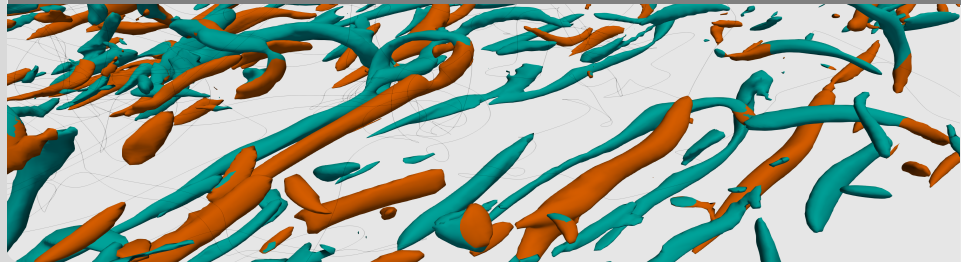


Production, transport and dissipation of turbulent stresses across scales and space

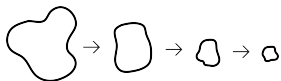
Davide Gatti, A. Chiarini, A. Cimarelli, M. Quadrio | September 6, 2018

INTERNATIONAL TURBULENCE INITIATIVE , 5 – 7 SEPTEMBER 2018, BERTINORO, ITALY

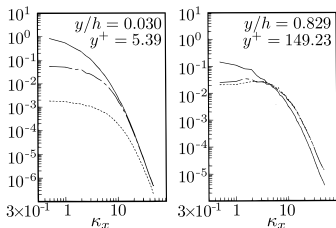
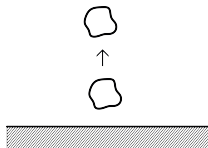


(Two) Classic approaches to turbulence

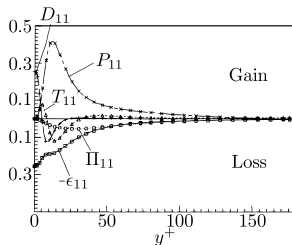
Space of scales



Physical space



Kim *et al.* JFM 1987



Mansour *et al.* JFM 1988

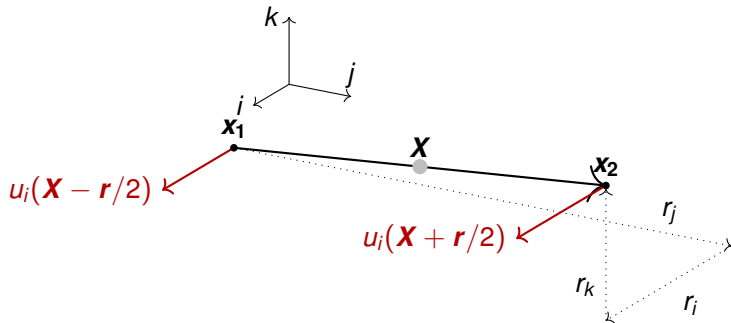
Generalized Kolmogorov Equation (GKE)

Exact budget equation for $\langle \delta u_i \delta u_j \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$

Generalized Kolmogorov Equation (GKE)

Exact budget equation for $\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$

$$\delta u_i = (u_i(\mathbf{X} + \mathbf{r}/2, t) - u_i(\mathbf{X} - \mathbf{r}/2, t))$$



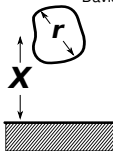
dependent on: $\left\{ \begin{array}{l} \mathbf{X} = (\mathbf{x}_1 + \mathbf{x}_2)/2 \\ \mathbf{r} = \mathbf{x}_2 - \mathbf{x}_1 \end{array} \right.$

Generalized Kolmogorov Equation (GKE)

Amount of turbulent energy
at location \mathbf{X} and scale (up to) r

Davidson *et al.* JFM 2006

■ $\langle \delta u_i \delta u_i \rangle(\mathbf{X}, r)$

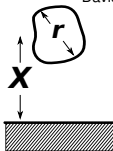


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Davidson *et al.* JFM 2006

- $\langle \delta u_i \delta u_i \rangle(\mathbf{X}, r)$



Production, dissipation and transport
of turbulent **energy**
in both the

Space of scales & Physical space

Cimarelli *et al.* JFM 2013, 2016

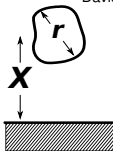
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Generalized Kolmogorov Equation (GKE)

