Observation of $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \chi_{b12}(1P)$ and search for $e^+ e^- \rightarrow \phi \chi_{b12}(1P)$ at $\sqrt{s} = 10.96 - 11.05$ GeV


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We report searches for the processes $e^+e^-\rightarrow \pi^+\pi^-\pi^0\chi_{b1,2}(1P)$ and $e^+e^-\rightarrow \phi\chi_{b,j}$ ($j = 1, 2$) based on data samples collected by the Belle experiment at the KEKB collider. We report the first observation of the process $e^+e^-\rightarrow (\pi^+\pi^-\pi^0)_{\text{non-}\chi_{b,j}}$ and first evidence for $e^+e^-\rightarrow \omega\chi_{b,j}$ in the vicinity of the $\Upsilon(11020)$ resonance, with center-of-mass energies from 10.96 to 11.05 GeV. The significances for $(\pi^+\pi^-\pi^0)_{\text{non-}\chi_{b1}}$ and $\omega\chi_{b,j}$ are greater than 5.3$\sigma$ and 4.0$\sigma$, respectively. We also investigate the energy dependence of the $e^+e^-\rightarrow \pi^+\pi^-\pi^0\chi_{b,j}$ cross section, but we cannot determine whether the contributions are from the $\Upsilon(10860)$ and $\Upsilon(11020)$ resonances or nonresonant continuum processes. The signals for $e^+e^-\rightarrow \phi\chi_{b,j}$ are not significant, and the upper limits of the Born cross sections at the 90% confidence level are 0.7 and 1.0 pb for $e^+e^-\rightarrow \phi\chi_{b1}$ and $\phi\chi_{b2}$, respectively, for center-of-mass energies from 10.96 to 11.05 GeV.

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TABLE I. The predicted branching fractions of $\Upsilon(11020) \to \omega \chi_{bj}$ and $\phi \chi_{bj}$ [16], as well as the relative magnitudes, where $B_j = B(\Upsilon(11020) \to \omega(\phi) \chi_{bj})$, $R_{ij} = \frac{B_j}{B_i}$.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$B_0$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$R_{10}$</th>
<th>$R_{20}$</th>
<th>$R_{31}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega \chi_{bj}$</td>
<td>$(0.15-2.81) \times 10^{-3}$</td>
<td>$(0.63-11.68) \times 10^{-3}$</td>
<td>$(1.08-20.02) \times 10^{-3}$</td>
<td>$\approx 4.11$</td>
<td>$\approx 7.06$</td>
<td>$\approx 1.72$</td>
</tr>
<tr>
<td>$\phi \chi_{bj}$</td>
<td>$(0.68-4.62) \times 10^{-6}$</td>
<td>$(0.50-3.43) \times 10^{-6}$</td>
<td>$(2.22-15.18) \times 10^{-6}$</td>
<td>$\approx 0.74$</td>
<td>$\approx 3.28$</td>
<td>$\approx 4.43$</td>
</tr>
</tbody>
</table>

including all possible decays is used to study the possible background channels and investigate the background shape.

For charged tracks, the impact parameters perpendicular to and along the beam direction with respect to the interaction point are required to be less than 1.0 and 3.5 cm, respectively. The transverse momentum is restricted to be higher than 0.1 GeV/c. A particle identification (PID) hypothesis [24] $L(X)$ for each charged track is formed from different detector subsystems for particle $X \in e, \mu, \pi, K, p$. Tracks with a likelihood ratio $R(K) = L(K) / (L(K) + L(\pi)) < 0.4$ are identified as pions while those with $R(K) > 0.6$ are identified as kaons. Similarly, we define the likelihood ratios $R(e)$ and $R(\mu)$ for identification of electrons and muons, respectively, with $R(e) > 0.01$ and $R(\mu) > 0.1$. A neutral cluster in the electromagnetic calorimeter is reconstructed as a photon if it does not match the extrapolated position of any charged track and its energy is greater than 30 MeV.

