Integral energy budgets in turbulent channels with and without drag reduction

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Today’s goal

“how do dissipation and production of turbulent kinetic energy relate to turbulent friction drag?”
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Reynolds decomposition
\[ u = \langle u \rangle + u' \]

turbulent $\epsilon$ + mean $\phi$
dissipation rate

$\Pi_p$
pumping power

turbulent production
Today’s goal

“how do dissipation and production of turbulent kinetic energy relate to turbulent friction drag in drag-reduced flows?”

Seemingly trivial, nontrivial problem!

Example turbulent dissipation in drag-reduced flow:

- Ricco et al., JFM (2012): it increases
- Agostini, et al., JFM14: it decreases
Constant Power Input (CPI)
**Constant Power Input (CPI)**

**Definitions and characteristic quantities**

- Total power $\Pi_t$ is kept constant: $\Pi_t = \Pi_p + \Pi_c$  
  - Control power $\Pi_c = \gamma \Pi_t$
  - Pumping power $\Pi_p = (1 - \gamma) \Pi_t$

- „Drag reduction“ increases flow rate: $U_b/U_{b,0} > 1$

- A power-based velocity scale:
  $$U_\Pi = \sqrt{\frac{\Pi_t h}{3\mu}}$$

  “The Stokes flow minimizes the power consumption for given flow rate”  
  Bewley (JFM, 2009), Fukagata et al. (Physica D, 2009)

- A power-based Reynolds number:
  $$Re_\Pi = \frac{U_\Pi h}{\nu}$$  
  Hasegawa, Frohnapfel, Quadrio (JFM, 2009)
The wind of turbulence
The “wind decomposition” of turbulence
A triple decomposition with analytical advantages  Eckhardt et al, JFM 2007

\[ u = \langle u \rangle + u' \]
The “wind decomposition” of turbulence
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\[ u = U_\ell + U_\Delta + u' \]
Production and mean dissipation

Mean dissipation decouples!

\[ P = P_\ell + P_\Delta \]

\[ \phi = \phi_\ell + \phi_\Delta + \phi_\ell_\Delta \]
Analytical derivations
A fair amount of cumbersome algebra

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D. Gatti et al., J. Fluid Mech. (submitted)
The new description

pumping power $\Pi_p$

MKE

TKE
The new description

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- laminar dissipation \( \phi_\ell \)
- laminar production \( P_\ell \)
- pumping power \( \Pi_p \)

MKE \rightarrow TKE
The new description

pumping power $\Pi_p$

laminar dissipation $\phi_{\ell}$

control power $\Pi_c$

MKE

laminar production $P_{\ell}$

TKE
The new description

- Pumping power $\Pi_p$
- Laminar dissipation $\phi_\ell$
- Control power $\Pi_c$
- Laminar production $\mathcal{P}_\ell$
- Production by deviation $-\mathcal{P}_\Delta$
- Turbulent dissipation $\epsilon$

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The new description

pumping power $\Pi_p$

laminar dissipation $\phi_\ell$

control power $\Pi_c$

MKE

laminar production $\mathcal{P}_\ell$

dissipation by deviation $\phi_\Delta = -\mathcal{P}_\Delta$

production by deviation $-\mathcal{P}_\Delta$

TKE

turbulent dissipation $\epsilon$

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Two integrals of the turbulent shear stress

Via FIK-like derivations, it is discovered that $\alpha$ and $\beta$ parametrize all the fluxes

$$\alpha = \int_0^1 (1 - y)r(y)\,dy$$

$$\beta = \int_0^1 r^2(y)\,dy \geq 3\alpha^2$$

E.g.

$$P_\Delta = -\phi_\Delta = Re_\Pi(3\alpha^2 - \beta^2) \leq 0$$
Key results
Every flux has a physical meaning

- $\phi_\ell$ is the best way to dissipate pumping power
- $P_\ell$ is the fraction of pumping power wasted to produce turbulence
  - it decreases when control is successful
  - it can be negative as $P_\ell \sim \alpha$
- $\phi_\Delta$ is the penalty for not being laminar
- $\phi_\Delta + \epsilon$ is the fraction of total power wasted by turbulence
  - it cannot be negative
A drag reduction model

- Control effect parametrized through $\Delta B$

> applicable to:

- riblets and roughness
- superhydrophobic surfaces
- spanwise wall forcing
- some feedback controls
A drag reduction model

- Control effect parametrized through $\Delta B$
- CPI constraint $3\text{Re}_\Pi^2 (1 - \gamma) = \text{Re}_\tau^2 \text{Re}_B$

![Graphs showing drag-reduced and natural conditions](image)
How do dissipations change with control?

Back to our initial question
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\[ \nu_w = -\nu(x, y_s, z, t) \]
How do dissipations change with control?

Back to our initial question
Conclusion and outlook

- “Wind” decomposition and CPI introduced
- Theoretical framework for the flow control problem from energy perspective…
- …relevant also for uncontrolled flows: FIK-like identity for $\epsilon$
- Optimal control theory: better choice of cost function
- Development of drag-reduction-aware RANS turbulence models
- CPI-enabled scale-energy analysis of drag reduced flows
European Drag Reduction and Flow Control Meeting

Bad Herrenalb (near Karlsruhe, Germany)

26—29 March 2019
European Drag Reduction and Flow Control Meeting

Topics:
- all laminar and turbulent drag reduction
- flow & noise control studies

Attended by all major scientists in the field

Young contributors are invited to submit abstracts

Conference fee ~ 400€ including accommodation!

More info: www.edrfcm.science
THANKS
for your kind attention!

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