A Scalable Port-Hamiltonian Approach to Plug-and-Play Voltage Stabilization in DC Microgrids

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Abstract: One of the major challenges of voltage stabilization in DC microgrids are the large system scale and multiple interacting components. In recent publications, this has been addressed by a scalable and decentralized control design method referred to as plug-and-play. In contrast to the existing approaches, the method presented in this paper does not require the proposition of a Lyapunov function for control design and subsequent stability analysis, and can handle unknown loads (disturbances). It is based on passivity techniques, namely interconnection and damping assignment passivity-based control (IDA-PBC), and port-Hamiltonian systems (PHSs). The Hamiltonian being a byproduct of the modeling step, serves as natural Lyapunov function in our approach, which obviates the proposition step. An extension of the controller with integral action (IA) robustifies the design against unknown loads. A simulation which illustrates the functionality of the proposed voltage controller concludes our contribution.

Keywords: voltage stabilization, DC microgrids, plug-and-play, scalable control, port-Hamiltonian systems, passivity-based control, integral action

1. INTRODUCTION

Microgrids have been established as a central concept for future electrical energy supply systems. They are local entities comprising flexible loads and distributed generation units (DGUs) that encompass also storage devices. These DGUs are commonly connected to the electrical network via controllable power inverter interfaces functioning as voltage sources (Schiffer et al., 2016) (Guerrero et al., 2011) (Meng et al., 2017).

Regarding their modeling and control, microgrids provide a manifold of challenges. A central one is the stabilization of the bus voltages (Dragičević et al., 2016, p. 4877) (Schiffer et al., 2016), which in DC microgrids directly determine the currents and thus power flows through the network to cover the spatially distributed load demands. In principle, the voltage stabilization is structured hierarchically with the basic control task being performed by local voltage controllers at primary level. In order to cope with the complexity of microgrids, these controllers are usually implemented in a decentralized or distributed manner (cf. fig. 2 Meng et al. (2017)). The voltage references for the primary controllers are then set by higher level controls to achieve control goals that require more system information such as overall energy management via power flow control or optimization tasks (Meng et al., 2017, pp. 929-930) (Guerrero et al., 2011, p. 160) (Schiffer et al., 2016, p. 9) (Dragičević et al., 2016).

The most popular control method at primary level is decentralized, droop-based voltage control that works without communication on top of inner current and voltage control loops (Guerrero et al., 2011, p. 160) (Dragičević et al., 2016, p. 4877). However, it suffers from load-dependent voltage deviations and propagation of voltage errors along resistive transmission lines. Thus, it must be supplemented with at least a secondary control, which in turn requires some communication system, just to stabilize the voltage (Dragičević et al., 2016) (Meng et al., 2017). An alternative approach to decentralized, primary voltage control is plug-and-play control as proposed in (Tucci et al., 2016) and extended in (Tucci et al., 2018). It allows an offset-free voltage stabilization already at primary level as well as adding or removing DGUs without modifying existing controllers. Furthermore, its local control design requires only local information about the considered DGU inverter interface making it a fully decentralized, scalable control design method which is independent of the overall microgrid size. Regarding an economic perspective, the pure local information requirement is additionally compatible with high privacy standards in market systems. However, this approach is based on solving linear matrix inequalities and quasi-stationary approximations of line dynamics that neglect line inductances. While the latter is only valid in low-voltage networks, the former may suffer from numerical infeasibility. This lead to Nahata et al. (2018) which addresses the plug-and-play voltage stabilization problem in DC microgrids by applying passivity theory and its close link to Lyapunov stability (cf. (Sepulchre et al., 1997, pp. 40)).
Hamiltonians are of the quadratic form (Duindam, V.)
energy in the system. Throughout this paper we consider
smooth function of the states representing the total stored
semidefinite). The Hamiltonian
g
put vector (Duindam, V. et al., 2009, p. 69).
control output vector, and
put vector,
In (1),
In the first part, we formulate the subsystems
control design and a subsequent proof of global (asymp-
totic) Lyapunov stability of the DC microgrid bus voltage
can be concluded. However, the approach of Nahata et al.
(2018) (i) necessitates to find an appropriate Lyapunov
function which (i) can tackle DC microgrids with unknown
loads and (ii) directly delivers a Lyapunov function for
the desired reference voltages. From this, overall asymp-
totic Lyapunov stability of the DC microgrid stability
is generally not fully available.
In this paper, we address these issues by proposing an IDA-
PBC approach extended by IA on the basis of PHSs for
voltage stabilization. To the best of the authors’ knowledge
such an approach has not been proposed in literature yet.
In summary, our main contributions comprise the devel-
oping of a scalable plug-and-play voltage stabilization
method which (i) can tackle DC microgrids with unknown
loads and (ii) directly delivers a Lyapunov function for
control design and a subsequent proof of global (asym-
totic) voltage stability.

