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ABSTRACT

Energy transfer along finite curvature plasma-metal interfaces in cylindrical metallic waveguides entirely filled by magnetoactive plasma is studied. Angular phase velocity, angular velocity of energy transfer, and angular group velocity are introduced and analyzed as functions of the waveguide parameters: radius, plasma particle density, azimuthal wave number, and external static axial magnetic field.

I. INTRODUCTION

The difference between "velocity of transmission of phase" v_{ph} and "velocity of propagation of vibratory motion" v_g was established for the first time in Ref. 1 as one of the conclusions of research in the field of theory of light.

Thirty four years later, in studies of elastic waves in continuous media, the average wave energy flux density was found to be proportional to the average wave energy density.[2]

The brief and complete overview of the history of implementation of the term "group velocity" in Ref. 3 makes it possible not to repeat it herein. However, the significance of the group velocity as a tool for studying a wide scope of physical problems keeps it in the focus of many investigations until now.

One can find three different definitions of the group velocity, e.g., in Ref. 4. The first one is the group velocity $v_g = d\omega/dk$. In Ref. 4, Rayleigh was specified as the person who was the first to introduce this definition while studying the sound waves. However, as it is already mentioned above, Hamilton had operated with the group velocity already in 1839[1] while studying the theory of light. The second definition is the signal velocity v_s of Sommerfeld[4] who introduced this value while ascertaining in a one-dimensional approach how a well-defined signal propagates in a material medium. The third one is the velocity of energy transfer v_{en}, which is the proportionality coefficient between the energy flux surface density averaged over the time period (in electrodynamics, this is the well-known Poynting vector) and the averaged wave energy volume density. For instance, a rigorous identity between the group velocity v_g and the velocity of energy transfer v_{en} in non-uniform non-dissipative media with or without anomalous dispersion was demonstrated under very general conditions in Refs. 5 and 6. The precise physical meaning of the complex group velocity in absorbing and active media is a matter of extensive research (see, e.g., Ref. 7 and references therein). These three velocities were shown to be identical for non-absorbing media and differ considerably in an absorption band.

The group velocity also plays an important role in the theory of linear particle accelerators.[8] In particular, the group velocity was shown to be a significant characteristic observable in studies of power flow in the waveguide.[9]

They use to introduce group velocity in classical textbooks on electrodynamics, including plasma electrodynamics. It was demonstrated on pages 246–248 in Ref. 10 that in vacuum waveguides, the axial energy flux density is in general case proportional to the energy density with the group velocity $v_{gz} \equiv \partial\omega/\partial k_z$ as the coefficient of proportionality.

The term group velocity is also widely used in plasma electrodynamics. The group velocity $v_g = d\omega/dk$ was introduced, for example, on page 33 in the textbook[11] as the quantity which "characterizes the velocity of amplitude (and with that energy) displacement" without any reasoning. However, the authors of this textbook almost did not apply the term in presenting selected chapters of plasma electrodynamics. This explains the fact that this mistake in the physical sense of the group velocity did not interfere with the authors of the textbook in obtaining correct physical results in the field of plasma electrodynamics and avoiding mistakes in physical issues.

The other popular comprehensive textbook in plasma electrodynamics[12] does not discuss the relation between the group velocity $\upsilon_g = d\omega/dk$ and velocity of energy transfer υ_{en}. In studies on plane electromagnetic waves in infinite homogeneous plasmas, the group velocity was applied there for explaining the phenomenon of conical refraction[13] in magnetohydrodynamics which is analogous to that in crystal optics.[14]

However, the energy transfer is usually considered in non-bounded media. Even the waveguide theory considers usually the energy transfer along the waveguide axis only (along the boundary). In this case, the axial wave vector k_z is obtained from the dispersion relation. Then, both phase and group wave velocities are defined in a usual way but using the axial component of the wave vector instead of complete wave vector in the medium. However, it is clear that the electromagnetic energy in waveguides with arbitrary cross section can rotate around the axis but not only propagate along the axis.

The objective of this paper is to introduce an angular velocity of energy transfer ω_{en} for description of electromagnetic energy rotation around the axis of cylindrical plasma waveguides, as well as an angular group velocity ω_g, and angular phase velocity ω_{ph}. Propagation of azimuthal surface waves (ASWs) of **ex**traordinary polarization (XASWs) along the plasma-metal interface in cylindrical waveguides is considered as an example.

ASWs are well-known to be eigenwaves of these waveguides.[15–17] They propagate strictly across the axial external static magnetic field $\vec{B}_0 \| \vec{z}$. Their fields depend on time and coordinates as follows: $\vec{E}, \vec{H} = f(r)\exp(im\varphi - i\omega t)$. Here, φ is the azimuthal angle, and m and ω are the azimuthal wavenumber and angular frequency of ASWs, respectively. The waveguide parameters are assumed not to depend on the axial coordinate, so that $\partial/\partial z = 0$. In this case, the Maxwell equations split into two subsets. One describes the waves of **o**rdinary polarization (OASWs) with the field components E_z, H_r, H_φ. The other corresponds to the extraordinary polarized waves (XASWs) with the field components H_z, E_r, E_φ.

The case of a dense plasma, for which the inequality $\Omega_e^2 > \omega_e^2$ is valid, here ω_e and Ω_e are the electron cyclotron and plasma (Langmuir) frequencies, respectively, is considered. In this case, XASWs propagate along the plasma-metal interface in the following two frequency ranges:

$$\omega_{\rm LH} < \omega < |\omega_e|, \tag{1}$$

$$\omega_{\rm UH} < \omega < \omega_2, \tag{2}$$

where $\omega_2 = |\omega_e|/2 + \sqrt{\Omega_e^2 + \omega_e^2/4}$ is the cut-off frequency for bulk modes, and $\omega_{\rm LH}$ and $\omega_{\rm UH}$ are the lower and upper hybrid frequencies, respectively. The frequency range (1) is referred hereinafter as the low frequency (LF) region and (2) as the high frequency (HF) range.

