To obtain the Gordon Identity, we sandwich the result between the two spinors $u(p, m)$ and $\gamma_5\bar{u}(p', m')$ and obtain

$$(p^\mu + p^{\mu\prime})\bar{u}(p', m')\gamma_5 u(p, m) = \bar{u}(p', m')\left[\gamma^\mu\not{p} + \not{p}'\gamma^\mu + i\sigma^{\mu\nu}(p_\nu - p'_\nu)\right]\gamma_5 u(p, m) \tag{B.6}$$

Now we use the fact, that we want p to be an incoming momentum and p' to be an outgoing one, namely use the Dirac equations (B.1). Remember the anticommuting relation $\{\gamma_5, \gamma^\mu\}$, we get (suppressing the arguments of the spinors)

$$(p^\mu + p^{\mu\prime})\bar{u}\gamma_5 u = -m\cdot\bar{u}\gamma^\mu\gamma_5 u + m'\cdot\bar{u}\gamma^\mu\gamma_5 u + (p_\nu - p'_\nu)i\bar{u}\sigma^{\mu\nu}\gamma_5 u \tag{B.7}$$

rearranging this expression, we obtain the Gordon Identity with γ_5

$$\bar{u}\left[(p^\mu + p^{\mu\prime}) - i\sigma^{\mu\nu}(p_\nu - p'_\nu)\right]\gamma_5 u = \bar{u}\gamma^\mu\gamma_5 u\cdot(m' - m) \tag{B.8}$$

where $\bar{u} = \bar{u}(p', m')$ and $u = u(p, m)$ with p incoming and p' outgoing.

Looking at the momentum flow through the diagrams, we have $\bar{s}(p)$ and $d(p - q)$ and thus the Gordon Identity becomes

$$(2p - q)^{\mu} \bar{s} \gamma_5 d = i \bar{s} \sigma^{\mu\nu} \gamma_5 d q_{\nu} + \bar{s} \gamma^{\mu} \gamma_5 d \cdot (m_s - m_d) \tag{B.9}$$

Colour

$$T^a_{\alpha\beta} f_{abc} T^b_{\beta\gamma} = i \frac{3}{2} T^c_{\alpha\gamma} \tag{B.10}$$

Field Strength Tensor

$$G_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha} \pm i g_3 \left[A_{\alpha}, A_{\beta} \right] \tag{B.11}$$

is through colour algebra, $A_{\mu} = T^a A^a_{\mu}$ and $G_{\mu\nu} = T^a G^a_{\mu\nu}$ equivalent to

$$G^a_{\alpha\beta} = \partial_{\alpha} A^a_{\beta} - \partial_{\beta} A^a_{\alpha} \mp g_3 f^{abc} A^b_{\alpha} A^c_{\beta} \tag{B.12}$$

NB: the non-abelian term comes from the $sg \to dg$ penguin diagram

Squark Vertex Mixing

$$\begin{aligned} Z_D^{Ii} &= \Gamma_{DL}^{iI} \\ Z_D^{(I+3)i} &= \Gamma_{DR}^{iI} \\ Z_U^{Ii} &= \Gamma_{UL}^{iI*} \\ Z_U^{(I+3)i} &= \Gamma_{UR}^{iI*} \end{aligned} \tag{B.13}$$

B.1.2. Diagram 1: Gluon Attached to the Gluino Line

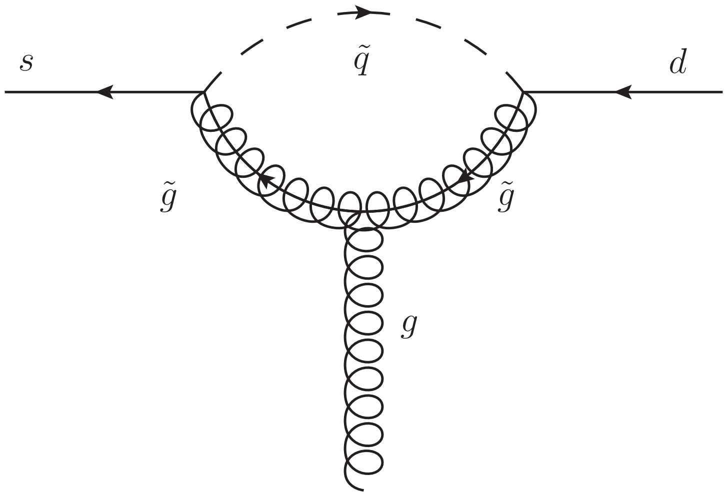

Figure B.1.: *The gluino gluon chromomagnetic penguin, gluon attached to the gluino line.*

Feynman Rules

Using Feynman rules [52] on the diagram and denoting the vertex mixing matrices by [54]

$$Z_D^{Ii} = \Gamma_{DL}^{iI} \qquad \text{and} \qquad Z_D^{(I+3)i} = \Gamma_{DR}^{iI} \tag{B.14}$$

we obtain

$$\begin{aligned} i\mathcal{M} =& \bar{s}_\alpha \left[ig_3\sqrt{2}T^a_{\alpha\beta}\left(-\Gamma_{DL}^{I2*}P_R + \Gamma_{DR}^{I2*}P_L\right)\right] \\ & \left(i\frac{(-1)\gamma^\lambda(p+k)_\lambda + m_{\tilde{g}}}{(p+k)^2 - m_{\tilde{g}}^2}\right)(-g_3 f_{abc}\gamma^\mu)\left(i\frac{(-1)\gamma^\epsilon(p-q+k)_\epsilon + m_{\tilde{g}}}{(p-q+k)^2 - m_{\tilde{g}}^2}\right) \\ & \left[ig_3\sqrt{2}T^b_{\beta\gamma}\left(-\Gamma_{DL}^{I1}P_L + \Gamma_{DR}^{I1}P_R\right)\right] d_\gamma \left(i\frac{1}{k^2 - m_{\tilde{d}_I}}\right)\epsilon_\mu^{*c}(q) \end{aligned} \tag{B.15}$$

rearranging this expression a bit while taking only the Dirac structure with two γ-matrices and discarding the rest, we obtain

$$i\mathcal{M} = (-1)^2 i^5 2g_3^3 \cdot T^a_{\alpha\beta} f_{abc} T^b_{\beta\gamma} \cdot \epsilon_\mu^{*c}(q)\left(D_V^{(1),\lambda\mu} I_{V,\lambda}^{(1)} + D_V^{(2),\mu\epsilon} I_{V,\epsilon}^{(2)}\right) \tag{B.16}$$

with Integrals $I_{V,\alpha}^{(i)}$ and fermion chains $D_V^{(i),\alpha}$

Dirac Structure

The Dirac structure is in both cases given by

$$D_V^{(i),\alpha\beta} = \bar{s}_\alpha \left(-\Gamma_{DL}^{I2*} P_R + \Gamma_{DR}^{I2*} P_L\right) \gamma^\alpha \gamma^\beta \left(-\Gamma_{DL}^{I1} P_L + \Gamma_{DR}^{I1} P_R\right) d_\gamma m_{\tilde{g}} \tag{B.17}$$

where $\gamma^\alpha\gamma^\beta = \gamma^\lambda\gamma^\mu$ for $D_V^{(1),\lambda}$ and $\gamma^\alpha\gamma^\beta = \gamma^\mu\gamma^\epsilon$ for $D_V^{(2),\epsilon}$