2. BASIC PROCEDURE

In the first part, we formulate the subsystems DGU and
electrical line comprising any DC microgrid as PHSs of the
form
\[
\dot{x} = [J(x) - R(x)] \frac{\partial H(x)}{\partial x} + g(x)u + k(x)d,
\]
\[
y = g^T(x) \frac{\partial H(x)}{\partial x},
\]
\[
z = k^T(x) \frac{\partial H(x)}{\partial x}.
\]
In (1), \(x \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}^m\) is the control
input vector, \(d \in \mathbb{R}^d\) is the disturbance vector, \(y \in \mathbb{R}^m\) is the
control output vector, and \(z \in \mathbb{R}^d\) is the disturbance output
vector (Duindam, V. et al., 2009, p. 69). \(J(x), R(x), g(x),\) and \(k(x)\) are real-valued matrices of respective sizes
with \(J(x) = -J^T(x)\) and \(R(x) = R^T(x) \succ 0\) (positive
semidefinite). The Hamiltonian \(H(x) : \mathbb{R}^n \rightarrow \mathbb{R}\) is a
smooth function of the states representing the total stored
energy in the system. Throughout this paper we consider
electric circuits with linear storage elements. Thus, the
Hamiltonians are of the quadratic form (Duindam, V.
et al., 2009, p. 109)
\[H(x) = \frac{1}{2} x^T Q x,\]
with \(Q = Q^T\). The minimum of such Hamiltonians is the
system’s equilibrium \(x^*\). Based on representation (1)
and by employing the Hamiltonian as Lyapunov function,
we then design an IDA-PBC for a general DGU inverter
interface to achieve voltage reference tracking. In a last
step, the IDA-PBC is extended by an IA which preserves
the port-Hamiltonian form. This robustifies the IDA-
PBC and ensures zero steady-state voltage errors in the
presences of unknown loads.

For a later voltage stability analysis, we establish the following lemma:

**Lemma 1.** If \(Q \succ 0\) and \(R(x) = R^T(x) \succ 0\) (positive
definite), then a PHS (1) is strictly passive and globally
asymptotically stable in the equilibrium \(x^*\) defined by
the minimum of its Hamiltonian and Lyapunov function,
respectively.

**Remark 1.** Variables referring to equilibria will be denoted by “*”.

**Proof.** (i) If \(Q \succ 0\), then \(H(x)\) is a positive definite
function (cf. (Khalil, 2002, p. 117)) at \(x^*\) (in this case
\(x^* = 0\)) that is \(H(x^*) = 0\) and \(H(x) > 0\) for all \(x \neq x^*\).
(ii) Furthermore, it is
\[
\frac{dH(x)}{dt} = \frac{\partial T H(x)}{\partial x} \dot{x}
\]
\[
= -\frac{\partial T H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} < 0 \quad \forall x \neq x^*.
\]
as \(R(x) \succ 0\). (i) and (ii) then imply strict passivity
(Khalil, 2002, p. 236) and \(H(x)\) is a Lyapunov function.
Radially unboundedness and positive definiteness of
\(H(x)\) at \(x^*\) and strict passivity then imply the global
asymptotic stability of the equilibrium \(x^*\) of (1) for \((d \equiv 0) u \equiv 0\)
by Lyapunov’s direct method (van der Schaft, 2017, pp.
44–45).