Ordinarily polarized azimuthal waves (AWs) are of surface nature if their frequency is lower than the Langmuir frequency ($\omega < \Omega_e$).[16,17] They are out of scope of detailed analysis in the present paper. That is why their properties are mentioned only briefly here.

Electrodynamic properties of plasmas are described by the dielectric permittivity tensor ε_{ik} of a cold collisionless plasma.[18] The radial distributions of the axial electric field of OASWs and axial magnetic field of XASWs are described by second order differential equations of Bessel type. The solutions of these equations are linear combinations of modified Bessel functions $I_m(\xi)$ and MacDonald functions $K_m(\xi)$.[19] In particular, for the XASWs

$$H_z = A_1 I_m(k_\perp r) + A_2 K_m(k_\perp r). \tag{3}$$

In (3), $k_\perp = k\sqrt{(\mu^2 - 1)\varepsilon_1}$, $\mu = \varepsilon_2/\varepsilon_1$, $k = \omega/c$, $\varepsilon_{1,2}$ are the components of the permittivity tensor ε_{ik}, and $A_{1,2}$ are constants of integration. The value $k_\perp^{-1}$ has the following clear physical sense: it is a radial scale at which the amplitude of the XASW field sufficiently decreases with going away from the plasma interface.

Radial distributions of the radial $E_r(r)$ and azimuthal $E_\varphi(r)$ electric fields of XASWs can be derived from the known distribution of the wave axial magnetic field $H_z(r)$

$$E_r = \frac{\omega}{ck_\perp^2}\left[\mu\frac{\partial H_z}{\partial r} + \frac{m}{r}H_z\right], \quad E_\varphi = \frac{i\omega}{ck_\perp^2}\left[\frac{\partial H_z}{\partial r} + \mu\frac{m}{r}H_z\right]. \tag{4}$$

The interior problem of ASW propagation along the plasma-metal interface of a cylindrical waveguide is considered. The cylindrical metallic chamber of radius a is entirely filled by a plasma column (see Fig. 1). The boundary condition of finite magnitude of the wave fields allows us to determine the

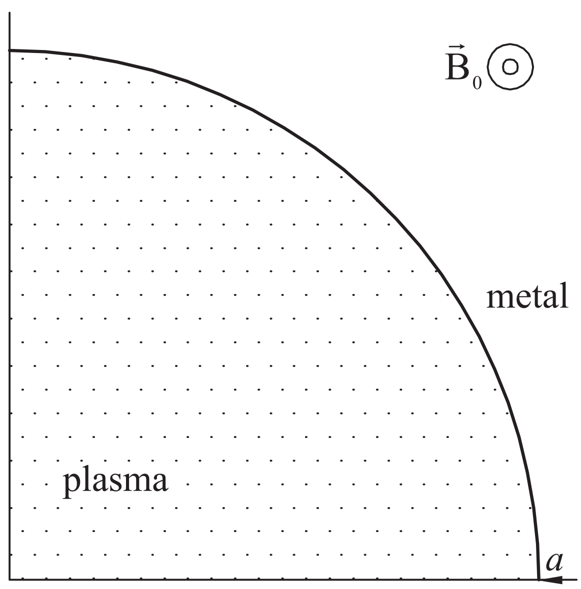

FIG. 1. Scheme of the problem: a metal chamber is entirely filled with magnetoactive plasma.

constant of integration A_2. This integration constant is set to $A_2 = 0$ to eliminate the pole of $K_m(\xi)$, which is known to diverge at the axis ($r = 0$), $\lim_{\xi\to 0} K_m(\xi) = \infty$.

XASWs are unidirectional waves,[15,17] which means that in specific frequency ranges, their dispersion relation allows solutions with only one sign of azimuthal wavenumber. In other words, XASWs propagate in specific frequency ranges in a definite direction without the reflected signal. This feature can be of interest for radio frequency engineering. The azimuthal wavenumbers of XASWs can be only positive, $m > 0$, in the LF frequency range (1). In this range, XASWs propagate in the same direction in which electrons gyrate in an external static axial magnetic field. In the HF range (2), XASW azimuthal wavenumbers are only negative, $m < 0$.

Since the waveguide metal walls are assumed to have infinite electric conductivity, the application of non-dissipative boundary condition at the plasma-metal interface, $r = a$, allows us to derive the dispersion relation of XASWs in the following form:

$$D(\omega, m) \equiv k_\perp R F'_m(k_\perp a) + m\mu F_m(k_\perp a) = 0, \tag{5}$$

where $F_m(\xi) = I_m(\xi)$ for the considered problem. In the approximation of large azimuthal wavenumber, $|m| \gg 1$, Eq. (5) transforms into the dispersion relation for surface waves which propagate exactly across the external static magnetic field along a planar plasma-metal boundary.[20]

The dispersion relation (5) also describes the dispersion properties of extraordinarily polarized electromagnetic azimuthal bulk waves (XABWs) whose frequency real parts lie outside of the frequency ranges (1) and (2), which are not the subject of detailed analysis of the present paper. In this case, $F_m(\xi) = J_m(\xi)$, where $J_m(\xi)$ is the Bessel function of the first kind.[19]

Angular phase velocity ω_{ph}, angular group velocity ω_g, and angular velocity of energy transfer ω_{en} of both LF and HF XASWs are introduced and studied in the present paper with respect to their dependence on plasma waveguide parameters: azimuthal wavenumber, radius of the interface, plasma particle density, and external static magnetic field.

The paper is arranged as follows. An analytical study of electromagnetic energy rotation and treatment of the mentioned three angular velocities is given in Sec. II. The dependencies of the three angular velocities are numerically analyzed in detail in Sec. III. Section IV contains the discussion of the obtained results and conclusions.

