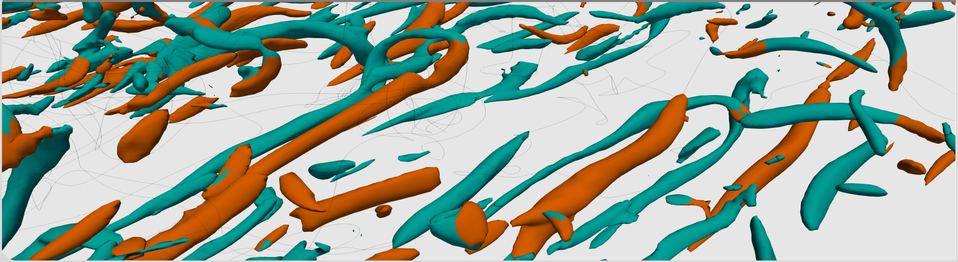
Alternative ways of looking at scale interactions in wall turbulence

Davide Gatti | October 18, 2018

LINNÉ FLOW SEMINAR, KTH, STOCKHOLM, SWEDEN



Global energy budgets

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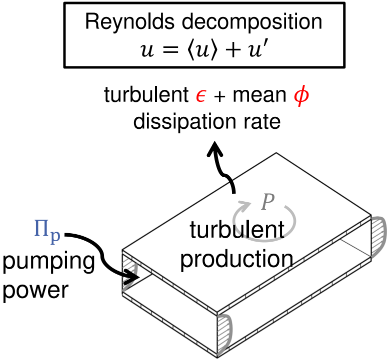
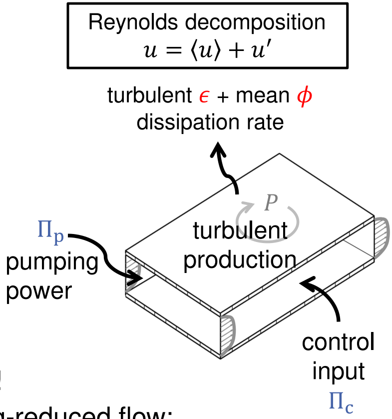


Global energy budgets

”how do dissipation and production of turbulent kinetic energy relate to turbulent friction drag in drag reduced flows?”



Seemingly trivial, nontrivial question!
Example turbulent dissipation in drag-reduced flow:

- Ricco *et al.*, JFM (2012): it increases
- Agostini *et al.*, JFM (2014): it decreases

Constant Power Input (CPI)

Definitions and characteristic quantities

- Total power Π_t is kept constant $\Pi_t = \Pi_p + \Pi_c \rightarrow$ control power $\Pi_c = \gamma \Pi_t$

  $\Pi_p = (1-\gamma)\,\Pi_t$ pumping power

- "Drag reduction" increases flow rate $U_b / U_{b,0} > 1$

- Power-based velocity scale $U_\Pi = \sqrt{\dfrac{\Pi_t h}{3\mu}}$

  "Stokes flow minimises the power consumption for given flow rate"

  Bewley, JFM (2009), Fukagata *et al.*, Physica D. (2009)

- Power-based Reynolds number $Re_\Pi = \dfrac{U_\pi h}{\nu}$

  Hasegawa, Quadrio, Frohnapfel, JFM (2014)

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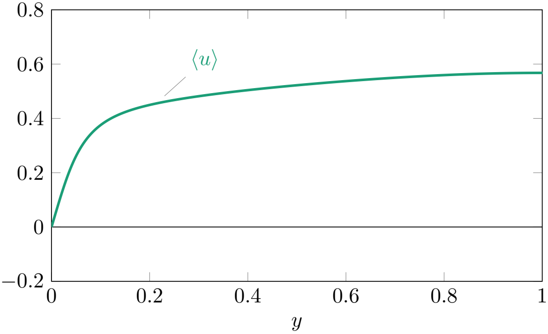
The ”wind decomposition” of turbulence

A triple decomposition with analytical advanages Eckhardt *et al.*, JFM 2007

$$u = \langle u \rangle + u'$$



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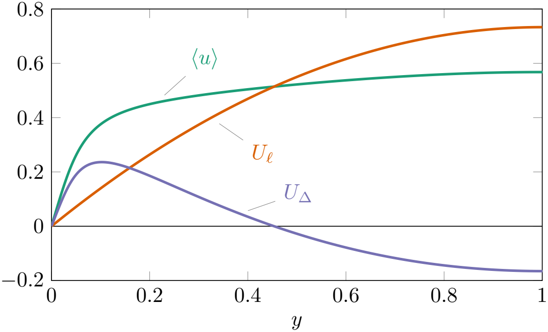
$$u = \overbrace{U_\ell + U_\Delta}^{\langle u \rangle} + u'$$

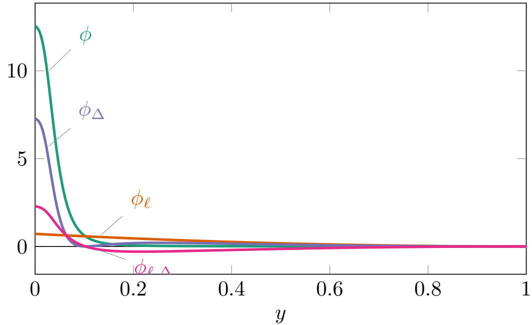


Production and mean dissipation

Mean dissipation decouples!

$$P = P_\ell + P_\Delta$$

$$\phi = \phi_\ell + \phi_\Delta + \phi_{\ell\Delta} \to 0$$



Analytical derivations

A fair amount of cumbersome algebra

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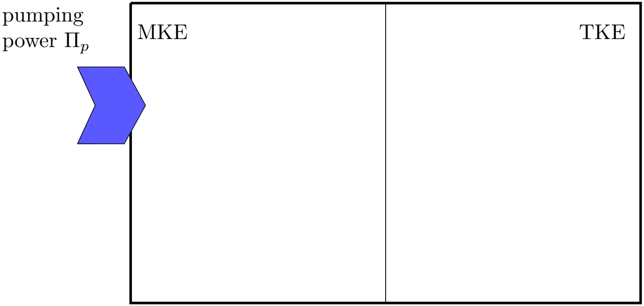
Gatti *et al.*, JFM (2018)