3. MODELING

In section 3.1 we introduce the microgrid to be modeled
and identify relevant subsystems. Sections 3.2 and 3.3
present port-Hamiltonian models of the two main sub-
systems, i.e., DGU and line. Passivity and voltage stability
of the overall model is discussed in section 3.4.

3.1 System description

We consider a DC microgrid in islanded mode. The
microgrid consists of DGUs, loads, and lines. The loads are
mapped to the terminals of the DGU inverter interfaces
by Kron reduction (Dörfler and Bullo, 2013). This allows
for a bipartite graph representation of the DC microgrid
as shown in fig. 1. The bipartite graph consists of multiple
instances of two subsystems, i.e., DGUs and lines. Note
that each DGU can be connected to an arbitrary number of
electrical lines. The terminals of the DGUs are denoted as
point of common coupling (PCC) where PCC, is the PCC
for the i-th DGU. In the sequel, port-Hamiltonian models
for the two subsystems DGU and line are presented.

3.2 DGU inverter interface model

Fig. 2 depicts the structure of any DGU \(i\). It consists of
a DC voltage source, which may represent a renewable
energy source or a storage, a buck converter, and a series
RLC filter. In order to avoid additional constraints in
the subsequent control design, the DC voltage source is
assumed to represent an infinite power source and the
buck converter is considered as ideal transformer without
operational constraints. A current source providing \( I_{Li}(U_i) \) describes the aggregated loads from Kron reduction. \( I_{Ni} \) is a network exchange current at \( PCC_i \) that is the net-current injected into the microgrid which equals the accumulated incoming and outgoing line currents

\[
I_{Ni} = - \left( \sum I_{l,in} - \sum I_{l,out} \right) \tag{4}
\]

at \( PCC_i \). From a modeling point of view, \( I_{Li}(U_i) \) and \( I_{Ni} \) are neither controllable nor measured and thus represent unknown disturbances. Based on fundamental electrical network theory, a port-Hamiltonian model of form (1) of the DGU inverter interface is defined by

\[
\dot{x}_i = \begin{bmatrix} \frac{-R_{Li} - 1}{L_{Li}} & 1 & 0 \end{bmatrix} \frac{\partial H_i(x_i)}{\partial x_i} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_i + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \xi_i, \tag{5a}
\]

\[
y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\partial H_i(x_i)}{\partial x_i}, \tag{5b}
\]

\[
\zeta_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\partial H_i(x_i)}{\partial x_i}, \tag{5c}
\]

\[
H_i(x_i) = \frac{1}{2L_{Li}} x_{1,i}^2 + \frac{1}{2C_{Li}} x_{2,i}^2 = \frac{1}{2} x_i^T \begin{bmatrix} \frac{1}{L_{Li}} & 0 \\ 0 & \frac{1}{C_{Li}} \end{bmatrix} x_i, \tag{5d}
\]

with \( x_i = [\psi_{ti},Q_{ti}]^T, u_i = U_{ti}, y_i = I_{ti}, \) and disturbances \( d_i = [-I_{Ni} - I_{Li}(U_i)]^T \) and \( \xi_i = [U_{ti},U_{ti}]^T \) due to interaction with the lines and unknown load. The variables \( \psi_{ti} \) and \( Q_{ti} \) are the the flux linkage of \( L_{li} \) and the charge of \( C_{li} \), respectively. For the sake of IDA-PBC design, an undisturbed DGU inverter interface model is required (cf. Ortega and García-Cansco (2004)). The undisturbed model consists of

\[
\dot{x}_i = \begin{bmatrix} \frac{-R_{Li} - 1}{L_{Li}} & 1 & 0 \end{bmatrix} \frac{\partial H_i(x_i)}{\partial x_i} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_i, \tag{6a}
\]

\[
y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\partial H_i(x_i)}{\partial x_i}, \tag{6b}
\]

\[
H_i(x_i) = \frac{1}{2L_{Li}} x_{1,i}^2 + \frac{1}{2C_{Li}} x_{2,i}^2 = \frac{1}{2} x_i^T \begin{bmatrix} \frac{1}{L_{Li}} & 0 \\ 0 & \frac{1}{C_{Li}} \end{bmatrix} x_i. \tag{6c}
\]