To select $e^+ e^- \to \pi^+ \pi^- \chi_{bj}$ candidates, we require that there be exactly four tracks with zero net charge, of which two are positively identified as pions and the other two as leptons. At least three photons are required in the event, and a $\pi^0$ list is created with the invariant mass of the photon pairs satisfying $M_{\gamma\gamma} \in [0.12, 0.15]$ GeV/c$^2$, which covers nearly $\pm 3\sigma$ around the $\pi^0$ peak. To improve the track momentum and photon energy resolutions, and to suppress the background, a five-constraint (5C) kinematic fit is performed for the $\gamma \pi^+ \pi^- \pi^0 e^+ e^-$ candidates enforcing energy and momentum conservation and constraining the invariant mass of $\pi^0$ candidates. The four momenta of the final-state particles after the 5C kinematic fit are kept for further analysis. The $\chi^2_{5C}/\text{ndf}$ is required to be less than 20, where $\chi^2_{5C}$ is the resulting $\chi^2$ of the kinematic fit, and ndf is the number of degrees of freedom. If there are multiple $\pi^0$ candidates surviving the kinematic fit in an event, the one with the smallest $\chi^2_{5C}$ is kept. The lepton pair is taken as an $\Upsilon(1S)$ candidate if its invariant mass is in the region $[9.42, 9.60]$ GeV/c$^2$.

The $\chi_{bj}$ candidates are reconstructed with the selected $\Upsilon(1S)$ and the photon not used to form a $\pi^0$ candidate. The invariant mass of $\pi^+ \pi^- \pi^0$ ($M(\pi^+ \pi^- \pi^0)$) versus the corrected invariant mass of $\gamma \Upsilon(1S)$ ($M(\gamma \Upsilon(1S)) = M(\gamma e^+ e^-) - M(e^+ e^-) + m_{\Upsilon(1S)}$) is shown in Fig. 1 for the sum of the data samples in the $\Upsilon(11020)$ energy region, which is defined as $E_{c.m.} > 10.96$ GeV. Clusters of events for the production of $\chi_{bj}$ can be seen both when $M(\pi^+ \pi^- \pi^0)$ is in the $\omega$ mass region ($[0.75, 0.81]$ GeV/c$^2$) and at higher masses ($> 0.81$ GeV/c$^2$). For events having $M(\pi^+ \pi^- \pi^0)$ in the $\omega$ mass region, the $\chi_{bj}$ signal is dominant while for signal events with higher $\pi^+ \pi^- \pi^0$ masses, the $\chi_{bj}$ signal is dominant. The background in this case comes predominantly from false $\pi^0$ candidates produced by combinatorial photons.

An unbinned two-dimensional (2D) extended maximum likelihood fit to the $M(\pi^+ \pi^- \pi^0)$ and $M(\gamma \Upsilon(1S))$ distributions of the candidate events is applied to determine the numbers of $\omega \chi_{bj}$ and $\pi^+ \pi^- \pi^0 \chi_{bj}$ events. In the fit, the shapes of $\omega \chi_{bj}$ and $\pi^+ \pi^- \pi^0 \chi_{bj}$ obtained from MC simulation are used to describe the signals, and a 2D function $f(x, y) = ax + by$ ($x = M(\gamma \Upsilon(1S))$ and $y = M(\pi^+ \pi^- \pi^0)$) is used to fit the background. Here $\pi^+ \pi^- \pi^0 \chi_{bj}$ MC sample is generated following a four-body phase space
(PHSP) distribution, and this process is denoted as \(\pi^+\pi^-\pi^0\chi_{b1}\). The projections of the fit results for events in the \(\chi_{b1}\) signal region (top right), in the \(\omega\) signal region (bottom left), and out of the \(\omega\) signal region (bottom right).

