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# Pull-in actuation in hybrid micro-machined contactless suspension

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**Abstract.** This paper presents a result of study of the pull-in phenomenon in the hybrid micro-machined contactless suspension ( $\mu$ -HCS), combining inductive suspension and electrostatic actuation, reported at PowerMEMS 2015 [1]. Assuming the quasi-static behavior of a levitated proof mass, a non-linear analytical model describing the pull-in actuation along the vertical direction is developed. The developed model allows us to predict the static pull-in parameters of the suspension and to show a dependence of these parameters on suspension design. It is shown that the pull-in displacement can be larger by almost a factor of two than one occurring in a spring-mass system with constant stiffness (classic pull-in). The model is verified by using numerical estimations as well as experimental data and agrees well with measurements and calculations.

## 1. Introduction

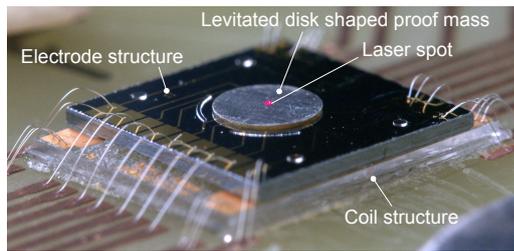
Micro-machined contactless suspensions ( $\mu$ -CS), employing the phenomena of electromagnetic levitation, eliminate mechanical attachments between stationary and moving parts in Micro-Electro-Mechanical Systems (MEMS) and as a result provide the solution of fundamental issue in MEMS related to the domination of friction over inertial forces in the micro-world. As a result, a new generation of micro-sensors and actuators based on levitation have been demonstrated.

Depending on a source of force field,  $\mu$ -CS can be simply classified as electrostatic, magnetic, and hybrid. For instance, electrostatic suspensions ( $\mu$ -ECS) were successfully used in micro-inertial sensors [2]. Magnetic suspensions ( $\mu$ -MCS) can be also further classified as inductive, diamagnetic and superconducting suspensions, which found applications in micro-bearings [3, 4], micro-inertial sensors [5, 6], bistable switches [7] and nano-force sensors [8]. Hybrid suspensions ( $\mu$ -HCS) combine different force fields, which make the main difference of  $\mu$ -HCS from both  $\mu$ -ECS and -MCS.

In particular, capabilities of  $\mu$ -HCS were demonstrated in applications as micro-motors [9, 10] and micro-accelerators [11]. A wide range of different operation modes such as the linear and angular positioning, bistable linear and angular actuations and the adjustment of stiffness components of  $\mu$ -HCS were demonstrated and experimentally studied in the prototypes reported in [1, 12, 13]. Thus,  $\mu$ -HCS establish a promising direction for further improvements in performance and operation capabilities of micro-sensors and -actuators.

In this work, the pull-in phenomenon in  $\mu$ -HCS shown in Fig. 1 is analytically and numerically studied. In order to model micro-machined inductive CSs, the qualitative technique developed in [14], where the induced eddy current within a levitated micro-object is approximated by a magnetic dipole, is used. Note that this technique has been recently generalized in [15, 16], where the induced eddy current is approximated by a system of dipoles. Then, a reduced analytical





**Figure 1.** Fully assembled prototype of hybrid suspension under testing (the detailed description of the device can be found in [1, 12]).

model of  $\mu$ -HCS, which describes the behaviour of levitated proof mass in a vertical direction, is derived. Using the obtained analytical model, the static pull-in behavior of  $\mu$ -HCS is studied and verified by experimental result published in [1, 12].

## 2. A reduced model

In order to develop a model of  $\mu$ -HCS shown in Fig. 1, let us consider its scheme shown in Fig. 2. It consists of stabilization and levitation coils to provide the stable levitation of a dick shaped proof mass (PM) and the embedded electrodes as shown in Fig. 2. Changing the equilibrium position of PM by means of electrostatic force acting on its bottom surface and keeping the same current in the coils the dynamics of  $\mu$ -HCS is adjusted. Also, applying the pull-in voltage to the electrodes the bi-stable actuation is performed [1, 12].