II. ANALYTICAL TREATMENT OF XASW ELECTROMAGNETIC ENERGY TRANSFER

To the best of our knowledge, the terms angular phase velocity, angular group velocity, and angular velocity of energy transfer are introduced in this paper for the first time. However, the terms which correspond to angular rotation are well-known long ago. For example, orbital angular momentum (OAM) carried by light beams (vortex beams) was discovered in Ref. 21 and has been widely employed in many technological applications like optical tweezers, optical drives of micro-machines, atom trapping, and optical communications.

Higher-order mode gyrotrons were shown to be natural sources of high-power OAM millimeter wave beams.[22] While OAM describes the phase twisting, the so-called spin angular momentum is used for studying the twisting of the wave electric field (polarization).[22]

The well-defined OAM of rotating cavity modes operating near the cutoff frequency excited by gyrating electrons in a high-power electron cyclotron maser (ECM)—a gyrotron—was derived by photonic and electromagnetic wave approaches in Ref. 23. A mode generator, built with a high-precision 3D printing technique to mimic the rotating gyrotron modes for precise low-power measurements, showed clear natural production of higher-order OAM modes. Cold-test measurements of higher-order OAM mode generation promised the realization towards wireless long-range communications using high-power ECMs.

In studying a capability of XASWs to sustain a gas discharge in cylindrical waveguides, the angular discharge "length" was used in Ref. 24 rather than the linear discharge length commonly used for the case of planar discharge chambers.

To study electromagnetic energy rotation carried by XASWs, one has to get use of the Poynting vector[2,10,25]

$$\vec{S} \equiv \frac{c}{4\pi}\vec{E} \times \vec{H}. \tag{6}$$

Since azimuthal waves (AWs, both bulk and surface) do not propagate in the axial direction, $k_z = 0$, they naturally do not transfer any energy along the $\vec{z}$ axis. This is confirmed via the direct calculation of the axial component of the Poynting vector

$$S_z \equiv \frac{c}{4\pi}(E_r H_\varphi - E_\varphi H_r). \tag{7}$$

For ordinarily polarized AWs, the electric wave field components E_r and E_φ are equal to zero, and the Poynting vector component S_z is hence equal to zero. Extraordinarily polarized AWs do contain neither radial H_r nor azimuthal H_φ magnetic fields, and their axial energy flux is also zero.

The absence of axial component of energy flux of azimuthal waves is also clear in terms of group velocity

$$\upsilon_{gz} \equiv \frac{\partial\omega}{\partial k_z} = -\frac{\partial D(\omega, m)/\partial k_z}{\partial D(\omega, m)/\partial\omega}. \tag{8}$$

Indeed, since the axial wavenumber k_z does not come, e.g., into the XASW dispersion relation (5), $D(\omega,m)$ is independent of k_z, and hence, $\upsilon_{gz} = 0$.

This result for the axial group velocity was expected in advance since the axial power flux in the waveguide of arbitrary cross-section is proportional to the axial wavenumber.[26] Generally speaking, the dispersion relation of electromagnetic waves in cylindrical waveguides with the axial magnetic field can depend on the axial wavenumber k_z. But if the wave group velocity in the axial direction is zero, one can conclude that the dispersion relation of electromagnetic waves is an even function of k_z. In this case, the first derivative of the dispersion relation in respect of k_z is an odd function of k_z. As a sequence, the electromagnetic wave power does not propagate axially if $k_z = 0$ like it should be from the physical point of view. In other words, the spectra of electromagnetic waves in cylindrical waveguides with the axial magnetic field are degenerated in respect of the k_z sign. This is well-known, but it is very pleasant to see that the present theory is self-coordinated and internally consistent.

Since azimuthal waves (both bulk and surface) propagate in cylindrical waveguides, their energy flux in the radial direction is also zero. This is obvious since eigenwaves of cylindrical waveguides are standing waves in the radial direction. This can also be checked by the calculation of the Poynting vector. The radial component of the Poynting vector is

$$S_r \equiv \frac{c}{4\pi} E_\varphi H_z. \tag{9}$$

In the non-dissipative approach, $\varepsilon_{1,2}$ are real values, and the phase shift of E_φ compared to H_z is $\pi/2$ [see Eq. (4)]. This means that averaging of S_r over the time period T gives zero

$$\langle S_r \rangle_T \equiv \frac{c}{4\pi} E_\varphi(r) H_z(r) \times \frac{\int_0^T (-\cos(m\varphi - \omega t + \varphi_0)\sin(m\varphi - \omega t + \varphi_0))dt}{T} = 0. \tag{10}$$

Finally, the energy flux of azimuthal waves (both bulk and surface) in the azimuthal direction is nonzero. This is expected from the point of view of general physics, but the application of the Poynting vector directly confirms this statement. The azimuthal component of the Poynting vector is

$$S_\varphi \equiv \frac{-c}{4\pi} E_r H_z. \tag{11}$$

In the non-dissipative approach, the phase shift of E_r compared to H_z is 0 [see Eq. (4)]. This means that averaging of S_φ over the period T does not give zero

$$\langle S_\varphi \rangle_T \equiv \frac{-c}{4\pi} E_r(r) H_z(r) \frac{\int_0^T \cos^2(m\varphi - \omega t + \varphi_0)dt}{T} = \frac{-cE_r(r)H_z(r)}{8\pi} \neq 0. \tag{12}$$