This can be simplified [90] to

$$D_V^{(i),\alpha\beta} = -\frac{1}{2} m_{\tilde{g}} \left[\Gamma^+ \bar{s}_\alpha \gamma^\alpha \gamma^\beta d_\gamma + \Gamma^- \bar{s}_\alpha \gamma^\alpha \gamma^\beta \gamma_5 d_\gamma\right] \tag{B.18}$$

with

$$\left(\Gamma_{DL}^{I2*}\Gamma_{DR}^{I1} \pm \Gamma_{DR}^{I2*}\Gamma_{DL}^{I1}\right) =: \Gamma^\pm \tag{B.19}$$

Integrals: Expansion in small momenta

We use the following Taylor expansion

$$\frac{1}{(p+k)^2 - m^2} = \frac{1}{k^2 - m^2} - 2\frac{k \cdot p}{(k^2 - m^2)^2} + \mathcal{O}(p^2) \tag{B.20}$$

the first denominator turns into

$$\begin{aligned}
\mathrm{I}^\lambda &= \frac{(p+k)^\lambda}{(p+k)^2 - m_{\tilde{g}}^2} \frac{1}{(p-q+k)^2 - m_{\tilde{g}}^2} \frac{1}{k^2 - m_{\tilde{d}}^2} \\
&= (p+k)^\lambda \left[\frac{1}{k^2 - m_{\tilde{g}}^2} - 2\frac{k \cdot p}{(k^2 - m_{\tilde{g}}^2)^2}\right] \left[\frac{1}{k^2 - m_{\tilde{g}}^2} - 2\frac{k \cdot (p-q)}{(k^2 - m_{\tilde{g}}^2)^2}\right] \frac{1}{k^2 - m_{\tilde{d}}^2} + \mathcal{O}(p^2) \\
&= \frac{(p+k)^\lambda}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{d}}^2)} - 2\frac{k^\lambda \left(k \cdot p + k \cdot (p-q)\right)}{(k^2 - m_{\tilde{g}}^2)^3 (k^2 - m_{\tilde{d}}^2)} \\
&= \frac{k^\lambda}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{d}}^2)} + p^\lambda \frac{1}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{d}}^2)} - (4p - 2q)_\rho \frac{k^\lambda k^\rho}{(k^2 - m_{\tilde{g}}^2)^3 (k^2 - m_{\tilde{d}}^2)}
\end{aligned} \tag{B.21}$$

Integrating over k gives

$$\begin{aligned}
\mathrm{I}^{\lambda} &= p^{\lambda} C_0 - 2(2p-q)_{\rho} D_0^{\lambda\rho} \\
&= p^{\lambda} C_0 - 2(2p-q)_{\rho} g^{\lambda\rho} D_{0,T} \\
&= p^{\lambda} C_0 - 2(2p-q)^{\lambda} D_{0,T}
\end{aligned} \tag{B.22}$$

where the arguments of the C and D Passarino-Veltman functions [91] are $C_0(m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{d}})$ and $D_{0,T}(m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{d}})$.
We proceed analogously for the second denominator

$$\begin{aligned}
\mathrm{I}^{\epsilon} &= \frac{(p-q+k)^{\epsilon}}{(p+k)^2 - m_{\tilde{g}}^2} \frac{1}{(p-q+k)^2 - m_{\tilde{g}}^2} \frac{1}{k^2 - m_{\tilde{d}}^2} \\
&= (p-q+k)^{\epsilon} \left[\frac{1}{k^2 - m_{\tilde{g}}^2} - 2\frac{k \cdot p}{(k^2 - m_{\tilde{g}}^2)^2} \right] \left[\frac{1}{k^2 - m_{\tilde{g}}^2} - 2\frac{k \cdot (p-q)}{(k^2 - m_{\tilde{g}}^2)^2} \right] \frac{1}{k^2 - m_{\tilde{d}}^2} + \mathcal{O}(p^2) \\
&= \frac{(p-q+k)^{\epsilon}}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{d}}^2)} - 2\frac{k^{\epsilon}\,(k \cdot p + k \cdot (p-q))}{(k^2 - m_{\tilde{g}}^2)^3 (k^2 - m_{\tilde{d}}^2)} \\
&= \frac{k^{\epsilon}}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{d}}^2)} + (p-q)^{\epsilon} \frac{1}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{d}}^2)} - (4p-2q)_{\rho} \frac{k^{\epsilon} k^{\rho}}{(k^2 - m_{\tilde{g}}^2)^3 (k^2 - m_{\tilde{d}}^2)}
\end{aligned} \tag{B.23}$$

Integrating over k gives

$$\begin{aligned}
\mathrm{I}^{\epsilon} &= (p-q)^{\epsilon} C_0 - 2(2p-q)_{\rho} D_0^{\epsilon\rho} \\
&= (p-q)^{\epsilon} C_0 - 2(2p-q)_{\rho} g^{\epsilon\rho} D_{0,T} \\
&= (p-q)^{\epsilon} C_0 - 2(2p-q)^{\epsilon} D_{0,T}
\end{aligned} \tag{B.24}$$

Combining Dirac Structure and Integral Solutions

We take only the axial part of the Dirac structure and rename the summation index $\epsilon \to \lambda$. The piece proportional to $D_{0,T}$ is

$$\begin{aligned}
D_V^{(1),\lambda\mu} I_{V,\lambda}^{(1)} + D_V^{(2),\mu\epsilon} I_{V,\epsilon}^{(2)} &= -2(2p-q)_{\lambda} D_{0,T} \left(-\frac{1}{2} m_{\tilde{g}} \Gamma^{-} \bar{s}_{\alpha} \gamma^{\lambda} \gamma^{\mu} \gamma_5 d_{\gamma} \right) \\
&\quad - 2(2p-q)_{\lambda} D_{0,T} \left(-\frac{1}{2} m_{\tilde{g}} \Gamma^{-} \bar{s}_{\alpha} \gamma^{\mu} \gamma^{\lambda} \gamma_5 d_{\gamma} \right) \\
&= m_{\tilde{g}} (2p-q)_{\lambda} D_{0,T} \Gamma^{-} \left(\bar{s}_{\alpha} \underbrace{\left(\gamma^{\lambda} \gamma^{\mu} + \gamma^{\mu} \gamma^{\lambda} \right)}_{2g^{\lambda\mu}} \gamma_5 d_{\gamma} \right) \\
&= 2 m_{\tilde{g}} (2p-q)^{\mu} D_{0,T} \Gamma^{-} \bar{s}_{\alpha} \gamma_5 d_{\gamma}
\end{aligned} \tag{B.25}$$

