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Validation of a 5-zone-car-cabin model to predict the energy saving potentials of a battery electric vehicle's HVAC system

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Abstract. This paper presents a 5-zone-car-cabin model which is able to simulate the car cabin's thermal condition depending on several influencing parameters. This includes the solar radiation, to which special attention is paid in this paper. In addition, a generic methodology for parameter optimization considering measurements on a reference vehicle is presented. Thus, a very high degree of determination of the model was achieved. This paper is motivated by the impact of auxiliary loads on overall energy consumption in battery electric vehicles. The further use of the model is intended to calculate the energy saving potentials of the heating, ventilation, and air conditioning system by reducing or increasing the target interior temperature. This is necessary for a predictive control of secondary consumers.

1. Introduction

Most of the potential car buyers do not trust in the vehicle range of battery electric vehicles (BEVs), as they are often smaller than the manufacturer indicates. In addition, the so-called range anxiety (The fear that a vehicle cannot reach its destination because of insufficient range.) is still widespread. In order to counteract the range disadvantage, an efficient use of the available energy is necessary. This requires the knowledge of the overall energy requirement to cover the present route. Each route has different energy loads in terms of propulsion and secondary consumers. The heating, ventilation, and air-conditioning (HVAC) system can be the largest auxiliary load. It is shown that the range of electric vehicles significantly reduces when the HVAC system is activated [1], [2]. Depending on the weather conditions, up to 25% of the required energy is used by secondary consumers [3]. Thus, the HVAC system offers a considerable energy saving potential by reducing or increasing the target interior temperature in case of a predictive control. However, possible energy savings quickly lead to the conflict with passenger's comfort. To identify the energy saving potentials of an HVAC system, some preparation steps are necessary, according to figure 1.

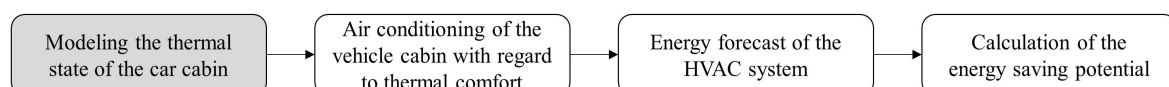


Figure 1. The objective of the research project is shown in partial steps. In this paper, only the first step is discussed.



In the first step, detailed knowledge of the condition within the car cabin depending on relevant influencing variables is acquired. Therefore, the thermal behavior is simulated by a 5-zone-car-cabin model considering relevant variables. In the second step, an automatic climate control will be used to estimate how much thermal energy must be supplied to the vehicle interior by the HVAC system. In the next step, the required electrical energy can be calculated using the necessary heat and mass flow. Finally, the energy saving potential of the HVAC system can be calculated for a given route. This paper covers only the first step by validating a 5-zone-car-cabin model.

2. Related work

In previous work the advantages and disadvantages of empirical, single-zone, multi-zone, and CFD (computational fluid dynamics) simulation methods have been discussed [4]. The model presented in this paper is intended to predict the energy demand of an HVAC system, considering the passenger's comfort aspects. This requires a classification of the car cabin into different volumes is necessary since each body part has a different temperature sensation. The temperature distribution and heat flows in the car cabin can be investigated in detail with CFD, but the modelling effort is very high and the simulation speed for fast predictions is too slow [5][6]. As a compromise between computation time and accuracy, a 5-zone model was chosen. In literature, several different models for car cabin thermal calculations have been presented. An overview of the models has been shown in [7] and [8]. Other related works mainly have focused on a single-zone car cabin model [9][10]. When considering solar radiation, a multi-volume model provides advantages compared to a single zone. The partial heating of a zone results in thermodynamic effects that cannot be represented by a single-volume model. Especially in summer scenarios, the resulting predictive energy calculation can achieve a higher accuracy. In other related works only the ambient temperature have been used as input variable [11], [12]. Lajunen et. al has used the simulation software AMESim and considered the flow velocity, but has not taken solar irradiance into account, which results in a less powerful model [13].