A. Angular phase velocity

The linear phase velocity $v_{ph} = \omega/k_\varphi$ cannot be considered as an appropriate tool for describing the "velocity of phase transmission" of azimuthal waves since one has to choose an azimuthal wave vector k_φ in some way. The simplest solution would be $k_\varphi = m/r$. However, since it depends on the radial coordinate r, it does not fit. One has to introduce the angular phase velocity to describe the azimuthal wave propagation in the azimuthal direction. It can be easily derived from differentiating the phase $\zeta = (\zeta_0 + m\varphi - \omega t) = \text{Const}$, where $d\zeta = (md\varphi - \omega dt) = 0 \Rightarrow$

$$\omega_{ph} = \omega/m. \tag{13}$$

B. Angular group velocity

In the case of the cylindrical plasma waveguide structure, the angular group velocity

$$\omega_g \equiv \frac{\partial \omega}{\partial m} = -\frac{\partial D(\omega, m)/\partial m}{\partial D(\omega, m)/\partial \omega} \tag{14}$$

can be introduced. The definition (14) cannot be applied for studying ASW properties in neither plasma-filled coaxial metallic waveguides nor for the exterior problem which is ASWs propagating around a metal rod in infinite magneto-active plasma since the dispersion relation contains McDonald functions. The partial derivative of $K_m(\xi)$ with respect to the order m is not defined by mathematics. However, the definition (14) can be applied for studying the properties of XABWs mentioned above since the derivative $\partial J_m(\xi)/\partial m$ is known in mathematics. The following numerical analysis confirms that, in general, the angular velocity ω_{en} of energy transfer (18) is not equal to the angular group velocity (14) though their signs coincide.

C. Angular velocity of energy transfer

In contradiction to the angular phase velocity ω_{ph} given in (13) and the angular group velocity ω_g of Eq. (14) which are integral characteristics of the azimuthal energy transfer, the angular velocity of energy transfer $\omega_{en}(r)$ can be introduced as the radial dependence of the ratio of the averaged azimuthal Poynting flux $S_\varphi(r) = -(c/8\pi)E_r(a)H_z(a)\beta(r)$ to the angular electromagnetic energy density $d^3W/(drdzd\varphi) = (r/16\pi)(|E(r)|^2 + |H(r)|^2) = (a/16\pi)(|E(a)|^2 + |H(a)|^2)\chi(r)$ which is also averaged over the wave period. Both values are presented already in normalized forms, where their radial dependencies given in Eqs. (3) and (4) are contained in the functions $\beta(r)$ and $\chi(r)$.

Since, as it was mentioned above, in the simplest cases of Cartesian coordinates the velocity of energy transfer is equal to the group velocity, it is convenient to introduce the integral azimuthal velocity of energy transfer ω_{en}. First, one has to calculate the time averaged energy flux (not energy flux density, which is the Poynting vector) which is carried by the AWs through the rectangle ABCD (see Fig. 2). One side is a segment on the axis of the waveguide; its length is arbitrary, let it be DC $= dz$. The opposite side is the axial segment of the same length AB $= dz$ at the metallic wall of the waveguide. The other two sides are the radii DA and CB which connect the ends of the first and second sides. This flux is

$$dz \int_0^a S_\varphi dr. \tag{15}$$

This flux can also be calculated as the product of the angular velocity of energy transfer ω_{en} and the time averaged energy of the electromagnetic wave, which is contained inside the elementary volume drawn in Fig. 2.

Note that this elementary volume is not (!) a rectangular parallelepiped even in the limit $d\varphi \ll 1$. The volume is restricted by the rectangle ABCD described above and the other one, which can be obtained by rotating the first one by the elementary angle $d\varphi$ (see Fig. 2). This electromagnetic energy dW is given by

$$dW = dz\, d\varphi \int_0^a r u dr, \tag{16}$$

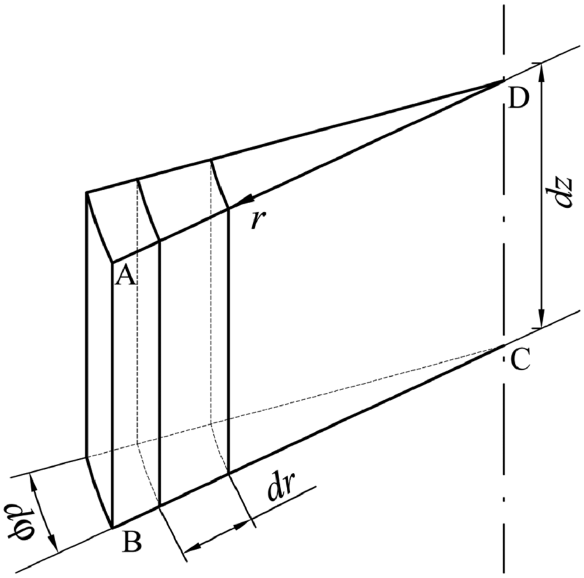

FIG. 2. Schematic of the definition of angular velocity of energy transfer.

where u is the electromagnetic energy density[6]

$$u = \frac{1}{8\pi}\left(|\mathrm{E}|^2 + |\mathrm{H}|^2\right). \tag{17}$$

Then, the definition of the integral angular velocity of energy transfer reads as

$$\omega_{en} = \int_0^a S_\varphi dr \bigg/ \int_0^a r u dr. \tag{18}$$

In general, monochromatic electromagnetic azimuthal waves (both bulk and surface) propagate in cylindrical waveguides with azimuthal phase velocity (13) and carry on their energy with the angular velocity of energy transfer (18). Azimuthal waves do not carry their energy along the $\vec{z}$ axis, $v_{enz} = 0$.

