Energy-optimized bipedal running of a simple humanoid robot

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A method to optimize energy efficiency for bipedal running robots is presented. A running model of a simple bipedal robot consisting of five rigid bodies connected by actuated revolute joints is introduced. The actuators' torques are generated by a trajectory tracking controller to produce periodic running gaits. The controller's reference trajectories are parameterized by Bézier polynomials. A numerical optimization is used to employ reference trajectories with optimal energy efficiency for average velocities in the range of 1.5 to 5.5 m/s.

1 Introduction

Exoskeletons and humanoid robots offer the opportunity to support disabled people like paraplegics and substitute humans in dangerous situations like disaster control. Thus, they can save and significantly improve human lives. One of the main challenges of bipedal locomotion is energy efficiency which affects both actuator design and range and so applicability.

In this paper a planar model of a simple bipedal robot is used to compute periodic running gaits. The model has point feet and running gaits thus consist of a single support and a flight phase as well as the transitions between them. This results in one of the most simple models for bipedal locomotion with transitions between two continuous phases.

2 Method

To model running a five-link robot with rigid links for trunk, thighs and shanks connected by actuated revolute joints is introduced (Fig. 1). The lengths, masses and inertias of the robot's links are set to match an average human with a height of $h = 1.8$ m and a mass of $m = 80$ kg [1]. The actuators are characterized by the motor's coefficient of static power $c_{P0} = 1.81 \cdot 10^{-3}$ W/(Nm)^2 [2]. The model has point feet, meaning it cannot directly transmit any torque between the legs and the ground. The controller for the actuated joints can be derived employing input-output linearization to track desired joint angle trajectories [3]. For computation of periodic running gaits the hybrid zero dynamics of the controlled system, which corresponds to a model with a perfect controller, is regarded.

The model's running gaits consist of two phases: the single support phase where the stance leg is in contact with the ground and the flight phase where the model is detached from the ground. Since the robot has point feet, the contact between stance foot and ground in the single support phase is modeled as unactuated ideal revolute joint (Fig. 1). The transition from single support to flight is modeled as instantaneous lift-off with smooth changes in configuration and velocities. The landing at the end of the flight phase is modeled as instantaneous plastic impact of the former swing leg on the ground resulting in smooth changes of the configuration but jumps in the velocities.

The model for the flight phase is described by seven generalized coordinates, the absolute coordinates $q_{a,f} = [x_{cm}\ z_{cm}\ \theta]^T$ and four body coordinates q_b (the actuated joint angles), whereas the single support phase requires only one absolute angle $q_{a,s} = \theta$ (Fig. 1).

The reference trajectories for the body coordinates are parameterized as $q_{b,s} = q_{b,s}(\theta)$ in the single support phase and $q_{b,f} = q_{b,f}(x_{cm})$ in the flight phase yielding autonomous systems in both cases. All reference trajectories are parameterized by Bézier polynomials. The conditions requested for the phase transitions are formulated as constraints to the Bézier parameters.

Controlling the model to the reference trajectory it still retains one degree of freedom in the single support phase and three degrees of freedom in the flight phase. This is equivalent to the hybrid zero dynamics described in [3]. During the flight phase the robot's center of mass describes a parabolic path which can be solved analytically. It is thus sufficient to compute the evolution of the absolute orientation θ in both phases. This can be formulated as state space differential equation

$$z = \begin{bmatrix} \theta \\ L \end{bmatrix} , \qquad \dot{z} = \begin{bmatrix} \kappa_1(\theta)\ L \\ \kappa_2(\theta) \end{bmatrix} \tag{1}$$

where L is the robot's angular momentum and κ_1 and κ_2 are functions of θ only. Equation (1) allows for further reduction of the computational model through a transformation to a scalar, time free equation which can be solved by quadrature:

$$\frac{\mathrm{d}\theta}{\mathrm{d}L} = \frac{\kappa_1(\theta)\ L}{\kappa_2(\theta)} \qquad \Rightarrow \qquad (L^-)^2 = (L^+)^2 + \int_{\theta^+}^{\theta^-} \frac{\kappa_2(\theta)}{\kappa_1(\theta)} \mathrm{d}\theta \ . \tag{2}$$