Based on all relevant heat transfer mechanisms, a MATLAB/Simulink model was developed, which captures the car cabin as a 5-volume model and calculates the average air volume temperature, see Chapter 3. Further, the model is validated by measurements on a reference vehicle. In addition, a generic methodology for parameter optimization has been applied to increase the accuracy of the model using special selected reference measurements, see Chapter 4. Finally, we summarize our work in Chapter 5, where simulation results of the model are compared with the measurements on the vehicle.

3. Structure of the 5-zone-car-cabin model

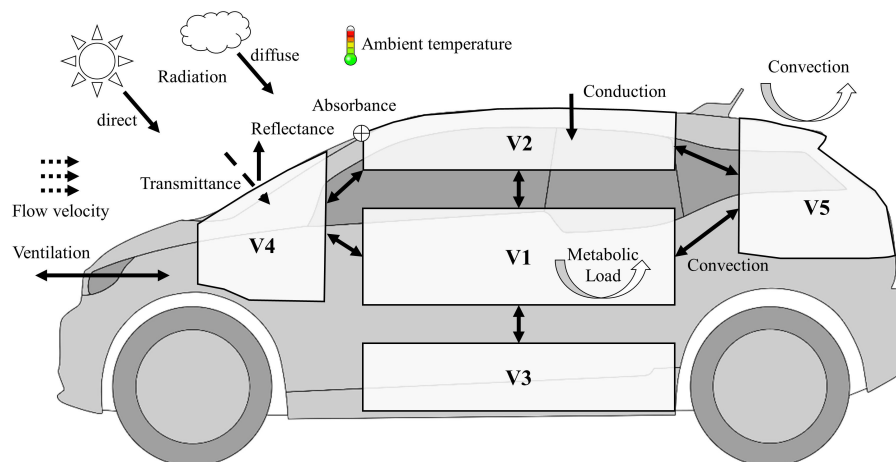


Figure 2. Classification of the vehicle interior into five zones and representation of the influences (ambient temperature, sun, HVAC system, ...) and heat transfer mechanisms which are considered in the model.

3.1. Environmental influences

In the model, ambient temperature, solar radiation, and the exchange of the cabin with the air-conditioning system of the vehicle are considered. This results in heat flows, which influence the condition of the "car cabin, according to figure 2. 'Ventilation' represents the bidirectional heat flow of the air conditioning system. The general equation describing the heat flow of a body part $\dot{Q}_{body\ part,i}$, is calculated by the sum of heat transfer as well as diffuse and direct radiation:

$$\dot{Q}_{body\ part,i} = S \cdot U \cdot (T_{out} - T_{in}) + \varepsilon \cdot \sigma \cdot S \cdot (T^4 - T_{ambient}^4) + \tau \cdot S \cdot \theta \cdot \dot{I}_{sun} \quad (1)$$

The heat transfer $\dot{Q}_{conduction}$ depends on ambient T_{out} and internal T_{in} temperatures, the surface area S , as well as the overall heat transfer coefficient U which depends on the convection coefficients h_i, h_o , the material thickness λ , and the thermal conductivity k . The solar radiation enters the system through absorption and transmittance. The vehicle's windows hardly absorb radiation energy, which is why thermal energy is transmitted directly in the car cabin.

3.2. Coupling of the 5 zones and chassis parts

The chassis parts roof, underbody, side plates and the windows describe the system boundary. They are considered separately and are connected to the vehicle interior by an associated heat flow. The following assumptions were made for the model: All body surfaces are assumed to be flat surfaces with a simple geometry. The heat input, which enters through the boot and the front wall, is neglected. The air volumes in the interior are connected to each other by convection alone, as are the body panels and thermal masses with the adjacent air volumes. The body elements are considered as layered flat panels. All mentioned components are similarly modeled, except the vehicle windows. These consist only of a single layer. Figure 3 shows the heat transfer mechanisms convection, radiation and transmittance during the passive heating or cooling, but without consideration of the forced convection.