III. RESULTS OF NUMERICAL ANALYSES

The radial dependencies of normalized azimuthal Poynting flux $\beta(r)$, normalized angular electromagnetic energy density $\chi(r)$, angular velocity of energy rotation $\omega_{en}(r)/|\omega_e|$, and linear velocity of energy transfer $r\omega_{en}(r)/c$ are plotted in Figs. 3 and 4 for $Z \equiv \Omega_e/|\omega_e| = 5$ and $m = 2$. The electromagnetic energy is concentrated mainly near the plasma-metal interface and the electromagnetic energy flux is also the largest near the interface. However, the angular velocity of the energy transfer has its maximum near the waveguide axis. The latter means that the efficiency of energy rotation is the largest just there: the full energy of elementary cylindrical volume dr makes the full circle faster than in the other elementary cylindrical layers. Figure 4 presents the linear velocity of the energy rotation to confirm the fact that it does not exceed the velocity of light. The solid and dotted curves in Figs. 3 and 4 correspond to the case of small wave penetration depth into the plasma: $k_{ef} = 0.649$, $k_\perp a = 3.0$, $\omega/|\omega_e| = 0.594$, and the dashed and dashed-dotted curves describe the case of larger wave penetration depth: $k_{ef} = 5.77$, $k_\perp a = 0.333$, $\omega/|\omega_e| = 0.99$.

The XASW phase velocity ω_{ph} given in Eq. (13) is a very simple function of the XASW eigenfrequency ω. The dependence of ω on the plasma waveguide parameters was analyzed in detail in Refs. 15 and 17. That is why not too much attention is paid to ω_{ph} in the present paper. It is only used to be compared with the angular velocity of energy transfer ω_{en} in Figs. 5 and 6. The dependence of ω_{en} on the plasma waveguide parameters is studied in detail in Figs. 5–8. Thereafter, the dependence of the

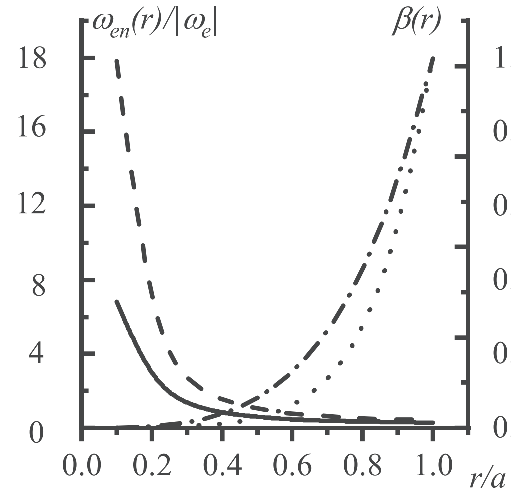

FIG. 3. Radial profile of normalized azimuthal energy flux density $\beta(r)$—right ordinate axis (dashed-dotted and dotted curves) and that of angular velocity of energy rotation, $\omega_{en}(r)/|\omega_e|$—left ordinate axis (dashed and solid curves).

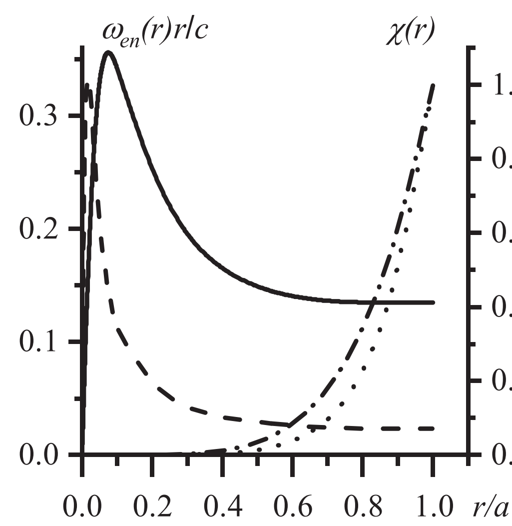

FIG. 4. Radial profile of normalized angular electromagnetic energy density $\chi(r)$—right ordinate axis (dashed-dotted and dotted curves) and that of local linear velocity of energy transfer, $r\omega_{en}(r)/c$—left ordinate axis (dashed and solid curves).

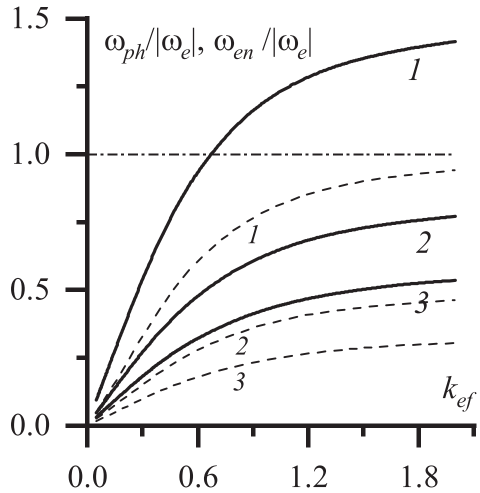

FIG. 5. LF XASW angular phase velocity (dashed lines) and angular velocity of energy transfer (solid lines) for $Z = 5$.

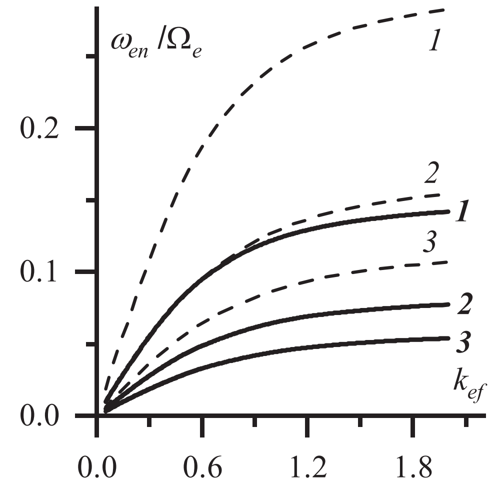

FIG. 7. LF XASW angular velocity of energy transfer for two different strengths of the external static axial magnetic field B_0: $Z = 5$ (dashed lines) and $Z = 10$ (solid lines).

angular group velocity ω_g is analyzed and compared with ω_{en} in Figs. 9–12. The numbers nearby the curves mark the azimuthal wavenumber m.