The new description



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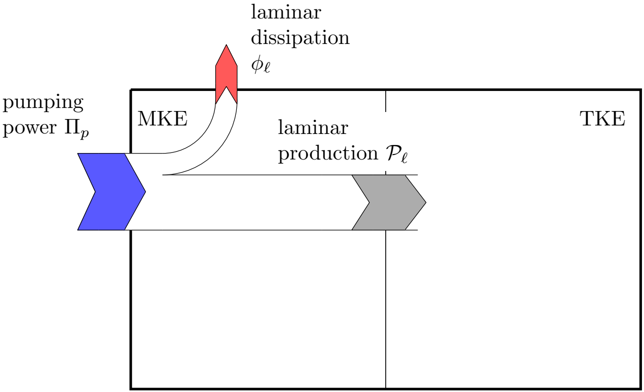

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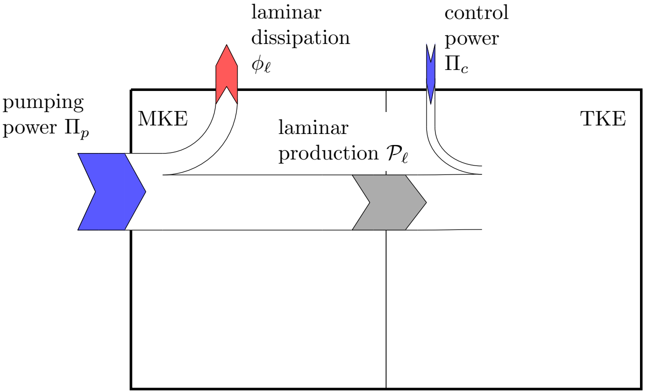

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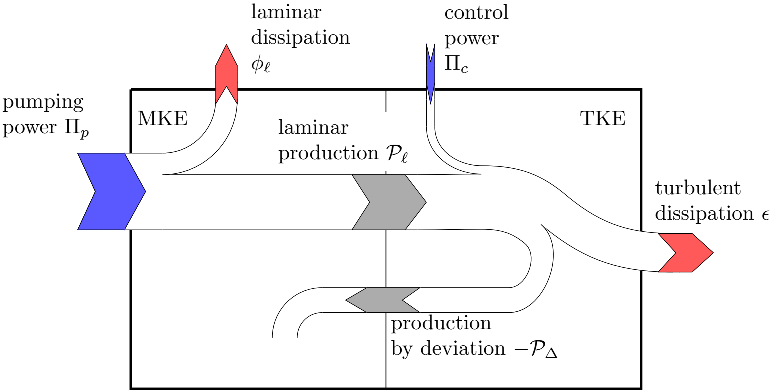

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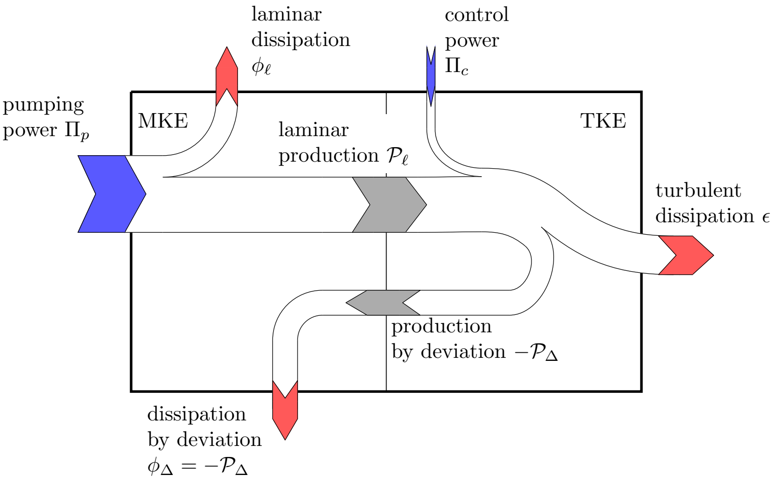

Two integrals of the turbulent shear stress

Via FIK-like CPI derivations, it is discovered that α and β parametrize all the fluxes

$$\alpha = \int_0^1 (1-y)\, r(y) dy$$

$$\beta = \int_0^2 r(y)^2 dy \geq 3\alpha^2$$

E.g.

$$P_\Delta = -\phi_\Delta = Re_\Pi \left(3\alpha^2 - \beta\right) \leq 0$$

$$\epsilon = \left\{ \frac{(\alpha Re_\Pi)^2}{2} \left(1 + \sqrt{1 + \frac{4(1-\gamma)}{(\alpha Re_\Pi)^2}}\right) - \frac{\beta Re_\Pi^2}{3} + \gamma \right\}$$

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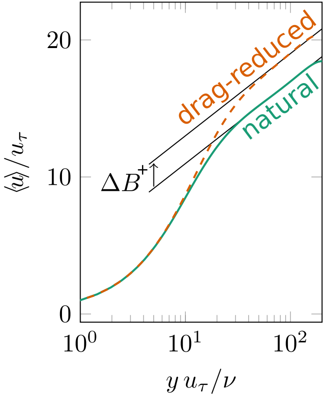
Every flux has a physical meaning!

- ϕ_ℓ is the best way to dissipate pumping power
- P_ℓ is the fraction of the pumping power wasted to produce turbulence
  - it decreases when control is successful (U_b increses for given Π_t)
  - it can be negative as $P_\ell \propto \alpha$
- ϕ_Δ is the penalty for not being laminar
- $\phi_\Delta + \epsilon$ is the fraction of the total power wasted by turbulence
  - it cannot be negative

A drag reduction model

- Control effect on $r(y)$ parametrised through ΔB^+
- Empirical description of velocity profile Luchini, PRL (2017)
- $3Re_\Pi^2(1-\gamma) = Re_\tau^2 Re_B$



- riblets and roughness
- superhydrophobic surfaces
- spanwise wall forcing
- some feedback controls, etc.

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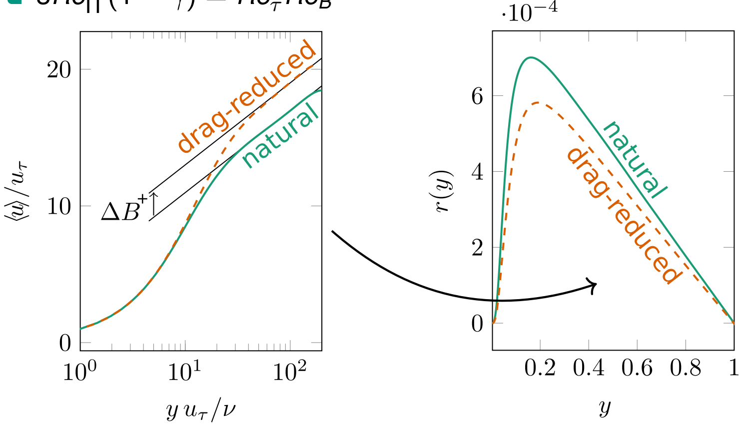
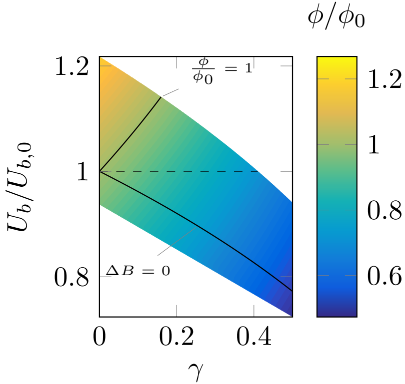


Ho do dissipations change with control?