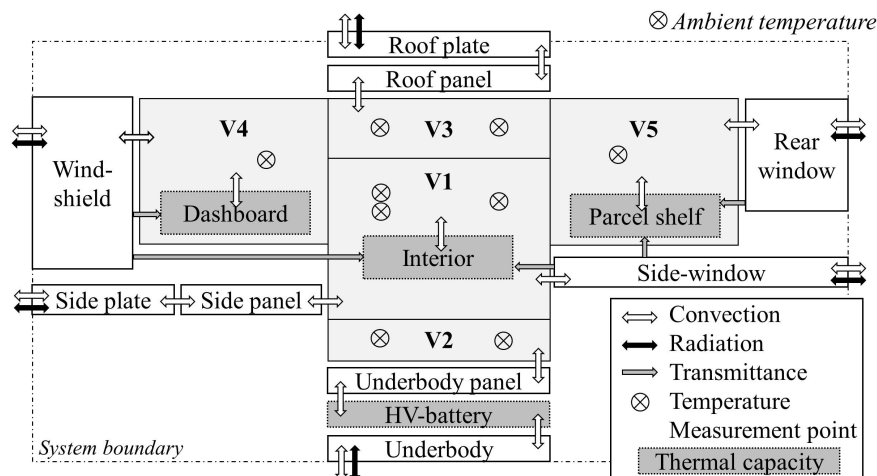


Figure 3. All chassis parts considered in the model and their coupling among each other.

Representation of the occurring heat transfer mechanisms with passive heating or cooling. The forced convection is not considered here.

The components with a thermal capacity (also referred to as thermal mass), like the parcel shelf, the dashboard and interior are handled identical except for material parameters and considered ambient influences. Sun radiation acts on the parcel shelf only through the rear window and on the dashboard only through the windshield. The interior, on the other hand, will be irradiated through all windows. Each thermal mass is connected by a heat convection to the volume in which it is located. This is described by the following formula [14]:

$$\dot{Q}_{convection} = \alpha(Nu(Re, Pr)) \cdot S \cdot \Delta T \quad (2)$$

The lithium-ion battery in the reference vehicle's underbody represents an additional thermal mass with 230 kg and affects the thermal condition of the car cabin, in particular by the delayed temperature rise of the footwell during the heating process [15]. In addition, the reference vehicle's body shape (hatchback sedan) influences the heat input into the parcel shelf zone, since heat is also exchanged via side windows. Therefore, heat flow, which enters through the side windows, is divided between the main zone (V1) and the parcel shelf zone (V5). In a conventional sedan, this heat flow completely enters the main zone (V1).

In case of the convective coupling of the volumes six virtual contact surfaces exist, according to figure 2. In the passive state, i.e. heating and cooling of the car cabin only by external conditions, the coupling is exclusively based on natural convection. The heat flow is calculated according to equation 2. In case of an active ventilation of the car cabin by the HVAC system, the coupling of these six contact surfaces by forced air-mass flows is relevant. The resulting heat flow is dependent on the inflow temperature and the interior temperature. The choice of the ventilation mode determines how the mass flows are blown through the car cabin. Since a constant pressure of the car cabin is assumed, the in- and output of air of each volume has to be equal. The amount of the heat flow depends on the temperature difference ΔT of adjacent volumes and can be described by the following formula:

$$\dot{Q}_{ventilation} = \dot{m} \cdot c \cdot \Delta T \quad (3)$$

Upward flows above the dashboard and the parcel shelf result in circulation flows, which cause an air exchange in the entire car cabin. [4] They arise above the thermal mass because the components heat up faster than the adjacent air volumes due to solar radiation. The airflow is deflected from the windows into the head zone (V3), from there into the main zone (V1) and finally back into the parcel shelf (V5) or dashboard zone (V4). The magnitude of the heat flows depends on the temperature difference between adjacent volumes and on resulting air mass flow above the thermal masses. A further influence caused by the sun is also considered: the solar radiation passes through the windshield and is absorbed by the dashboard. The heat radiation of the heated dashboard, however, does not transmit back through the windshield due to the wavelength of the radiation. This effect is negligible when solar radiation is not considered. All heat flows mentioned are added to a resulting heat flow for each volume $\dot{Q}_{V,i}$, which is generally calculated as follows:

$$\dot{Q}_{V,i} = \dot{Q}_{windows} + \dot{Q}_{body\ part} + \dot{Q}_{thermal\ masses} + \dot{Q}_{convection} + \dot{Q}_{ventilation} \quad (4)$$