They present the dispersion dependencies as a function of eigenfrequency vs wave vector, $\omega = \omega(k)$. The ratio m/a can be chosen as a characteristic azimuthal wave vector in the present problem. That is why the effective wavenumber $k_{ef} = |m|\delta/a$ is chosen as abscissa in Figs. 5–12, where $\delta = c/\Omega_e$ is the skin-depth. For LF XASWs, the value of k_{ef} is closely related to the magnitude of the argument $k_\perp a$ of the Bessel functions in the dispersion relation (5): $k_{ef} \approx |m|/(k_\perp a)$. According to this definition, increasing k_{ef} corresponds to decreasing waveguide radius a and decreasing plasma particle density. However, the effective wavenumber k_{ef} is not a proper abscissa if one studies the dependencies on azimuthal wavenumber m since k_{ef} is proportional to $|m|$. To investigate these dependencies, separate numerical runs were carried out which results are given below in the text but not presented in the form of figures.

The ranges of k_{ef} for which numerical analyses were performed are chosen based on the following reasons. For LF XASWs, the calculations are stopped for those values of k_{ef} for which the eigenfrequency becomes close to $|\omega_e|$ which is the upper limit of the LF frequency range (1). This range is $0 < k_{ef} < 2$.

In Figs. 6, 8, 10, and 12, in which the dependencies of the HF XASW angular phase velocity, angular velocity of energy transfer, and angular group velocity on k_{ef} are presented, the curves

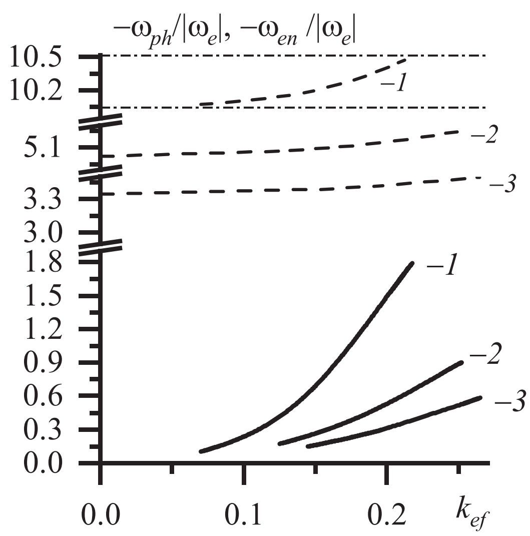

FIG. 6. HF XASW angular phase velocity (dashed lines) and angular velocity of energy transfer (solid lines) for $Z = 10$.

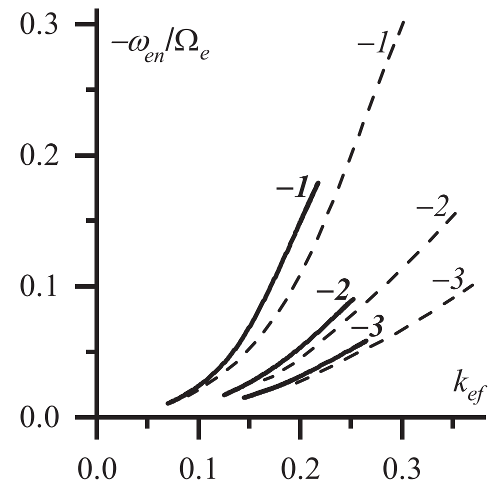

FIG. 8. HF XASW angular velocity of energy transfer for two different strengths of the external static axial magnetic field B_0: $Z = 5$ (dashed lines) and $Z = 10$ (solid lines).

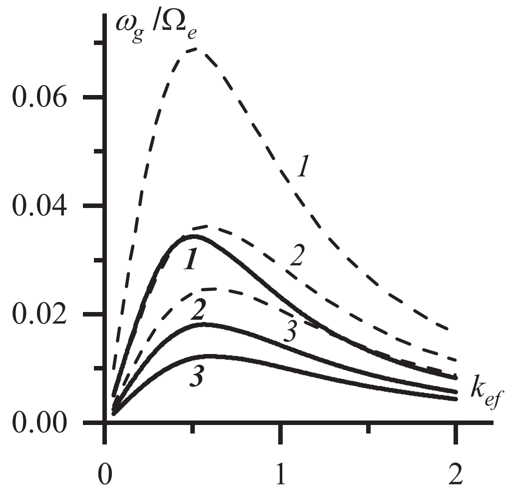

FIG. 9. Angular group velocity of LF XASWs vs effective wavenumber for $Z = 5$ (dashed lines) and $Z = 10$ (solid lines).

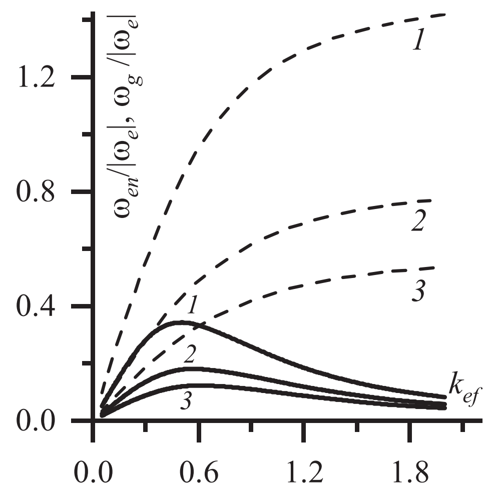

FIG. 11. Angular group velocity (solid curves) and angular velocity of energy transfer (dashed curves) of LF XASWs vs effective wavenumber for $Z = 10$.

are calculated for those ranges of k_{ef} where these waves can propagate. These ranges are known to be limited by zero on the left side and a moderate value of k_{ef} on the right side.[15,17] For HF XASWs, small values of k_{ef} also correspond to plasma waveguides with thin penetration depths ($k_{\perp}a \gg 1$), and the values of k_{ef} close to the right limit of the range correspond to plasmas wherein HF XASWs penetrate deeply ($k_{\perp}a \ll 1$).

