ECM Models for Li-Ion Batteries – A Short Mathematical Survey and Simulations

Andreas Melcher, Carlos Ziebert, Magnus Rohde, Boxia Lei, Hans Jürgen Seifert

Institute of Applied Materials - Applied Materials Physics (IAM-AWP), Karlsruhe Institute of Technology

Abstract

Accurate models and simulation results of advanced Li-ion batteries and cells are of vital interest in powertrain electrification demands, stationary storage applications as well as in the experimental quantification of Li-ion batteries. Various models with different complexities are possible. The range of such models varies from the fully coupled electrochemical thermal model describing the dynamics of the battery in terms of partial differential equations (PDE) over Reduced Order Models (ROM) as simplification of the PDE model up to Equivalent Circuit Models (ECM) which use a phenomenological description of the electrical behavior in terms of ordinary differential equations. This work presents a short overview of some ECMs followed by a first implementation of an extended ECM with a simplified thermal model in Matlab®/Simulink®/Simscape™[1]. The identification problem of the structure and the parameters of an ECM is discussed in terms of the Current Interruption Technique (CIT). Several test cases are simulated with an example of an ECM.

Equivalent Circuit Models for Li-ion Batteries

Li-ion batteries are widely used in today's electric vehicles (EV), hybrid electric vehicles (HEV), cell phones and many other industrial applications. Very often it is necessary to control the battery or whole battery packs with a Battery Management System (BMS) to provide useful and important information about the battery system like the State of charge (*SOC*) to predict the range of an EV/ HEV or the temperature to avoid thermal runaway as the worst case scenario in battery applications. Therefore accurate models and simulations are of high importance. In the following a short overview of mathematical models for Li-ion batteries is given. This work is focused on Equivalent Circuits Models (ECM) which are the state of the art in modern BMS, but can also be used for the extraction of experimental data of Li-ion batteries.

For the description of the dynamical behavior of Li-ion batteries a large range of mathematical-physical models exists. Starting with the electrochemical-thermal model, see [1], [2] and the references therein, a spatio-temporal description of the concentration of the Li-ions and potentials in the electrodes as well as the temperature distribution of the battery is given in terms of a multi-scale, multiphysics system of elliptic and parabolic differential equations using homogenization methods. This model is able to capture almost all physical relevant phenomena at the expense of

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[1] Matlab®/Simulink®/Simscape™ are registered trademarks of The Mathworks Inc.

corresponding complexity with respect to spatial and temporal resolution. As a consequence it cannot be applied in BMS today due to its high complexity.
A possible ansatz to reduce this complexity is the use of Reduced Order Models (ROM) which can be derived from the previous model using some simplification assumptions like the eigenfunction expansion method [3]. One possible candidate is the Single Particle Model (SPM) which is able to reproduce the dynamics of the original model in some ranges with good agreement and with less complexity. The corresponding partial differential equations can be converted into ordinary differential equations by the methods of lines (MOL) to reach less complexity. But these equations are also complex in comparison to the Equivalent Circuit Models (ECM).

An ECM model describes the pure electrical behavior of the battery in terms of voltages, currents, resistances and capacitances. Using the theory of passive electrical networks the resulting equations are a low dimensional system of ordinary differential equations which is in general nonlinear, since the system parameter depends on temperature T and state of charge *SOC*. Due to these dependencies an extended ECM model is used together with an additional differential equation for the *SOC* and a simplified temperature model in form of a linear inhomogeneous differential equation. These models are able to capture the essential electrical phenomena with good accuracy and less complexity. They consist only of passive electrical elements and are easy to embed in BMS for real time application to ensure safe and reliable operation conditions. An overview of ECMs can be found in [4, 5].

In general most of the ECMs can be described by the Thevenin-Model. The general structure of a Thevenin-ECM is shown in Figure 1. It contains two voltage sources. The first one describes the open circuit voltage of the battery; the second one describes parasitic voltages. Furthermore is contains several parallel resistors and capacitors which represent polarizations like electrochemical or concentration polarization mechanisms in the battery. The resistor R_0 describes the ohmic resistance of the battery and R_p describes additional parasitic effects. This can be used to model thermal runaway with an ECM, where R_p becomes of low impedance for rising temperatures, which can result in a short circuit in the battery. All parameter of this circuit depend on SOC and temperature T. So a simplified temperature model in form of an ordinary differential equation is used for the SOC.

