A novel analytical foot rollover model for planar walking
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Motivation: established foot rollover model

- Rigid (convex) feet model rolling kinematics
- Established parameterization: radius \( r(\varphi) \)
- Contact point: implicit problem \( n(\varphi) \cdot e_z = -1 \)
- Dynamics: differential algebraic equation (DAE) of index 2

Simulation:
- Contact point iteration
- Time integration of DAE

Novel foot rollover model

- Parameterization: ankle trajectory \( r_P(\theta) \)
  - with orientation \( \beta \)
- Velocity: in tangential trajectory \( v_P(\theta) \)
- Contact point: explicit solution \( x_C = x_P + z'_c \)
- Dynamics: ordinary differential equation (ODE)

Simulation:
- Contact point via explicit equation
- Time integration of ODE

Example parameterizations

- Seven segment walker with rigid feet and hybrid zero dynamics-based controller*
- Single support phase + impact of swing foot
- Gait generation via optimization of cost of transport (input of mechanical work)

\[
CT = \frac{1}{m g L} \int_0^T \max(u_i q_i, 0) dt
\]

- Two ankle trajectory parameterizations:
  - circular foot contour
  - Bézier polynomial (fit to human data**)

Comparison of both parameterizations:
- Model & controller complexity identical
- Contours/rollover shapes very similar
- Significant influence on \( C_T \)
- Bézier fit (human data) \( \approx 45\% \) better