From this, the temperature difference can be calculated in each volume using the general gas equation:

$$\Delta T = \int_0^t \frac{\dot{Q}_{V,i}}{n \cdot c_p} dt = \int_0^t \frac{\dot{Q}_{V,i} \cdot R \cdot T}{p \cdot V \cdot c_p} dt \quad (5)$$

4. Reference measurements and parameter optimization

Measurements on a reference vehicle are used for the comparison between model and reality. Most of the variables are assumed to be constant (at 20°C and 1 bar air pressure) including environment parameters, material-specific properties, and geometric quantities. The choice of vehicle and interior colors also has an influence on the thermal behavior. The reference vehicle uses carbon, the natural fiber kenaf and eucalyptus, [16]. Most of the vehicle components are composed of several materials, so that lumped material properties are assumed for these components. The parameter optimization applied here is intended to compensate the deviations from the real value. The geometric dimensions of the reference vehicle were measured and transferred to simple geometric shapes in the model. All specified parameters are listed in the measurement record [17].

In addition to the measuring points in the vehicle, a further temperature sensor records the ambient temperature. The mean value of the reference vehicle's left and right solar sensors and the vehicle

speed were recorded. The output variables, which were recorded for a target-actual comparison, are the five average volume temperatures.

After recording a reference measurement, it is compared with the results of the model. If measured values and simulation are close enough, the next process step can be continued, which means a further measurement with an additional variable will be compared with the model. However, if the model quality is unacceptable, a parameter optimization is performed. Afterwards measurement and model are compared again. This procedure works iterative until the evaluation criterion is fulfilled, which means the reality is represented sufficiently well, see figure 4.

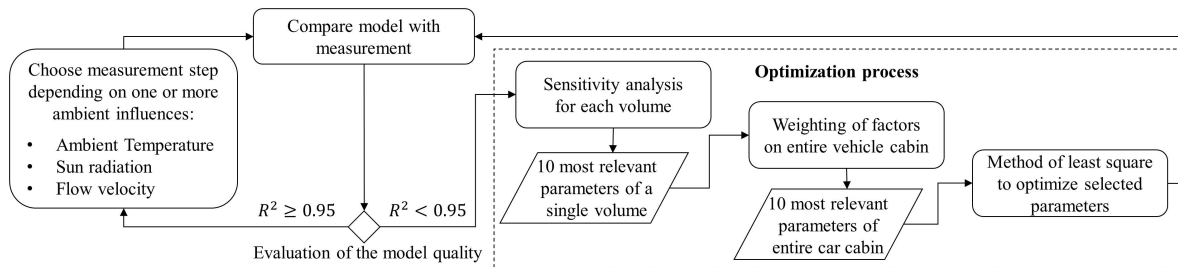


Figure 4. Methodology of optimization process to increase model accuracy. In each step, the measurement is extended by an additional environmental influence and is compared with the model. Finally, the model quality is evaluated.

In order to evaluate the model quality, the coefficient of determination R^2 is used, which indicates the relationship between declared variance and observed variance. R^2 is always between 0¹ (no relationship between the results of the model and the reality) and 1 (the model perfectly reflects reality). For $R^2 \geq 0.9$, there is a strong link between the model and the reality. The objective criterion of optimization was $R^2 \geq 0.95$. The coefficient of determination is calculated as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (6)$$

For the general setting of the reference measurements, the three input variables can be combined with two selected states to a total of eight (2^3) cases, which are shown in table 1. A consideration of both simple cases (Case 1 and 2) makes sense, since different thermodynamic effects come into effect. Therefore, both were considered and summarized. For each iteration, an additional environmental influence (flow velocity or solar radiation) should be added.

Table 1. 8 combinations of the three environmental influences: ambient temperature, flow velocity and solar radiation.