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Davidson *et al.* JFM 2006

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Production, dissipation and transport
of turbulent **energy**
in both the

Space of scales & Physical space

Cimarelli *et al.* JFM 2013, 2016

- GKE

$$\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle \rightarrow \dots \text{anisotropy?}$$

GKE: budget for $\langle \delta u_i \delta u_i \rangle$

$$\frac{\partial \phi_k}{\partial r_k} + \frac{\partial \psi_k}{\partial X_k} = \xi$$

scale flux $\phi_k = \underbrace{\delta U_k \langle \delta u_i \delta u_i \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_i \rangle}_{\text{turbulent transport}} - \underbrace{2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_i \rangle}_{\text{viscous diffusion}}$

space flux $\psi_k = \underbrace{\langle u_k^* \delta u_i \delta u_i \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho} \langle \delta p \delta u_i \rangle}_{\text{pressure transport}} - \underbrace{\frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_i \rangle}_{\text{viscous diffusion}}$

source $\xi = \underbrace{-2 \langle u_k^* \delta u_i \rangle \delta \left(\frac{\partial U_i}{\partial X_k} \right)}_{\text{production}} - \underbrace{2 \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_i}{\partial X_k} \right)^*}_{\text{production}} - \underbrace{4 \epsilon_{ii}^*}_{\text{dissipation}}$

Anisotropic GKEs (AGKEs): budget for $\langle \delta u_i \delta u_j \rangle$

$$\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}$$

scale flux $\phi_{k,ij} = \underbrace{\delta U_k \langle \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} - \underbrace{2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$

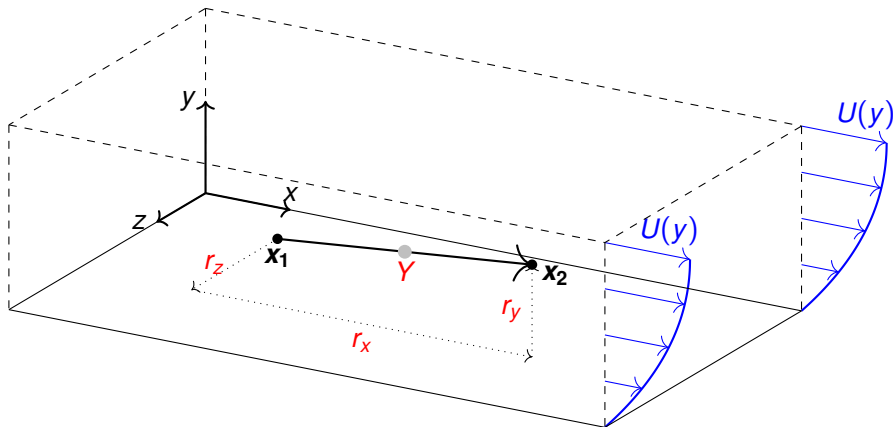
space flux $\psi_{k,ij} = \underbrace{\langle u_k^* \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho} \langle \delta p \delta u_i \rangle \delta_{kj} + \frac{1}{\rho} \langle \delta p \delta u_j \rangle \delta_{ki}}_{\text{pressure transport}} - \underbrace{\frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$

source $\xi_{ij} = \underbrace{-\langle u_k^* \delta u_j \rangle \delta \left(\frac{\partial U_i}{\partial X_k} \right) - \langle u_k^* \delta u_i \rangle \delta \left(\frac{\partial U_j}{\partial X_k} \right)}_{\text{production}} +$

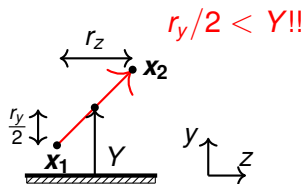
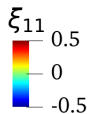
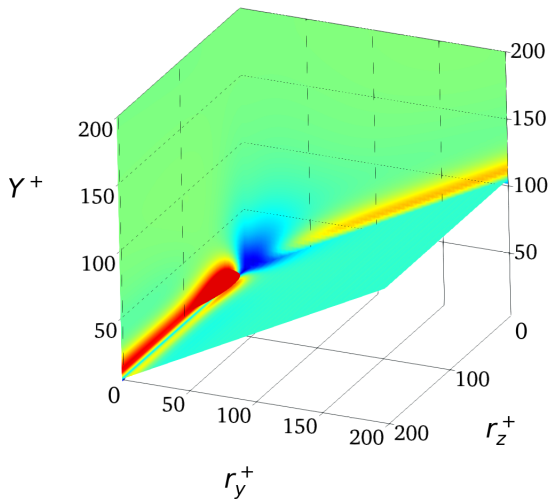
$\underbrace{-\langle \delta u_k \delta u_j \rangle \left(\frac{\partial U_i}{\partial X_k} \right)^* - \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_j}{\partial X_k} \right)^*}_{\text{production}} + \underbrace{\frac{1}{\rho} \left\langle \delta p \frac{\partial \delta u_i}{\partial X_j} \right\rangle + \frac{1}{\rho} \left\langle \delta p \frac{\partial \delta u_j}{\partial X_i} \right\rangle}_{\text{pressure strain}} - \underbrace{4\epsilon_{ij}^*}_{\text{dissipation}}$

