Space and scale fluxes of Reynolds stresses in turbulent channel flows

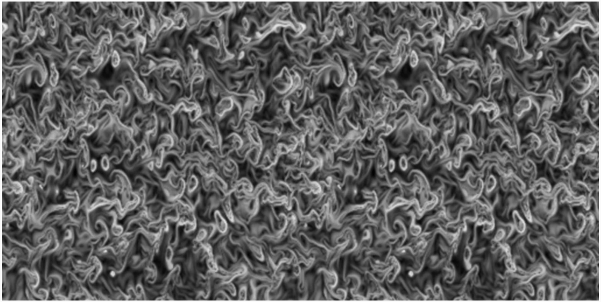
Alessandro Chiarini[1,2], Bettina Frohnapfel[2], Maurizio Quadrio[1], Andrea Cimarelli[3], Yosuke Hasegawa[4], Davide Gatti[2]

GAMM Annual Meeting 2018, March 19-23, Munich, Germany

[1]Politecnico di Milano, [2]Karlsruhe Institute of Technology-KIT, [3]Università Politecinca delle Marche, [4]Univerisity of Tokyo

Wall bounded flows

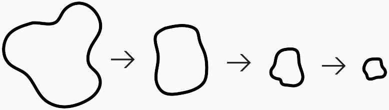
Chaotic motion ...



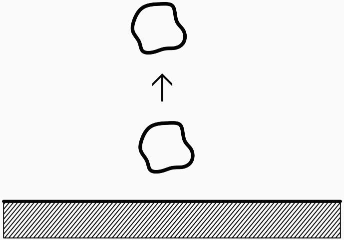
... production, dissipation and transport of turbulent energy!

Wall bounded flows

Space of scales

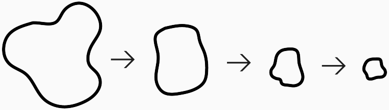


Physical space

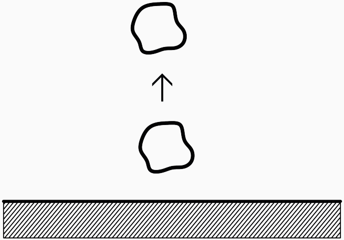


Wall bounded flows

Space of scales



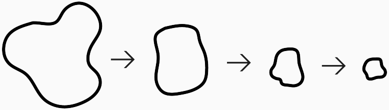
Physical space



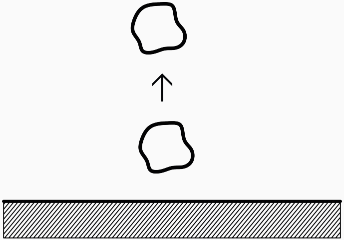
Difficult to consider both spaces concurrently ...
But essential!

Wall bounded flows

Space of scales



Physical space



Difficult to consider both spaces concurrently ...

But essential!

For instance: Modeling (Large-eddies simulations)

In the present work ...

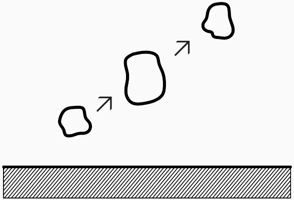
Production, Dissipation and transport

of turbulent stresses

in both the

Space of scales & Physical space

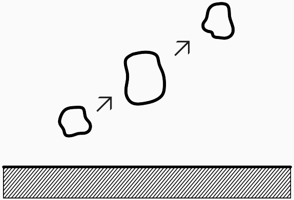
- Aim



In the present work ...

- Aim

Production, Dissipation and transport
of turbulent stresses
in both the
Space of scales & Physical space

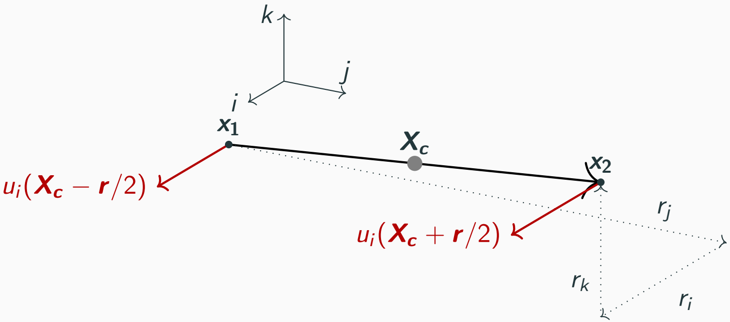


- How?