Changes in the signal yields and significances are shown in Fig. 1. The projections of the fit results for events in the \(\chi_{b1}\) signal region, in the \(\omega\) signal region, and in the region above the \(\omega\) mass are also shown in Fig. 1. The statistical significances for \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\), \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\), \(\omega\), and \(\omega\) are 5.5\%, 0.6\%, 0.6\%, and 2.5\%, respectively. The significances are calculated based on the change in likelihood when the signal yield is set to zero in the fit [25]. The signal yields for \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\) and \(\omega\) are 19.6 \pm 5.3 and 7.8 \pm 3.2, respectively, and the signal yields for \(\omega\) and \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\) are consistent with zero. Then we assume that the processes \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\) and \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\) exist at the same time, or the processes \(\omega\) and \(\omega\) exist at the same time, and the fit is repeated. The statistical significances for \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\) and \(\omega\) are 6.1\% and 4.6\%, respectively. The changes on the significances arise from the similarity in signal shapes between \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\) and \((\pi^+\pi^-\pi^0)_{\text{non-}\omega}\), and between \(\omega\) and \(\omega\). Thus, evidence for \(\omega\) has been found, but we cannot determine whether the events are from \(\omega\) or \(\omega\). We also use other forms of background descriptions as systematics. Changes in the signal yields and significances are negligible.

In order to study the energy dependent cross section of \(\pi^+\pi^-\pi^0\chi_{b1}\) and \(\pi^+\pi^-\pi^0\chi_{b2}\) events, we extract the observed signal yields \(N_{\text{obs}}\) with data samples listed in Table II. Because of the limited statistics for most energy points, we do not perform a 2D fit as for the summed sample, nor do we separate \(\pi^+\pi^-\pi^0\) into \(\omega\) and \(\omega\), nor \(\gamma\) into \(\chi_{b1}\) and \(\chi_{b2}\). The number of \(\chi_{b1}\) signal events in each sample is computed using the formula: \(N_{\text{obs}} = N_{\text{sig}} - N_{\text{side}}\), where \(N_{\text{sig}}\) is the number of events in the \(\chi_{b1}\) signal region and \(N_{\text{side}}\) is that in the sideband region. Here the signal region is defined as \(M(\gamma_{\chi(1S)}) \in [9.852, 9.952] \text{ GeV/c}^2\), while the sideband region is \([9.77, 9.82]\) and \([9.98, 10.03] \text{ GeV/c}^2\).

The Born cross sections are calculated with

\[
\sigma_{\text{Born}} = \frac{N_{\text{obs}}}{eB\delta\mp \sqrt{1 - \Pi^2}},
\]

where \(e\) is the reconstructed efficiency, \(B\) is the corresponding product of intermediate decay branching fractions, \(\delta\) is the integrated luminosity, \((1 - \delta)\) is the ISR correction factor, and \((1/|1 - \Pi^2|)\) is the vacuum polarization factor [26]. We use the weighted branching fraction \(B_{\text{inter}} = \frac{B(\chi_{b1} \rightarrow \gamma(Y(1S)), \gamma + \gamma(Y(1S)) \rightarrow \gamma(1S)) - (1 - f)}{f}\), where \(f = N_1/(N_1 + N_2) = 0.74 \pm 0.06\) is the fraction of \(\chi_{b1}\) in the process \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\chi_{b1}\) near the \(Y(10860)\) peak [4]. In order to estimate the ISR correction factors, we use

\[
1 + \delta = \int_0^{1/m_0} \frac{G_{\text{BW}}(s)(1 - x)}{G_{\text{BW}}(s)} F(x, s) dx,
\]

where \(m_0\) is the mass threshold of \(\pi^+\pi^-\pi^0\chi_{b1}\), \(F(x, s)\) is the radiative function [23] and \(G_{\text{BW}}(s)\) is the Born cross section.

The energy-dependent cross sections for \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\chi_{b1}\) are listed in Table II and plotted in Fig. 2. A maximum likelihood fit of the cross sections is performed.