Back to our initial question



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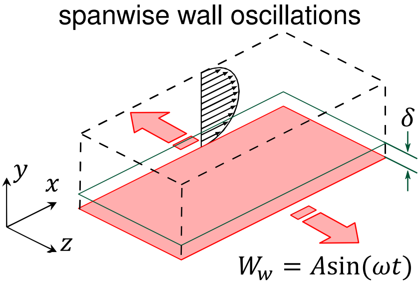

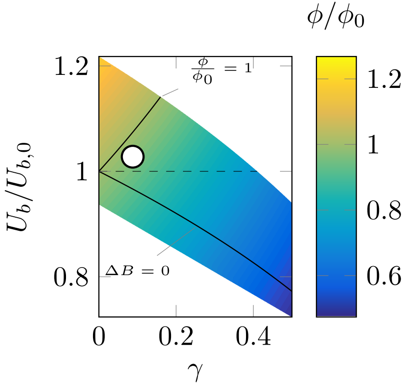

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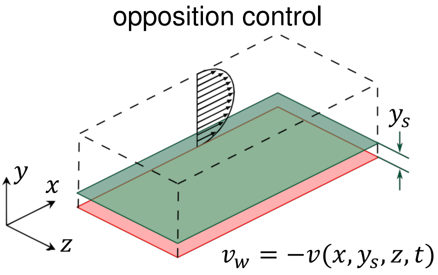

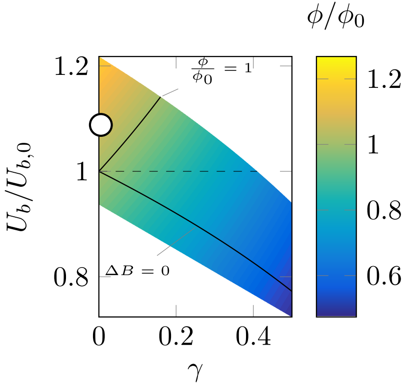

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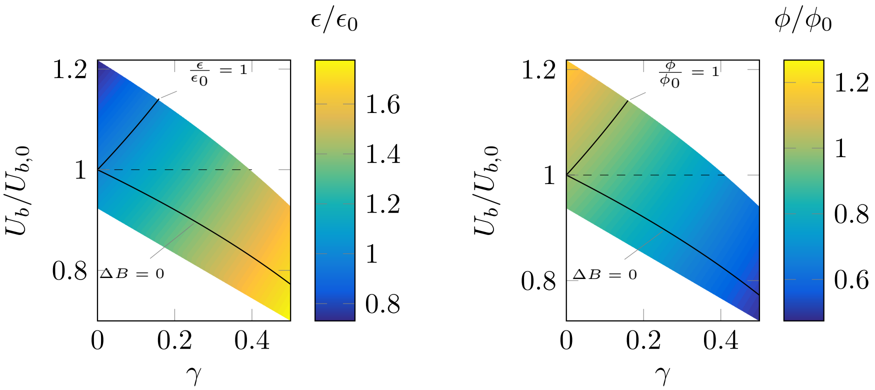

Key results

- ”Wind” decomposition and power nondimensionalisation introduced
- Theoretical framework for the flow control problem
  from energy perspective...
- ...relevant also for uncontrolled flows: FIK-like identity for ϵ
- Understand how changes of $r(y)$ modify ϕ, ϵ,...
- Potential for drag-reduction-aware RANS turbulence models?
- This is the global perspective: what about more detailed physics?

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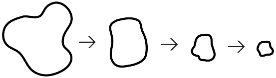
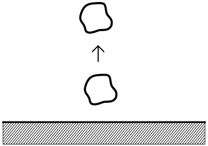
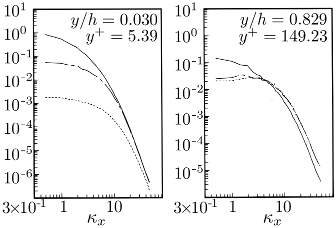
(Two) Classic approaches to turbulence

Space of scales

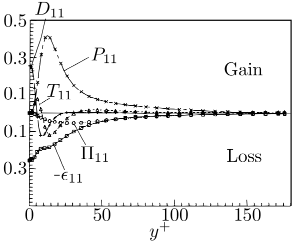


Physical space





Kim *et al.* JFM 1987



Mansour *et al.* JFM 1988

Generalized Kolmogorov Equation (GKE)

Exact budget equation for $\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$

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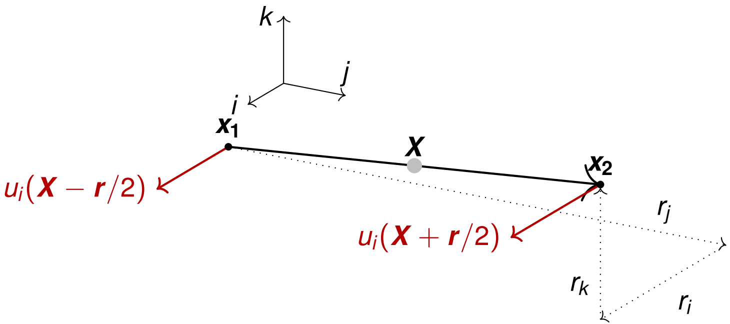
$$\delta u_i = \left(u_i \left(\boldsymbol{X} + \boldsymbol{r}/2, t \right) - u_i \left(\boldsymbol{X} - \boldsymbol{r}/2, t \right) \right)$$



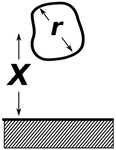
dependent on: $\begin{cases} \boldsymbol{X} = (\boldsymbol{x_1} + \boldsymbol{x_2})/2 \\ \boldsymbol{r} = \boldsymbol{x_2} - \boldsymbol{x_1} \end{cases}$