AGKEs for indefinite plane channels

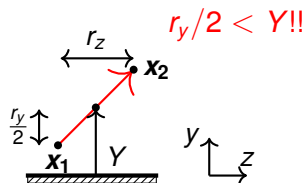
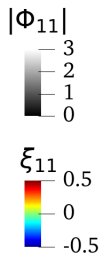
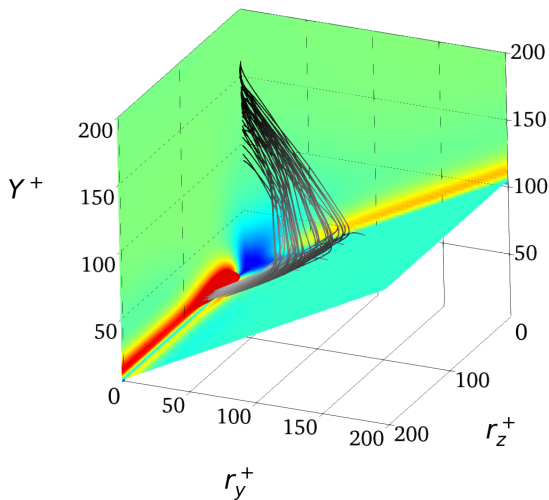
$$\langle \delta u_i \delta u_j \rangle(\mathbf{X}, \mathbf{r}) \rightarrow \langle \delta u_i \delta u_j \rangle(\mathbf{Y}, r_x, r_y, r_z)$$



Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$ space



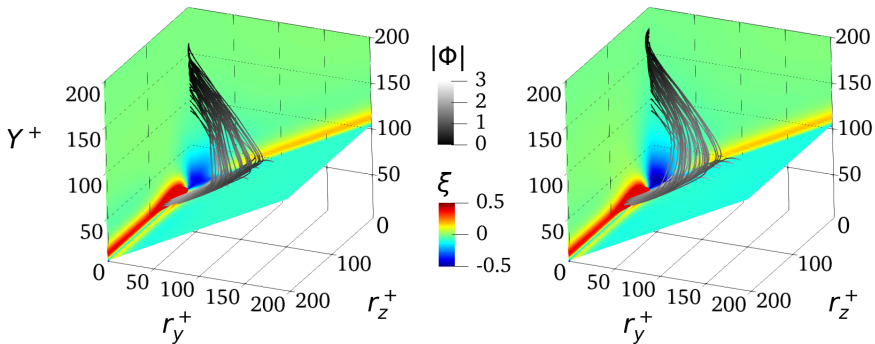
Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$ space