The Integral function is given by

$$
\begin{aligned}
D_{0,T}(m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{d}}) &= \frac{i}{16\pi^2}\left(-\frac{1}{8}\right)\frac{1}{m_{\tilde{d}}^2}\underbrace{\frac{3-4x+x^2+2\log x}{(x-1)^3}}_{2F_3[x]} \\
&= \frac{i}{16\pi^2}\left(-\frac{1}{4}\right)\frac{1}{m_{\tilde{d}}^2}F_3[x] \qquad \text{(B.26)}
\end{aligned}
$$

The piece proportional to C_0 is

$$
\begin{aligned}
D_V^{(1),\lambda\mu} I_{V,\lambda}^{(1)} + D_V^{(2),\mu\epsilon} I_{V,\epsilon}^{(2)} &= (p-q)^\lambda C_0 \left(-\frac{1}{2} m_{\tilde{g}} \Gamma^- \bar{s}_\alpha \gamma^\mu \gamma^\lambda \gamma_5 d_\gamma\right) \\
&\quad + p^\lambda C_0 \left(-\frac{1}{2} m_{\tilde{g}} \Gamma^- \bar{s}_\alpha \gamma^\lambda \gamma^\mu \gamma_5 d_\gamma\right) \\
&= -\frac{1}{2} m_{\tilde{g}} \Gamma^- C_0 \left[(p-q)^\lambda \bar{s}_\alpha \gamma^\mu \gamma^\lambda \gamma_5 d_\gamma + p^\lambda \bar{s}_\alpha \gamma^\lambda \gamma^\mu \gamma_5 d_\gamma\right] \qquad \text{(B.27)}
\end{aligned}
$$

We use the relation $\gamma^\mu \gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}$ and exploit the antisymmetry of $\sigma^{\mu\nu}$

$$
\begin{aligned}
&= -\frac{1}{2} m_{\tilde{g}} \Gamma^- C_0 \Big[(p-q)^\lambda \bar{s}_\alpha g^{\mu\lambda} \gamma_5 d_\gamma - i(p-q)^\lambda \bar{s}_\alpha \sigma^{\mu\lambda} \gamma_5 d_\gamma \\
&\qquad + p^\lambda \bar{s}_\alpha g^{\lambda\mu} \gamma_5 d_\gamma - i p^\lambda \bar{s}_\alpha \sigma^{\lambda\mu} \gamma_5 d_\gamma\Big] \\
&= -\frac{1}{2} m_{\tilde{g}} \Gamma^- C_0 \Big[(p-q)^\mu \bar{s}_\alpha \gamma_5 d_\gamma + p^\mu \bar{s}_\alpha \gamma_5 d_\gamma \\
&\qquad - i(p-q)^\lambda \bar{s}_\alpha \sigma^{\mu\lambda} \gamma_5 d_\gamma + i p^\lambda \bar{s}_\alpha \sigma^{\mu\lambda} \gamma_5 d_\gamma\Big] \\
&= -\frac{1}{2} m_{\tilde{g}} \Gamma^- C_0 \left[(2p-q)^\mu \bar{s}_\alpha \gamma_5 d_\gamma + i q^\lambda \bar{s}_\alpha \sigma^{\mu\lambda} \gamma_5 d_\gamma\right]
\end{aligned}
\qquad \text{(B.28)}
$$

Using the Gordon Identity in reverse on the second part, we obtain

$$
- m_{\tilde{g}} \Gamma^- C_0 (2p-q)^\mu \bar{s}_\alpha \gamma_5 d_\gamma \qquad \text{(B.29)}
$$

(We omitted the $\frac{i}{16\pi^2}$ factor so far for readability)

The whole expression for the integral and Dirac structure combined is then

$$D_V^{(1),\lambda\mu} I_{V,\lambda}^{(1)} + D_V^{(2),\mu\epsilon} I_{V,\epsilon}^{(2)} = -\frac{1}{2}\frac{i}{16\pi^2} m_{\tilde{g}} \Gamma^- \left[(2p-q)^\mu \frac{1}{m_{\tilde{d}}^2} F_3[x] \bar{s}_\alpha \gamma_5 d_\gamma + 2C_0 (2p-q)^\mu \bar{s}_\alpha \gamma_5 d_\gamma\right] \tag{B.30}$$

We find that $C_0(m,m,M)$ can be expressed as

$$C_0(m,m,M) = 1/M^2(-F_4[x] - F_3[x]) \tag{B.31}$$

with $x = \frac{m^2}{M^2}$, this leads to

$$D_V^{(1),\lambda\mu} I_{V,\lambda}^{(1)} + D_V^{(2),\mu\epsilon} I_{V,\epsilon}^{(2)} = -\frac{1}{2}\frac{i}{16\pi^2} m_{\tilde{g}} \Gamma^- \left[(2p-q)^\mu \frac{1}{m_{\tilde{d}}^2} \left(F_3[x] - 2F_3[x] - 2F_4[x]\right) \bar{s}_\alpha \gamma_5 d_\gamma\right] \tag{B.32}$$

Rearranging these terms a little, the final expression is

$$D_V^{(1),\lambda\mu} I_{V,\lambda}^{(1)} + D_V^{(2),\mu\epsilon} I_{V,\epsilon}^{(2)} = \frac{1}{2}\frac{i}{16\pi^2}\frac{m_{\tilde{g}}}{m_{\tilde{d}}^2} \Gamma^- (2p-q)^\mu \bar{s}_\alpha \gamma_5 d_\gamma \left[F_3[x] + 2F_4[x]\right] \tag{B.33}$$

The $F_4[x]$ piece is cancelled by the self energy diagrams and thus we can discard it in the following steps.

Putting the pieces together

The matrix element is given by

$$i\mathcal{M} = (-1)^2 i^5 2g_3^3 \cdot T^a_{\alpha\beta} f_{abc} T^b_{\beta\gamma} \cdot \epsilon_\mu^{*c}(q) \left(\frac{1}{2}\frac{i}{16\pi^2} m_{\tilde{g}} \Gamma^- (2p-q)^\mu \frac{1}{m_{\tilde{d}}^2} F_3[x] \bar{s}_\alpha \gamma_5 d_\gamma\right) \tag{B.34}$$

The color factor is given by

$$T^a_{\alpha\beta} f_{abc} T^b_{\beta\gamma} = i\frac{3}{2} T^c_{\alpha\gamma} \tag{B.35}$$

Next, we use the Gordon-Identity (Equation (B.9)) to substitute

$$(2p-q)^\mu \bar{s}\gamma_5 d = i\bar{s}\sigma^{\mu\nu}\gamma_5 d q_\nu + \bar{s}\gamma^\mu \gamma_5 d \cdot (m_s - m_d) \tag{B.36}$$

To obtain the appropriate form for the operator, we also revert the Feynman rules for the polarization vector and the (outgoing) momentum q

$$q_\nu \to -i\partial_\nu \qquad\qquad \epsilon_\mu^{*c}(q) \to A_\mu^c \tag{B.37}$$