3.3 Electrical line model

Fig. 3 shows the structure of an exemplary electrical line \( l \). The line is described by a \( \pi \)-section equivalent circuit with parameters \( R_l, L_l \). Shunt conductances are neglected. Line capacitances are not considered explicitly but may be lumped with the filter capacitance \( C_l \) in the DGU model. The voltage control design reduces to controlling the DGU inverter interfaces.

\[
J_l = 0, R_l = R_l, g_1 = 1, Q_l = \frac{1}{2L_l} \tag{7}
\]

where \( x_l = \psi_l, u_l = U_l - U_j, \) and \( y_l = I_l \). The variable \( \psi_l \) is the flux linkage of the line inductance. Note that (7) has no disturbance input and output.

3.4 Microgrid Voltage Stability

The interconnection of the models from sections 3.2 and 3.3 in the microgrid is accomplished according to (4), (5), and (7) by

\[
u_i = U_i - U_j, \tag{8a}
\]

\[
y_i = I_t, \tag{8b}
\]

\[
d_{1,i} = -I_{Ni} = \sum I_{l,in} - \sum I_{l,out}. \tag{8c}
\]

\[
\zeta_{1,i} = U_i, \tag{8d}
\]

Such interconnections are skew-symmetric and consequently passivity-preserving (cf. Nahata et al. (2018), lemma 1). Furthermore, (7) defines a strictly passive system according to lemma 1. Thus, arbitrary interconnections of DGUs and lines, as indicated in fig. 1, always yield an overall strictly passive DC microgrid model with globally asymptotically stable bus voltages, if the \( i = 1, \ldots, N \) DGU models are strictly passive and the minima of their Hamiltonians specify such equilibria that the desired references \( U_{ref,i} \) are established. Thus, the following control design reduces to controlling the DGU inverter interfaces.

4. PLUG-AND-PLAY VOLTAGE CONTROL DESIGN

The voltage control design follows two steps: First, we design a passivity-based voltage controller for an undisturbed DGU model (6) by an IDA-PBC technique. This establishes an equilibrium of the controlled system at the reference voltage \( U_{ref,i} \) for each \( PCC_i \) by a higher-level control, instead of at the natural equilibrium \( x_i^* = 0 \) implying the impractical operating point

\[
x_{2,i}^* = Q_{ref,i}^* = C_{ti} U_{ref,i}^* = 0 \Rightarrow U_i^* = 0. \tag{9}
\]

Furthermore, the IDA allows for adjusting the closed-loop system dynamics. Secondly, an IA, which preserves the PHS form of the previous IDA-PBC, is added to compensate the disturbance currents. The result is an
IDA-PBC law with IA which is robust against unknown disturbances.