Generalized Kolmogorov Equation (GKE)

Amount of turbulent energy
at location $\boldsymbol{X}$ and scale (up to) $\boldsymbol{r}$

Davidson *et al.* JFM 2006

- $\langle \delta u_i \delta u_i \rangle(\boldsymbol{X}, \boldsymbol{r})$



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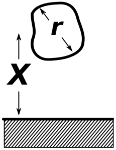

- GKE

Production, dissipation and transport
of turbulent **energy**
in both the
Space of scales & Physical space

Cimarelli *et al.* JFM 2013, 2016

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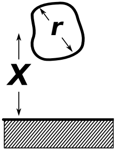

- GKE

Production, dissipation and transport
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Cimarelli *et al.* JFM 2013, 2016

$$\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle \rightarrow$$ **...anisotropy?**

GKE: budget for $\langle \delta u_i \delta u_i \rangle$

$$\boxed{\frac{\partial \phi_k}{\partial r_k} + \frac{\partial \psi_k}{\partial X_k} = \xi}$$

scale flux $$\phi_k = \underbrace{\delta U_k \langle \delta u_i \delta u_i \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_i \rangle}_{\text{turbulent transport}} \underbrace{-2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_i \rangle}_{\text{viscous diffusion}}$$

space flux $$\psi_k = \underbrace{\langle {u_k}^* \delta u_i \delta u_i \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho} \langle \delta p \delta u_i \rangle}_{\text{pressure transport}} \underbrace{-\frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_i \rangle}_{\text{viscous diffusion}}$$

source $$\xi = \underbrace{-2 \langle {u_k}^* \delta u_i \rangle \delta \left(\frac{\partial U_i}{\partial x_k} \right)}_{\text{production}} \underbrace{-2 \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_i}{\partial x_k} \right)^*}_{\text{production}} \underbrace{-4 {\epsilon_{ii}}^*}_{\text{dissipation}}$$

Anisotropic GKEs (AGKEs): budget for $\langle \delta u_i \delta u_j \rangle$

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

scale flux
$$\phi_{k,ij} = \underbrace{\delta U_k \langle \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} \underbrace{-2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$$

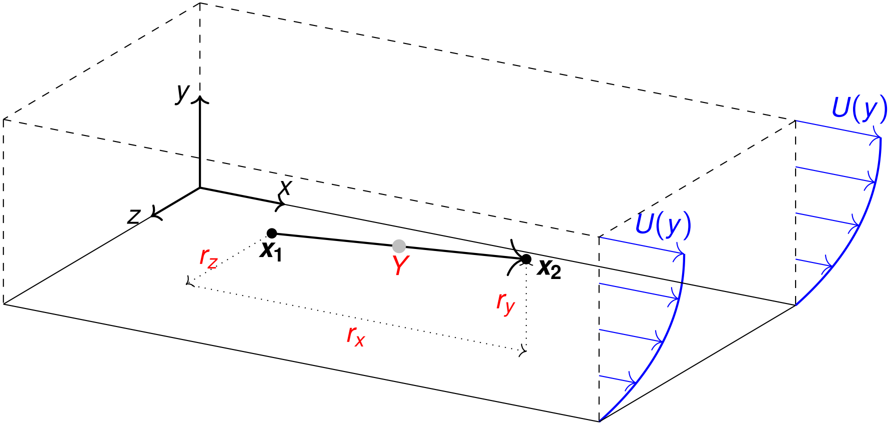
space flux
$$\psi_{k,ij} = \underbrace{\langle u_k{}^* \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho} \langle \delta p \delta u_i \rangle \delta_{kj} + \frac{1}{\rho} \langle \delta p \delta u_j \rangle \delta_{ki}}_{\text{pressure transport}} \underbrace{- \frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$$

source
$$\xi_{ij} = \underbrace{-\langle u_k{}^* \delta u_j \rangle \delta \left(\frac{\partial U_i}{\partial x_k} \right) - \langle u_k{}^* \delta u_i \rangle \delta \left(\frac{\partial U_j}{\partial x_k} \right)}_{\text{production}} +$$

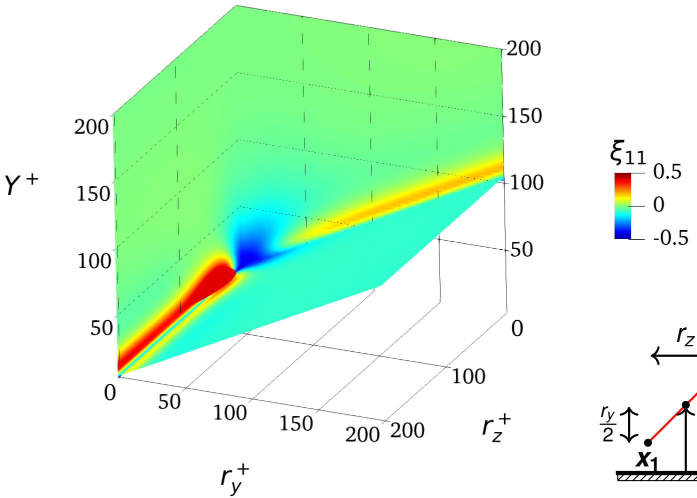
$$\underbrace{-\langle \delta u_k \delta u_j \rangle \left(\frac{\partial U_i}{\partial x_k} \right)^* - \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_j}{\partial x_k} \right)^*}_{\text{production}} \underbrace{+ \frac{1}{\rho} \left\langle \delta p \frac{\partial \delta u_i}{\partial X_j} \right\rangle + \frac{1}{\rho} \left\langle \delta p \frac{\partial \delta u_j}{\partial X_i} \right\rangle}_{\text{pressure strain}} \underbrace{-4 \epsilon_{ij}{}^*}_{\text{dissipation}}$$

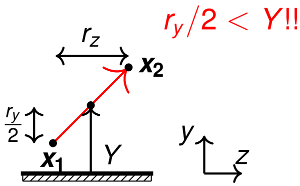
AGKEs for indefinite plane channels

$$\langle \delta u_i \delta u_j \rangle (\boldsymbol{X}, \boldsymbol{r}) \rightarrow \langle \delta u_i \delta u_j \rangle (Y, r_x, r_y, r_z)$$



Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$ space





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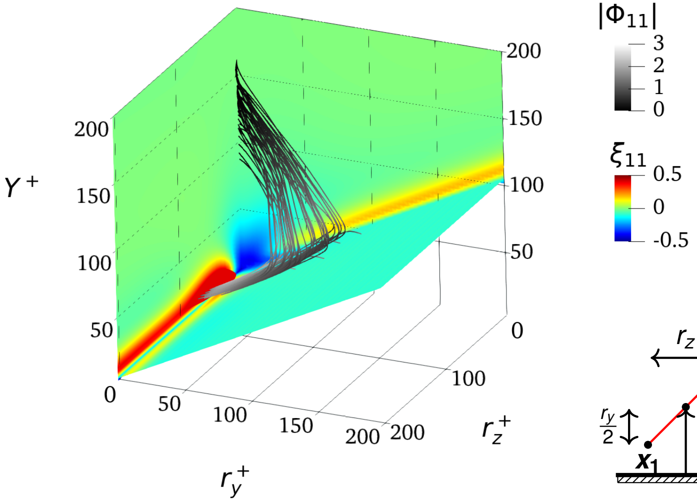
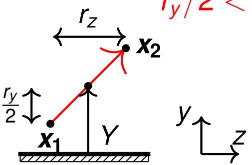


$r_y/2 < Y$!!



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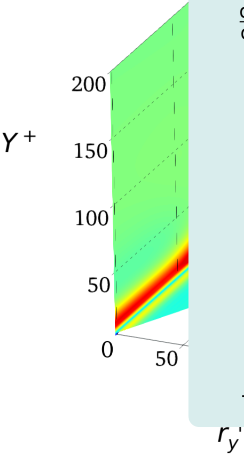
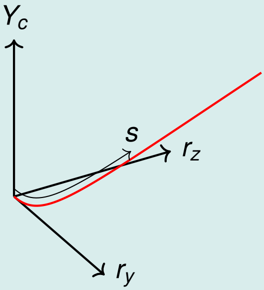
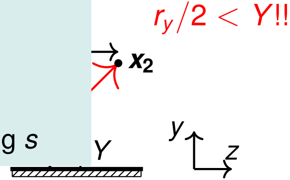


Field lines

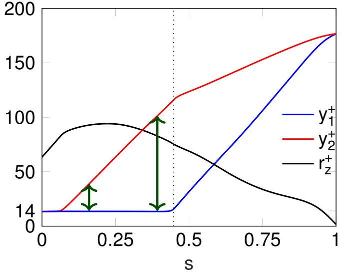
$$\frac{d\boldsymbol{x}}{ds} = \phi_{i,j} \qquad \text{where } \boldsymbol{x} = (Y_c, r_y, r_z)$$

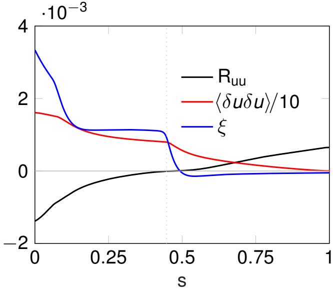


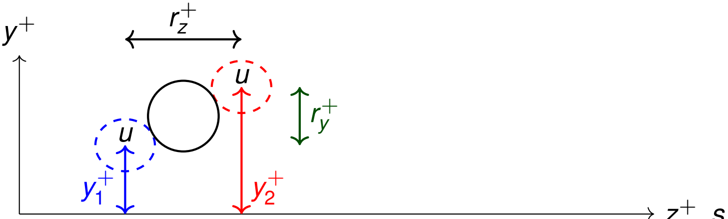
track AGKE terms and R_{ij} along s



Fluxes, field lines: attached & detached scales







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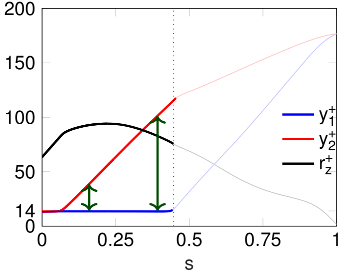
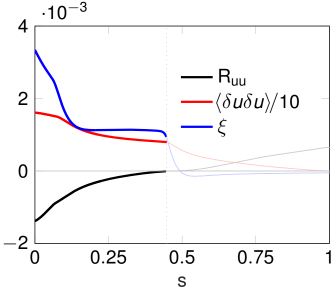
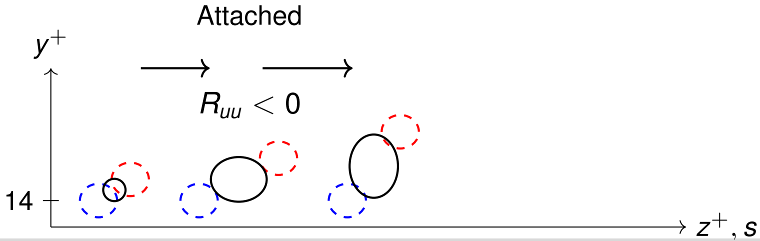
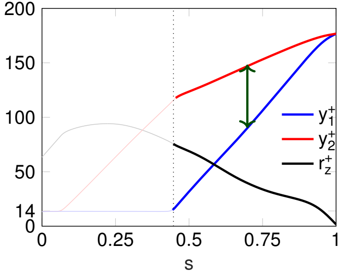
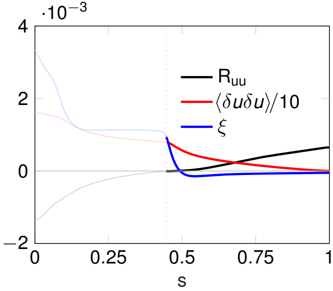


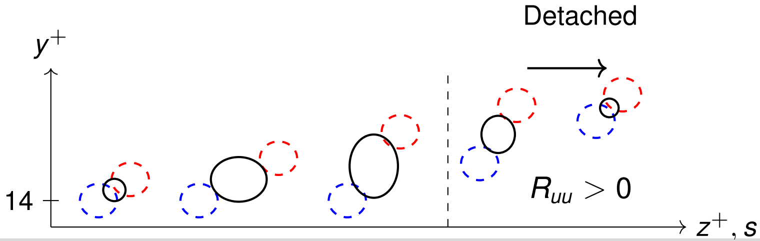




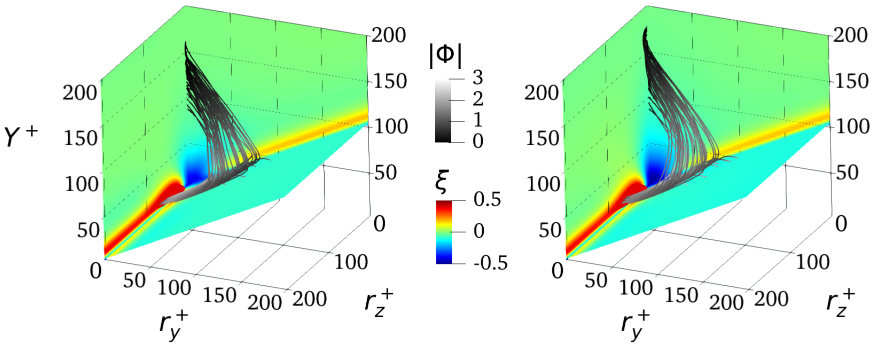
Fluxes, field lines: attached & detached scales