We also use $g_3^2 = \alpha_s 4\pi$. Now plugging all this in, we get

$$\begin{aligned} i\mathcal{M} &= (-1)^2 i^5 2 g_3 \alpha_s 4\pi \cdot i\frac{3}{2} T^c_{\alpha\gamma} \cdot A^c_\mu \left(\frac{1}{2}\frac{i}{16\pi^2} m_{\tilde{g}} \Gamma^- \frac{1}{m^2_{\tilde{d}}} F_3[x] \left(i\bar{s}\sigma^{\mu\nu}\gamma_5 d \left(-i\partial_\nu\right)\right)\right) \\ &= (-1)^3 i^9 6 g_3 \alpha_s \pi \cdot T^c_{\alpha\gamma} \cdot \left(\frac{1}{16\pi^2} m_{\tilde{g}} \Gamma^- \frac{1}{m^2_{\tilde{d}}} F_3[x] \bar{s}\sigma^{\mu\nu}\gamma_5 d \partial_\nu A^c_\mu \right) \end{aligned} \tag{B.38}$$

We obtain the gluon field strength tensor by

$$\begin{aligned} \sigma^{\mu\nu}\partial_\nu A^c_\mu &= -\sigma^{\nu\mu}\partial_\nu A^c_\mu \\ &= -\sigma^{\mu\nu}\partial_\mu A^c_\nu \\ &= -\frac{1}{2}\sigma^{\mu\nu}\left(\partial_\mu A^c_\nu - \partial_\nu A^c_\mu\right) \\ &= -\frac{1}{2}\sigma^{\mu\nu} G^c_{\mu\nu} \end{aligned} \tag{B.39}$$

Where we used the antisymmetry of $\sigma^{\mu\nu}$ and renamed the summation indices. The field strength tensor then arises when we include the non-abelian part coming from the ($sg \to dg$) penguin.

Inserting this into the matrix element, we obtain

$$i\mathcal{M} = (-1)^4 i^9 3 g_3 \alpha_s \pi \cdot T^c_{\alpha\gamma} \cdot \left(\frac{1}{16\pi^2} m_{\tilde{g}} \Gamma^- \frac{1}{m^2_{\tilde{d}}} F_3[x] \bar{s}\sigma^{\mu\nu}\gamma_5 d G^c_{\mu\nu} \right) \tag{B.40}$$

Rearranging the terms, we can identify the chromomagnetic operator Q_g^- [92]

$$i\mathcal{M} = -i3\alpha_s\pi\Gamma^- \underbrace{\left(-\frac{g_3}{16\pi^2} \bar{s}\sigma^{\mu\nu}\gamma_5 T^c d G^c_{\mu\nu} \right)}_{Q_g^-} \frac{m_{\tilde{g}}}{m^2_{\tilde{d}}} F_3[x] \tag{B.41}$$

With $\mathcal{H} = -\mathcal{M}$, the final result for this diagram is

$$\mathcal{H} = 3\alpha_s\pi \left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right) \frac{m_{\tilde{g}}}{m^2_{\tilde{d}}} F_3[x] Q^-_g \tag{B.42}$$

Therefore, the contribution to the Wilson Coefficient is

$$\boxed{C^-_g = 3\alpha_s\pi \left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right) \frac{m_{\tilde{g}}}{m^2_{\tilde{d}}} F_3[x]} \tag{B.43}$$

B.1.3. Diagram 2: Gluon Attached to the Squark Line

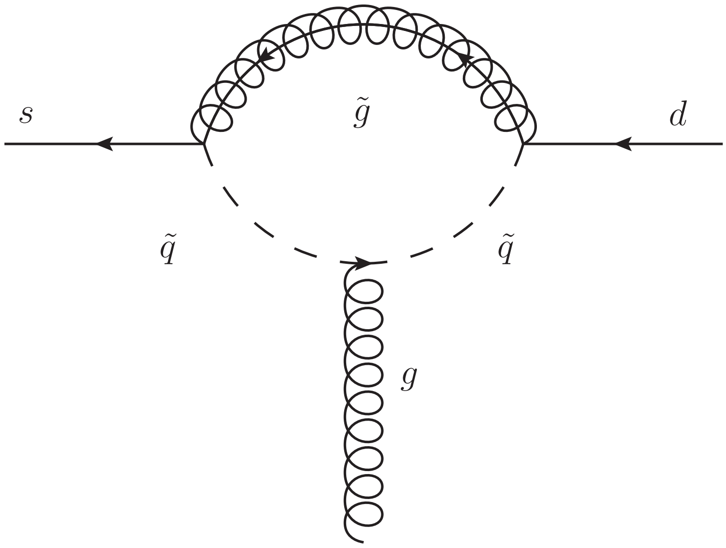

Figure B.2.: *The gluino gluon chromomagnetic penguin, gluon attached to the squark line.*

Feynman Rules

Like with the first diagram, we use Feynman rules [52] on the diagram with the Γ notation for the vertex mixing matrices [54]

$$Z_D^{Ii} = \Gamma_{DL}^{iI} \qquad \text{and} \qquad Z_D^{(I+3)i} = \Gamma_{DR}^{iI} \tag{B.44}$$

This gives us

$$\begin{aligned} i\mathcal{M} =& \bar{s}_\alpha \left[i g_3 \sqrt{2} T^a_{\alpha\beta} \left(-\Gamma_{DL}^{I2*} P_R + \Gamma_{DR}^{I2*} P_L \right) \right] \\ & \left(i \frac{\gamma^\lambda k_\lambda + m_{\tilde{g}}}{k^2 - m_{\tilde{g}}^2} \delta_{ab} \right) \left[i g_3 \sqrt{2} T^b_{\delta\gamma} \left(-\Gamma_{DL}^{I1} P_L + \Gamma_{DR}^{I1} P_R \right) \right] d_\gamma \\ & \left(i \frac{1}{(p+k)^2 - m_{\tilde{d}_I}^2} \right) \left(i \frac{1}{(p-q+k)^2 - m_{\tilde{d}_I}^2} \right) \left[-i g_3 (2p - q + 2k)^\mu T^c_{\beta\delta} \right] \epsilon_\mu^{*c}(q) \end{aligned} \tag{B.45}$$