Flow velocity [km/h]		0		available	
Solar radiation [W/m ²]		0		available	
$T_{interior} - T_{ambient}$ [°C]	Positive	Case 1	Case 3	Case 5	Case 7
	Negative	Case 2	Case 4	Case 6	Case 8

If the model quality is not sufficient, a sensitivity analysis with all factors used in the model will be performed so that their respective share of the variance of the output variable can be determined. Each parameter is hereby multiplied with a correction factor, which is initialized with the value 1 and receive upper and lower limits ($\pm 10\%$). Subsequently, for each correction factor datasets are generated from random values in the permitted interval. For each dataset, a simulation is performed and the results are evaluated using the correlation between the correction factor and the curve of the

¹ R^2 is intended to evaluate a linear correlation so that negative values can be calculated for a very large deviation between model and measurement.

volume temperature. The correlation coefficient R between $x^{(i)}$ and $y^{(j)}$, which is in the interval $[-1 \ 1]$, is calculated from the covariance matrix \underline{C} [18]:

$$R(i,j) = \frac{\underline{C}(i,j)}{\sqrt{\underline{C}(i,i)\underline{C}(j,j)}} \quad (7)$$

with

$$C = cov(x,y) = E[(x - \mu_x)(y - \mu_y)] \quad (8)$$

where $\mu_x = E[x]$ and $\mu_y = E[y]$.

All factors are ranked according to their influence [19]. A change in the parameters can have the effect that the temperature profile of one volume is influenced positively, while a different volume is negatively influenced. Therefore, the five volumes are weighted according to their importance in terms of thermal comfort: V1 (0.4), V2 + V3 (each 0.2) and V4 + V5 (each 0.1). The ten most important factors of each volume are given the scores 10 to 1, all others are scored with 0. As a result, the volume-specific influences can be transferred on the entire car cabin. From the weighted parameters, the ten most important are selected for optimization. The cost function used within the optimization is the method of least squares, in which the sum of deviations between the model curve $f(x_i)$ and the values of the measurement \hat{y}_i are minimized [20]:

$$\min \sum_{i=1}^n (f(x_i) - \hat{y}_i)^2 \quad (9)$$

5. Simulation results

Including the general criteria for selecting reference measurements according to table 1, four reference measurements were selected, see table 2, which are discussed here.

Table 2. Setup for the four selected reference measurements.

Reference measurement:	Case 1	Case 2	Case 3	Case 8
Flow velocity [<i>km/h</i>]	0	0	0	$v_{veh.} + v_{wind}$
Solar radiation [<i>W/m²</i>]	0	0	400-700	200-800
Initial vehicle temperature [<i>°C</i>]	20	4	20	20
Ambient temperature ² [<i>°C</i>]	0 to -8	20	20	20
Measurement duration [<i>h</i>]	15	22	3	2
Number of passengers [-]	0	0	0	2

5.1. Results of heating (Case 1) and cooling (Case 2) without sun radiation and flow velocity

Except for the specific heat capacity of the parcel shelf and the external heat transfer coefficients, the top 10 of the heating process (case 1) and the cooling process (case 2) have the same parameters, so that 12 parameters were relevant for the optimization. A detailed description of all optimization results is available in the measurement record [14].

After about 15 hours of cooling, a constant temperature level in all 5 volumes was achieved, see figure 5. The temperature of the dashboard zone (V4) drops to the lowest level. The other volume temperatures are about 1-1.5°C above this level. During the heating, the highest temperature is achieved in the head zone (V3). The difference in temperature levels is caused by temperature stratification in the car cabin, since warm air has lower density and rises as much as possible. After about 10 hours of heating, constant temperatures in all volumes occur. They are only slightly increased until the end of the measurement. The process of cooling is significantly faster compared to the heating process, since the effect of natural convection in the cabin is fully utilized. The cooling of the air on the component's cold surfaces causes a direct increase in density. As a result, the air drops

² The ambient temperature cooled down, because Case 1 was measured overnight.

quickly to the lower parts of the car cabin. This decrease causes a continuous air mass exchange. Whereas, in case of a heating process without sun radiation, there is almost stagnant air.