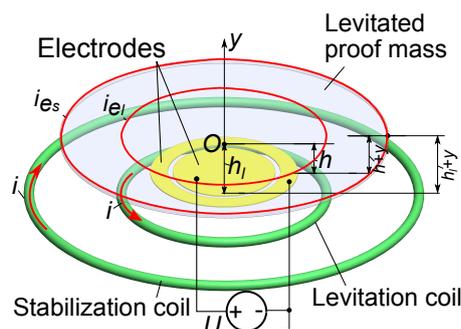
Assigning an origin to the equilibrium point  $O$  and assuming that the resistivity of conducting PM, and its linear and angular velocities are small, then the exact quasi-static nonlinear model, which describes the behavior of PM along the vertical  $y$ -axis, is [16, 17]

$$m \frac{d^2 y}{dt^2} + mg + \frac{I^2}{L} \frac{dM}{dy} M + \frac{A}{4} \frac{U^2}{(h+y)^2} = F, \quad (1)$$

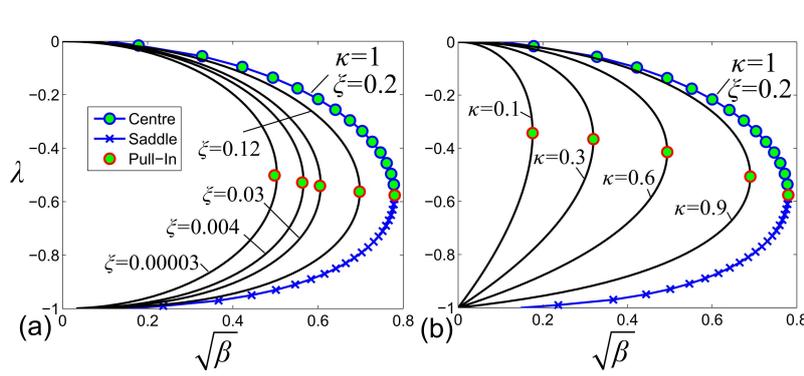
where  $m$  is the mass of levitated PM,  $g$  is the gravity acceleration,  $I$  is the amplitude of a harmonic current  $i$  of coils,  $L$  is the self inductance of the PM,  $M$  is the mutual inductance between the PM and coils,  $U$  is the applied voltage to the electrodes both of each which has the same area of  $A_e$ ,  $A = \varepsilon_0 A_e$ ,  $\varepsilon_0$  is the permeability of free space,  $F$  is the force acting on the PM along the  $y$ -axis.

In a general case, the mutual inductance,  $M$ , is a complex nonanalytic function. This fact becomes the main difficulty for analytical study of suspension model (1). However, accounting for particularities of micro-machined performance of device [15] such that the linear sizes of coils and PM is much larger than the levitation height,  $h_l$ , and the distribution of density of induced eddy current is not homogenous. Due to these particularities of the device, the force interaction along vertical direction is reduced to interaction between eddy current,  $i_{el}$  and levitation coil current [18]. Considering both the levitation coil and the eddy current circuit as filamentary circles, the mutual inductance between the levitation coil and eddy current can be described by the Maxwell formula.

Moreover, upon holding a certain condition described in [13], the Maxwell formula can be well approximated by the logarithmical function [14]. Hence, accounting for (1), the following



**Figure 2.** Scheme for modelling hybrid contactless suspension:  $U$  is the potential applied to electrodes,  $h$  is the space between an electrodes plane and equilibrium point of proof mass,  $h_l$  is the levitation height between a plane of coils and equilibrium point of proof mass,  $i_{el}$  and  $i_{es}$  are the eddy currents corresponding to the maximum current density [16].



**Figure 3.** Bifurcation diagrams and their evolution in depending on the suspension design parameters: a) the effect of  $\xi$  changing in a range from  $3 \times 10^{-5}$  to 0.2; b) the effect of  $\kappa$  in a range from 0.1 to 1.0 (centre and saddle are corresponding to stable and unstable equilibrium, respectively).

reduced analytical model of suspension in the dimensionless form can be proposed as follows

$$\frac{d^2\lambda}{d\tau^2} + 1 - \eta \frac{\kappa}{1 + \kappa\lambda} \left[ \ln \frac{4}{\xi(1 + \kappa\lambda)} - 2 \right] + \frac{\beta}{(1 + \lambda)^2} = \tilde{F}, \quad (2)$$

where  $\eta = I^2 a^2 / (mghL)$ ,  $a = r_l \mu_0$ ,  $\mu_0$  is the magnetic permeability of free space,  $r_l$  is the radius of levitation coil,  $\xi = h_l / (2r_l)$ ,  $\tau = \sqrt{g/h\tau}$ ,  $\lambda = y/h$ ,  $\beta = AU^2 / (4mgh^2)$ ,  $\kappa = h/h_l$  and  $\tilde{F} = F/mg$ . As it is shown in [14], the accuracy of approximation of modelling the electromagnetic force is dependent on the parameter  $\xi$ . If the parameter  $\xi$  is less than 0.3, the electromagnetic force is approximated by the logarithmic function with the error less than six percentages. Upon trending the parameter  $\xi$  to the zero, the error between the exact equation and approximation as well trends to the zero. Worth noting that in all known prototypes of  $\mu$ -HCS published in the literature the parameter  $\xi$  is less than 0.25. This fact provides the applicability of the reduced model for further analytical study of  $\mu$ -HCS.