![Image 1](image1.png)

**FIG. 1.** A scatter plot of \(M(\pi^+\pi^-\pi^0)\) versus \(M(\gamma_{\chi(1S)})\) from data (top left), and the projections of the 2D fit for events in the \(\chi_{b1}\) signal region (top right), in the \(\omega\) signal region (bottom left), and out of the \(\omega\) signal region (bottom right).

![Image 2](image2.png)

**FIG. 2.** Fit to the cross sections of \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\chi_{b1}\) as described in the text. The red boxes with error bars are the cross sections of \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\chi_{b1}\) and the solid blue curve is the fit.
The likelihood for the three data samples of larger integrated luminosity around 10.865 GeV is calculated assuming the number of signal events follows the Gaussian distribution:

\[
L(\mu_{\text{sig}}; N_{\text{obs}}, \sigma) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(\mu_{\text{sig}} - N_{\text{obs}})^2}{2\sigma^2}},
\]

where \(\mu_{\text{sig}}\) is the number of expected signal events, and \(\sigma\) is the statistical uncertainty of \(N_{\text{obs}}\). For the other samples, the likelihood is calculated assuming the number of signal events follows the Poisson distribution:

\[
L(\mu_{\text{sig}}; N_{\text{sig}}, N_{\text{side}}) = \int_0^\infty P(N_{\text{sig}}; \mu_{\text{sig}} + \mu_{\text{bkg}}) P(N_{\text{side}}; \mu_{\text{bkg}}) d\mu_{\text{bkg}},
\]

where \(P(N; \mu) = \frac{1}{\sqrt{2\pi}\mu^N e^{-\mu}}\) is the probability density function of the Poisson distribution, and \(\mu_{\text{bkg}}\) is the number of expected background events. Since the known cross section energy dependences, i.e., those of \(\pi\pi \Upsilon(nS)\) \[8\] and \(\pi\pi\phi(mP)\) \[9\], exhibit \(\Upsilon(10860)\) and \(\Upsilon(11020)\) peaks but no nonresonant contributions. The fit function here is also a coherent sum of two BW amplitudes in the form of Eq. (3) for \(\Upsilon(10860)\) and \(\Upsilon(11020)\), and the masses and widths are fixed to their world average values \[22\] while the corresponding products \(\Gamma_{\pi\pi} \cdot B\) are left free. The fit results are shown in Fig. 2. Two solutions are found that differ in phase, but the resulting \(\Gamma_{\pi\pi} \cdot B\) are consistent with each other. The obtained product branching fractions are \(B(\Upsilon(10860) \rightarrow e^+e^-) \cdot B(\Upsilon(10860) \rightarrow \pi^+\pi^-\chi_{bJ})(15.3 \pm 3.7) \times 10^{-9}\), \(B(\Upsilon(11020) \rightarrow e^+e^-) \cdot B(\Upsilon(11020) \rightarrow \pi^+\pi^-\pi^0\chi_{bJ}) = (18.3 \pm 9.0) \times 10^{-9}\), where the errors are statistical. We also try to introduce a coherent continuum component into the fit, but the significance of this hypothesis is only 1.4\(\sigma\). The introduction of the continuum term results in a change of the \(\Upsilon(10860)\) product branching fraction of \(12.6 \times 10^{-9}\) and that of the \(\Upsilon(11020)\) product branching fraction of \(12.8 \times 10^{-9}\), which are taken as systematic uncertainty due to “continuum contribution.”