4.1 Passivity-based voltage controller

From the desired equilibrium at \( U_{\text{ref},i} \), follows
\[
U_i^* = U_{\text{ref},i} \Rightarrow x_{2,i} = Q_{\text{ref},i} = C_{ui} U_{\text{ref},i},
\]
for \( x_{2,i} \), i.e. the charge stored in the capacitor \( C_{ui} \). In order to adjust the closed-loop equilibrium to this reference value of
\[
x_i^* = [0, x_{2,i}^*]^T,
\]
we use algebraic IDA (Ortega and García-Canesco, 2004). Proposition 1. The nonlinear state feedback controller obtained from algebraic IDA with the desired Hamiltonian
\[
H_{d,i}(x_i) = \frac{1}{2T_{u,i}} x_{1,i}^2 + \frac{1}{2C_{ui}} (x_{2,i} - x_{2,i}^*)^2,
\]
the parameterization
\[
J_{d,i} = \begin{bmatrix} 0 & -J_{12}(x_i) \\ J_{12}(x_i) & 0 \end{bmatrix}, \quad R_{d,i} = \begin{bmatrix} r_{1,i}(x_i) & 0 \\ 0 & r_{2,i}(x_i) \end{bmatrix},
\]
where \( r_{1,i}(x_i), r_{2,i}(x_i) \geq 0 \forall x_i \), and the full-rank left annihilator
\[
g_i^+ = g_{2,i}^+ \left[ 0, 1 \right], \quad g_{2,i}^+ \neq 0 \forall x_i
\]
is given by
\[
u_i = \beta_i(x_i) = \left[ g_i^+ g_i^+ \right]^{-1} g_i^T \left( [J_{d,i} - R_{d,i}] \frac{\partial H_{d,i}(x_i)}{\partial x_i} - [J_i - R_i] \frac{\partial H_i(x_i)}{\partial x_i} \right)
\]
\[
= x_{2,i}^* + \frac{1}{L_{ti}} (R_{d,i} - r_{1,i}(x_i)) x_{1,i}. \tag{15b}
\]

Proof. In the first step, we perform the so-called energy shaping by applying the simplest possible shift to (6c) such that its new minimum and thus the systems equilibrium is established at \( x_i^* \) which yields the desired Hamiltonian (12). Then, (12) is inserted into the IDA-PBC matching equation (cf. Ortega and García-Canesco (2004))
\[
g_{2,i}^+(x_i) \left[ J_i(x_i) - R_i(x_i) \right] \frac{\partial H_i(x_i)}{\partial x_i} = 0,
\]
\[
g_{2,i}^+(x_i) \left[ J_{d,i}(x_i) - R_{d,i}(x_i) \right] \frac{\partial H_{d,i}(x_i)}{\partial x_i},
\]
where \( g_i^+(x_i) \) is a full-rank left annihilator of \( g_i(x_i) \), i.e. \( g_i^+(x_i) g_i^-(x_i) = 0 \) and
\[
x_i = [J_{d,i}(x_i) - R_{d,i}(x_i)] \frac{\partial H_{d,i}(x_i)}{\partial x_i} \tag{17a}
\]
\[
y_i = g_i - \frac{\partial H_{d,i}(x_i)}{\partial x_i} \tag{17b}
\]
is the desired, closed-loop port-Hamiltonian DGU model. In this case, this yields an algebraic system of equations in \( J_{d,i}, R_{d,i}, g_i^+ \) whose solution establishes the so-called IDA and provides together with (12) the desired passivity-based voltage controller.

Remark 2. In general (16) is a system of linear, first order PDEs which can be simplified to an algebraic system of equations by specifying the desired Hamiltonian as done in (12).

For the solution of these algebraic equations, we parameterize the interconnection and damping matrices of the desired PHS from (16) with (13). Furthermore, we select the most general full-rank left annihilator (14) since \( g_i = [1, 0] \). Inserting (6), (12), (13), and (14) into the matching equation (16) then results in
\[
g_{2,i}^+(x_i) \left[ 0, 1 \right] \left[ \begin{array}{c} x_{2,i} \\ \frac{x_{2,i}}{C_{ui}} \end{array} \right] = \frac{g_{2,i}^+(x_i) \left[ J_{12}(x_i) r_{2,i}(x_i) \right]}{L_{ti}} \left( \frac{x_{2,i}}{x_{2,i} - x_{2,i}^*} \right).
\]
Comparing the coefficients for \( x_{1,i} \) and \( x_{2,i} \) in (18) yields
\[
g_{2,i}^+(x_i) \frac{x_{1,i}}{L_{ti}} = g_{2,i}^+(x_i) J_{12}(x_i) \frac{x_{1,i}}{L_{ti}} \tag{19}
\]
\[
0 = g_{2,i}^+(x_i) r_{2,i}(x_i) \frac{1}{C_{ui}} (x_{2,i} - x_{2,i}^*) \tag{20}
\]
from which \( J_{12}(x_i) = -1 \) and \( r_{2,i}(x_i) = 0 \) follow. Finally, with (6), (12), and (13), the nonlinear state feedback controller (15) can be calculated according to (Ortega and García-Canesco, 2004, p. 433).