Rearranging this expression a bit while taking only the scalar Dirac structure and discarding the rest, we obtain

$$i\mathcal{M} = (-1) i^6 2 g_3^3 \cdot T^a_{\alpha\beta} T^c_{\beta\delta} T^a_{\delta\gamma} \cdot \epsilon_\mu^{*c}(q) \cdot D_S I_S^\mu \tag{B.46}$$

with Integral I_S^μ and fermion chain D_S

Dirac Structure

The Dirac structure is given by

$$\begin{aligned}
D_S &= \bar{s}_\alpha \left(-\Gamma^{I2*}_{DL}P_R + \Gamma^{I2*}_{DR}P_L\right)\left(-\Gamma^{I1}_{DL}P_L + \Gamma^{I1}_{DR}P_R\right) d_\gamma m_{\tilde{g}} \\
&= \bar{s}_\alpha \left(-\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR}P_R - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}P_L\right) d_\gamma m_{\tilde{g}} \\
&= -m_{\tilde{g}} \left[\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR}\bar{s}_\alpha P_R d_\gamma + \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\bar{s}_\alpha P_L d_\gamma\right] \\
&= -\frac{1}{2}m_{\tilde{g}} \left[\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR}\bar{s}_\alpha(1+\gamma_5)d_\gamma + \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\bar{s}_\alpha(1-\gamma_5)d_\gamma\right] \\
&= -\frac{1}{2}m_{\tilde{g}} \left[\underbrace{\left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} + \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right)}_{\Gamma^+}\bar{s}_\alpha d_\gamma + \underbrace{\left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right)}_{\Gamma^-}\bar{s}_\alpha \gamma_5 d_\gamma\right] \\
&= -\frac{1}{2}m_{\tilde{g}} \left[\Gamma^+\bar{s}_\alpha d_\gamma + \Gamma^-\bar{s}_\alpha\gamma_5 d_\gamma\right] \qquad \text{(B.47)}
\end{aligned}$$

where only the axial part contributes to the chromomagnetic operator and we can therefore discard the first term

Integral

The Integral is given by

$$I^\mu_S = \int \frac{d^4k}{(2\pi)^4}\frac{(2p-q+2k)^\mu}{k^2-m^2_{\tilde{g}}}\frac{1}{(p+k)^2-m^2_{\tilde{d}}}\frac{1}{(p-q+k)^2-m^2_{\tilde{d}}} \qquad \text{(B.48)}$$

Expanding the denominators in small momenta (which is the same as in diagram one but with the gluino mass and the squark mass interchanged), we get

$$\begin{aligned}
I^\mu_S &= \int \frac{d^4k}{(2\pi)^4}\frac{(2p-q+2k)^\mu}{(k^2-m^2_{\tilde{d}_I})^2}\frac{1}{k^2-m^2_{\tilde{g}}} - 2\int \frac{d^4k}{(2\pi)^4}\frac{k_\lambda(2p-q)^\lambda}{(k^2-m^2_{\tilde{d}_I})^3}\frac{(2p-q+2k)^\mu}{k^2-m^2_{\tilde{g}}} + \mathcal{O}(p^2) \\
&= (2p-q)^\mu \underbrace{\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2_{\tilde{d}_I})^2}\frac{1}{k^2-m^2_{\tilde{g}}}}_{=\frac{i}{16\pi^2}C_0(m_{\tilde{d}_I},m_{\tilde{d}_I},m_{\tilde{g}})} + 2\underbrace{\int \frac{d^4k}{(2\pi)^4}\frac{k^\mu}{(k^2-m^2_{\tilde{d}_I})^2}\frac{1}{k^2-m^2_{\tilde{g}}}}_{\to 0 \text{ because of symmetry}} \\
&\quad - 2(2p-q)^\lambda(2p-q)^\mu \underbrace{\int \frac{d^4k}{(2\pi)^4}\frac{k^\lambda}{(k^2-m^2_{\tilde{d}_I})^2}\frac{1}{k^2-m^2_{\tilde{g}}}}_{\to 0 \text{ because of symmetry}} - 4(2p-q)^\lambda \underbrace{\int \frac{d^4k}{(2\pi)^4}\frac{k^\lambda k^\mu}{(k^2-m^2_{\tilde{d}_I})^2}\frac{1}{k^2-m^2_{\tilde{g}}}}_{=\frac{i}{16\pi^2}D_0^{\lambda\mu}(m_{\tilde{d}_I},m_{\tilde{d}_I},m_{\tilde{d}_I},m_{\tilde{g}})} \\
&= \frac{i}{16\pi^2}(2p-q)^\mu\left(C_0(m_{\tilde{d}_I},m_{\tilde{d}_I},m_{\tilde{g}}) - 4D_{0,T}(m_{\tilde{d}_I},m_{\tilde{d}_I},m_{\tilde{d}_I},m_{\tilde{g}})\right) \qquad \text{(B.49)}
\end{aligned}$$

where we defined $D_{0,T}$ by

$$D_0^{\lambda\mu}(m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{g}}) = g^{\lambda\mu} D_{0,T}(m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{g}}) \tag{B.50}$$

These integral functions are given by

$$\begin{aligned} C_0(m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{g}}) &= \frac{1}{m_{\tilde{d}_I}^2} \frac{-1 + x - x\log x}{(1-x)^2} \\ D_{0,T}(m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{g}}) &= \frac{1}{8} \frac{1}{m_{\tilde{d}_I}^2} \frac{1 - 4x + 3x^2 - 2x^2 \log x}{(x-1)^3} \end{aligned} \tag{B.51}$$

with

$$x = \frac{m_{\tilde{g}}^2}{m_{\tilde{d}_I}^2} \tag{B.52}$$

The difference appearing in the full expression is then

$$C_0(m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{g}}) - 4\, D_{0,T}(m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{d}_I}, m_{\tilde{g}}) = -\frac{1}{m_{\tilde{d}_I}^2} \underbrace{\frac{x^2 - 1 - 2x\log x}{2(x-1)^3}}_{F_4[x]} \tag{B.53}$$

Putting the pieces together, we obtain the final expression for the integral

$$I_S^\mu = -\frac{1}{m_{\tilde{d}_I}^2} \frac{i}{16\pi^2} (2p - q)^\mu F_4[x] \tag{B.54}$$

with the integral function

$$F_4[x] = \frac{x^2 - 1 - 2x\log x}{2(x-1)^3} \tag{B.55}$$

Putting the pieces together

The matrix element is given by

$$i\mathcal{M} = (-1) i^6 2 g_3^3 \cdot T^a_{\alpha\beta} T^c_{\beta\delta} T^a_{\delta\gamma} \cdot \epsilon^{*c}_\mu(q) \cdot \left(-\frac{1}{2} m_{\tilde{g}} \Gamma^- \bar{s}_\alpha \gamma_5 d_\gamma \right) \left(-\frac{1}{m_{\tilde{d}_I}^2} \frac{i}{16\pi^2} (2p-q)^\mu F_4[x] \right) \tag{B.56}$$

The color factor is given by

$$T^a_{\alpha\beta}T^c_{\beta\delta}T^a_{\delta\gamma} = \left(\frac{1}{2}\delta_{\alpha\gamma}\delta_{\delta\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\delta\gamma}\right)T^c_{\beta\delta} = \frac{1}{2}\delta_{\alpha\gamma}T^c_{\beta\beta} - \frac{1}{6}T^c_{\alpha\gamma} \tag{B.57}$$

The first part vanishes because $SU(N)$ generators are traceless.