Budget equation for $\langle \delta u_i \delta u_k \rangle$

$\langle \delta u_i \delta u_k \rangle$

$$\langle \delta u_i \delta u_k \rangle \rightarrow \boxed{\delta u_i = \left(u_i\left(\boldsymbol{X_c} + \boldsymbol{r}/2, t\right) - u_i\left(\boldsymbol{X_c} - \boldsymbol{r}/2, t\right)\right)}$$

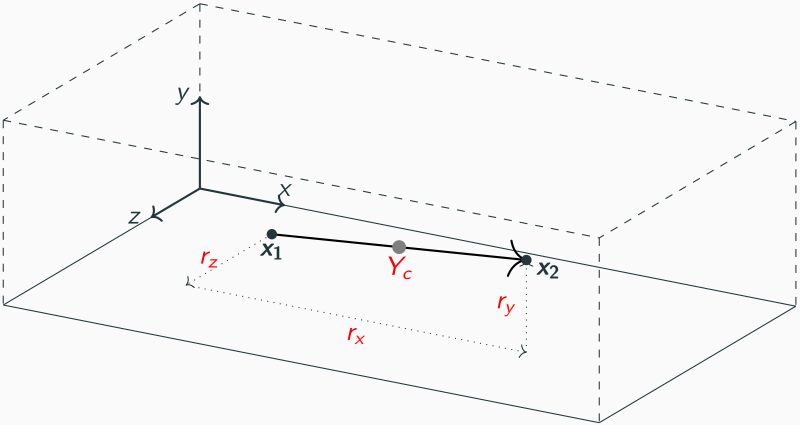


Dependent on:

$$\begin{cases} \boldsymbol{X_c} = (\boldsymbol{x_1} + \boldsymbol{x_2})/2 \\ \boldsymbol{r} = \boldsymbol{x_2} - \boldsymbol{x_1} \end{cases}$$

$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

$$\langle \delta u_i \delta u_k \rangle \rightarrow \langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z)$$



$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

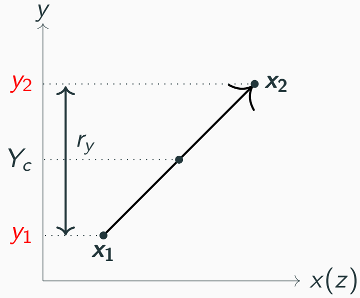
$$\langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_k \rangle\big|_{y_1} + \langle u_i u_k \rangle\big|_{y_2}}_{sum\ \ of\ \ variances} - \underbrace{2R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$

$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

$$\langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_k \rangle|_{y_1} + \langle u_i u_k \rangle|_{y_2}}_{sum\ \ of\ \ variances} \underbrace{-2R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$

$$y_1 = Y_c - r_y/2$$
$$y_2 = Y_c + r_y/2$$

$\rightarrow$



$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

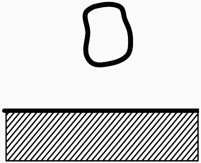
$$\langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_k \rangle|_{y_1} + \langle u_i u_k \rangle|_{y_2}}_{sum\ \ of\ \ variances} - \underbrace{2 R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$

$$\langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z) \neq \langle u_i u_k \rangle|_{y_1} + \langle u_i u_k \rangle|_{y_2}$$

$$\downarrow$$

$$R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z) \neq 0$$

Coherent structures!

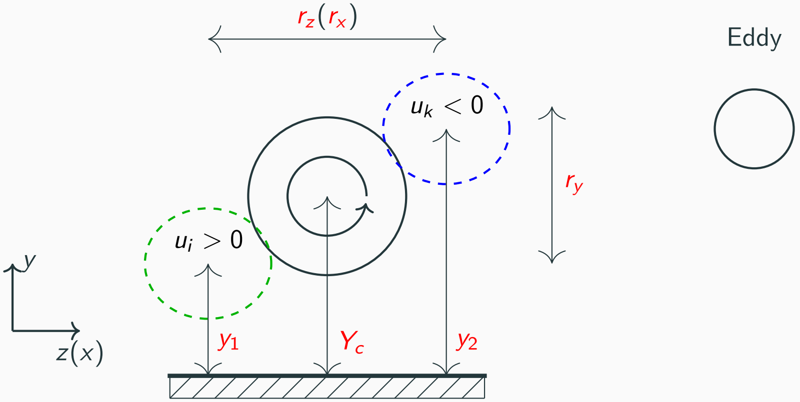


$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

$$\langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z) > \underbrace{\langle u_i u_k \rangle|_{y_1} + \langle u_i u_k \rangle|_{y_2}}_{sum\ \ of\ \ variances} - \underbrace{2R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$

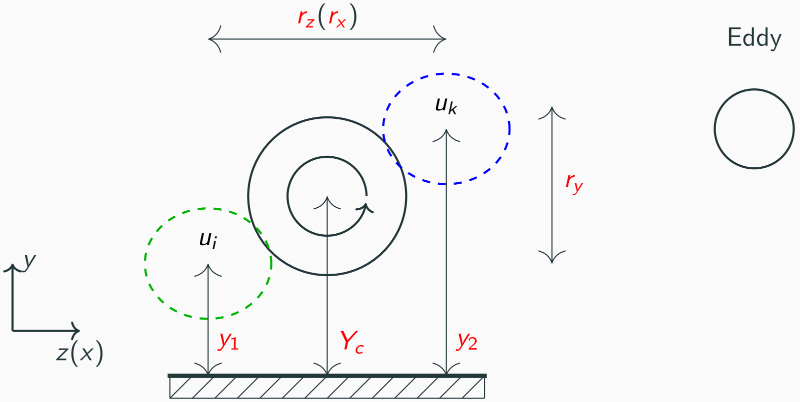
$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

$$\langle \delta u_i \delta u_k \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_k \rangle|_{y_1} + \langle u_i u_k \rangle|_{y_2}}_{sum\ \ of\ \ variances} - \underbrace{2R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z)}_{Cross-correlation} < 0$$