The dynamics of the closed-loop DGU inverter interface which is obtained by inserting (12) and (15b) into (17), are specified by the PHS
\[
\dot{x}_i = \begin{bmatrix} -r_{1,i}(x_i) -1 \\ 1 \end{bmatrix} \left[ \begin{array}{c} \frac{x_{2,i}}{L_{ti}} \\ \frac{1}{C_{ui}} (x_{2,i} - x_{2,i}^*) \end{array} \right], \tag{21a}
\]
\[
y_i = \begin{bmatrix} 0 \\ \frac{x_{1,i}}{L_{ti}} \end{bmatrix}, \tag{21b}
\]
\[
H_{d,i}(x_i) = \frac{1}{2T_{u,i}} x_{1,i}^2 + \frac{1}{2C_{ui}} (x_{2,i} - x_{2,i}^*)^2. \tag{21c}
\]
The damping parameter \( r_{1,i}(x_i) \geq 0 \) represents a degree of freedom in the control design which allows to adjust the damping of the \( x_{1,i} \) dynamics. Without loss of generality we assume \( r_{1,i}(x_i) > 0 \forall x_i \neq x_i^* \) in the following, since \( r_{1,i}(x_i) = 0 \) for any \( x_i \neq x_i^* \) introduces a vanishing resistance – and consequently damping – which is impractical. With this, the strict passivity and global asymptotic stability of the closed-loop DGU PHS (21) regarding the equilibrium (11), which establishes \( U_{\text{ref},i} \) for any DGU \( i \), follows immediately from lemma 1.

4.2 Integral action

In the previous subsections, the control design and stability analysis has been based on the undisturbed DGU PHS (6). In order to robustify the control and guarantee a zero-steady state error of the voltage set point, the control input is extended by IA via \( v_i \) to
\[
u_i(x_i) = \beta_i(x_i) + v_i = U_i \tag{22}
\]
(cf. (Ortega and García-Canesco, 2004, p. 445)) yielding the disturbed, closed-loop DGU dynamics
\[
\dot{x}_i = \begin{bmatrix} -r_{1,i}(x_i) -1 \\ 1 \end{bmatrix} \frac{\partial H_{d,i}(x_i)}{\partial x_i} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d_i + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i \tag{23}
\]
Remark 3. In the sequel, we assume the disturbances \( d_i \) to be piecewise constant over time.

Since the disturbances \( d_i \) act on the state variable \( x_{2,i} \), which is not directly influenced by \( u_i(x_i) \), the standard IA in the PHS framework via output feedback of passive outputs as suggested in Ortega and García-Canesco (2004) is not applicable and the method via state transformation from Donaire and Junco (2009) is employed.
Proposition 2. The IA after Donaera and Junco (2009) for the closed-loop DGU dynamics (23) is given by

\[ v_i = [r_1, \dot{r}_1, \dot{r}_1, \dot{r}_2] \frac{x_{1,i}}{L_{ti}} + \frac{k_{1,i}}{C_{ti}} \int \left( x_{2,i} - x_{1,i} \right) dt + L_{ti} (x_{2,i}^* - x_{2,i}) \]  

(24)

Proof. For the IA design, the disturbances in the closed-loop DGU dynamics (23) are neglected and (23) is factorized into its canonical form

\[ \dot{x}_h = \begin{bmatrix} J_1 - R_1 & 0 & 0 \\ J_{ti} - R_{ti} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{z}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g_1 \end{bmatrix} v_i \]

(25)

with \( x_1 = x_{1,i} \) and \( x_h = x_{2,i} \) (cf. Donaera and Junco (2009); Kotyczka (2013)). This separates the state vector \( x_i \) into one \((n_1 = 1) \) actuated or relative-degree-one (RD1) state \( x_1 \), which is directly influenced by the controlled input (here \( v_i \)), and one \((n_h = 1) \) higher relative-degree (HRD) or unactuated state \( x_h \) that receives no direct action through \( v_i \).