$$\langle \delta u \delta u \rangle$$

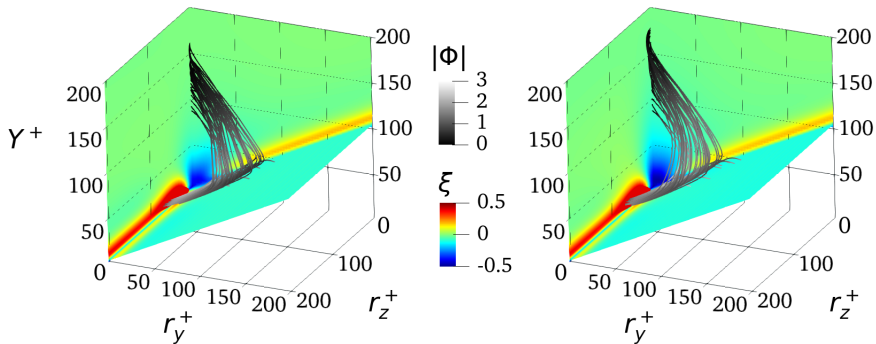
versus

$$\langle \delta u_i \delta u_i \rangle$$



$\langle \delta u \delta u \rangle$

versus

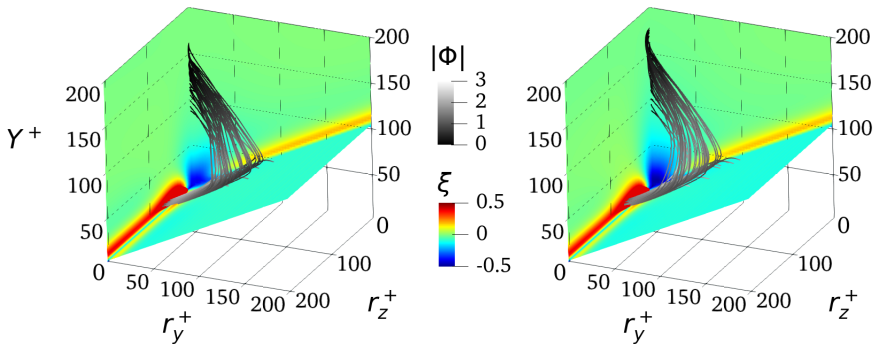
 $\langle \delta u_i \delta u_i \rangle$ 

$$\langle \delta u \delta u \rangle \approx \langle \delta u_i \delta u_i \rangle$$

$$\langle \delta u \delta u \rangle$$

versus

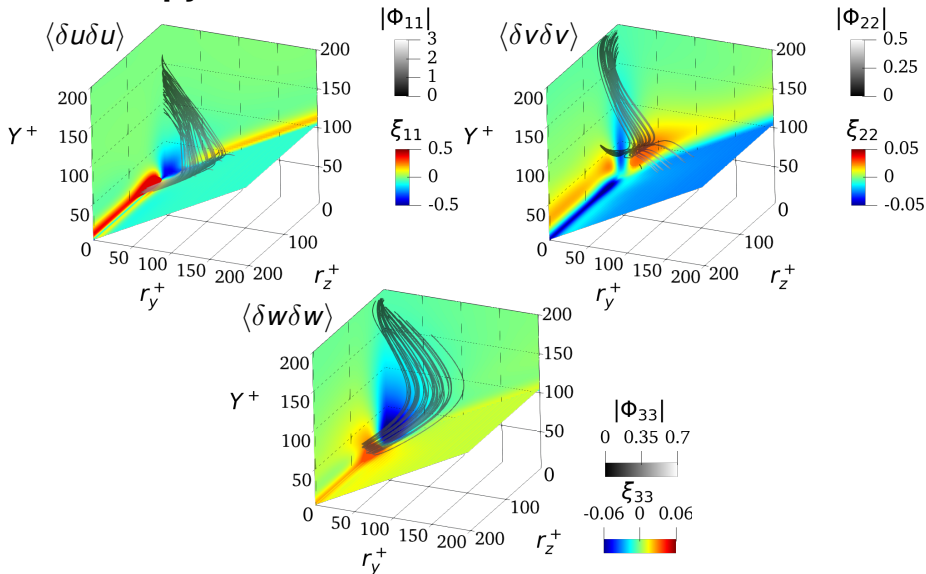
$$\langle \delta u_i \delta u_i \rangle$$



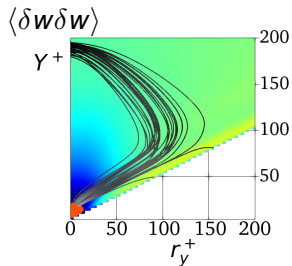
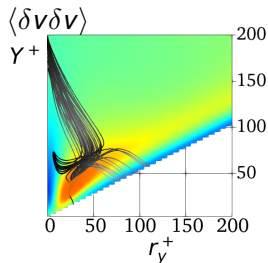
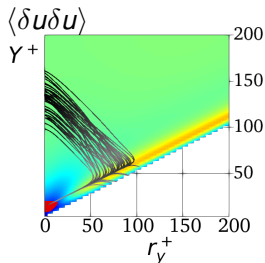
$$\langle \delta u \delta u \rangle \approx \langle \delta u_i \delta u_i \rangle$$

what does AGKE add to GKE?