$\langle \delta u_i \delta u_k \rangle$: indefinite plane channel

$$\langle \delta u_i \delta u_k \rangle(Y_c, r_x, r_y, r_z) \leftrightarrow \underbrace{\langle u_i u_k \rangle|_{y_1} + \langle u_i u_k \rangle|_{y_2}}_{sum\ \ of\ \ variances} \underbrace{-2R_{u_i|_{y_1} u_k|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$



Budget equation for $\langle \delta u_i \delta u_k \rangle$

$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \xi_{ik}$$

Budget equation for $\langle \delta u_i \delta u_k \rangle$: Source ξ_{ik}

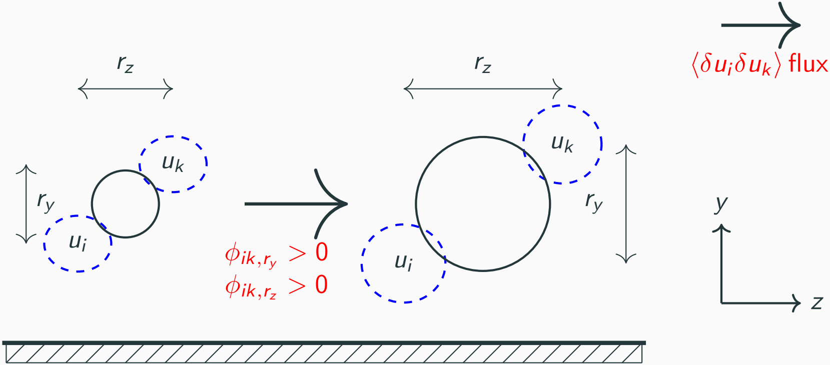
$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \xi_{ik}$$

$$\xi_{ik} = \underbrace{\Pi_{ik}}_{Production} + \underbrace{P_{\text{strain},ik}}_{Pressure\ \ Strain} + \underbrace{\epsilon_{ik}}_{Dissipation}$$

- $\xi(Y_c, \boldsymbol{r}) \rightarrow$ Net production of $\langle \delta u_i \delta u_k \rangle$ at scale $\boldsymbol{r}$ and position Y_c

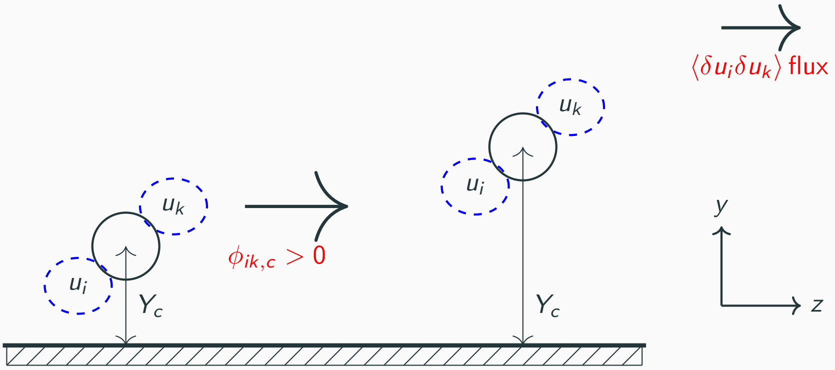
Budget equation for $\langle \delta u_i \delta u_k \rangle$: Scale flux ϕ_{ik,r_j}

$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \xi_{ik}$$

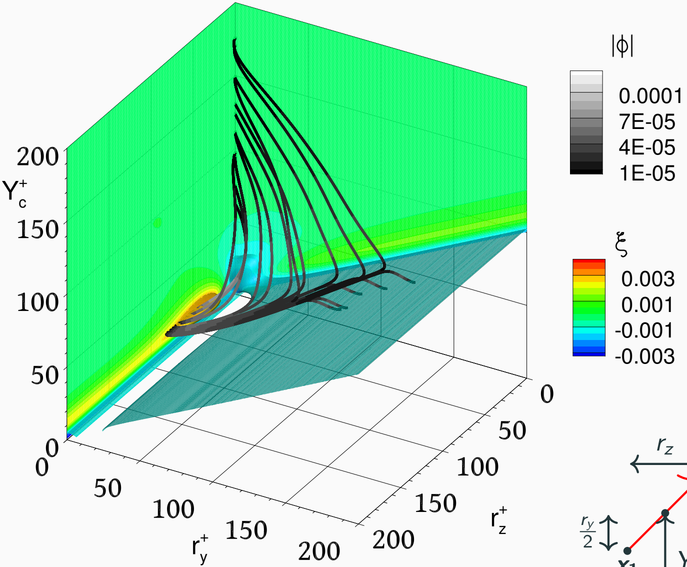


Budget equation for $\langle \delta u_i \delta u_k \rangle$: Physical flux $\phi_{ik,c}$

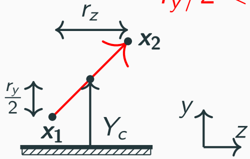
$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \xi_{ik}$$



Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$-space

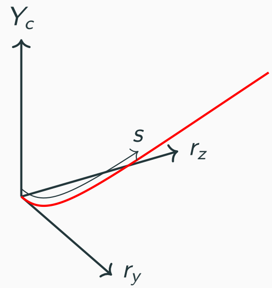


$r_y/2 < Y_c$!