There are several sources of systematic error in the cross section measurements, and most of the uncertainties are similar to the previous work \[4\], including tracking efficiency (1.0% per pion and kaon track and 0.35% per lepton), PID efficiency (1.3% per pion and 1.6% per lepton), photon energy resolution calibration (1.1%), \(\pi^0\) selection (2.2%), 5C kinematic fit (4.2%), and trigger simulation (3.0%). The uncertainty from luminosity is 1.5% \[9\]. Comparing the reconstruction efficiency with the ISR process in EVTGEN with the efficiency without the ISR process added to EVTGEN, but still corrected for with the ISR correction factor, yields an uncertainty of 1.0%. The corresponding uncertainty from the branching fractions of \(\chi_{bJ} \rightarrow \gamma\Upsilon(1S), \Upsilon(1S) \rightarrow e^+e^-\) is 8.2\% \[22\].

The total systematic uncertainty, 11.9\%, is obtained by adding all the above results in quadrature.

The systematic uncertainty in the measured branching fractions rises from the cross section measurements and the fit to those cross sections. The systematic uncertainties in the fit to the cross sections mainly come from the parameterization of the BW function, PHSP factor, resonance parameters, and the possible continuum contribution. The first is estimated by replacing the constant width with an energy dependent width \(\Gamma_{\text{tot}} = \Gamma_{\text{tot}} \cdot \Phi(\sqrt{s})/\Phi(M)\). The second source is estimated by replacing the PHSP factor of \(\pi^+\pi^-\pi^0\chi_{bJ}\) with the two-body PHSP factor of \(\rho\chi_{bJ}\). The third source is estimated by varying the resonance parameters \(\Upsilon(10860)\) and \(\Upsilon(11020)\) within \(\pm 1\sigma\). The final systematic uncertainty is estimated by adding a coherent continuum contribution to the fit function. The changes of the branching fractions are taken as the symmetrized systematic uncertainty. The details are listed in Table III.

| TABLE III. Summary of the absolute systematic uncertainties in product branching fractions ($\times 10^{-6}$), where $B(\Upsilon(10860), 11020)$ represent $B(\Upsilon(10860), 11020) \rightarrow e^+e^-$, $B(\Upsilon(10860), 11020) \rightarrow \pi^+\pi^-\pi^0\chi_{bJ}$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$B(\Upsilon(10860))$</th>
<th>$B(\Upsilon(11020))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross sections</td>
<td>1.8</td>
<td>2.1</td>
</tr>
<tr>
<td>BW parameterization</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>PHSP factor</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>Resonance parameters</td>
<td>2.4</td>
<td>1.6</td>
</tr>
<tr>
<td>Continuum contribution</td>
<td>12.2</td>
<td>12.6</td>
</tr>
<tr>
<td>Sum</td>
<td>12.6</td>
<td>12.8</td>
</tr>
</tbody>
</table>
We use the figure of merit, $S/\sqrt{S + B}$, to optimize the $K^+K^-$ invariant mass window requirement. Here $S$ is the reconstructed number of signal events obtained from MC simulation of the signal process, $\Upsilon(11020) \to \phi\chi_{bJ}$ with $\phi \to K^+K^-$, $\chi_{bJ}$ anything, in the signal region, $[9.88, 9.93]$ GeV/c$^2$. The number is normalized according to the theoretical calculation of the branching fraction of $\Upsilon(11020) \to \phi\chi_{bJ}$ [16] and the total $\Upsilon(11020)$ events in our data sample. $B$ is the number of background events in the signal region in the generic MC sample with the c.m. energy shifted to 11.022 GeV. We require $M(K^+K^-)$ to be within $m_\phi \pm 7.5(7.0)$ MeV/c$^2$ for category one (two), where $m_\phi$ is the nominal mass of $\phi$ [22]. The $\phi$ mass sideband region is defined as $M(K^+K^-) \in [1.000, 1.005]$ or [1.035, 1.040] GeV/c$^2$. There is no evidence for the $\chi_{bJ}$ signal in the $\phi$ mass sideband events, nor in the generic MC sample (significance is less than 0.1$\sigma$ from the fit) mentioned above.

