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A DAE formulation for geared rotor dynamics including frictional contact between the teeth

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A DAE approach is presented for geared rotor dynamics simulations with rigid helical evolvent gears. It includes the normal contact force between the teeth as well as tangential components.

Given the evolvent tooth flank geometry of gear 1 and gear 2 [1], the contact line and the velocity difference in the contact are found. The requirement of no penetration of the teeth yields a non-holonomic constraint and the contact normal force. The friction caused force and moment are obtained by applying Coulomb's friction model.

This approach is used to investigate the dynamics of two ideal rotors with translational DoFs, which are connected by gears to one another. The driving rotor has a given angular speed, while the driven rotates unrestrainedly and is connected to a rotational damper. Because of the periodic friction terms, the solution is periodic. A direct time integration or a harmonic approach can be used for the numerical computation.

1 Kinematics

A general position on an evolvent flank surface on gear 1 or 2 (cf. fig. 1) is given in the inertial space $\{\boldsymbol{e}_x^I, \boldsymbol{e}_y^I, \boldsymbol{e}_z^I\}$

$$\begin{aligned} \boldsymbol{x}^{(1)} &= \boldsymbol{u}^{(1)} + r_{b1}(\cos\gamma_1 + q^{(1)}\sin\gamma_1)\boldsymbol{e}_x^I + r_{b1}(q^{(1)}\cos\gamma_1 - \sin\gamma_1)\boldsymbol{e}_y^I + z^{(1)}\boldsymbol{e}_z^I, \\ \boldsymbol{x}^{(2)} &= \boldsymbol{u}^{(2)} + r_{b2}(\cos\gamma_2 + q^{(2)}\sin\gamma_2)\boldsymbol{e}_x^I + r_{b2}(q^{(2)}\cos\gamma_2 - \sin\gamma_2)\boldsymbol{e}_y^I + z^{(2)}\boldsymbol{e}_z^I + a_W\boldsymbol{e}_x^I, \end{aligned} \tag{1}$$

where the index $j \in \{1,2\}$ denotes the gear, r_{bj} is the respective radius of base circle, $q^{(j)}$ and $z^{(j)}$ body-fixed coordinates, $\boldsymbol{u}^{(j)}$ the displacement of the gears and a_W their distance. Moreover, the two abbreviations $\gamma^{(1)} = q^{(1)} + \frac{z^{(1)}\tan\beta_b}{r_{b1}} - \varphi^{(1)}$ and $\gamma_2 = q^{(2)} - \frac{z^{(2)}\tan\beta_b}{r_{b2}} + \varphi^{(2)}$ are introduced, with β_b the helical gearing angle and $\varphi^{(j)}$ the rotation about $\boldsymbol{e}_z^I$.

The vectors tangential to the flank surface are easy to calculate; the outer unit normal $\boldsymbol{n}^{(j)}$ is their normalized vector product. In contact, $\boldsymbol{n}^{(1)} = -\boldsymbol{n}^{(2)}$, which yields the body-fixed contact line parametrization. Thus, the spacial contact line $\boldsymbol{x}_c^{(j)}$ is found by inserting the parametrization into $\boldsymbol{x}^{(j)}$. In the same way, the velocity of particles on the contact line $\boldsymbol{v}_c^{(j)}$ is found. The tangential relative velocity in the contact is the projection of the relative velocity on the contact tangential plane. Depending on the location on the contact line, $\boldsymbol{v}_T$ may be positive, negative, or zero because of the distributed nature of the contact.

2 Application to geared rotor dynamics

The previously defined quantities are needed in order to calculate the friction components. Denoting $\boldsymbol{f}_N^{(j)}$ the contact normal force, μ the friction coefficient, ℓ the contact length and $n_{fl,j}$ the number of flanks, with Coulomb's model [2]

$$\begin{aligned} \boldsymbol{f}_{T,0}^{(j)} &= -\mu \int_{z_{min}^{(j)}}^{z_{max}^{(j)}} \frac{\left|\boldsymbol{f}_N^{(j)}\right|}{\ell} \frac{\boldsymbol{v}_T}{|\boldsymbol{v}_T|} \frac{dz^{(j)}}{\cos\beta_b}, & \boldsymbol{f}_T^{(j)} &= \sum_{i=0}^{n_{fl,j}-1} \boldsymbol{f}_{T,i}^{(j)}, \\ \boldsymbol{m}_{T,0}^{(j)} &= -\mu \int_{z_{min}^{(j)}}^{z_{max}^{(j)}} \frac{\left|\boldsymbol{f}_N^{(j)}\right|}{\ell} \left(\boldsymbol{x}_c^{(j)} - \boldsymbol{u}^{(j)}\right) \times \frac{\boldsymbol{v}_T}{|\boldsymbol{v}_T|} \frac{dz^{(j)}}{\cos\beta_b}, & \boldsymbol{m}_T^{(j)} &= \sum_{i=0}^{n_{fl,j}-1} \boldsymbol{m}_{T,i}^{(j)}, \end{aligned} \tag{2}$$

with $\boldsymbol{f}_{T,0}^{(j)}$ the friction force on a single tooth and $\boldsymbol{f}_T^{(j)}$ the total friction force, and same for the torque $\boldsymbol{m}_T^{(j)}$. The integration limits $z_{min}^{(j)}$ and $z_{max}^{(j)}$ are the limitation points of the contact line.