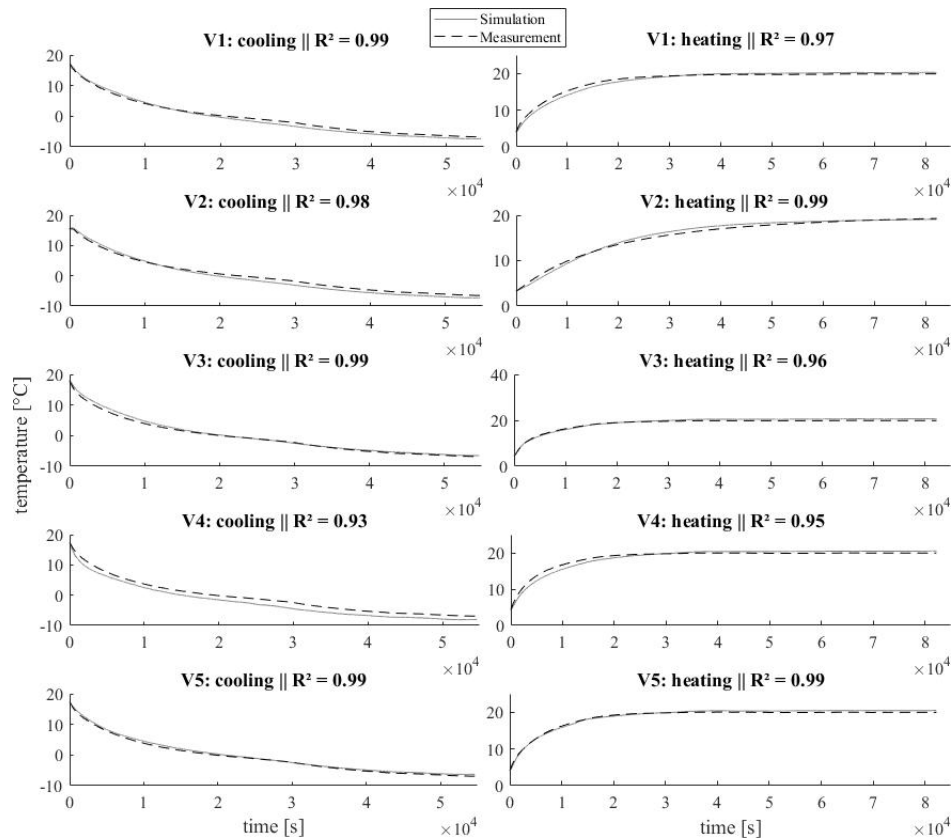


Figure 5. Comparison of simulated and measured temperatures for each volume. The left column shows the cooling process (Case 1) during an ambient temperature of 0°C , which falls to -8°C during the measurement period of 15 hours. The right column shows the heating process (Case 2) during an ambient temperature of 20°C over a period of 22 hours. Both cases are without solar radiation and flow velocity. The accuracy of the model is given by the coefficient of determination R^2 .

Through the optimization of parameters for the heating and cooling process without sun and flow influence, all volumes reach the objective criterion with the exception of the dashboard (V4) during the cooling process. However, this can be tolerated, since all other volumes have a very high coefficient of determination and the dashboard zone has only the fourth priority.

5.2. Results involving solar radiation (Case 3 and Case 8)

The second step focused on the factors that specifically reflect the influence of the sun on the car cabin. In this case, a heating process under the influence of solar radiation was considered. The measurement was recorded in Germany in the spring around noon.

The mean value from the two intensity curves was used as input variable for the simulation via zones V1, V4 and V5 by transmittance as well as indirectly through absorption via V3. The target temperatures were almost reached after 1.5 hours with the exception of V5, but heating up was much too slow, according to figure 6. On the one hand, this is due to the division of the heat flows between the chassis parts and the car cabin, and, on the other hand, due to the consideration of the irradiated surfaces. These miscalculations clearly show that the correct representation of the car cabin condition is only possible under consideration of the sun's position. This depends on the driving direction, but also on location, time of day and season. In addition, the used measurement equipment (the reference vehicle's two built-in solar sensors) is not sufficient to validate the effect of the sun on the 5 zones.

The solar radiation angle has a major influence. Therefore, the sun intensity cannot be considered as a lumped value.