In Figs. 5 and 6, absolute values of angular phase velocity and angular velocity of energy transfer normalized by the electron cyclotron frequency, $|\omega_{en,ph}/\omega_e|$, are chosen as the ordinate axis. The dashed curves correspond to the angular phase velocities and the solid curves to angular velocities of energy transfer.

In Fig. 5, one can see that the absolute value of the angular velocity of energy transfer of LF XASWs (solid curves) is larger than that of the corresponding angular phase velocity (dashed curves). The horizontal dashed-dotted line relates to the upper limit of the LF frequency range (1), $Z = 5$. The absolute value of the angular phase velocity is known[15,17] to approach $\omega_{LH}/|m| \ll |\omega_e|$ with the decreasing effective wavenumber, $k_{ef} \to 0$. The dispersion of LF XASWs is normal: the signs of their angular velocities of energy transfer and phase velocities coincide. The absolute value of the angular velocity of energy transfer $|\omega_{en}|$ of LF XASWs decreases with increasing curvature radius a of the plasma-metal interface and increasing plasma particle density. The dependence of $|\omega_{en}|$ on the absolute value $|m|$ of the azimuthal wavenumber is not clear from Fig. 5 since the effective wavenumber k_{ef} also depends on $|m|$.

Additional numerical analysis proves that $|\omega_{en}|$ of LF XASWs decreases with increasing $|m|$. For small values of k_{ef}, both $|\omega_{en}|$

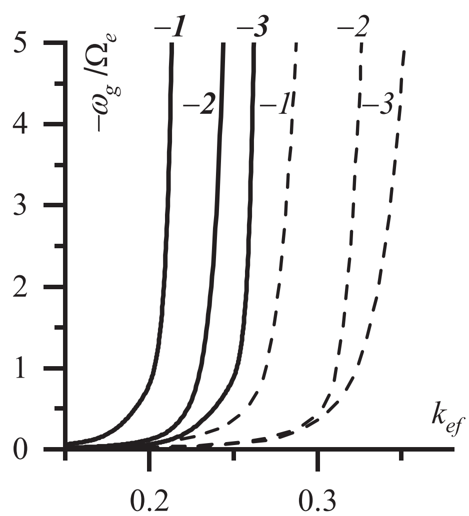

FIG. 10. Angular group velocity of HF XASWs vs effective wavenumber for $Z = 5$ (dashed lines) and $Z = 10$ (solid lines).

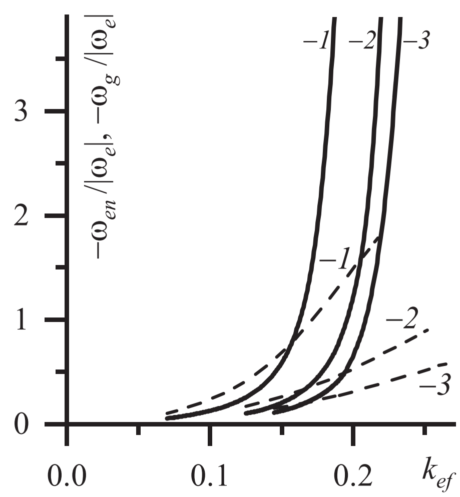

FIG. 12. Angular group velocity (solid curves) and angular velocity of energy transfer (dashed curves) of HF XASWs vs effective wavenumber for $Z = 10$.

and $|\omega_{ph}|$ are approximately proportional to k_{ef}, and for large values of k_{ef}, they saturate.

Figure 6 shows that angular phase velocities (dashed curves) of HF XASWs are larger than their velocities of energy transfer (solid curves). The dashed-dotted lines indicate the boundaries of the frequency range (2) for HF XASWs. With the decreasing effective wavenumber ($k_{ef} \to 0$), the absolute values of the phase velocities approach the finite values $\omega_{UH}/|m|$, and $|\omega_{en}|$ of HF XASWs decreases to a value which is much lower than $|\omega_e|$. The absolute values of ω_{ph} approach the finite values $\omega_2/|m|$ for moderate values of Max$\{k_{ef}\} < 1$,[15–17] and HF XASWs do not propagate in the waveguides with parameters which correspond to the values of $k_{ef} >$ Max$\{k_{ef}\}$. For $0 < k_{ef} <$ Max$\{k_{ef}\}$, $|\omega_{en}|$ monotonously increases. The absolute values of both angular phase velocity and angular velocity of energy transfer of HF XASWs decrease with increasing $|m|$. The dispersion of HF XASWs also appears to be normal.

The dispersion properties of XASWs are known to strongly depend on the external static magnetic field B_0.[15–17] Figures 7 and 8 present the results of studying the influence of an external static magnetic field on the angular velocity of energy transfer. That is why the absolute value of ω_{en} normalized by the Langmuir frequency is used in these figures as the ordinate axis. In this case, both abscissa and ordinate contain the Langmuir frequency in the denominator. In Figs. 7 and 8, the dashed curves are used for the dependencies plotted for $Z \equiv \Omega_e/|\omega_e| = 5$, and the solid curves for the weaker external static magnetic field, $Z = 10$. In Fig. 7, one can see that $|\omega_{en}|$ of LF XASWs increases with increasing B_0. The opposite behavior is shown in Fig. 8 for HF XASWs, where $|\omega_{en}|$ decreases with increasing B_0.