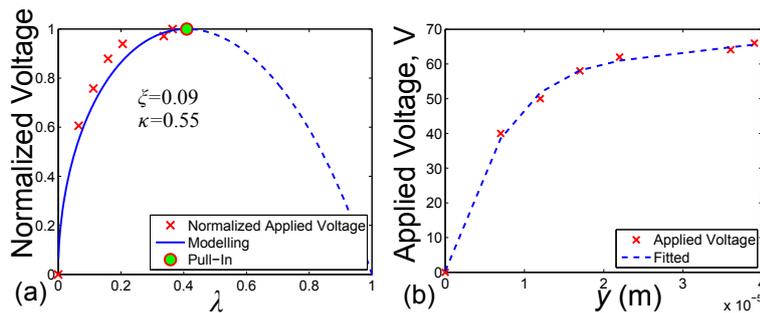
### 3. Static pull-in

At the equilibrium point ( $\lambda = \beta = 0$ ),  $\eta$  must be equal to  $D = \ln 4/\xi - 2$ . Hence, the equilibrium state is defined by the following equation

$$f(\lambda, \beta) \equiv -\frac{\kappa\lambda}{1 + \kappa\lambda} - \frac{\ln(1 + \kappa\lambda)}{D(1 + \kappa\lambda)} - \frac{\beta}{(1 + \lambda)^2} = 0. \quad (3)$$

Using (3), the bifurcation diagram can be mapped as shown in Fig. 3, which depicts the distribution of centre and saddle as well as pull-in points depending on the design parameters. The pull-in points correspond to the transient state, in which the sign of  $f(\lambda, \beta)$  is unchangeable in the vertical direction [19]. For a case of  $\kappa = 1$ , pull-in has the following parameters: displacement is  $\lambda_{pi} = (1 - e - D)/(2D + e)$ , the square of voltage is  $\beta_{pi} = -(\lambda_{pi} + \ln(1 + \lambda_{pi})/D)(1 + \lambda_{pi})$ . For a case, when  $\kappa$  is small ( $\kappa \ll 1$ ), pull-in parameters can be approximated as  $\lambda_{pi} \approx -1/(3 - 2\kappa)$  and  $\beta_{pi} \approx \kappa(1 + 1/D)(2 - 2\kappa)^2/(3 - 2\kappa)^3$ . Once  $\kappa$  tends to zero, the pull-in displacement becomes the same as in classic static pull-in occurring in the spring-mass system with electrostatic actuation [20] and corresponds to  $\lambda_{pi} = 1/3$ . However, the square of pull-in voltage is different from the classic static pull-in and becomes  $\beta_{pi} = \kappa(1 + 1/D)4/27$ .

Fig. 4 provides comparison of results of experimental measurements of static pull-in behavior along vertical direction reported in [1] and modelling. Table 1 shows the parameters of the device



**Figure 4.** Static pull-in behavior for experiment reported in [1]: a) comparison of modelling and experiment (parameters of device are shown in Table 1; b) measurement of applied voltage vs the proof mass displacement [1].

**Table 1.** Results of measurements and modelling of the static pull-in displacement.

	Parameters	Device [1]	Device [12]
Measured	Levitation height, $h_l$	180 $\mu\text{m}$	200 $\mu\text{m}$
	Spacing, $h$	100 $\mu\text{m}$	120 $\mu\text{m}$
	Diameter of levitation coil, $d_l$	2 mm	2 mm
	<b>Pull-In displacement</b>	<b>39 <math>\mu\text{m}</math></b>	<b>45 <math>\mu\text{m}</math></b>
Modelled	$\xi$	0.09	0.1
	$\kappa$	0.55	0.6
	<b>Pull-In displacement</b>	<b>41 <math>\mu\text{m}</math></b>	<b>49 <math>\mu\text{m}</math></b>

and sums up results of study of the static pull-in displacements taking from Fig. 4. In addition, Tab. 1 provides results for  $\mu$ -HCS reported in [12]. As it is seen from analysis of Fig. 4 and Tab. 1 the modelling agrees well with experimental data.

#### 4. Conclusion

In this work the static pull-in behavior of  $\mu$ -HCSs have been studied. Pull-in parameters were defined analytically based on the developed model. Results of modelling were verified by experimental data. This fact provides the applicability of the reduced model for further analytical study of pull-in phenomenon in  $\mu$ -HCS. Moreover, we showed that the pull-in displacement in  $\mu$ -HCS is larger in comparing with classic static pull-in displacement. In particular, for a  $\mu$ -HCS having design parameters such as  $\kappa = 1$  and  $\xi = 0.2$  the pull-in displacement can be larger by almost a factor of two in compared to classic one.

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