Search for \( CP \) violation with kinematic asymmetries in the \( D^0 \to K^+K^-\pi^+\pi^- \) decay


(The Belle Collaboration)
We search for CP violation in the singly-Cabibbo-suppressed decay $D^0 \rightarrow K^+K^-\pi^+\pi^-$ using data corresponding to an integrated luminosity of 988 fb$^{-1}$ collected by the Belle detector at the KEKB $e^+e^-$ collider. We measure a set of five kinematically dependent CP asymmetries, of which four asymmetries are measured for the first time. The set of asymmetry measurements can be sensitive to CP violation via interference between the different partial-wave contributions to the decay and performed on other pseudoscalar decays. We find no evidence of CP violation.

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Charge-conjugation and parity (CP) symmetry violation has been observed in various weak decays involving strange and beauty quarks [1] and is well described in the standard model (SM) by the Cabibbo-Kobayashi-Maskawa matrix [2]. But the magnitude of CP violation in the SM is too small to explain the baryon asymmetry in the visible universe [3]. Therefore, the search for additional processes that violate CP symmetry, which are not described by the SM, is of great interest to explain the matter-dominant universe. CP violation in the charm sector is expected to be small, less than $\mathcal{O}(10^{-3})$ in the SM [4,5], which makes it an excellent probe for CP violation beyond that of the SM [1].

CP violation in the singly-Cabibbo-suppressed decay $D^0 \rightarrow K^+K^-\pi^+\pi^-$ was searched for using $\hat{T}$-odd correlations [6–8], where $\hat{T}$ reverses the direction of momenta and spin, which is different from the usual time reversal operator $T$ [9]. No CP violation is observed up to now, but the $\hat{T}$-odd correlation measured may be weakly sensitive to CP violation in this decay [9]. In this paper, we report the first measurement of a set of CP-violating kinematic asymmetries in $D^0 \rightarrow K^+K^-\pi^+\pi^-$ decays. The set of kinematic asymmetries probes the rich variety of interfering contributions in a decay, which can be sensitive to non-SM CP-violating phases.

Assuming CPT symmetry, we construct a CP-violating asymmetry by comparing amplitudes of the decay with their CP-conjugate amplitudes. Amplitudes of the decay can be extracted from $A_X$, which we define as

$$A_X = \frac{\Gamma(X > 0) - \Gamma(X < 0)}{\Gamma(X > 0) + \Gamma(X < 0)},$$

where $X$ is a kinematic variable, such as the vector triple product of the final-state momenta used in Refs. [6–8], $\Gamma(X > 0)$ is the rate for $D^0$ decays in which $X > 0$; and $\Gamma(X < 0)$, for $D^0$ decays in which $X < 0$. The CP-conjugated amplitudes can be extracted similarly for $\bar{D}^0$ decays using $\bar{X}$. We can then define our CP-violating kinematic asymmetry as

$$a_X^{CP} = \frac{1}{2} (A_X - \eta_X^{CP} \bar{A}_{\bar{X}}),$$

where $\eta_X^{CP}$ is a CP eigenvalue specific to $X$.

We measure a set of kinematic asymmetries for five different $X$, where four asymmetries are measured for the first time and one asymmetry is proportional to the $\hat{T}$-odd correlation using the vector triple product of the final-state momenta, which has been measured previously [6–8]. The set can be sensitive to CP violation in the interference between the $S$-wave and $P$-wave production of the $K^+K^-$ and $\pi^+\pi^-$ pairs in the $D^0 \rightarrow K^+K^-\pi^+\pi^-$ decay, where the process of a quasi-two-body decay to a dikaon system and dipion system contributes to over 40% of the decay rate [1]. It covers the asymmetries that can be measured without considering the mass of the intermediate particles. The kinematic variables are constructed from the angles $\theta_1$, $\theta_2$, and $\Phi$, which are shown in Fig. 1. The $\theta_1$ is the angle between the $K^+$ momentum and the direction opposite to that of the $D^0$ momentum in the center-of-mass (CM) frame of the $K^+K^-$ system. The $\theta_2$ is defined in the same way as $\theta_1$ substituting $K^+$ with $\pi^+$ and $K^-$ with $\pi^-$. The $\Phi$ is the angle between the decay planes of the $K^+K^-$ and $\pi^+\pi^-$ pairs in the CM frame of $D^0$. Three kinematic variables have $\eta_X^{CP} = -1$: $\sin 2\Phi$, $\cos \theta_1 \cos \theta_2 \sin \Phi$, and $\sin \Phi$; the last variable is proportional to the vector triple product of the final-state momenta. The remaining two kinematic

