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On the influence of the lamella's elasticity on self-excited vibrations in gearboxes

Georg Jehle[1,*] and **Alexander Fidlin**[1]

[1] Karlsruhe Institute of Technology, Institute of Engineering Mechanics, Kaiserstraße 10, 76131 Karlsruhe

Eek noise in a gearbox of a vehicle drivetrain is a phenomenon, which can arise while shifting between gears and which is not accepted by customers. Beneath audible squeaking, it can cause damage of mechanical components. There is a wide range of possible reasons for the occurrence of this effect, which strongly depends on properties of the considered gearbox (physical parameters, geometry, operation, ...). From the mathematical point of view, the occurrence can be predicted using linear stability analysis of the stationary behaviour of a physically motivated gearbox model.
The components of a gearbox are clutch discs being in contact, gears and elastically supported shafts. In this contribution, a rigid multibody model of the device [4] is extended by the elastic modelling of the motor's side disc (rotating Kirchhoff plate). The aim of the overall system is to analyze the shifting process.
The analysis reveals that beneath instability mechanisms which are known from systems with rigid bodies, new instabilities occur incorporating of out-of-plane vibrations of the plate. In a reasonable parameter region, the first two unsymmetrical modes of the lamella have the main contribution to the instability.

1 Introduction

Friction induced vibrations constitute a wide research field. They have been studied in models of simple structure like 1-DoF-oscillators as well as in more complex ones like automotive disc brakes [1]. Low-frequency vibrations often arise because of the Stribeck effect, whereas the reason for higher frequency vibrations usually is a mode-coupling flutter instability. In vehicle drivetrains, there are several friction sources with the clutch disc the most important representative. The eek instability at frequencies in the range of 200Hz and 1000Hz (mode-coupling flutter), which arises between clutch discs, has extensively been studied [2–4]. In particular in [4], an eek instability was found in a rigid body model of the clutch disc, shafts and gears, where the effect arises because of the interaction of friction between the discs and coupling of DoFs in the gears. Beneath those observations, the literature of brake squeal (e.g. [5–7]) reveals that the interaction of elastic friction discs can also introduce mode-coupling flutter instabilities.
The objective of this contribution is to investigate to which extent elastic plate modes can be relevant for vibration phenomena in shift gearboxes. For this reason, in section 2 the gearbox model is discussed. In section 3, the stationary behaviour during the shifting process is investigated.

2 Modelling

The model of investigation (cf. fig. 1) consists of a crankshaft rigidly connected with a clutch pad (lamella), a gear input shaft, a clutch disc, a gear pair on input shaft and a layshaft and bearings. To derive differential equations which describe the behaviour, first a Hamilton functional H is set up which takes into account all participating components.
During the shifting process, the gear input shaft, the clutch disc and the gear stick together, thus they are one rigid body $\mathcal{B}^{(1)}$ (mass m_1, inertia J_1). This component has a rotational DoF $\dot{\varphi}^{(1)}\boldsymbol{e}_z$ and all translational DoFs $\boldsymbol{u}^{(1)}$. The clutch disc is rigid. Thus, for this body, the kinetic energy T_G, the stored energy in the bearing U_B and the dissipation δW^B_{Diss} contribute to H.
Gear flanks on $\mathcal{B}^{(1)}$ and $\mathcal{B}^{(2)}$ have no penetration. This condition results in a nonholonomic constraint $0 = \boldsymbol{n}_c^{(1)} \cdot \boldsymbol{v}_{rel}$ as a simple but adequate gear model [4], where the Lagrange multiplier is the contact normal force. Friction effects in the gears and the influence of the flank stiffness are neglected. Here, the virtual work of the constraint force contributes to H, which vanishes when the constriant is fulfilled.
The clutch primary side (lamella on body $\mathcal{B}^{(3)}$) is modelled using Kirchhoff's plate theory in a frame rotating with Ω. It is hollow-cylindrical (thickness h, outer radius r_o, inner radius r_i) and hinged at $r = r_o$. For the transition between the material-fixed description of velocity, strain and stress of material points to an inertial-fixed description, the coordinate transformation and discretization

$$u(R,\phi,t) = w(r,\varphi,t) \approx \sum_{k=1}^{n} R_k(r)\Phi(\varphi)a_k(t), \qquad r = R, \qquad \varphi = \phi + \Omega t \tag{1}$$