After applying all the selection criteria, the recoil mass spectra of $\phi$ as a function of the initial beam four momenta from both data categories are shown in Fig. 3 for the sum of data in the energy region $\sqrt{s} = 10.96$–11.05 GeV. We perform a simultaneously unbinned maximum likelihood fit to the $\phi$ recoil mass spectra with the signal shapes from the simulated signal MC shapes, and a background shape obtained from data with the following procedure: a series of shapes are obtained from $\Upsilon(5S)$ data, where, in calculating the $\phi$ recoil mass, the c.m. energy is changed to that of each individual data point, and summing up the shapes according to the luminosity. The ratios of the numbers of $\chi_{b1}$ or $\chi_{b2}$ events in the two categories are fixed according to the expected branching fractions of $\chi_{b1}$ or $\chi_{b2} \to \gamma\Upsilon(1S)$ [22] and the efficiencies. The fit results, which yield $\chi^2/\text{ndf} = 104.2/55 = 1.9$, are shown in Fig. 3. According to the fit, $(1.5 \pm 0.5) \times 10^3 \chi_{b1}$ and $(2.4 \pm 0.5) \times 10^3 \chi_{b2}$ events are produced. The statistical significances are found to be 3.3$\sigma$ and 4.8$\sigma$ for $\chi_{b1}$ and $\chi_{b2}$, respectively.

When we vary the background shape by multiplying the nominal background shape with a first-, second-, or third-order polynomial, the smallest significances of the $\chi_{b1}$ and $\chi_{b2}$ signals are found to be 2.6$\sigma$ and 2.1$\sigma$, respectively (multiplying by the third-order polynomial), yielding $\chi^2/\text{ndf} = 43.6/49 = 0.89$. The most conservative upper limits on the numbers of produced signal events in all the above tests are reported. After considering the systematic uncertainty which we discuss later, the upper limits for the produced numbers of $\phi\chi_{b1}$ and $\phi\chi_{b2}$ signal events are determined to be $2.2 \times 10^3$ and $3.1 \times 10^3$ at 90% confidence level (C.L.), respectively. The upper limits on the Born cross sections of $e^+e^- \to \phi\chi_{b1}$ and $\phi\chi_{b2}$ are 0.7 and 1.0 pb, respectively, averaged over the $\Upsilon(11020)$ region, specifically $\sqrt{s} = 10.96$–11.05 GeV. The calculation is based on Eq. (1), where the reconstruction efficiency, ISR correction factor, and vacuum polarization factor are averaged with weights according to the luminosity of each sample.

The sources of systematic uncertainties in the $\phi\chi_{bJ}$ cross section measurement are similar to those of the $\pi^+\pi^-\pi^0\chi_{bJ}$ modes, including the tracking efficiency, PID, photon detection, luminosity, trigger simulation, ISR correction, $\phi$ mass window, and intermediate branching fraction. Most of these have been discussed in the $\pi^+\pi^-\pi^0\chi_{bJ}$ analysis. The uncertainty from the $\phi$ mass window requirement is found to be negligible by studying the consistency of the $K^+K^-$ invariant mass between data and MC simulation. The uncertainty from the branching fraction of $\phi \to K^+K^-$ is 1.0% [22]. The total systematic uncertainty for the cross section measurement is thus, combining all uncertainties in quadrature, 5.5% for either $e^+e^- \to \phi\chi_{b1}$ or $\phi\chi_{b2}$.