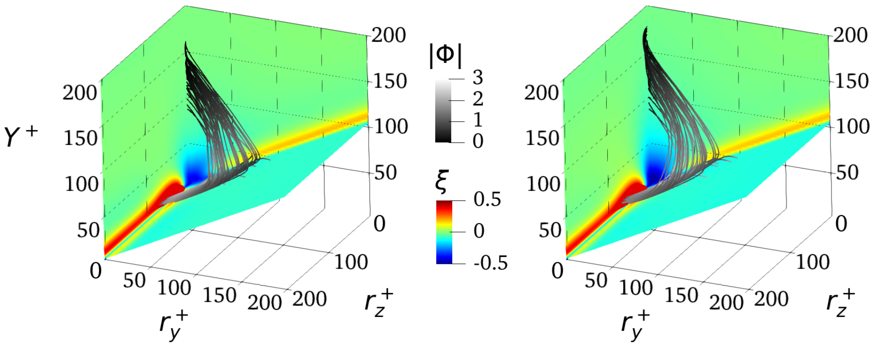




$\langle \delta u \delta u \rangle$ **versus** $\langle \delta u_i \delta u_i \rangle$

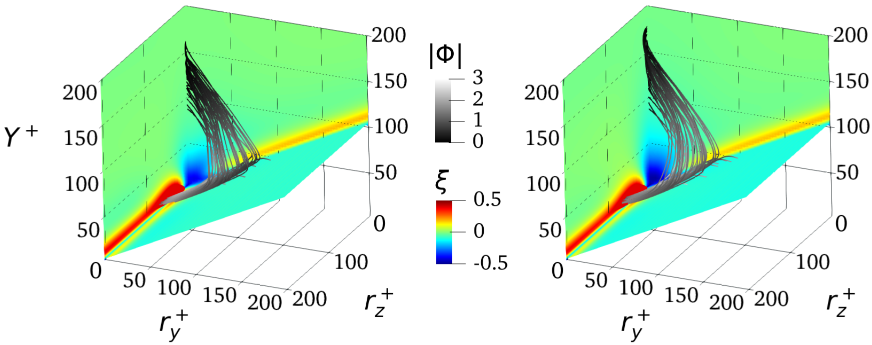


$\langle \delta u \delta u \rangle$ **versus** $\langle \delta u_i \delta u_i \rangle$



$$\langle \delta u \delta u \rangle \approx \langle \delta u_i \delta u_i \rangle$$

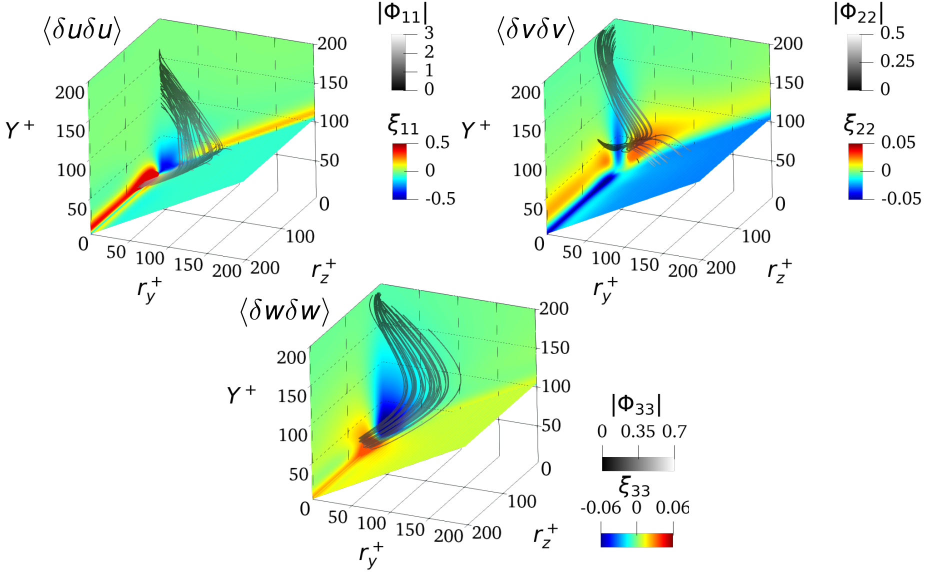
$\langle \delta u \delta u \rangle$ **versus** $\langle \delta u_i \delta u_i \rangle$



$$\langle \delta u \delta u \rangle \approx \langle \delta u_i \delta u_i \rangle$$

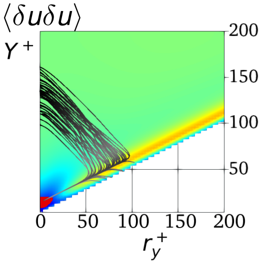
what does AGKE add to GKE?

Anisotropy

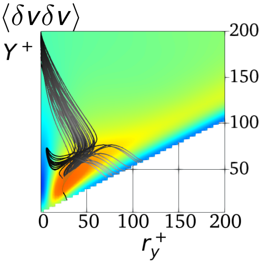


$\langle \delta u \delta u \rangle$
$|\Phi_{11}|$
3
2
1
0
ξ_{11}
0.5
0
-0.5
$\langle \delta v \delta v \rangle$
$|\Phi_{22}|$
0.5
0.25
0
ξ_{22}
0.05
0
-0.05
$\langle \delta w \delta w \rangle$
$|\Phi_{33}|$
0 0.35 0.7
ξ_{33}
-0.06 0 0.06
Y^+
r_y^+
r_z^+
0 50 100 150 200