* Corresponding author: e-mail georg.jehle@kit.edu

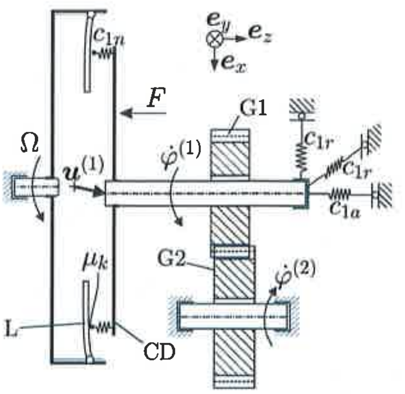

Fig. 1: Gearbox model (side view)

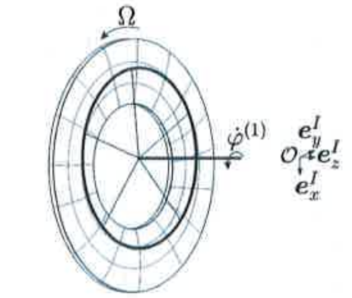

Fig. 2: Discs of the gearbox model

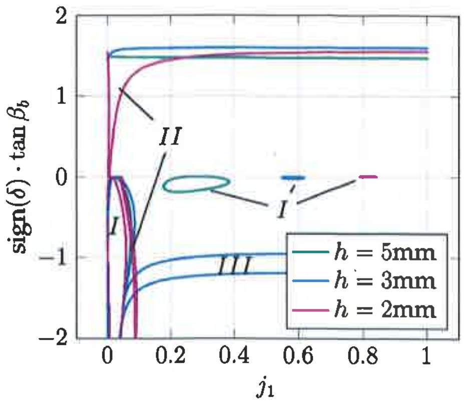

Fig. 3: Map of stability

is applied. Here, R and Φ are material-fixed coordinates and u the deformation in material-fixed coordinates, whereas r and φ are space-fixed coordinates and w the space-fixed deformation. The contribution of the plate to H consists of kinetic (T_L) and potential (U_L) energy as well as structural dissipation δW^L_{Diss}.
The last stage of the modelling is the contact between the discs, which is idealized as a ring (radius R). The contact itself incorporates both normal and tangential forces and torques. Thus, a contact potential ($U_{C,N}$) and dissipative energy (δW^C_{Diss}) as well as virtual work of friction forces ($\delta W_{C,T}$) contribute to H.
In total, the constrained variational problem is given through the Hamilton functional and the constraint

$$0 = \int_{t_1}^{t_2} \{\delta\,[T_L + T_G - U_L - U_{C,N} - U_B] + \delta W_{C,T} + \delta W_{Diss} + \delta W_\lambda\}\,dt, \qquad 0 = \boldsymbol{n}_c^{(1)} \cdot \boldsymbol{v}_{rel}, \tag{2}$$

Together with a discretization (eq. (1)) using an appropriate functional basis, the fundamental lemma of variational calculus yields a DAE. This system of equations is linearized around the stationary solution in order to investigate stability by analyzing the generalized eigenvalues.

3 Results

In the stationary case, the trajectory of the geometrical center of $\mathcal{B}^{(1)}$ is deflected mainly in direction $\boldsymbol{n}_c^{(1)}$ (normal direction of the active tooth flank). This is a result of the eccentrically acting contact force between the teeth. The thinner the lamella, the larger its deflection, which is not symmetric because of the eccentricity of $\mathcal{B}^{(1)}$.
In the spectrum of eigenvalues, 3 rigid body modes are found, which include the coupled eigenmotion of $\mathcal{B}^{(1)}$ and a symmetric lamella mode. Higher modes are attributed to non-symmetric plate modes (synchronous and reverse rotation). The thicker the lamella, the higher are the frequencies of these modes. A frequency jump is found when the slip δ between the plates vanishes. The maximum real part of eigenvalues in the $(J_1/(m_1 R^2) - \tan\beta_b \cdot \mathrm{sign}(\delta))$-parameter space reveals that the plate thickness has a strong influence on the stability of the solution. Here, β_b is the helical gearing angle. The different domains of instable stationary solutions in the parameter space (cf. fig. 3) feature translational rigid body and symmetric plate modes (I), circumferential rigid body and plate modes (II), and a combination of both (III).

4 Conclusions

A minimal model of a shift gearbox was presented, which incorporates an elastic friction disc, and in which new instability mechanisms were found, which incorporate the interaction of elastic and rigid body modes. Though the model is a simplified approach, it is a first and effective step in order to find more phenomena which are related to elastic plate dynamics.

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