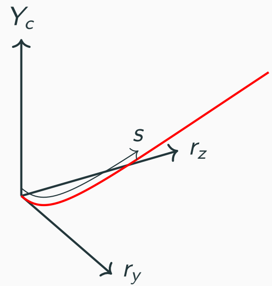
Budget equation for $\langle \delta u_i \delta u_k \rangle$: Field lines

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \phi_{i,k} \qquad \text{where } \boldsymbol{x} = (Y_c, r_y, r_z)$$

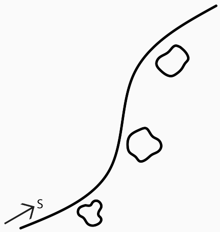


Budget equation for $\langle \delta u_i \delta u_k \rangle$: Field lines

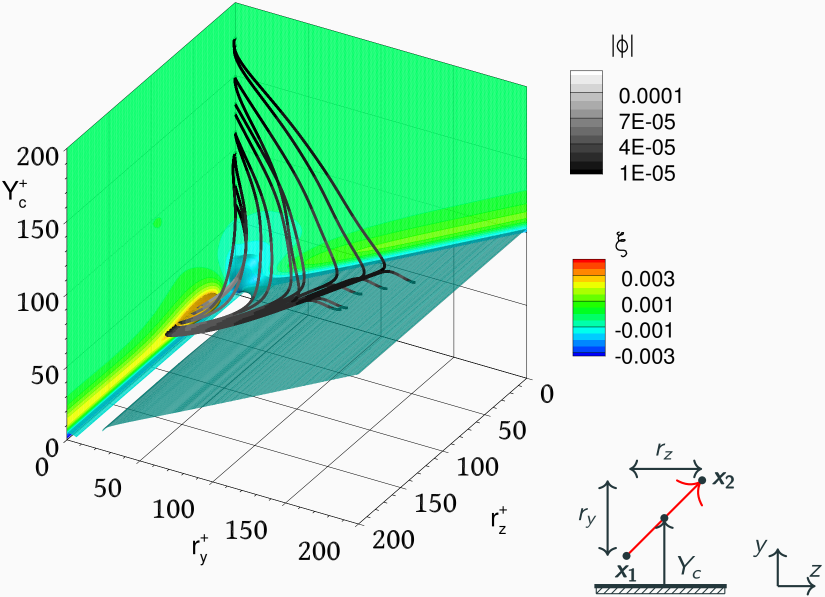
$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \phi_{i,k} \qquad \text{where } \boldsymbol{x} = (Y_c, r_y, r_z)$$



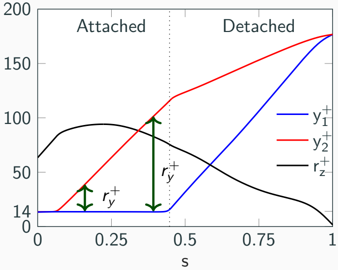
Coherent structures
in the $\langle \delta u_i \delta u_k \rangle$ transfers!

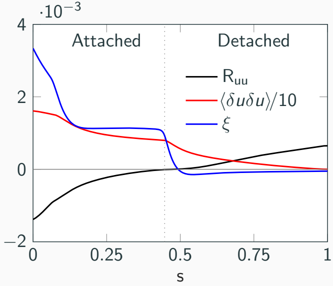


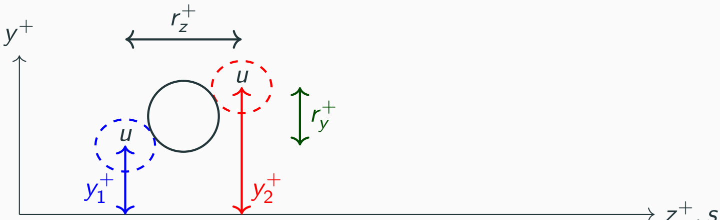
Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$-space



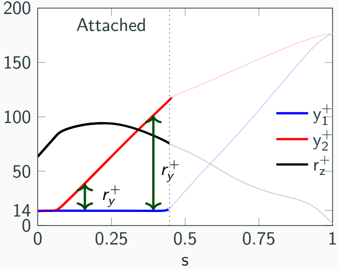
Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$-space