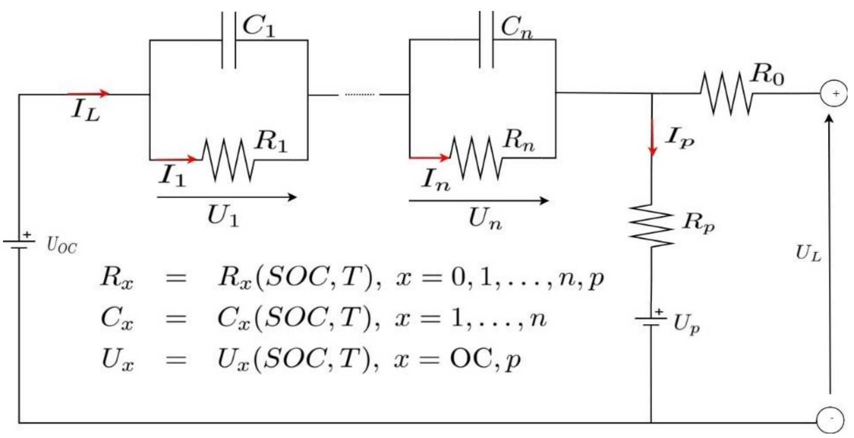

Fig. 1: General Thevenin model.

Mathematical Description and Implementation

Using the passive electrical network theory the general Thevenin model can be described as a system of $n + 2$ nonlinear ordinary differential equations and one algebraic equation:

$$\frac{dU_i}{dt} = \left(-\frac{1}{R_i C_i} + \left(\frac{1}{C_Q}\left(-\frac{1}{R_T C_T}(T - T_a) + \frac{1}{C_T} Q\right)\right)\right) U_i + \frac{1}{C_i} I_L, i = 1, \dots, n, \tag{1}$$

$$\frac{dT}{dt} = -\frac{1}{R_T C_T}(T - T_a) + \frac{1}{C_T} Q, \tag{2}$$

$$\frac{dSOC}{dt} = \frac{1}{C_Q} I_L, \tag{3}$$

$$U_L = U_{OC} - I_L R_0 - \sum_{i=1}^{n} U_i, \tag{4}$$

where R_i, C_i, U_i, I_i, $i = 1, \dots n$ denote the corresponding resistors, capacitors, voltages and currents of each *RC*-branch. U_{OC} is the open circuit voltage, I_L the load current and U_L the terminal voltage, R_0 is the ohmic resistance of the battery and the current I_P, the voltage U_P and the resistor R_P describe parasitic effects in the battery. T, T_a are the temperature and the ambient temperature, R_T, C_T the convection resistance and the heat capacity respectively, Q the power dissipated inside the cell and SOC the state of charge and C_Q the total charge of the battery.