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*Corresponding author. eunil@hep.korea.ac.kr

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variables have $\eta^{CP}_{X} = +1$: $\cos \Phi$ and $\cos \theta_1 \cos \theta_2 \cos \Phi$. The kinematic asymmetries where $\eta^{CP}_{X}$ is $-1$, commonly known as $\hat{T}$-odd correlations, are dependent on the imaginary part of the interference of amplitudes for production of the $K^+K^-$ and $\pi^+\pi^-$ states in different spin configurations [10–13]. The asymmetries where $\eta^{CP}_{X}$ is $+1$, are dependent on the real part of the interference of amplitudes. Both types of asymmetries are nonzero in the case of $CP$ violation. This set is measured for the first time for any four-body final state; these measurements can be performed for any other pseudoscalar meson that decays to four pseudoscalar mesons.

This analysis uses the data sample recorded by the Belle detector [14] at the $e^+e^-$ asymmetric-energy collider KEKB [15], where the CM energy of the collisions was varied from the mass of the $Y(1S)$ resonance up to that of the $Y(6S)$ resonance. The total data sample corresponds to an integrated luminosity of 988 fb$^{-1}$ [16].

Monte Carlo (MC) samples are used to devise the selection criteria, identify the different sources of background, model the data, validate the fit procedure, and determine systematic uncertainties. Inclusive MC samples were generated with EVTGEN [17], where the number of generated events corresponds to six times the integrated luminosity of the data sample. The detector response was simulated with GEANT3 [18]. To simulate the effect of beam-induced background, the generated events have data solely due to the beam backgrounds overlaid.

Since the final state is self-conjugate, the flavor of the $D^0$ mesons is identified by reconstructing the decay chains $D^{++} \rightarrow D^0\pi^+_s$, with $D^0$ decaying into $K^+K^-\pi^+\pi^-$, where $\pi^+_s$ is referred to as the slow pion. Here, and elsewhere in this paper, charge-conjugate states are implied unless stated explicitly otherwise.

Using MC simulated data, we developed the selection criteria to maximize a figure of merit of $S/\sqrt{S+B}$, where $S$ is the signal yield and $B$ is the background yield in a signal enhanced region, which is defined to be within 1.5 MeV/$c^2$ of the known $D^0$ mass [1] and within 0.25 MeV/$c^2$ of the known mass difference ($\Delta m$) between the $D^{++}$ candidate and its daughter $D^0$ [1].

We select charged tracks that originate from close to the $e^+e^-$ interaction point (IP) by requiring the impact parameters to be less than 4 cm in the beam direction and 2 cm in the plane transverse to the beam direction. To ensure the tracks are well reconstructed, we require they each have a transverse momentum greater than 0.1 GeV/$c$ and at least two associated hits in the silicon vertex detector in both the beam direction and azimuthal direction. Charged tracks are identified as pions or kaons depending on the ratio of particle identification likelihoods $L_K/(L_K + L_K)$, which are constructed from information recorded by the central drift chamber, time-of-flight scintillation counters, and aerogel threshold Cherenkov counter. We identify a charged track as a kaon when this ratio is above 0.6; otherwise it is assumed to be a pion. The kaon and pion identification efficiencies are typically over 80%, and the misidentification probabilities are below 10% [19].

We form a $D^0$ candidate from each combination of two oppositely charged kaon tracks and two oppositely charged pion tracks. We require each $D^0$ candidate to have an invariant mass within 30 MeV/$c^2$ of the known $D^0$ mass [1,20], where the range is larger than 7 times the mass resolution of the reconstructed $D^0$ candidate, and a momentum in the CM frame greater than 1.8 GeV/$c$. For each surviving candidate, we perform a vertex- and mass-constrained fit to the kaons and pions; we require the vertex fit to have a probability greater than 0.1%. We also perform a fit where each $D^0$ candidate is fit under the hypothesis that the trajectory of the candidate originates from the IP and require the fit to have a probability greater than 0.005%.