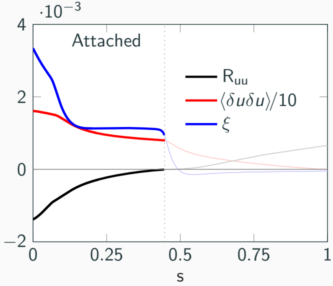


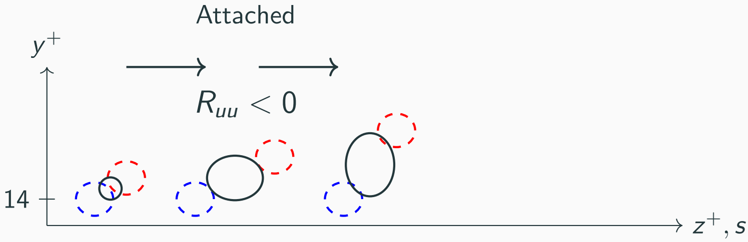




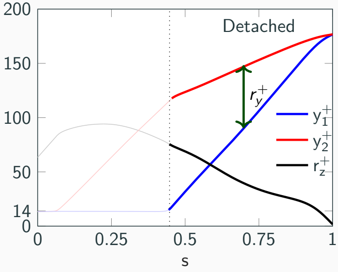
Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$-space

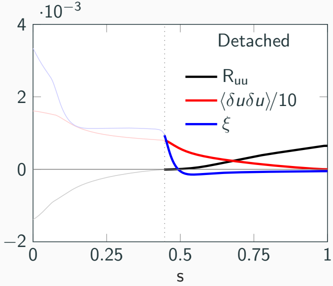


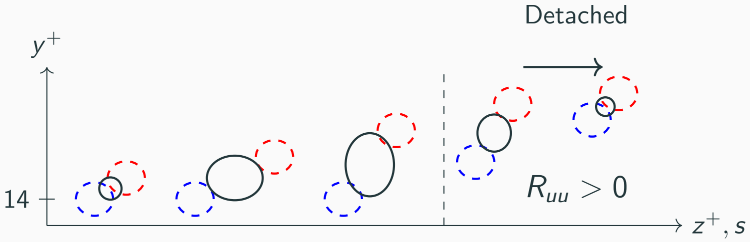




Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$-space





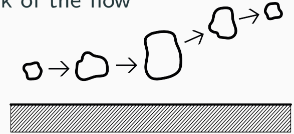


Conclusion

- Budget equation for $\langle \delta u_i \delta u_k \rangle$
    - Exact Derivation
    - Physical interpretation
        - Link to coherent structures

Conclusion

- Budget equation for $\langle \delta u_i \delta u_k \rangle$
  - Exact Derivation
  - Physical interpretation
    - Link to coherent structures
- Turbulent channel ($Re_\tau = 200$)
  - Energy transfer towards the bulk of the flow
    - Attached eddies
    - Detached eddies



Outlook

- Consider $\langle \delta u \delta v \rangle$

  $\langle \delta u \delta v \rangle \leftrightarrow \langle uv \rangle$ $\qquad$ $\langle uv \rangle \rightarrow$ increase C_f

- Drag reduced flow

  Highlight the main changes

THANKS

for your kind attention!

for questions and suggestions:

alessandro1.chiarini@mail.polimi.it

Backup slides

Something more ...

- Budget equation for $\langle \delta u_i \delta u_k \rangle$
- $\langle \delta v \delta v \rangle$ in $r_x = 0$-space
- $\langle \delta w \delta w \rangle$ in $r_x = 0$-space
- $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

Budget equation for $\langle \delta u_i \delta u_k \rangle$

$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \xi_{ik}$$

Budget equation for $\langle \delta u_i \delta u_k \rangle$

$$\frac{\partial \boxed{\phi_{ik,r_j}}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \xi_{ik}$$

- Flux of $\langle \delta u_i \delta u_k \rangle$ in the space of scales

$$\phi_{ik,r_j} = \underbrace{\langle \delta u_i \delta u_k \delta U \rangle \delta_{j,1}}_{mean\ \ transport}$$

$$\underbrace{+\langle \delta u_i \delta u_k \delta u_j \rangle}_{turbulent\ \ transport}$$

$$\underbrace{-2\nu \frac{\partial}{\partial r_j} \langle \delta u_i \delta u_k \rangle}_{viscous\ \ diffusion} \quad j = 1, 2, 3$$

Budget equation for $\langle \delta u_i \delta u_k \rangle$

$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \boxed{\phi_{ik,c}}}{\partial Y_c} = \xi_{ik}$$

- Flux of $\langle \delta u_i \delta u_k \rangle$ in the physical space

$$\phi_{ik,c} = \underbrace{\langle v^* \delta u_i \delta u_k \rangle}_{turbulent\ \ transport} \underbrace{+ \frac{1}{\rho} \langle \delta p \delta u_k \rangle \delta_{i,2} + \frac{1}{\rho} \langle \delta p \delta u_i \rangle \delta_{k,2}}_{pressure\ \ transport} \underbrace{- \frac{\nu}{2} \frac{\partial}{\partial Y_c} \langle \delta u_i \delta u_k \rangle}_{viscous\ \ diffusion}$$

Budget equation for $\langle \delta u_i \delta u_k \rangle$

$$\frac{\partial \phi_{ik,r_j}}{\partial r_j} + \frac{\partial \phi_{ik,c}}{\partial Y_c} = \boxed{\xi_{ik}}$$