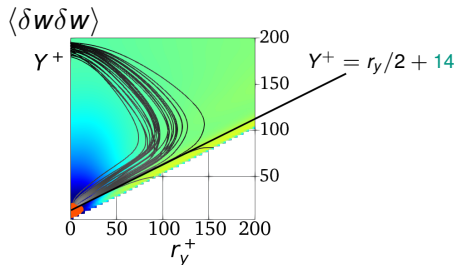
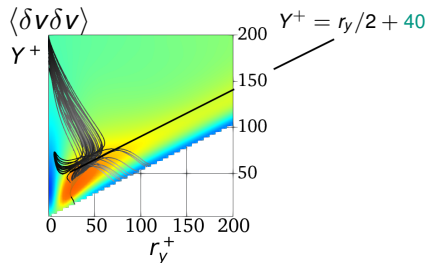
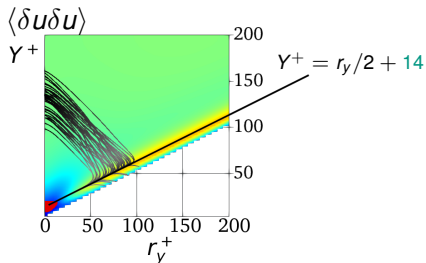
Anisotropy



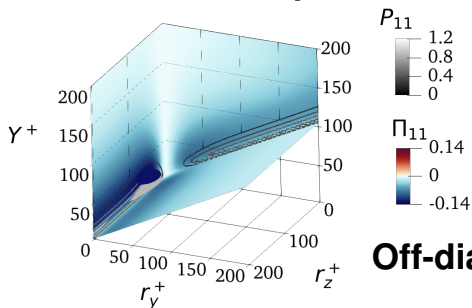
Anisotropy



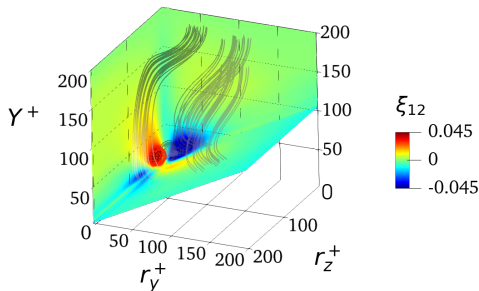
Anisotropy of attached scales

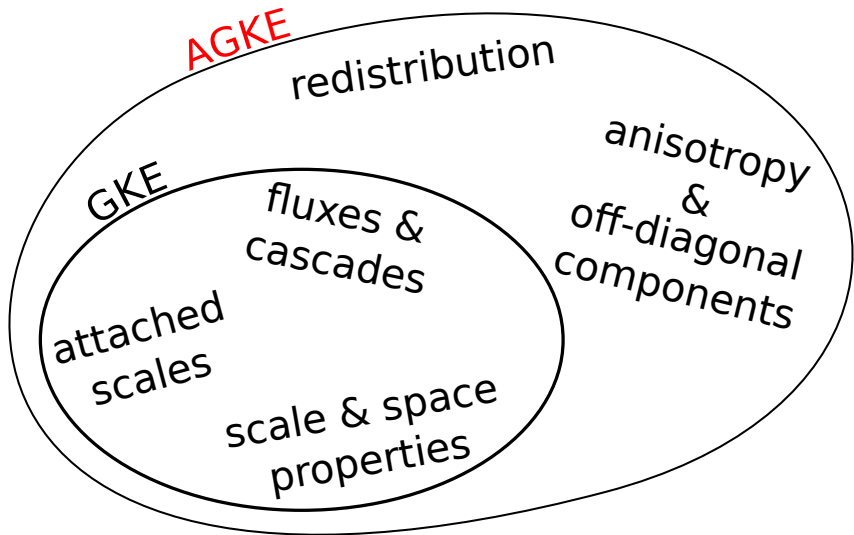


Redistribution: pressure strain



Off-diagonal component $\langle \delta u \delta v \rangle$

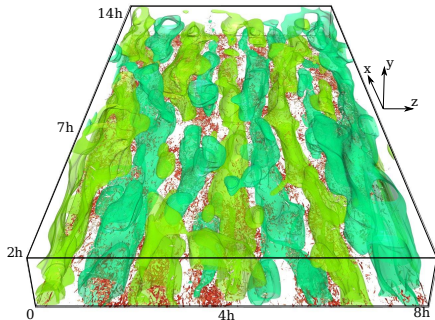




How will I use the AGKE?