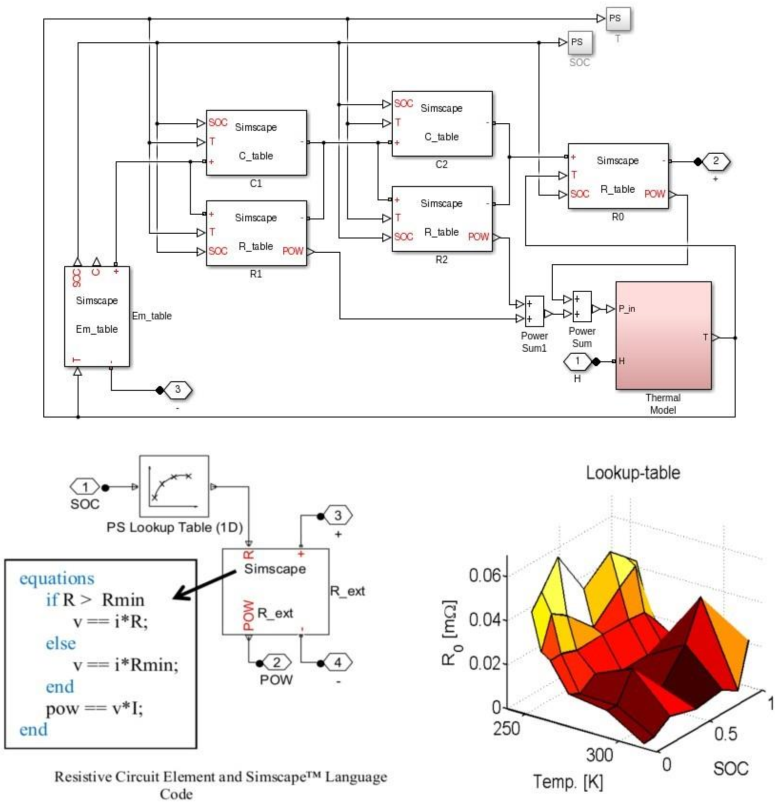

Fig. 2: The Simulink®/Simscape™ realization of the Thevenin model.

This system of equations can be rewritten in more general form using the state space approach:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}), \ \mathbf{x}(0) = \mathbf{x}_0 \tag{5}$$

$$y = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p}) \tag{6}$$

with $\mathbf{f} : \Omega_x \times \Omega_u \times \Omega_p \mapsto \Omega_f \subset \mathbb{R}^{n+2}$ and $\mathbf{g} : \Omega_x \times \Omega_u \times \Omega_p \to \Omega_g \subset \mathbb{R}$, where the manifolds $\Omega_x \subset \mathbb{R}^{n+2}$, $\Omega_u \subset \mathbb{R}^3$, $\Omega_p \subset \mathbb{R}^{2n+2}$ are suitable chosen. Furthermore $\mathbf{u} = (I_L, T_a, Q)^T \in \mathbb{R}^3$, $\mathbf{x} = (U_1, \ldots, U_n, SOC, T)^T \in \mathbb{R}^{n+2}$, $\mathbf{y} = U_L \in \mathbb{R}$ and $\mathbf{p} = (R_1, \ldots, R_n, C_1, \ldots, C_n, U_{OC})^T \in \mathbb{R}^{2n+2}$, $\mathbf{p} = \mathbf{p}(SOC, T)$ denotes the input, state-space, output and parameter vector respectively. In this description the parasitic effects are neglected, all other constants not mentioned in the parameter vector p are assumed to be constant.

The Matlab®/Simulink®/Simscape™ environment is an ideal tool to implement this system of differential and algebraic equations. For simulation purposes a Thevenin-model with two relaxation mechanisms is implemented. The corresponding electrical network in Simulink® can be seen in Figure 2. For the passive electrical elements a library is programmed using Simscape™ and lookup-tables are used in approximation of the nonlinear functions $\mathbf{f}$ and $\mathbf{g}$. The main difficulty is that the nonlinear functions $\mathbf{f}$ and $\mathbf{g}$ are in general not completely known due to the dependence of the passive electrical elements of the ECM on the *SOC* and the temperature *T*. This problem can be solved by using lookup-tables for the passive elements of the ECM. Lookup-tables are able to approximate these functions via interpolation and extrapolation in the simulation task. These tables can be filled via the experimental Current Interruption Technique (CIT), where for known *SOC* and temperature *T* charge and discharge curves of real batteries are measured and from these curves the polarization mechanism, relaxation times, numerical values for resistances and capacitances can be extracted. For more details see [6, 7, 8].

Simulation of a Pouch Cell

For the Simulink®/Simscape™ implementation of the model first simulations of a pouch cell with the dimensions 0.0084 × 0.215 × 0.22 m^3 at ambient temperature T_a = 25 °C were performed for the driving profiles Artemis Road and Artemis Urban [9]. Corresponding simulation results are shown in Figure 3 and statistical values in Table 1. The simulated battery is first charged to *SOC* = 1 over charging time t_L = 1200s and then the corresponding profile is applied in the simulation using the ode15s solver from Matlab® and the battery is completely discharged to *SOC* = 0.

Tab. 1: Mean value and standard deviation from simulation.