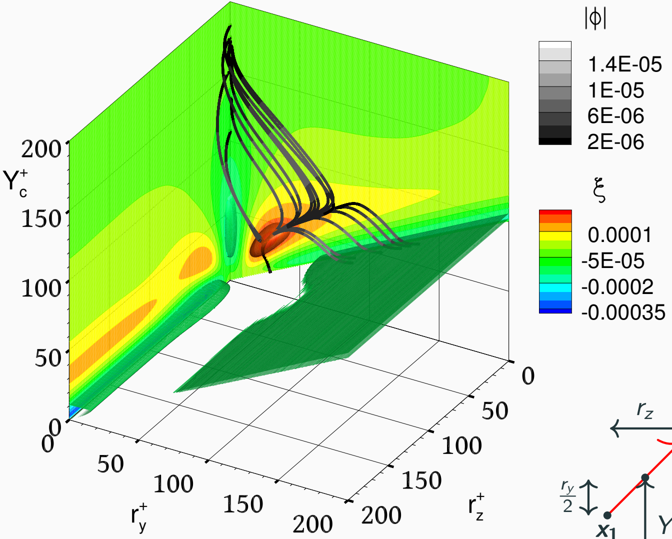
- Source term

$$\xi_{ik} = \underbrace{-\langle v^* \delta u_k \rangle \delta \left(\frac{\mathrm{d}U}{\mathrm{d}y}\right) \delta_{i,1} - \langle v^* \delta u_i \rangle \delta \left(\frac{\mathrm{d}U}{\mathrm{d}y}\right) \delta_{k,1}}_{production} +$$

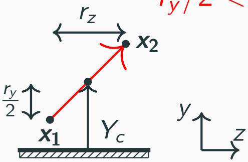
$$\underbrace{-\langle \delta v \delta u_k \rangle \left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^* \delta_{i,1} - \langle \delta v \delta u_i \rangle \left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^* \delta_{k,1}}_{production}$$

$$\underbrace{+\frac{1}{\rho}\left\langle \delta p \frac{\partial \delta u_i}{\partial X_k} \right\rangle + \frac{1}{\rho}\left\langle \delta p \frac{\partial \delta u_k}{\partial X_i} \right\rangle}_{pressure\ \ strain} - \underbrace{4\left(\epsilon^*_{i,k}\right)}_{dissipation}$$

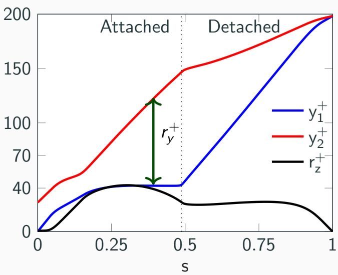
Turbulent channel ($Re_\tau = 200$): $\langle \delta v \delta v \rangle$ in $r_x = 0$-space

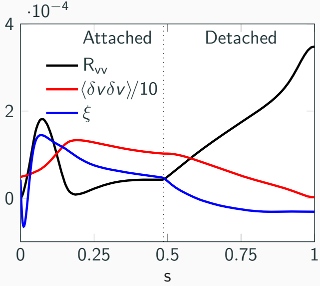


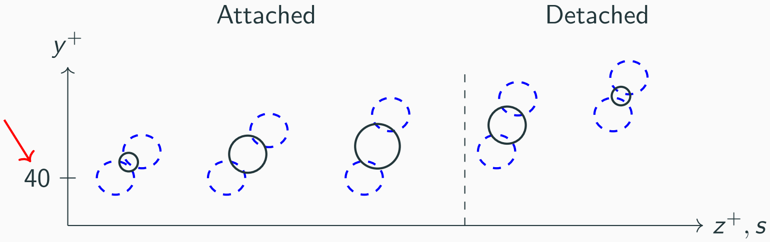
$r_y/2 < Y_c$!



Turbulent channel ($Re_\tau = 200$): $\langle \delta v \delta v \rangle$ in $r_x = 0$-space

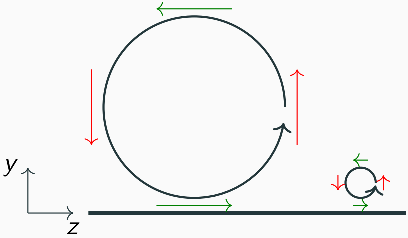






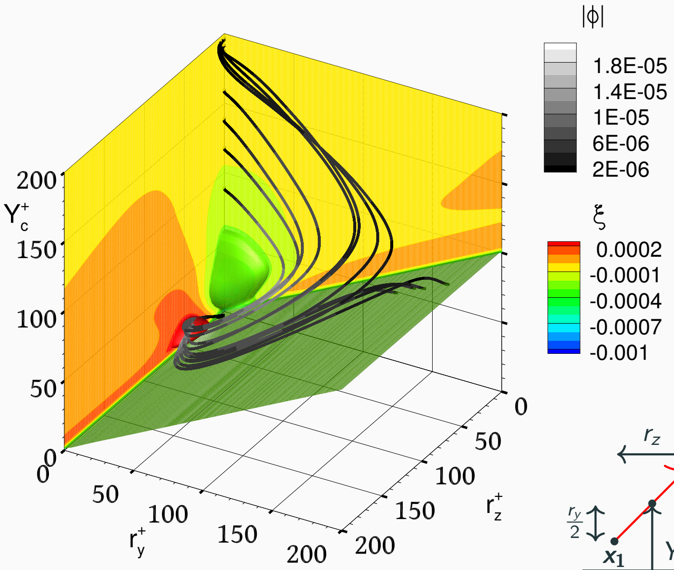
Turbulent channel ($Re_\tau = 200$): $\langle \delta v \delta v \rangle$ in $r_x = 0$-space