Next, we use the Gordon-Identity (Equation (B.9)) to substitute

$$(2p-q)^\mu \bar{s}\gamma_5 d = i\bar{s}\sigma^{\mu\nu}\gamma_5 d q_\nu + \bar{s}\gamma^\mu\gamma_5 d \cdot (m_s - m_d) \tag{B.58}$$

To obtain the form of the chromomagnetic operator, we also revert the Feynman rules for the polarization vector and the (outgoing) momentum q

$$q_\nu \to -i\partial_\nu \qquad\qquad \epsilon^{*c}_\mu(q) \to A^c_\mu \tag{B.59}$$

We also use $g_3^2 = \alpha_s 4\pi$. Now plugging all this in and rearranging the terms, the matrix elements becomes

$$\begin{aligned} i\mathcal{M} &= (-1)i^6 2g_3^3 \cdot \left(-\frac{1}{6}T^c_{\alpha\gamma}\right) \cdot A^c_\mu \cdot \left(-\frac{1}{2}m_{\tilde{g}}\Gamma^- i\bar{s}_\alpha\sigma^{\mu\nu}\gamma_5 d_\gamma\,(-i\partial_\nu)\right)\left(-\frac{1}{m^2_{\tilde{d}_I}}\frac{i}{16\pi^2}F_4[x]\right) \\ i\mathcal{M} &= (-1)^5 i^9 \frac{2}{3}\alpha_s\pi\Gamma^- \cdot \left(\frac{g_3}{16\pi^2}\bar{s}_\alpha\sigma^{\mu\nu}\gamma_5 T^c_{\alpha\gamma}d_\gamma\partial_\nu A^c_\mu\right)\left(\frac{m_{\tilde{g}}}{m^2_{\tilde{d}_I}}F_4[x]\right) \end{aligned} \tag{B.60}$$

Now we exploit the antisymmetry of $\sigma^{\mu\nu}$ in the following way

$$\begin{aligned} \sigma^{\mu\nu}\partial_\nu A^c_\mu &= \frac{1}{2}\left(\sigma^{\mu\nu}\partial_\nu A^c_\mu + \sigma^{\nu\mu}\partial_\mu A^c_\nu\right) \\ &= \frac{1}{2}\left(\sigma^{\mu\nu}\partial_\nu A^c_\mu - \sigma^{\mu\nu}\partial_\mu A^c_\nu\right) \\ &= \frac{1}{2}\sigma^{\mu\nu}\left(\partial_\nu A^c_\mu - \partial_\mu A^c_\nu\right) \\ &= -\frac{1}{2}\sigma^{\mu\nu}\left(\partial_\mu A^c_\nu - \partial_\nu A^c_\mu\right) \end{aligned} \tag{B.61}$$

With the non-abelian term coming from the $sg \to dg$ penguin, we can identify the gluon field strength tensor

$$\partial_\mu A^c_\nu - \partial_\nu A^c_\mu \to G^c_{\mu\nu} \tag{B.62}$$

The matrix element becomes

$$i\mathcal{M} = (-1)^6 i^9 \frac{1}{3}\alpha_s \pi \Gamma^- \cdot \left(\frac{g_3}{16\pi^2}\bar{s}_\alpha \sigma^{\mu\nu}\gamma_5 T^c_{\alpha\gamma} d_\gamma G^c_{\mu\nu}\right)\left(\frac{m_{\tilde{g}}}{m^2_{\tilde{d}_I}} F_4[x]\right) \tag{B.63}$$

Putting the squark mixing terms explicitly back in, taking away the i and suppressing fundamental color indices, with $\mathcal{M} = -\mathcal{H}$ we find

$$\mathcal{H} = \frac{1}{3}\alpha_s \pi \left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right)\frac{m_{\tilde{g}}}{m^2_{\tilde{d}_I}} F_4[x] \underbrace{\left(-\frac{g_3}{16\pi^2}\bar{s}\sigma^{\mu\nu}\gamma_5 T^c d G^c_{\mu\nu}\right)}_{Q_g^-} \tag{B.64}$$

and thereby the contribution from this diagram is given by

$$\boxed{C_g^- = \frac{1}{3}\alpha_s \pi \left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right)\frac{m_{\tilde{g}}}{m^2_{\tilde{d}_I}} F_4[x]} \tag{B.65}$$

B.1.4. Final Result

Putting the contributions from both diagrams together, see Equations (B.43) and (B.65), the final result for the Wilson coefficient of the chromomagnetic operator is given by

$$\boxed{C_g^- = \frac{\alpha_s \pi}{3}\left(\Gamma^{I2*}_{DL}\Gamma^{I1}_{DR} - \Gamma^{I2*}_{DR}\Gamma^{I1}_{DL}\right)\frac{m_{\tilde{g}}}{m^2_{\tilde{d}}}\left(9F_3[x] + F_4[x]\right)} \tag{B.66}$$

with $x = m_{\tilde{g}}/m_{\tilde{d}}$.

Appendix: Theoretical Background C

C.1. Derivation of relation between the branching ratios of $K_L \to \{\pi^0\pi^0, \pi^+\pi^-\}$ and ϵ_K

We want to derive the relation (3.28), that is relate the sum

$$\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \neq 0 \tag{C.1}$$

which is an intuitive measure of CP violation to the ηs and ϵ_K. These quantities are central observables in the phenomenology of Kaon CP violation, as discussed in Section 3.2.2.

The experimentally measured branching ratios for K_L and K_S to $\pi\pi$ final states are [29]

$$\begin{aligned}
K_L \to \pi^+\pi^- &\approx (1.967 \pm 0.010) \times 10^{-3} \\
K_L \to \pi^0\pi^0 &\approx (0.864 \pm 0.006) \times 10^{-3} \\
K_S \to \pi^+\pi^- &\approx (0.6920 \pm 0.0005) \\
K_S \to \pi^0\pi^0 &\approx (0.3069 \pm 0.0005)
\end{aligned} \tag{C.2}$$

We observe two things: the ratio decaying to two charged Pions versus two neutral Pions is nearly the same for K_L and K_S

$$\frac{\mathrm{BR}\left(K_L \to \pi^+\pi^-\right)}{\mathrm{BR}\left(K_L \to \pi^0\pi^0\right)} \approx \frac{\mathrm{BR}\left(K_S \to \pi^+\pi^-\right)}{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)} \approx \frac{7}{3} \tag{C.3}$$

and next to all K_S decay into $\pi\pi$

$$\mathrm{BR}\left(K_S \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_S \to \pi^0\pi^0\right) \approx 1 \tag{C.4}$$

we can use these two approximations to express the sum of the K_L to $\pi\pi$ branching ratios (3.27) only in decays to $\pi^0\pi^0$. We could also choose $\pi^+\pi^-$ instead, but this will yield the same result. The approximations we made so far levelled the difference between η_{00} and η_{+-}, which is fulfilled to good accuracy.

$$\begin{aligned} &\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \\ &= \frac{\mathrm{BR}\left(K_L \to \pi^0\pi^0\right)}{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)} \cdot \mathrm{BR}\left(K_S \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \\ &= \frac{\mathrm{BR}\left(K_L \to \pi^0\pi^0\right)}{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)} \cdot \left(1 - \mathrm{BR}\left(K_S \to \pi^0\pi^0\right)\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \\ &= \frac{\mathrm{BR}\left(K_L \to \pi^0\pi^0\right)}{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)} \end{aligned} \tag{C.5}$$