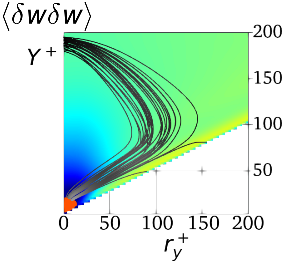
Anisotropy



$\langle \delta u \delta u \rangle$
Y^+
200
150
100
50
0
50
100
150
200
r_y^+

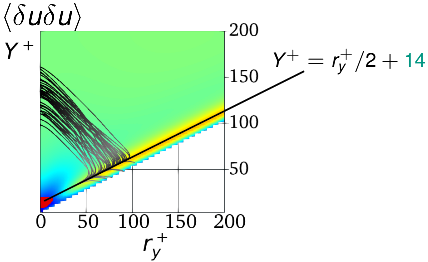


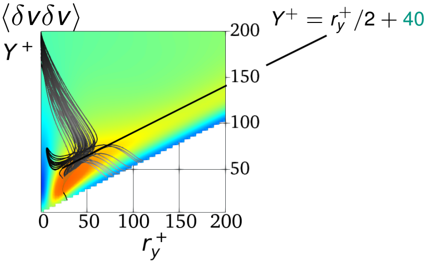
$\langle \delta v \delta v \rangle$
Y^+
200
150
100
50
0
50
100
150
200
r_y^+

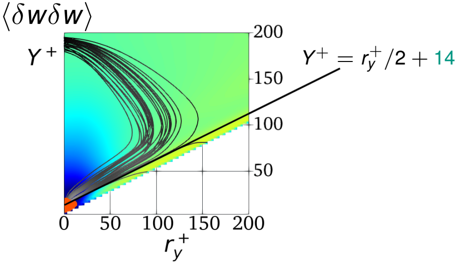


$\langle \delta w \delta w \rangle$
Y^+
200
150
100
50
0
50
100
150
200
r_y^+

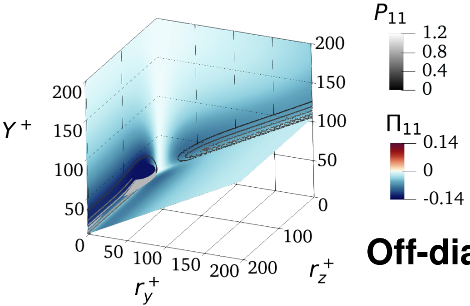
Anisotropy of attached scales



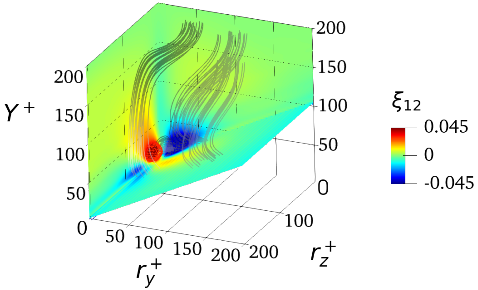




Redistribution: pressure strain

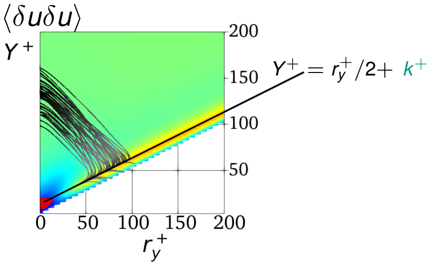


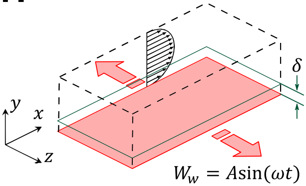
Off-diagonal component $\langle \delta u \delta v \rangle$



What happens with drag reduction?

Back to our original question, again





spanwise wall oscillations

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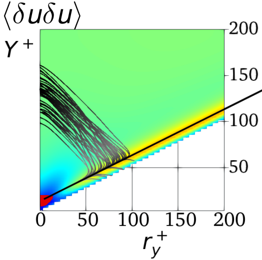
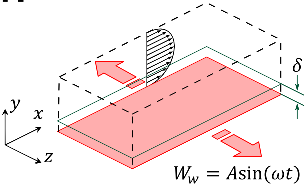
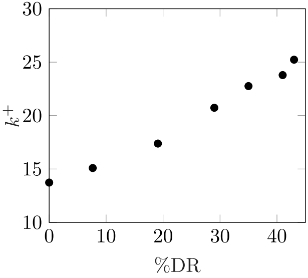
Back to our original question, again

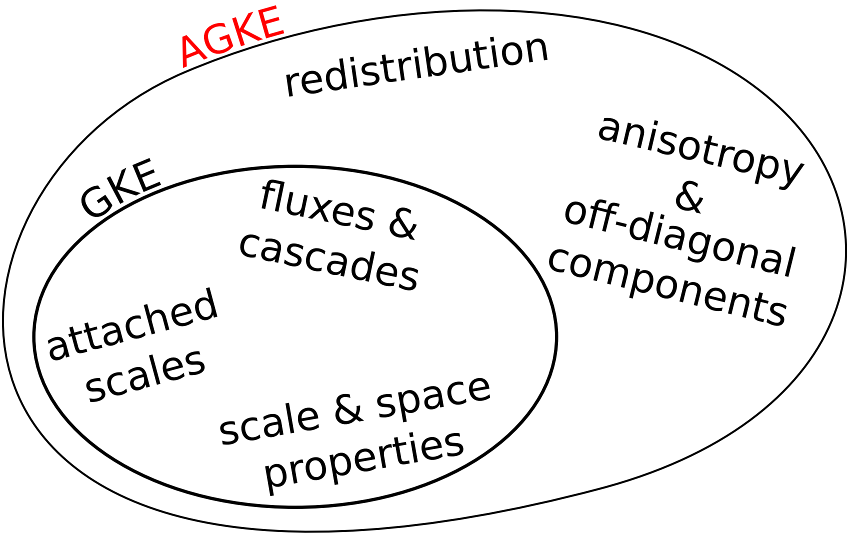


$Y^+ = r_y^+/2 + k^+$



spanwise wall oscillations

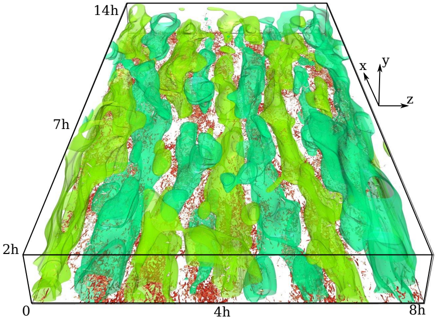



AGKE
redistribution
anisotropy
&
off-diagonal
components
GKE
fluxes &
cascades
attached
scales
scale & space
properties

How will I use the AGKE?