Why the wall-normal attached plane is shifted towards larger Y_c?

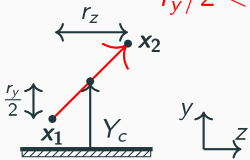


(Townsend 1976)

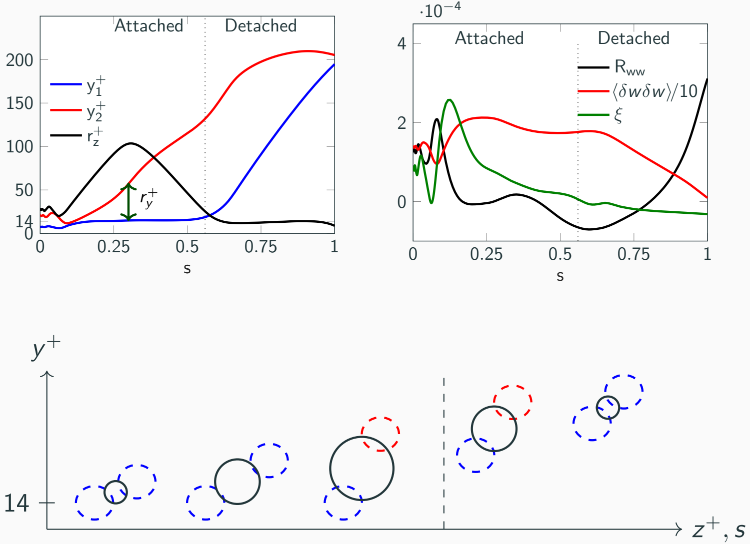
Turbulent channel ($Re_\tau = 200$): $\langle \delta w \delta w \rangle$ in $r_x = 0$-space



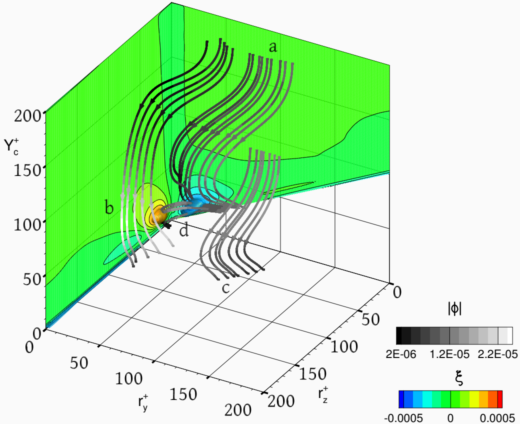
$r_y/2 < Y_c$!



Turbulent channel ($Re_\tau = 200$): $\langle \delta w \delta w \rangle$ in $r_x = 0$-space