To veto the Cabibbo-favored $D^0 \rightarrow K^+K^-\pi^0\pi^0$ decays, we remove $D^0$ candidates whose daughter pion pairs have invariant masses within 12.05 MeV/$c^2$ of the known $K^0_S$ mass [1], which is five times the mass resolution of the reconstructed $K^0_S$ candidate.

We form each combination of a positively charged pion track and $D^0$ candidate into a $D^++$ candidate and perform a vertex fit on the pion, where the fit is constrained to the intersection of the $D^0$ candidate trajectory with the IP region. We require each $D^{++}$ candidate to have a momentum in the CM frame greater than 2.5 GeV/$c$. We also require $\Delta m$ to be within $\pm7.6$ MeV/$c^2$ of the known $\Delta m$ [1], where the lower limit corresponds to the known $\pi^0$ mass.

In the signal region, 8.1% of events have multiple $D^{++}$ and/or $D^{+-}$ candidates, while the average multiple candidates per event is 1.1, which is comparable with Ref. [21]. We select either a $D^{++}$ or $D^{+-}$ candidate for each event, based on the smallest $\chi^2$ for the $D^0$ mass fit. If there are multiple $D^{++}$ and/or $D^{+-}$ candidates formed with this $D^0$, we select the one whose $\pi^+_s$ or $\pi^-_s$ has the smallest impact parameter in the transverse plane. Studies with the MC sample indicate that 93% of the multiple-candidate events are correctly selected. The efficiency for the $D^0 \rightarrow K^+K^-\pi^+\pi^-$ decay with the stated selections is 11%. A total of 474,971 events are reconstructed from the data sample.

After all selection criteria, our data sample contains events that fall into four different categories: correctly
reconstructed $D^0$ mesons coming from correctly reconstructed $D^{*+}$ mesons, which we call signal events; events with correctly reconstructed $D^0$ mesons coming from misreconstructed $D^{*+}$ candidates, which we call random-π_s events; events with a partially reconstructed $D^0$ candidate and the π_s from a $D^{*+}$, which we call partial-π_s events, which has a small peak in the signal region of $\Delta m$; and events with both $D^0$ and $D^{*+}$ candidates misreconstructed, which we call combinatorial events. Our selection criteria rejects over 99% of events with $D^{*+} \rightarrow K^+K^-\pi^+\pi^-\pi^-$, which could be confused for our signal, leaving a negligible number of such events.

We calculate the $CP$-violating kinematic asymmetry with the yield of the signal events for each flavor of $D^0$ and each sign of the relevant kinematic variable. To do this, we perform four separate fits to the data for each kinematic variable. Each fit is a binned two-dimensional extended maximum-likelihood fit to the reconstructed $D^0$ mass and $\Delta m$. The data are binned into 200 equal-width bins in each dimension. These additional requirements on $m(K^+K^-\pi^+\pi^-)$ and $\Delta m$ have a negligible effect on the selection efficiency.

One model is used for all fits. It contains components describing signal, random π_s, partial-π_s, and combinatorial events. The yield of each component is free in each fit, but parameters governing the shapes of the components are fixed from a single fit to all the data regardless of $D^0$ flavor and $X$.

The signal component is the product of a sum of bifurcated Gaussian and Gaussian probability density functions (PDFs) for $m(K^+K^-\pi^+\pi^-)$ and a sum of Gaussian and JohnsonSU [22] PDFs for $\Delta m$. The combinatorial component is the product of a Chebyshev function for $m(K^+K^-\pi^+\pi^-)$ and a Chebyshev function for $\Delta m$. The random-π_s component is the product of the signal shape for $m(K^+K^-\pi^+\pi^-)$ and the combinatorial shape for $\Delta m$. And the partial-π_s component is the product of a Chebyshev function for $m(K^+K^-\pi^+\pi^-)$ and a Bifurcated Gaussian PDF for $\Delta m$, where the shape parameters for the partial-π_s component are fixed to those obtained from a fit to an inclusive MC sample. The shape of the MC sample is validated by comparing it to data in the sidebands of the $m(K^+K^-\pi^+\pi^-)$ distribution; the shapes are compatible.