Role and occurrence of large scales

- Very Large Scale Motions at high *Re*



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Role and occurrence of large scales

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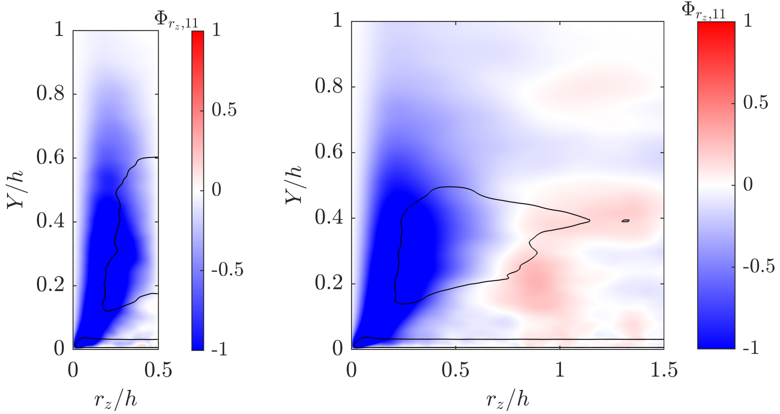
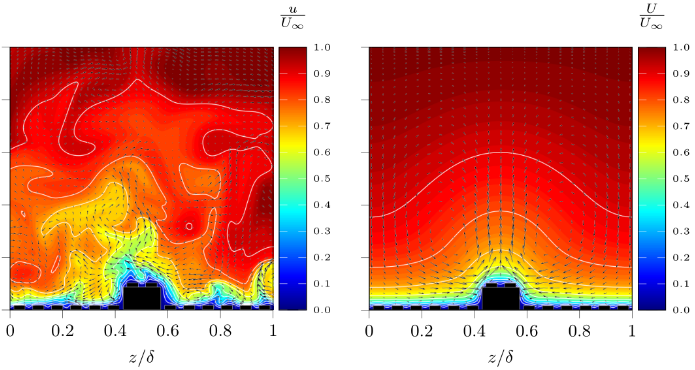


How will I use the AGKE?

Role and occurrence of large scales

- Very Large Scale Motions at high *Re*
- Secondary Motions of Prandtl second kind



Stroh *et al.* JFM submitted

THANKS

for your kind attention!

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European Drag Reduction and Flow Control Meeting

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Conclusion

We presented the AGKE: exact budget equations for $\langle \delta u_i \delta u_j \rangle$

- In addition to GKE:
  - anisotropy
  - off-diagonal components
  - redistribution
- In addition to spectral Reynolds stress budgets:
  - no need for homogeneity
  - allows scales in inhomogeneous directions
  - possible “fluxes” interpretation
- probably interesting for your research too!

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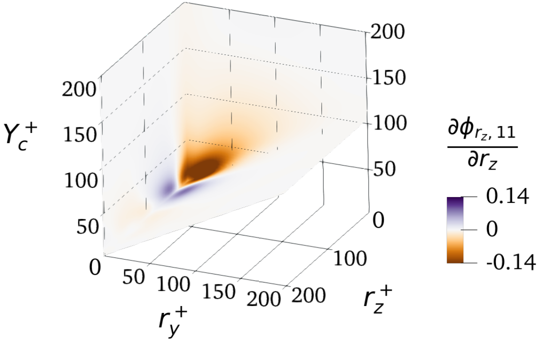
Fluxes, divergence: donor & receiver scales



contribution of various physical processes to $\langle \delta u_i \delta u_j \rangle$
(e.g. nonlinear turbulent transport)

$\langle \delta u_i \delta u_j \rangle$: relationship with correlation $R_i j$

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle |_{y_1} + \langle u_i u_j \rangle |_{y_2}}_{sum\ \ of\ \ variances} \underbrace{-2 R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z)}_{Cross-correlation}$$

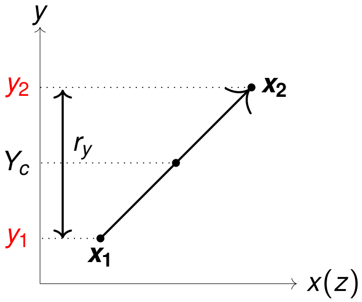
$\langle \delta u_i \delta u_j \rangle$: relationship with correlation $R_i j$

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2 R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z)}_{Cross-correlation}$$

$$y_1 = Y_c - r_y/2$$

$$y_2 = Y_c + r_y/2$$

$$(Y_c, r_x, r_y, r_z) \leftrightarrow (y_1, y_2, r_x, r_z)$$



$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_ij

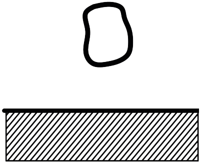
$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2 R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z)}_{Cross-correlation}$$

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) \neq \langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}$$

$$\downarrow$$

$$R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z) \neq 0$$

Coherent structures!

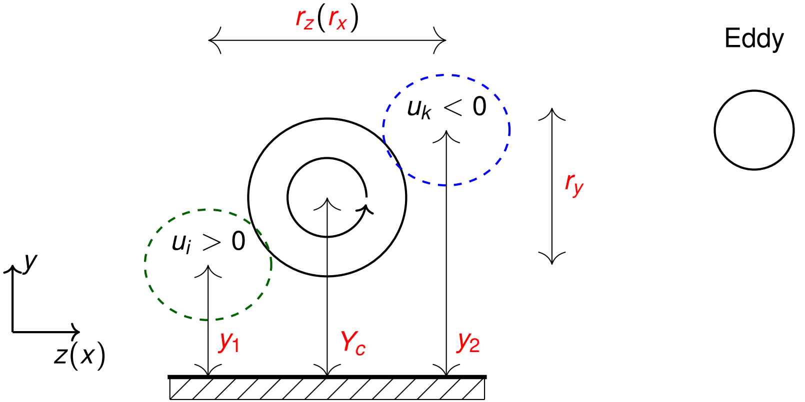


$\langle \delta u_i \delta u_j \rangle$: relationship with correlation $R_i j$

$$\langle \delta u \delta u \rangle (Y_c, 0, r_y, r_z) > \underbrace{\langle uu \rangle|_{y_1} + \langle uu \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2 R_{u|_{y_1} u|_{y_2}}(0, r_z)}_{Cross-correlation}$$

$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_ij

$$\langle \delta u \delta u \rangle (Y_c, 0, r_y, r_z) = \underbrace{\langle uu \rangle|_{y_1} + \langle uu \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2 R_{u|_{y_1} u|_{y_2}} (0, r_z)}_{Cross-correlation} < 0$$



$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_ij

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) \leftrightarrow \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{sum\ of\ variances} \underbrace{-2 R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z)}_{Cross-correlation}$$