Following Donaera and Junco (2009), (i) we initially transform the two-dimensional \((n = n_1 + n_h = 2) \), canonical DGU dynamics (25) into a three-dimensional \((m = n_1 + n_h + n_c = 3) \) extended PHS in new z-coordinates \n
\[ \begin{bmatrix} \dot{z}_{1,i} \\ \dot{z}_{2,i} \\ \dot{z}_{e,i} \end{bmatrix} = \begin{bmatrix} -r_1(z_i) - 1 & 0 & 0 \\ 0 & -k_{1,i} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial H_{x,i}(z_i)/\partial z_{1,i} \\ \partial H_{x,i}(z_i)/\partial z_{2,i} \\ \partial H_{x,i}(z_i)/\partial z_{e,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{C_{ti}} \end{bmatrix} v_i \]

(26)

\[ H_{x,i}(z_i) = H_{x,i}(z_{1,i}, z_{h,i}) + \frac{1}{2} z_{e,i}^2 \]

(27)

\[ = \frac{1}{2L_{ti}} z_{1,i}^2 + \frac{1}{2C_{ti}} (z_{h,i} - z_{e,i})^2 + \frac{1}{2} k_{1,i} z_{e,i}^2 \]

(28)

in which one \((n_c = 1) \) additional state realizes the IA

\[ z_{e,i} = k_{1,i} \int \frac{\partial H_{x,i}(z_i)}{\partial z_h} \, dt = k_{1,i} \frac{1}{C_{ti}} \int \left( z_{h,i} - z_{e,i}^* \right) \, dt. \]

(29)

The closed-loop DGU PHS (21) modified by IDA-PBC in form of the interconnection and damping matrices as well as the energy-shaped Hamiltonian (21c) are preserved by this transformation. Only the entries \( k_{1,i} \) in the interconnection matrix, which correspond to the IA, and a quadratic term in the extended state \( z_{e,i} \) in the Hamiltonian are added.

(ii) Afterwards, we establish the HRD state transformation

\[ z_{h,i} = x_{h,i} \]

(30)

such that the equilibrium in terms of \( z_{h,i} \) matches the desired one \( x_{h,i} \) implying \( U_{ref,i} \). Subsequently, we find the RD1 state transformation which satisfies requirement (30) by solving

\[ \frac{\partial H_{x,i}(z_i)}{\partial x_{1,i}} = \frac{\partial H_{x,i}(z_i)}{\partial z_{1,i}} - k_{1,i} \frac{\partial H_{x,i}(z_i)}{\partial z_{e,i}} \]

\[ x_{1,i} = \frac{z_{1,i}}{C_{ti}} - k_{1,i} \frac{z_{e,i}}{k} \]

(31)

(cf. Donaera and Junco (2009) (11)-(12)) for \( z_{1,i} \) which yields

\[ z_{1,i} = x_{1,i} + \frac{k_{1,i} L_{ti}}{k} z_{e,i} =: \Psi(x_{1,i}, x_{h,i}, z_{e,i}). \]

(32)

(iii) Finally, we compute the integral control law \( v_i \) from (cf. Donaera and Junco (2009) (13))

\[ \dot{z}_{h,i} = -r_1(z_i) \frac{z_{1,i}}{L_{ti}} - \frac{1}{C_{ti}} (z_{h,i} - z_{h,i}^*) \]

\[ = -\frac{1}{C_{ti}} (z_{h,i} - z_{e,i})^2 + \frac{k_{1,i} L_{ti}}{k} \left( z_{h,i} - z_{e,i}^* \right) \]

(33)

by inserting (29), (30), (32), \( x_1 = x_{1,i}, x_h = x_{2,i} \), and solving for \( v_i \) which yields (24). Note that we eliminated \( k^{-1} \) in (34) by setting \( k = k_{1,i} \) as it offers no more degrees of freedom in our case. \( \square \)