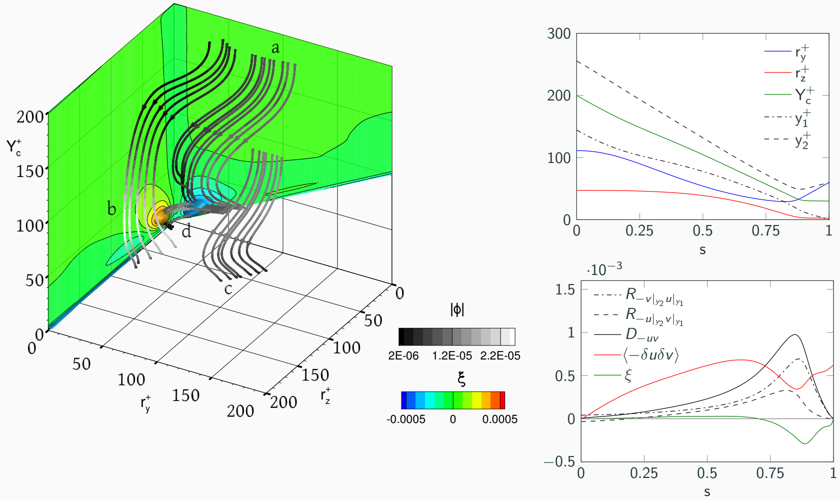


Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

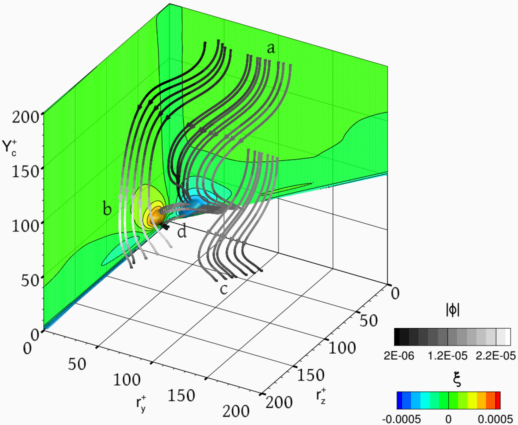


Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

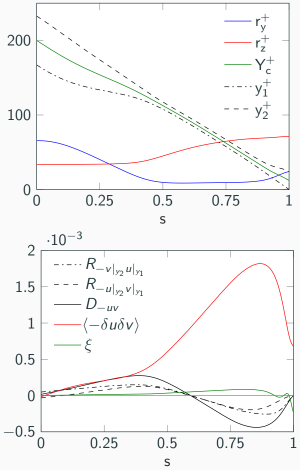
a)



Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

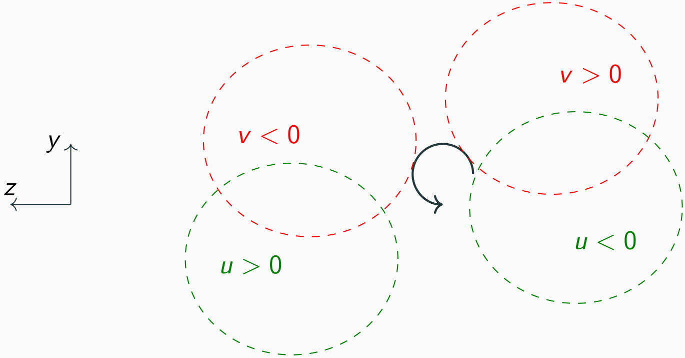


b)



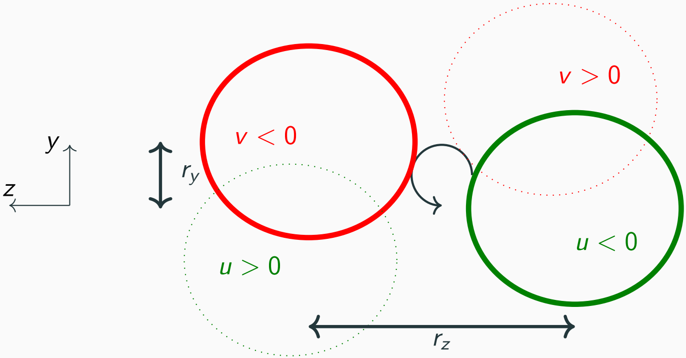
Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

- Which is its contribution to $\langle -\delta u \delta v \rangle$??



Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

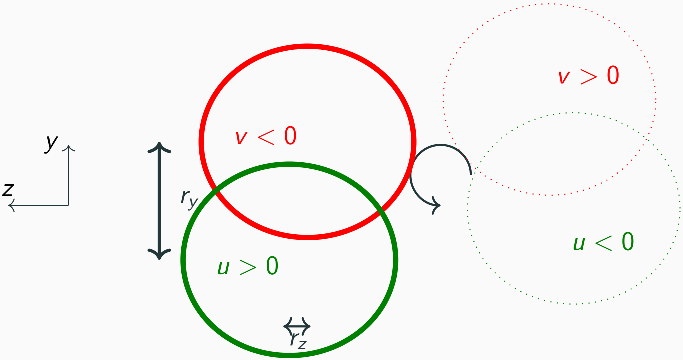
- $D_{-uv} < 0 \rightarrow$ Group b



Anticorrelation !!

Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

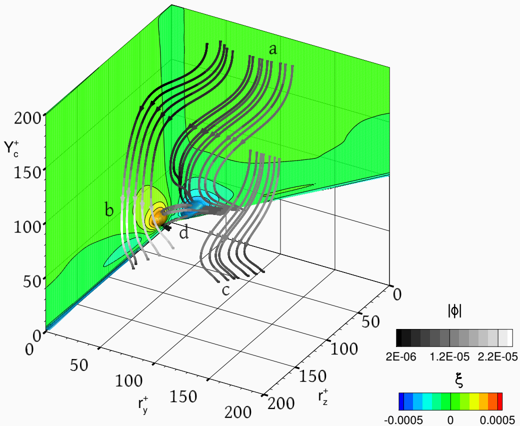
- $D_{-uv} > 0 \rightarrow$ Group a

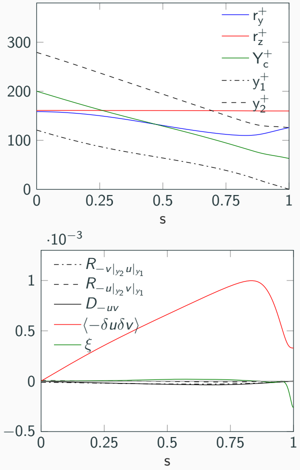


Positive correlation !!

Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

c)





Turbulent channel ($Re_\tau = 200$): $\langle -\delta u \delta v \rangle$ in $r_x = 0$-space

d)