Figure 2 shows the results of the fit to all the data, from which the shapes of all components are fixed for all remaining fits; the model agrees well with the data, as can be seen from the pulls, which are defined as the difference between the data points and the model expectation divided by the expected uncertainty. As an example of a set of fits used to determine the $CP$-violating kinematic asymmetry, we show separate fit results for positive and negative sin 2Φ for $D^0$ samples in Fig. 3. The signal yields determined by the fits are given in Table I for each $D^0$ flavor and kinematic variable sign.

![Image](image_url)

FIG. 2. Data (black points) and fit results (shaded regions) for the fit to all the data in projections of $m(K^+K^-\pi^+\pi^-)$ (left) and $\Delta m$ (right). The shaded regions are stacked upon each other and show, from lowest to highest, the combinatorial, partial-$D^*$, random-$\pi_s$, and signal components. The lower plots show the pulls for the fit; the unlabeled horizontal lines indicate ±3.

We perform several cross checks to validate our analysis:

To study the effect of the $D^0 \rightarrow K^+\bar{K}^*$, where $K^*$ decays to $K^+\pi^-$, we recalculate the asymmetries including a veto on $K^*$ and $\bar{K}^*$ that rejects the $D^0$ candidates with a $K^+\pi^-$ pair and $K^-\pi^+$ pair of an invariant mass within 80 MeV/$c^2$ of the known $K^*$ mass [1], which is twenty times the mass resolution of the reconstructed $K^*$ candidate. The recalculated asymmetries are consistent with the values without the veto.

To study the effects from the best candidate selection, we recalculate the asymmetries with no best candidate.

![Image](image_url)

FIG. 3. Two-dimensional fit results, distributions and pull of the data subsamples projected on the observables $m(K^+K^-\pi^+\pi^-)$ (left) and $\Delta m$ (right). The distribution sequence follows Fig. 2. Top histograms show the $D^0(\sin 2\Phi > 0)$ subsample while the bottom histograms show the $D^0(\sin 2\Phi < 0)$ subsample.
The recalculated asymmetries are consistent with those calculated including the best candidate selection.

The detector resolution of the kinematic variables could affect the asymmetries. We measure the fraction of cross-feed between signal events with $X > 0$ and $X < 0$ using an MC sample that has a similar shape to the data. The fraction of cross-feed is at the 1% level, making its effect negligible.

We estimate the effect of incorrectly assigning the flavor of the $D^0$ using an MC sample that has a similar integrated luminosity to the data. In the MC sample, incorrectly assigned events comprise less than 0.01% of the total number of events. Misassignment has a negligible effect.

There could be an effect due to an efficiency difference depending on the kinematic variable regions. Efficiencies depending on kinematic variable regions are measured using a MC sample. We find that the efficiency does not depend on the kinematic variables used to define the asymmetries.

Several sources of systematic uncertainty are considered. Individual uncertainties and the total systematic uncertainty are listed in Table II. The bias from the model PDF is estimated by changing the signal model, partial-$D^*$ model, and combinatorial model. We change the signal model and partial-$D^*$ model to products of one-dimensional Gaussian-kernel-estimated PDFs [23] and the combinatorial model to a product of one-dimensional PDFs obtained from an inclusive MC sample. The difference between the measured values is assigned as a systematic uncertainty.

The detector bias is estimated from a control sample of $D^0 \to K^-\pi^0_{\text{low}}\pi^-\pi^+_{\text{high}}$ events, where momentum is used to differentiate between the $\pi^+_{\text{high}}$ and $\pi^-_{\text{low}}$. This decay is Cabibbo-favored in which all kinematic asymmetries are expected to be much smaller than the measurement precision [4]. The kinematic variables are calculated in the same way as for the $K^+K^-\pi^+\pi^-$ final state, substituting $K^+$ with $\pi^+_{\text{low}}$. The kinematic asymmetries are found to be consistent with zero, and we assign their statistical uncertainties as the systematic uncertainties related to any detector bias.