Role and occurrence of large scales

- Very Large Scale Motions at high Re

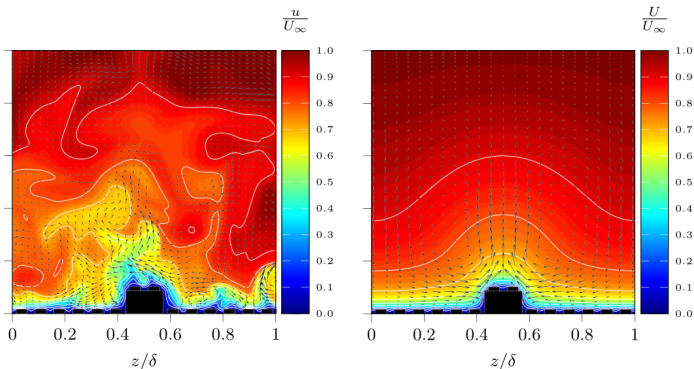


Gatti *et al.* FTaC 2018

How will I use the AGKE?

Role and occurrence of large scales

- Very Large Scale Motions at high Re
- Secondary Motions of Prandtl second kind



Stroh *et al.* JFM submitted

THANKS
for your kind attention!

for questions and suggestions:

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alessandro.chiarini@polimi.it
andrea.cimarelli@unimore.it

European Drag Reduction and Flow Control Meeting

26–29 March 2019, Bad Herrenalb, Germany



www.edrfcm.science

Conclusion

We presented the AGKE: exact budget equations for $\langle \delta u_i \delta u_j \rangle$

- In addition to GKE:
 - anisotropy
 - off-diagonal components
 - redistribution
- In addition to spectral Reynolds stress budgets:
 - no need for homogeneity
 - allows scales in inhomogeneous directions
 - possible “fluxes” interpretation
- probably interesting for your research too!

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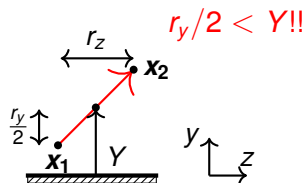
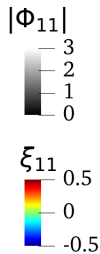
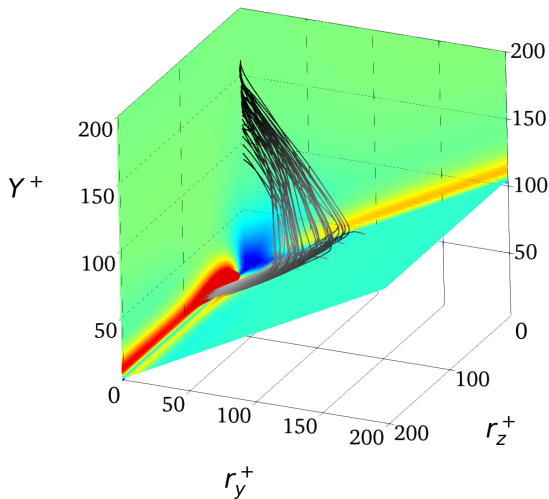
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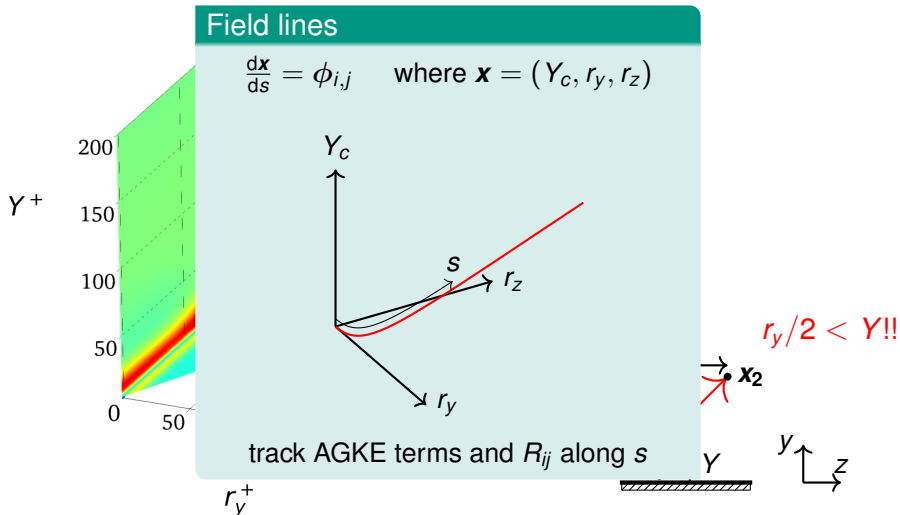
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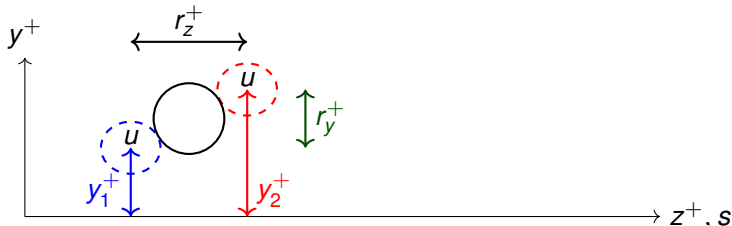
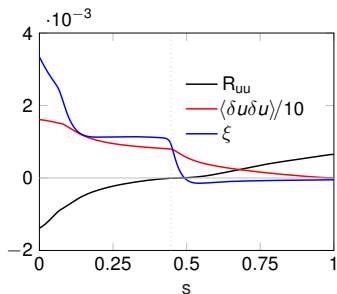
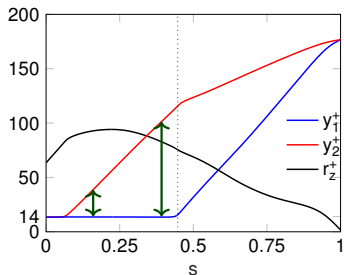
Fluxes, field lines