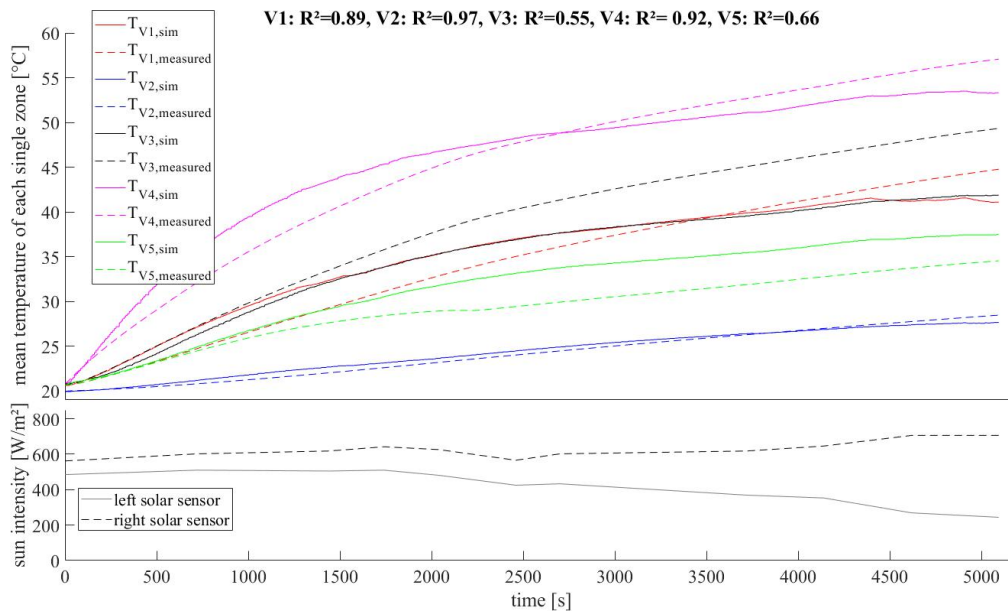


Figure 6. Top: Comparison of the heating curves of simulation and measurement at 20°C ambient temperature and negligible low flow velocity. Bottom: Reference vehicle's recorded solar intensity during the heating process.

This is also confirmed by the observation of the recorded test drive. The simulation results differ from the measured temperatures, as in the first part of the trip the sun was mostly shining in V5 and in the second part in V4 and partly in V1. For this reason, the recorded test drive (case 8) was not displayed. Similarly, the parameter optimizations for case 3 and case 8 were not carried out because it makes no sense to vary the relevant parameters without a correct representation of the sun's influence.

6. Conclusions and future work

The 5-zone interior model presented in this paper can depict the thermal condition of the car cabin depending on the outside temperature, the flow velocity and solar radiation. All heat transfer mechanisms such as heat radiation, heat conduction and heat transitions are taken into account and even natural phenomena caused by the sun's influence are implemented into the model. The achieved model accuracy for the heating and cooling process of the car cabin, which was achieved by parameter optimization, is very satisfactory since they reproduce the reality now very well. Nevertheless, the model is kept so generic, so that an adaptation to a reference vehicle can easily be implemented. The presented methodology allowed to adapt the model step by step to reality by including reference measurements, sensitivity analyzes and subsequent parameter optimization. Due to the realistic representation of the thermal condition in the car cabin, the model will be used as a basis for further investigations, e.g. how much air-conditioning is necessary in order to ensure the passengers' thermal comfort for a given route.

In future work, an automatic climate control for the car cabin, which was already implemented, will be validated. Based on the required heat and mass flow, the electrical energy of a HVAC system is calculated. The simulation results will be compared with the recorded energy consumption of reference vehicle's HVAC system. The 5-zone-car-cabin model can thus be used for further investigations with regard to potential energy saving potential of the HVAC system, which can lead to a direct increase in the range of BEVs. The investigation regarding the sun radiation has shown, that the model and the measurement of sun radiation has some weak points. Therefore, the model will be extended by the calculation of the sun's movement. Then, it will be validated with solar intensity

sensors for each volume instead of a left and right sensor, which could not correctly record the radiation direction and intensity of the sun for each volume.

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