The gear teeth must not penetrate in contact normal direction (cf. [3]), thus the relative velocity projection between two flanks $\boldsymbol{n}_c^{(1)} \cdot \left(\boldsymbol{v}_c^{(2)} - \boldsymbol{v}_c^{(1)}\right) = 0$. Because this no-penetration condition is independant of $\varphi^{(j)}$, it doesn't matter for which

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teeth pair it is formulated. For this reason and because of the assumption of a perfect geometry, the contact line load is equal in any position on $\boldsymbol{x}_c^{(j)}$. Now, $\boldsymbol{f}_N^{(j)} = \lambda \boldsymbol{n}_c^{(j)}$ with λ a Lagrange multiplier is specified; this leads to $\boldsymbol{f}_T^{(j)} = -\mu\lambda \boldsymbol{I}_f^{(j)}$ and $\boldsymbol{m}_T^{(j)} = -\mu\lambda \boldsymbol{I}_m^{(j)}$.

The approach is used for the setup of a model which consists of 2 ideal rotors connected by gears (cf. fig. 2), where rotor 1 has a given rotational speed $\dot{\varphi}^{(1)} = \omega_1$, and rotor 2 is connected to a damper. Both rotors have 3 translational DoFs. The equation of motion of the system is of the form

$$\begin{aligned} M\ddot{q} + D\dot{q} + Kq &= \lambda\left(G(q)^T + h(\dot{q}, q, t)\right) \\ G(q)\dot{q} &= 0 \end{aligned} \tag{3}$$

which is an index 2-DAE, $\lambda h(\dot{q}, q, t)$ periodic terms resulting from friction and $q = \left[u_x^{(1)}, u_y^{(1)}, u_z^{(1)}, u_x^{(2)}, u_y^{(2)}, u_z^{(2)}, \varphi^{(2)}\right]^T$ the state vector. The numerical solution of this system is possible in at least two ways: direct integration using an implicit solver (here: RADAU5) and a shooting method to find the periodic solution, or an n^{th} order harmonic approach. There is a very good match between both solutions for a minimum harmonic order $n = 4$ (cf. fig. 3).

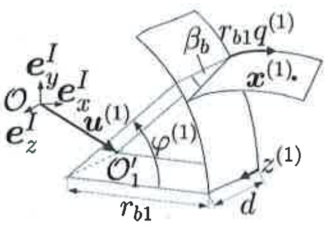

Fig. 1: Evolvent flank surface

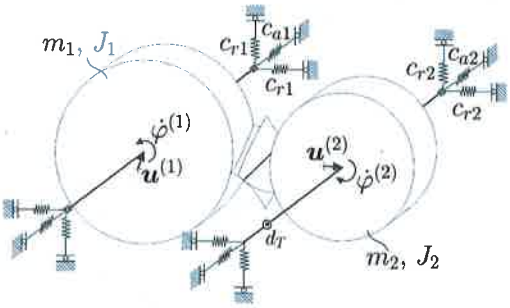

Fig. 2: Geared rotor dynamics model

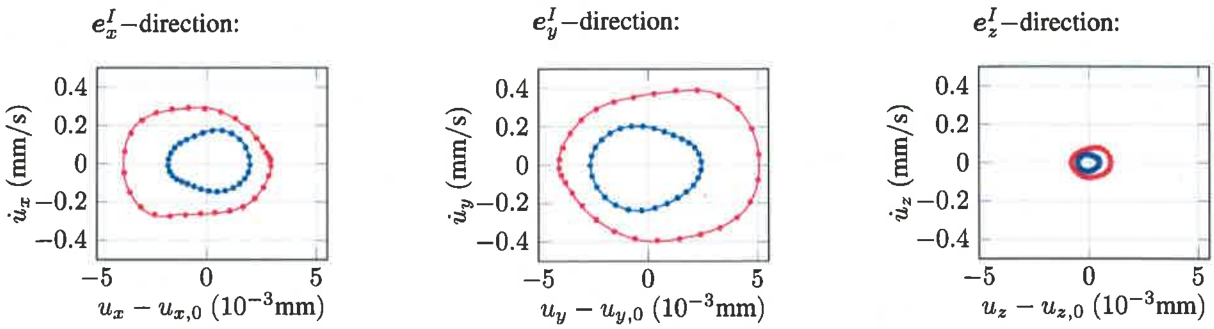

Fig. 3: Periodic orbit of the geared rotor dynamics model. Red – gear 1, blue – gear 2, solid line – RADAU5 solution, dots – 4th order harmonic approach.

3 Conclusions

In this communication, an approach of a friction model for the contact between tooth flanks was shown and successfully tested in a geared rotor dynamics model. Two methods to solve the equations were compared. This is the point of departure for the next steps, which are the analysis of the effect of friction on damping, the setup of a shift gearbox model, stability analysis of the periodic solution and analysis of the influence of the friction law (cf. [4–6]).

References

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