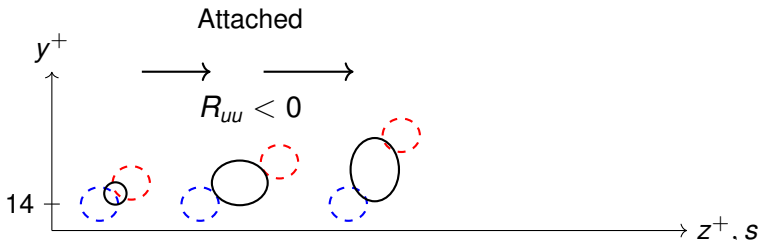
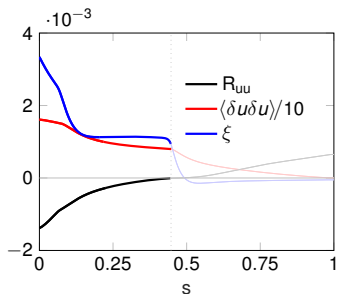
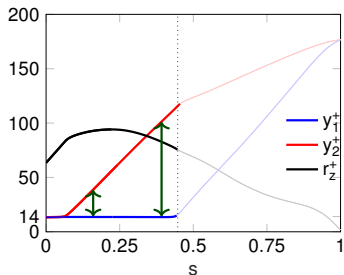
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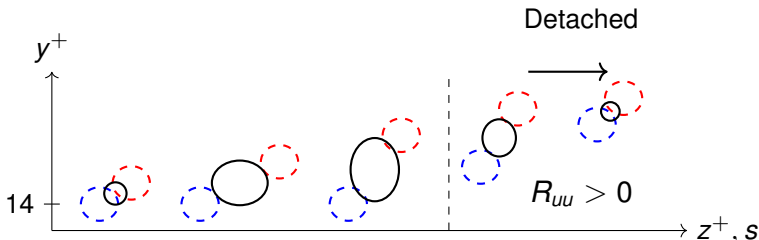
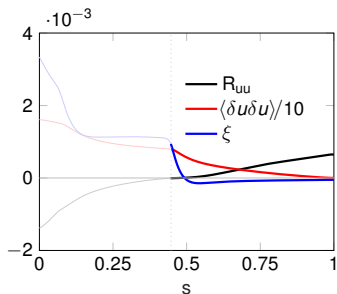
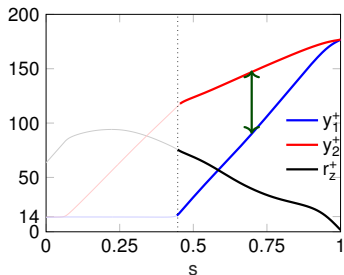
Fluxes, field lines: attached & detached scales