	T in [°C]	Std_T in [°C]	SOC	Std_C	U_L in [V]	Std_{U_L} in [V]	I_L in [A]	Std_{I_L} in [A]
Road :	29.48	1.96	0.59	0.29	2.87	0.30	−13.52	17.05
Urban:	36.29	4.14	0.49	0.30	2.83	0.41	−16.97	41.36

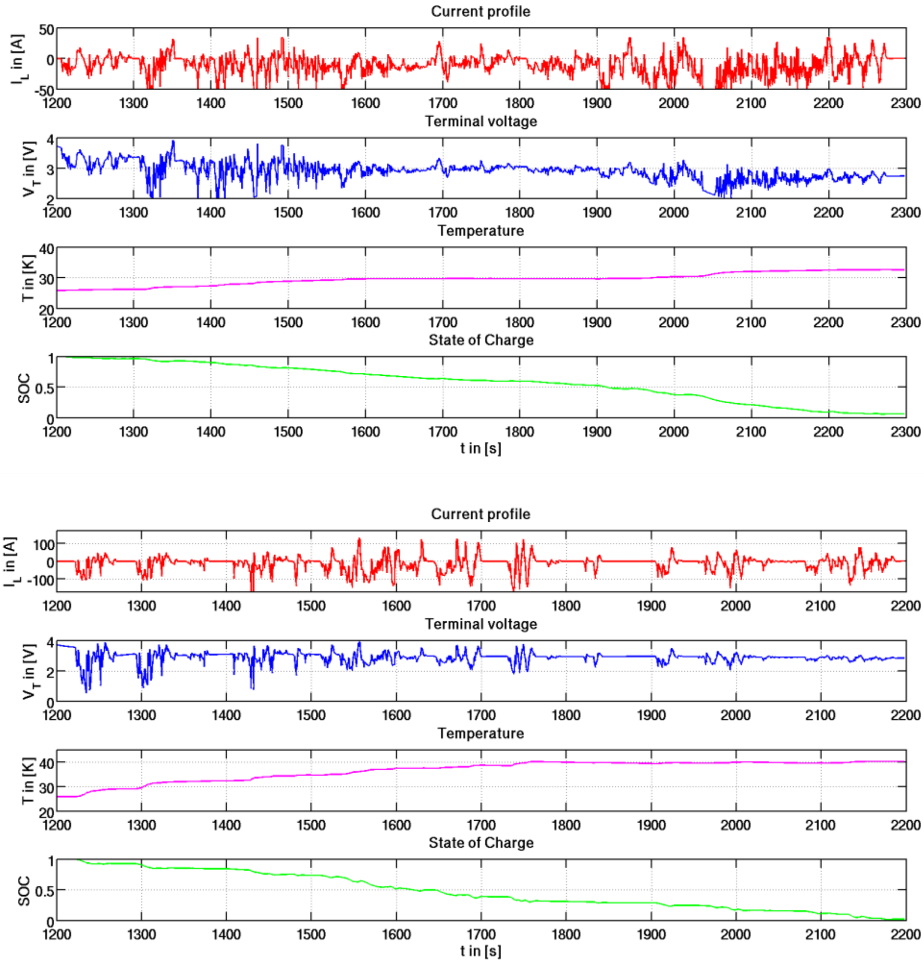

Fig. 3: Simulation of Artemis Road (upper graph) and Artemis Urban profile (lower graph).

Conclusion and Outlook

Using the Simulink®/Simscape™ implementation of an ECM model offers a quick and easy possibility to simulate the electrical-thermal dynamical behavior of Li-ion batteries based on look-up tables coming from experimental measurements. It is planned to measure more look-up tables for different types of Li-ion batteries and to perform corresponding simulations. Also parasitic effects will be taken into account for simulations of thermal runaway with ECM models in the near future. Furthermore, it is planned to extend the Simulink®/Simscape™ implementation for whole battery stacks and packs. Additionally, the Simulink®/ Simscape™ implementation will be extended in a way that an optimization of the model parameter in comparison to a specific experimental measurement with arbritary load profile will be possible due to an Least Squares fit.

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