To assess whether there is a bias introduced by the likelihood fit and to check the extraction of kinematic asymmetries from the two-dimensional binned fit, we generate MC samples with different asymmetries and compare the fit results with the generated values. The average difference between the measured and generated value is assigned as a systematic uncertainty.

The various sources of systematic uncertainty are independent of each other. Therefore we estimate the total systematic uncertainty by summing the uncertainties in quadrature. As a note, the kinematic asymmetries are constructed such that they are insensitive to the intrinsic production asymmetry [8].

The measured $A_X$ and $\bar{A}_X$ are listed in Table III with statistical errors. As in other experiments [7,8], final state interaction effects are observed with a similar amplitude for $A_{\sin \Phi}$ and $\bar{A}_{\sin \Phi}$. We find the $CP$-violating kinematic asymmetries to be

\begin{align}
    a_{\cos \Phi}^{CP} & = (3.4 \pm 3.6 \pm 0.6) \times 10^{-3}, \\
    a_{\sin \Phi}^{CP} & = (5.2 \pm 3.7 \pm 0.7) \times 10^{-3}, \\
    a_{\sin 2\Phi}^{CP} & = (3.9 \pm 3.6 \pm 0.7) \times 10^{-3}, \\
    a_{\cos \theta_1 \cos \theta_2 \cos \Phi}^{CP} & = (-0.2 \pm 3.6 \pm 0.7) \times 10^{-3},
\end{align}

### Table II. Contributions to the systematic uncertainty (in per mille) for each $CP$-violating kinematic asymmetry $a_i^{CP}$.

<table>
<thead>
<tr>
<th>Effect</th>
<th>$a_{\cos \Phi}^{CP}$</th>
<th>$a_{\sin \Phi}^{CP}$</th>
<th>$a_{\sin 2\Phi}^{CP}$</th>
<th>$a_{\cos \theta_1 \cos \theta_2 \cos \Phi}^{CP}$</th>
<th>$a_{\cos \theta_1 \cos \theta_2 \sin \Phi}^{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal model PDF</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Partial-$D^*$ model PDF</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Combinatorial model PDF</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Detector bias</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Likelihood fit bias</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Total</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
</tbody>
</table>
TABLE III. $A_X$ and $\tilde{A}_X$ (in per mille) for each kinematic variable $X$. The uncertainties are statistical only.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$A_X$</th>
<th>$\tilde{A}_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos \Phi$</td>
<td>$-195.2 \pm 5.1$</td>
<td>$202.0 \pm 5.1$</td>
</tr>
<tr>
<td>$\sin \Phi$</td>
<td>$71.6 \pm 5.2$</td>
<td>$61.3 \pm 5.2$</td>
</tr>
<tr>
<td>$\sin 2\Phi$</td>
<td>$-151.7 \pm 5.2$</td>
<td>$-159.4 \pm 5.1$</td>
</tr>
<tr>
<td>$\cos \theta_1 \cos \theta_2 \cos \Phi$</td>
<td>$140.8 \pm 5.1$</td>
<td>$-141.2 \pm 5.2$</td>
</tr>
<tr>
<td>$\cos \theta_1 \cos \theta_2 \sin \Phi$</td>
<td>$-44.5 \pm 5.2$</td>
<td>$-44.9 \pm 5.2$</td>
</tr>
</tbody>
</table>

where the first and second uncertainties are statistical and systematic, respectively. These results indicate that there is no CP violation within the statistical and systematic uncertainties for the interferences between the S-wave and P-wave production of the $K^+K^-$ and $\pi^+\pi^-$ pairs in this decay. No effects from new physics models can be observed within the experimental uncertainties. With more data from future experiments, it may be possible to measure the CP violation due to the SM in this decay.

In conclusion, we search for CP violation in $D^0 \to K^+K^0\pi^+\pi^-$ by measuring a set of five kinematic asymmetries. The set of measurements can be sensitive to CP violation via the rich variety of interference between the different partial-wave contributions to the decay. It can be performed on any other pseudoscalar meson that decays into four pseudoscalar mesons. Four asymmetries are measured for the first time. The set of CP-violating kinematic asymmetries is consistent with CP conservation and provide new constraints on new physics models [4,9,11].

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