The ratio of the total decay width of K_L and K_S is just the ratio of their lifetimes $\frac{\Gamma(\sum_f K_L \to f)}{\Gamma(\sum_f K_S \to f)} = \frac{\tau_S}{\tau_L}$. It follows for the branching ratio

$$\frac{\mathrm{BR}\left(K_L \to \pi^0\pi^0\right)}{\mathrm{BR}\left(K_S \to \pi^0\pi^0\right)} = \frac{\Gamma\left(K_L \to \pi^0\pi^0\right)/\Gamma\left(\sum_f K_L \to f\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)/\Gamma\left(\sum_f K_S \to f\right)} = \frac{\tau_L}{\tau_S} \cdot \frac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} \tag{C.6}$$

We noted before, that K_S and K_L have nearly equal masses and when decaying to the same final state, all integration constants in the computation of the decay rate are equal so only the matrix element remains to be calculated. Everything cancels in the ratio and we remain with

$$\frac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} = \frac{|\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle|^2}{|\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle|^2} \tag{C.7}$$

we define the ratios of these amplitudes as η

$$\begin{aligned} \eta_{00} &= \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle} \\ \eta_{+-} &= \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle} \end{aligned} \tag{C.8}$$

Remember from Section 3.2.2, that $|\epsilon_K| \approx |\eta_{00}| \approx |\eta_{+-}|$. So in the end we have produced Equation (3.28)

$$\boxed{\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \approx \tau_L/\tau_S \cdot |\epsilon_K|^2}$$

we could have done the same line of arguments with η_{+-} but since we have neglected the difference between the charged and neutral Pion final states in Equation (C.3), by assumption $\eta_{+-} = \eta_{00}$.

From the measurements [29]

$$\begin{aligned} \mathrm{BR}\left(K_L \to \pi^+\pi^-\right) &= (1.967 \pm 0.010) \times 10^{-3} \\ \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) &= (0.864 \pm 0.006) \times 10^{-3} \end{aligned} \tag{C.9}$$

we get

$$\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right) \approx 2.831 \times 10^{-3} \tag{C.10}$$

with [29]

$$\tau_L = 5.116 \times 10^{-8} \qquad \tau_S = 8.954 \times 10^{-11} \tag{C.11}$$

we have $\tau_L/\tau_S \approx 571$. The formula predicts $|\epsilon_K| \approx |\eta_{00}| \approx |\eta_{+-}| \approx 2.226 \times 10^{-3}$.

Measurements give [29]

$$|\eta_{00}| = (2.220 \pm 0.011) \times 10^{-3} \qquad |\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3} \tag{C.12}$$

$$|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3} \tag{C.13}$$

The essential information we were after is Equation (3.28)

$$|\epsilon_K| \approx |\eta_{00}| \approx |\eta_{+-}| \approx \sqrt{\tau_S/\tau_L} \cdot \sqrt{\mathrm{BR}\left(K_L \to \pi^+\pi^-\right) + \mathrm{BR}\left(K_L \to \pi^0\pi^0\right)} \tag{C.14}$$

which relates the ηs to the CP violating branching ratios and is accurate to $< 1\%$.

C.2. Explicit calculation of η_f in terms of observables

In this section we will expand on Reference [78] and show how they manage to express

$$\eta_f = \frac{\langle f|H_{eff}|K_L\rangle}{\langle f|H_{eff}|K_S\rangle} = \frac{1 - qg/ph}{1 + qg/ph} \tag{C.15}$$

by phase convention independent quantitites, cf. Equation (3.39). First we define the shorthand $r := qg/ph$ so that

$$\eta_f = \frac{1-r}{1+r} = \frac{1 - \text{Re}\, r - i\, \text{Im}\, r}{1 + \text{Re}\, r + i\, \text{Im}\, r} \tag{C.16}$$

we expand to make the denominator real

$$\eta_f = \frac{1 - \text{Re}\, r - i\, \text{Im}\, r}{1 + \text{Re}\, r + i\, \text{Im}\, r} \cdot \frac{1 + \text{Re}\, r - i\, \text{Im}\, r}{1 + \text{Re}\, r - i\, \text{Im}\, r} = \frac{1 - |r|^2 - 2i\, \text{Im}\, r}{1 + 2\, \text{Re}\, r + |r|^2} \tag{C.17}$$

now we split $|r|^2 = |q/p|^2 \cdot |g/h|^2$ and use the hint given in [78] to use the algebraic relation $(1 \pm ab) = 1/2\ \left[(1+a)(1 \pm b) + (1-a)(1 \mp b)\right]$ for $a = |q/p|^2$ and $b = |g/h|^2$

$$\begin{aligned} \eta_f &= \frac{1 - |r|^2 - 2i\, \text{Im}\, r}{1 + 2\, \text{Re}\, r + |r|^2} \\ &= \frac{(1+|q/p|^2)(1-|g/h|^2) + (1-|q/p|^2)(1+|g/h|^2) - 4i\, \text{Im}\, r}{(1+|q/p|^2)(1+|g/h|^2) + (1-|q/p|^2)(1-|g/h|^2) - 4\, \text{Re}\, r} \end{aligned} \tag{C.18}$$

Expanding by $(1+|q/p|^2)(1+|g/h|^2)$

$$\begin{aligned} \eta_f &= \frac{\left(\frac{(1+|q/p|^2)(1-|g/h|^2)}{(1+|q/p|^2)(1+|g/h|^2)}\right) + \left(\frac{(1-|q/p|^2)(1+|g/h|^2)}{(1+|q/p|^2)(1+|g/h|^2)}\right) + \left(\frac{-4i\, \text{Im}\, r}{(1+|q/p|^2)(1+|g/h|^2)}\right)}{\left(\frac{(1+|q/p|^2)(1+|g/h|^2)}{(1+|q/p|^2)(1+|g/h|^2)}\right) + \left(\frac{(1-|q/p|^2)(1-|g/h|^2)}{(1+|q/p|^2)(1+|g/h|^2)}\right) + \left(\frac{-4\, \text{Re}\, r}{(1+|q/p|^2)(1+|g/h|^2)}\right)} \\ &= \frac{\frac{(1-|g/h|^2)}{(1+|g/h|^2)} + \frac{(1-|q/p|^2)}{(1+|q/p|^2)} + \frac{-4i\, \text{Im}\, r}{(1+|q/p|^2)(1+|g/h|^2)}}{1 + \frac{(1-|q/p|^2)(1-|g/h|^2)}{(1+|q/p|^2)(1+|g/h|^2)} + \frac{-4\, \text{Re}\, r}{(1+|q/p|^2)(1+|g/h|^2)}} \end{aligned} \tag{C.19}$$

and thereby we found with the definitions given in Equation (3.39) the result of [78]

$$\eta_f = \frac{a_{\tilde{\epsilon}'_f} + a_{\tilde{\epsilon}} + i a_{\tilde{\epsilon}+\tilde{\epsilon}'_f}}{2 + a_{\tilde{\epsilon}} a_{\tilde{\epsilon}'_f} + a_{\tilde{\epsilon}\tilde{\epsilon}'_f}} \tag{C.20}$$