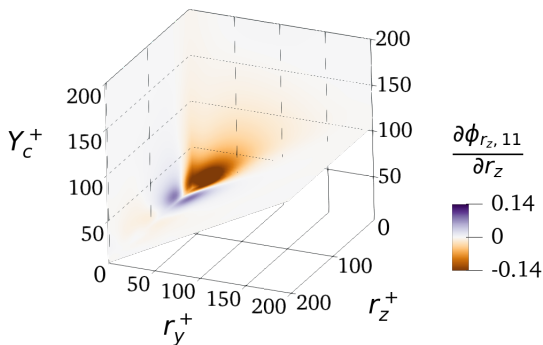
Fluxes, field lines: attached & detached scales



Fluxes, field lines: attached & detached scales



Fluxes, divergence: donor & receiver scales



contribution of various physical processes to $\langle \delta u_i \delta u_j \rangle$
(e.g. nonlinear turbulent transport)

$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_{ij}

$$\langle \delta u_i \delta u_j \rangle(Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{\text{sum of variances}} - \underbrace{2R_{u_i|y_1 u_j|y_2}(r_x, r_z)}_{\text{Cross-correlation}}$$

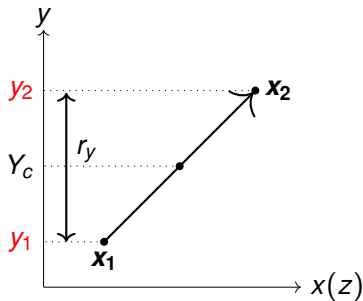
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$$y_1 = Y_c - r_y/2$$

$$y_2 = Y_c + r_y/2$$

$$(Y_c, r_x, r_y, r_z) \leftrightarrow (y_1, y_2, r_x, r_z)$$



$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_{ij}

$$\langle \delta u_i \delta u_j \rangle(Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{\text{sum of variances}} - \underbrace{2R_{u_i|y_1 u_j|y_2}(r_x, r_z)}_{\text{Cross-correlation}}$$

$$\langle \delta u_i \delta u_j \rangle(Y_c, r_x, r_y, r_z) \neq \langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}$$

↓

$$R_{u_i|y_1 u_j|y_2}(r_x, r_z) \neq 0$$

Coherent structures!

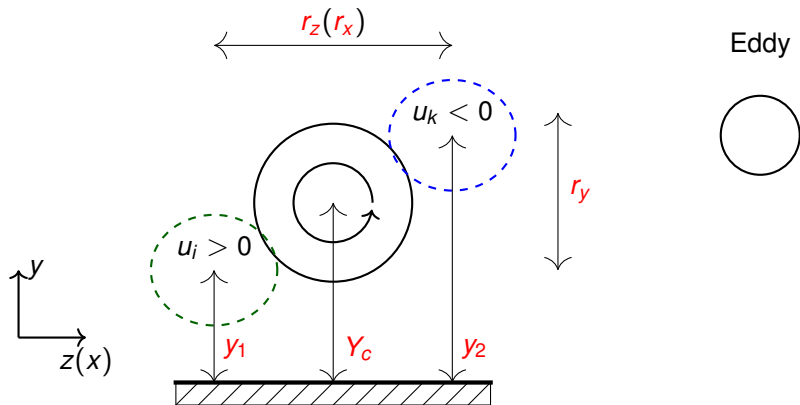


$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_{ij}

$$\langle \delta u \delta u \rangle (Y_c, 0, r_y, r_z) > \underbrace{\langle uu \rangle|_{y_1} + \langle uu \rangle|_{y_2}}_{\text{sum of variances}} - \underbrace{2R_{u|y_1 u|y_2}(0, r_z)}_{\text{Cross-correlation}}$$

$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_{ij}

$$\langle \delta u \delta u \rangle (Y_c, 0, r_y, r_z) = \underbrace{\langle uu \rangle|_{y_1} + \langle uu \rangle|_{y_2}}_{\text{sum of variances}} - \underbrace{2R_{u|y_1 u|y_2}(0, r_z)}_{\text{Cross-correlation}} < 0$$



$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_{ij}

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) \leftrightarrow \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{\text{sum of variances}} - \underbrace{2R_{u_i|y_1 u_j|y_2}}_{\text{Cross-correlation}}(r_x, r_z)$$