4.3 Passivity-based voltage controller with integral action

The overall control input (22) defining \( U_{ti} \) is now calculated with (15b) and (24) to

\[ u_i = \frac{x_{2,i}^2 L_{ti}}{C_{ti}} + (R_{ti} - r_{1,i}(x_{1,i}, z_{e,i})) \frac{x_{1,i}}{L_{ti}} \]

\[ + \frac{k_{1,i}}{C_{ti}} \int \left( x_{2,i}^* - x_{2,i} \right) dt + L_{ti} (x_{2,i}^* - x_{2,i}) \]

(35)

where \( k_{1,i}, r_{1,i}(x_{1,i}, z_{e,i}) \) are the control parameters, \( x_{2,i}^* \) is the reference and the states are the measurements. The block diagram of the control structure is illustrated in fig. 4. It can be seen that (35) amounts to a linear PI-controller for the HRD state \( x_{2,i} \) and a nonlinear state feedback law for the RD1 state \( x_{1,i} \). In case the degree of freedom in the damping assignment \( r_{1,i}(x_{1,i}, z_{e,i}) \) is not used and \( r_{1,i}(x_{1,i}, z_{e,i}) = R_{ti} \), the control structure becomes a linear PI-controller for the HRD state \( x_{2,i} \), which represents the IA, with a standard feedforward of the reference (voltage) which represents the energy shaping step for the undisturbed DGU PHS (see fig. 5).
5. SIMULATION

In order to assess the functionality of the designed passivity-based voltage controller with IA, we simulate the interaction of two controlled DGU inverter interfaces connected via one electrical line (see fig. 3) with SimScape in MATLAB/SIMULINK. Furthermore, both DGUs are initially connected to constant current loads of 4 A to investigate the IA behavior.

The DGUs are identically parameterized with $R_{t1} = 0.2 \ \Omega$, $L_{t1} = 1.8 \ \text{mH}$, $C_{t1} = 2.2 \ \text{mF}$ for $i = 1, 2$ and the electrical line with $R_{12} = 0.05 \ \Omega$, $L_{12} = 2.1 \ \mu\text{H}$ (cf. Tucci et al. (2016)). Reference voltages are set to $U_{\text{ref,1}} = 49.9 \ \text{V}$ and $U_{\text{ref,2}} = 50 \ \text{V}$, which causes in the stationary case a 2 A current flow from DGU 2 to DGU 1 over the electrical line. For convenience, the control parameters are identical for both DGUs: $k_{1,i} = 500 \ \text{s}^{-1}$, $g_{1,i}(x_{i}, z_{c}) = 5R_{t1} = 1 \ \Omega$ ($i = 1, 2$). The latter allows an intuitive interpretation of the damping assignment as it is tantamount to replacing the original filter resistance with a five times higher value.

At 3 s, the constant current load at DGU 1 changes from 4 A to 6 A and a constant power load of 250 W is added. The resulting bus voltages $U_1$ and $U_2$ as well as their references are depicted in fig. 6. Under constant current loads, i.e., from 2.98 s–3.00 s, and approximately 0.08 s after the load change, both voltages show zero-steady state errors which confirm the desired IA behavior and the expected equilibrium shift due to the voltage controllers. From around 3.05 s, both voltages remain within a ±0.1 V band around their references, demonstrating a reasonably fast disturbance behavior.

6. CONCLUSION

In this paper, we presented a new approach to scalable, plug-and-play voltage stabilization in DC microgrids which follows an IDA-PBC design with additional IA on the basis of PHSs. Due to the port-Hamiltonian modeling, we directly obtain the Lyapunov function necessary for control design and stability analysis and obviate its possibly cumbersome proposition which is mandatory in existing approaches. Furthermore, the IA enables our voltage controller to handle unknown disturbances, if they can be considered as piecewise constant over time. This limitation together with the consideration of operational constraints on the power inverters and DGUs, respectively, will be addressed in future work.

REFERENCES