C.3. Relating $\tilde{\epsilon}'_{+-}$ and $\tilde{\epsilon}'_{00}$

In this section, we derive the relation between $\tilde{\epsilon}'_{+-}$ and $\tilde{\epsilon}'_{00}$. To this end, we use the following notation

$$\begin{aligned}
\tilde{\omega} &= \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_{CP+}\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_{CP+}\rangle} = e^{i(\delta_2-\delta_0)} \cdot \frac{\text{Re } a_2}{\text{Re } a_0} \\
\tilde{\epsilon}'_0 &= \frac{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_{CP-}\rangle}{\langle(\pi\pi)_0|\mathcal{H}_{eff}|K_{CP+}\rangle} = i\frac{\text{Im } a_0}{\text{Re } a_0} \\
\tilde{\epsilon}'_2 &= \frac{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_{CP-}\rangle}{\langle(\pi\pi)_2|\mathcal{H}_{eff}|K_{CP+}\rangle} = i\frac{\text{Im } a_2}{\text{Re } a_2} \\
\tilde{\epsilon}'_{+-} &= \frac{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_{CP-}\rangle}{\langle\pi^+\pi^-|\mathcal{H}_{eff}|K_{CP+}\rangle} = \frac{\tilde{\epsilon}'_0 + \tilde{\epsilon}'_2\tilde{\omega}/\sqrt{2}}{1+\tilde{\omega}/\sqrt{2}} \\
\tilde{\epsilon}'_{00} &= \frac{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_{CP-}\rangle}{\langle\pi^0\pi^0|\mathcal{H}_{eff}|K_{CP+}\rangle} = \frac{\tilde{\epsilon}'_0 - \tilde{\epsilon}'_2\sqrt{2}\tilde{\omega}}{1-\sqrt{2}\tilde{\omega}}
\end{aligned} \tag{C.21}$$

together with the mixing parameter $\tilde{\epsilon}$ which parametrizes the admixture of the 'wrong' CP state in the K_L and K_S states. We work in the physical phase conventions, such that the phase dependence of $\tilde{\epsilon}$ is given by $\epsilon_K = \tilde{\epsilon} + \tilde{\epsilon}'_0$, with the above definitions. The convention independent quantities which we want to derive carry the same labels but without a tilde.

From Equation (3.40), we have the following relations between convention dependent and independent quantities. For the real parts, we have

$$\begin{aligned}
\text{Re } \tilde{\epsilon} &= \text{Re } \epsilon_K \\
\text{Re } \tilde{\epsilon}'_{+-} &= \text{Re } \epsilon'_{+-} \\
\text{Re } \tilde{\epsilon}'_{00} &= \text{Re } \epsilon'_{00}
\end{aligned} \tag{C.22}$$

From these we could directly derive the CPT relation of [69] between the real parts by explicitly calculating the real part of the convention dependent quantities. For the imaginary parts, cf. Equation (3.40), we have

$$\begin{aligned}
\text{Im } \tilde{\epsilon}'_{+-} + \text{Im } \tilde{\epsilon} &= \text{Im } \epsilon'_{+-} + \epsilon_K \\
\text{Im } \tilde{\epsilon}'_{00} + \text{Im } \tilde{\epsilon} &= \text{Im } \epsilon'_{00} + \epsilon_K
\end{aligned} \tag{C.23}$$

Using, that the phase convention dependence of $\tilde{\epsilon}$ within the class of physical phase conventions is given by $\epsilon_K = \tilde{\epsilon} + \tilde{\epsilon}'_0$, we find that

$$\begin{aligned}
\mathrm{Im}\ \epsilon'_{+-} &= \mathrm{Im}\ \tilde{\epsilon}'_{+-} - \mathrm{Im}\ \tilde{\epsilon}'_0 \\
\mathrm{Im}\ \epsilon'_{00} &= \mathrm{Im}\ \tilde{\epsilon}'_{00} - \mathrm{Im}\ \tilde{\epsilon}'_0
\end{aligned} \tag{C.24}$$

From Equation (C.21), we see that $\tilde{\epsilon}'_0$ is purely imaginary, thus we can summarize the Conditions (C.22) and (C.24) by

$$\begin{aligned}
\epsilon'_{+-} &= \tilde{\epsilon}'_{+-} - \tilde{\epsilon}'_0 \\
\epsilon'_{00} &= \tilde{\epsilon}'_{00} - \tilde{\epsilon}'_0
\end{aligned} \tag{C.25}$$

The appearance of $\tilde{\epsilon}'_0$ in the imaginary parts forces the relation between the imaginary parts of ϵ'_{+-} and ϵ'_{00} to be different than that of the real parts. The explicit expressions are

$$\begin{aligned}
\epsilon'_{+-} &= \tilde{\epsilon}'_{+-} - \tilde{\epsilon}'_0 \\
&= \frac{\tilde{\epsilon}'_0 + \tilde{\epsilon}'_2\tilde{\omega}/\sqrt{2}}{1+\tilde{\omega}/\sqrt{2}} - \tilde{\epsilon}'_0 \\
&= \frac{1}{\sqrt{2}}\tilde{\omega}\left(\tilde{\epsilon}'_2 - \tilde{\epsilon}'_0\right)\frac{1}{1+\tilde{\omega}/\sqrt{2}}
\end{aligned} \tag{C.26}$$

and

$$\begin{aligned}
\epsilon'_{00} &= \tilde{\epsilon}'_{00} - \tilde{\epsilon}'_0 \\
&= \frac{\tilde{\epsilon}'_0 - \tilde{\epsilon}'_2\sqrt{2}\tilde{\omega}}{1-\sqrt{2}\tilde{\omega}} - \tilde{\epsilon}'_0 \\
&= -2\frac{1}{\sqrt{2}}\tilde{\omega}\left(\tilde{\epsilon}'_2 - \tilde{\epsilon}'_0\right)\frac{1}{1-\sqrt{2}\tilde{\omega}}
\end{aligned} \tag{C.27}$$

Which is constructed to be convention independent, hence $\left(\tilde{\epsilon}'_2 - \tilde{\epsilon}'_0\right)$ has to be convention independent. We identify

$$\tilde{\epsilon}'_K = \frac{1}{\sqrt{2}}\tilde{\omega}\left(\tilde{\epsilon}'_2 - \tilde{\epsilon}'_0\right) \tag{C.28}$$

and thereby find

$$\begin{aligned}
\epsilon'_{+-} &= \tilde{\epsilon}'_K\frac{1}{1+\tilde{\omega}/\sqrt{2}} \\
\epsilon'_{00} &= -2\tilde{\epsilon}'_K\frac{1}{1-\sqrt{2}\tilde{\omega}}
\end{aligned} \tag{C.29}$$

Which gives the relation

$$\boxed{\epsilon'_{00} = -\frac{1}{2}\left(\frac{1-\sqrt{2}\tilde{\omega}}{1+\tilde{\omega}/\sqrt{2}}\right)\epsilon'_{+-}} \tag{C.30}$$

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