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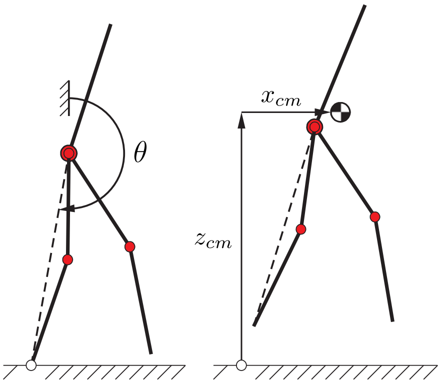

Fig. 1: Models for single support phase (left) and flight phase. Actuated joints in red.

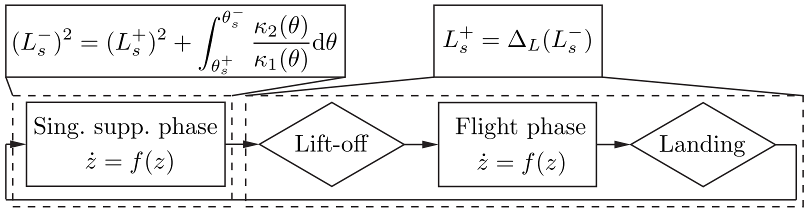

Fig. 2: Computation process for periodic running gaits.

The beginnings and endings of the phases are indexed by the superscripts "+"/"-" (before/after phase transition) respectively. Furthermore, since the angular momentum in the flight phase is preserved and the robots landing configuration is known from the beginning of the step for periodic gaits, an impact map $L_s^+ = \Delta_L(L_s^-)$ mapping the angular momentum from the end of the single support phase to its beginning, can be derived analytically. This allows for efficient computation of periodic running gaits in combination with Eq. (2). The process is depicted in Fig. 2.

Equation (2) in combination with the reference trajectories allows for computing of the most energy efficient gait using parameter optimization. For this purpose *MATLAB*'s `fmincon` solver with the included *SQP* algorithm is utilized. As objective function the dimensionless cost of transport

$$cot = \frac{\sum_{i=1}^{4} \int_{t^+}^{t^-} \max(c_P\, u_i^2 + u_i\, \dot{q}_{b,i}, 0)\mathrm{d}t}{m\, g\, \ell_{step}} \tag{3}$$

defined as the quotient of used energy in the i joints in one step and weight $m\,g$ times step length ℓ_{step} is introduced, where u_i are the actuator torques. This definition for the cost of transport assumes electric motors as actuators and calculates the supplied electric energy assuming no energy can be recuperated in generator mode. It is assumed that minimizing of the cost of transport means maximizing the energy efficiency of the robot.

Physical constraints, periodicity constraints and a desired average velocity are enforced by the *SQP* algorithm.

3 Results and conclusion

Figure 3 shows the computed optimum for the cost of transport *cot* of the model for average running velocities in a range of 1.5 to 5.5 m/s. The *cot* is monotonically increasing with the average velocity meaning faster running speeds are less energy efficient. The graph shows some minor spikes, for example at 3.5 m/s, which indicate bifurcations in the type of motion. The *SQP* algorithm reliably converges to the motion with optimal *cot*.

The proposed method uses numerical optimization in combination with a reduced model to compute energy efficient bipedal running gaits. It is suitable for further applications like parameter studies in the design process of bipedal robots. Multiple continuous phases are considered making the method applicable for future studies of walking with extended feet.

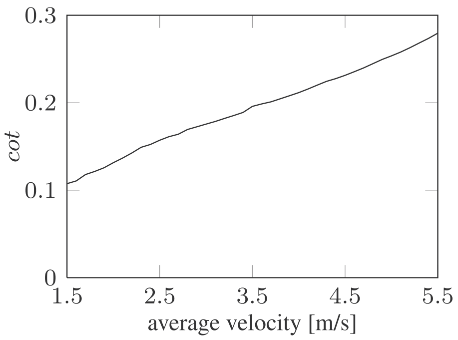

Fig. 3: Minimized cost of transport *cot* for the running model.

References

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