The results of numerical studies on the effect of the external static magnetic field B_0 on the dependence of the angular group velocity of XASWs vs effective wavenumber are presented in Figs. 9 and 10. By definition, larger Z corresponds to smaller B_0. The angular group velocity of LF XASWs is plotted in Fig. 9. It increases with the increasing external static magnetic field, unlike $|\omega_g|$ for HF XASWs, which is smaller for larger B_0 as shown in Fig. 10. This characteristic of the dependence of $|\omega_g|$ vs B_0 is in qualitative agreement with the dependence of the angular velocity of energy transfer $|\omega_{en}|$ (see Figs. 7 and 8). The angular group velocity of LF XASWs demonstrates non-monotonous behavior, unlike $|\omega_g|$ for HF XASWs which infinitely increases with the increasing effective wavenumber. This feature of the dependence of $|\omega_g|$ on the effective wavenumber has no analogy with that of $|\omega_{en}|$.

In Figs. 11 and 12, the dependence of the magnitude of the angular group velocity $|\omega_g|$ of XASWs on k_{ef} is compared with that of $|\omega_{en}|$. The absolute value $|\omega_g|$ of HF XASWs infinitely increases when k_{ef} approaches the right limit of the k_{ef} range within which HF XASWs can propagate. For sufficiently small values of k_{ef}, the absolute value of the angular group velocity is smaller than that of the angular velocity of energy transfer, $|\omega_g| < |\omega_{en}|$. The value of k_{ef} for which $|\omega_g| = |\omega_{en}|$ is larger for XASWs with larger absolute values of the azimuthal wavenumber m. The angular group velocity of LF XASWs is smaller than ω_{en} and depends non-monotonously on k_{ef}. The absolute value of the angular group velocity of both LF and HF XASWs decreases with the increasing absolute value of the azimuthal wavenumber.

IV. CONCLUSIONS

When non-axisymmetric electromagnetic waves (whose axial and azimuthal wavenumbers are nonzero, $k_z \neq 0$, $m \neq 0$) propagate in cylindrical waveguides, their energy is transferred in two directions. A part of the energy is transferred along the waveguide axis. This is described and studied elsewhere. The linear velocity of energy transfer, which is the proportionality coefficient between the Poynting vector given in Eq. (6) and spatial electromagnetic energy density Eq. (17), is the appropriate tool for description of this motion. The other part of the electromagnetic energy rotates around the waveguide axis. The linear velocity of energy transfer cannot be used for the description of such energy motion.

The radial dependence of the angular velocity of the energy rotation shows that the electromagnetic power rotates most effectively near the waveguide axis although the electromagnetic energy density and electromagnetic energy flux are the largest near the plasma-metal interface for XASWs. Since the radial dependence of the angular velocity of the energy transfer is strongly non-monotonous, the integral value of the angular velocity of the energy rotation ω_{en} is introduced.

The angular phase velocity ω_{ph}, angular group velocity ω_g, and angular velocity of energy transfer ω_{en} should be applied to study the electromagnetic energy rotation in cylindrical waveguides. The definitions of these physical values are introduced for the case of azimuthal electromagnetic waves ($k_z = 0$). These angular velocities are studied here in their dependence on the plasma waveguide parameters for a cylindrical metal chamber entirely filled by plasmas in two frequency ranges: low frequencies (LF) defined by Eq. (1) and high frequencies (HF) defined by (2).

The angular phase velocity ω_{ph} given by Eq. (13) is defined as the angular velocity with which the half-plane of the wave fixed phase ($m\varphi - \omega t = \text{Const}$) rotates around the waveguide axis.

The angular velocity of energy transfer ω_{en} described by Eq. (18) is defined as the proportionality coefficient between the averaged electromagnetic energy flux and the averaged electromagnetic energy contained in an angular unit volume (see Fig. 3, in which the angle $d\varphi$ should be put equal to unit).

The angular group velocity ω_g given by Eq. (14) is defined as the angular velocity with which the wavepacket consisting of the modes with different azimuthal wavenumbers m rotates around the waveguide axis without change in shape. This physical value cannot be considered as a universal one since it can be calculated only for a small scope of waveguide structures.

In the present paper, it is shown that the dispersion of XASWs is normal: the directions of the angular velocity of energy transfer ω_{en} and the angular phase velocity ω_{ph} coincide. For LF XASWs, the absolute value of the angular velocity of energy transfer is shown to be larger than that of the angular phase velocity ($|\omega_{en}| > |\omega_{ph}|$) or vice versa $|\omega_{en}| < |\omega_{ph}|$ for HF XASWs. Increasing the external static magnetic field results in an increase in the absolute value of the angular velocity of LF XASW energy transfer and decrease in the $|\omega_{en}|$ for HF XASWs. Both angular velocity of energy transfer and angular phase velocity increase with the increasing effective wavenumber.

The mathematical operation of differentiation over the order of Bessel functions is principally not defined for neither Neumann nor McDonald functions. That is why only for a few cylindrical plasma waveguide structures, like the interior problems studied in this paper, the angular group velocity can be theoretically calculated. The dependence of the angular group velocity ω_g of XASWs on plasma waveguide parameters is derived analytically and studied numerically. The difference between ω_g and the angular velocity of energy transfer ω_{en} is more pronounced for those plasma waveguide parameters for which the XASW eigenfrequency is close to the upper limits of these wave frequency ranges: electron cyclotron frequency $|\omega_e|$ and cut-off frequency for bulk modes ω_2. This result is in agreement with the conclusion of Ref. 4, where it is noticed that for electromagnetic waves, the group velocity can be considered as the velocity of energy transfer only if they propagate in media whose permittivity is a slowly varying function of the wave frequency.

The presented analysis of electromagnetic energy transfer in cylindrical plasma-filled waveguides can be of interest, first of all, for plasma electronics,[17] plasma production in gas discharges,[24,27,28] development of new nano-dimensional photonic elements, and surface enhanced Raman spectroscopy (see, e.g., Ref. 29 and references therein).

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