In summary, using the energy scan data in the vicinity of the $\Upsilon(11020)$ resonance, we observe the $e^+e^- \to (\pi^+\pi^-\pi^0)^{\text{non}-\omega}\chi_{b1}$ process with significance of 5.3$\sigma$. Evidence for $\omega\chi_{b1}$ or $\omega\chi_{b2}$. The limited statistics prevents us from drawing a conclusion concerning the origin of the signal events, that is, whether they arise from bottomonium decay, continuum production, or both. Since no continuum production of a multibody final state with a bottomonium is known, it is natural to assume that the origin of the signal is bottomonium. Under this assumption, the branching fractions are $B(\Upsilon(10860) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (2.5 \pm 0.6 \pm 2.1 \pm 0.7) \times 10^{-3}$, which is compatible with the previous measurement [4], and $B(\Upsilon(11020) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (8.7 \pm 4.3 \pm 6.1^{+4.3}_{-2.5}) \times 10^{-3}$.
which is compatible with the theoretical predictions [16]. Based on the 2D fit with summed data, the relative magnitude $R_{21}(\omega) \equiv \frac{\mathcal{B}(\Upsilon(11020) \to n\phi_{bJ})}{\mathcal{B}(\Upsilon(11020) \to a\phi_{bJ})}$ can be estimated to be $0.4 \pm 0.2$, where the common systematic uncertainties cancel.

The processes $e^+e^- \to \phi_{bJ}$ are also searched for in data within $\sqrt{s} = 10.96$–11.05 GeV, with no significant signals being observed. We report upper limits on the Born cross sections of $e^+e^- \to \phi_{bJ}$ and $\phi_{bK}$ as 0.7 and 1.0 pb at 90% C.L., respectively. Compared with the total cross section of $e^+e^- \to \Upsilon(11020)$, these upper limits correspond to $\Upsilon(11020)$ decay branching fractions of order $10^{-5}$, well above the theoretical predictions of order $10^{-6}$ [16].

Our measurement of the transition rate agrees with the expectation of Ref. [16], but the measured relative magnitudes $R_{21}(\omega)$ are significantly less than the theoretical predictions, which should be more reliable than the branching fraction predictions. This may inspire theorists to further investigate the discrepancy between the experimental measurement and the theoretical calculation.

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[2] K.F. Chen et al. (Belle Collaboration), Observation of Anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ Production Near the $\Upsilon(5S)$ Resonance, Phys. Rev. Lett. 100, 112001 (2008).
[4] X.H. He et al. (Belle Collaboration), Observation of $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ and Search for $X_b \to \omega\Upsilon(1S)$ at $\sqrt{s} = 10.867$ GeV, Phys. Rev. Lett. 113, 142001 (2014).
[6] P. Krotov et al. (Belle Collaboration), First observation of the $Z^{0}_{b}(10610)$ in a Dalitz analysis of $\Upsilon(10860) \to \Upsilon(nS)\pi^0\pi^0$, Phys. Rev. D 88, 052016 (2013).
[7] A. Garmash et al. (Belle Collaboration), Amplitude analysis of $e^+e^- \to \Upsilon(nS)\pi^+\pi^-$ at $\sqrt{s} = 10.865$ GeV, Phys. Rev. D 91, 072003 (2015).
[8] K.-F. Chen et al. (Belle Collaboration), Observation of an enhancement in $e^+e^- \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$, and $\Upsilon(3S)\pi^+\pi^-$ production around $\sqrt{s} = 10.89$ GeV at Belle, Phys. Rev. D 82, 091106 (2010).
[9] A. Abdesselam et al. (Belle Collaboration), Energy Scan of the $e^+e^- \to h_b(nP)\pi^+\pi^-$ ($n = 1, 2$) Cross Sections and Evidence for $\Upsilon(11020)$ Decays into Charged Bottomonium-Like States, Phys. Rev. Lett. 117, 142001 (2016).
[14] X. Li and M.B. Voloshin, Contribution of $Z_b$ resonances to $\Upsilon(5S) \to \pi\pi\gamma_{bJ}$, Phys. Rev. D 90, 014036 (2014).
[17] D. Santel et al. (Belle Collaboration), Measurements of the $\Upsilon(10860)$ and $\Upsilon(11020)$ resonances via $\sigma(e^+e^- \to \Upsilon(nS)\pi^+\pi^-)$, Phys. Rev. D 93, 011101 (2016).


