

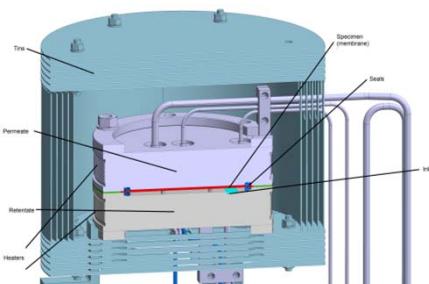
# Permeation data analysis including a nonzero hydrogen concentration on the low pressure detector side

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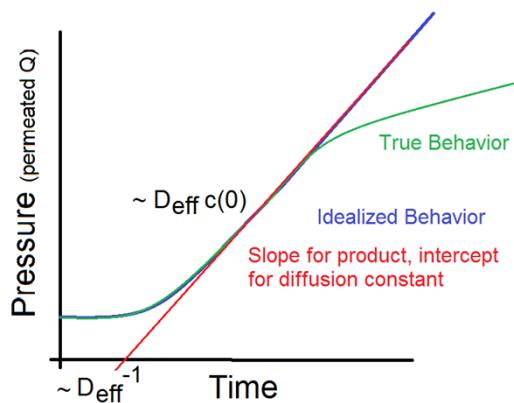
IKIT/INR/MET (Maschinenbau)

Determination of concentration dependent interstitial diffusion parameter regarding re-diffusion and small loading pressure: Situation of future fusion power plant 2 Pa tritium partial pressure (breeder unit) enriched to 1 Pa in purge gas system.



- 1.: Motivation
- 2.: Description of setup and simplification
- 3.: Status quo: FDM solver
- 4.: Status quo: Branch & Bond algorithm
- 5.: Desired Object: Gas release
- 6.: Other Projects

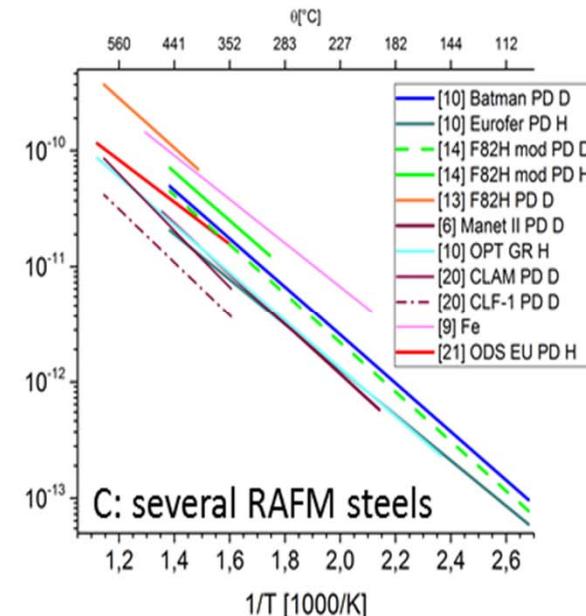
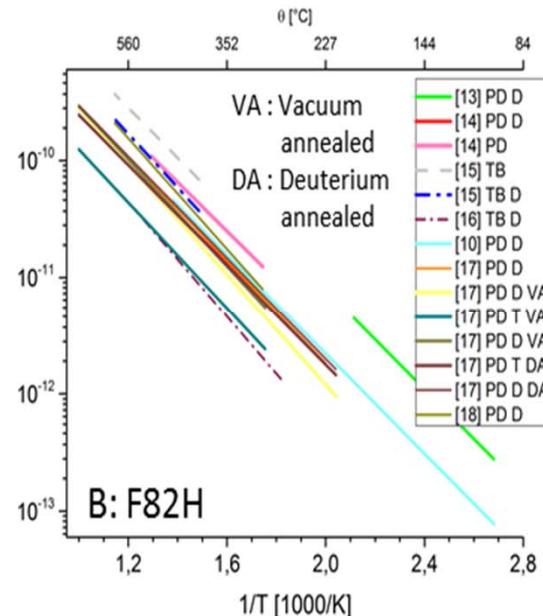
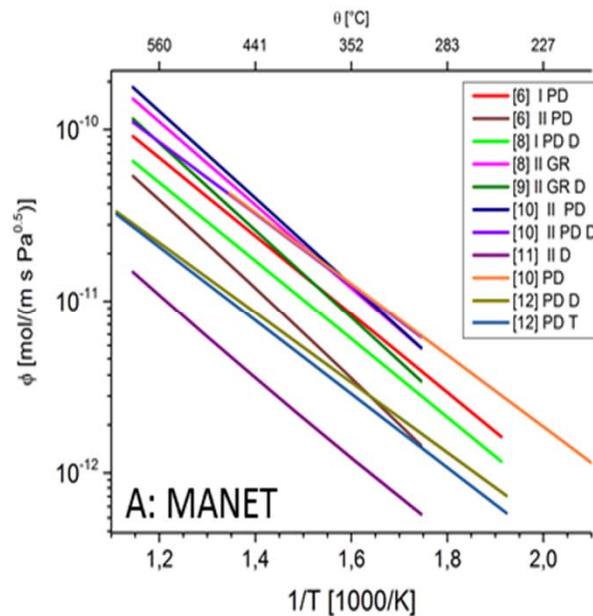
# 1.: Motivation



Time dependent non zero Q-concentration near measuring system (gauge or QMS) generates deviation from linear behavior.

$$j(t)_{measure} = \frac{\Phi}{\frac{D_{eff} k_s \sqrt{p_{load}} w_m^2}{d_m 4 \pi}} \left( 1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{\frac{-k^2 \pi^2 D_{eff} (t-t_{off})}{d_m^2}} \right)$$

Daynes, Forcey transport equation solution

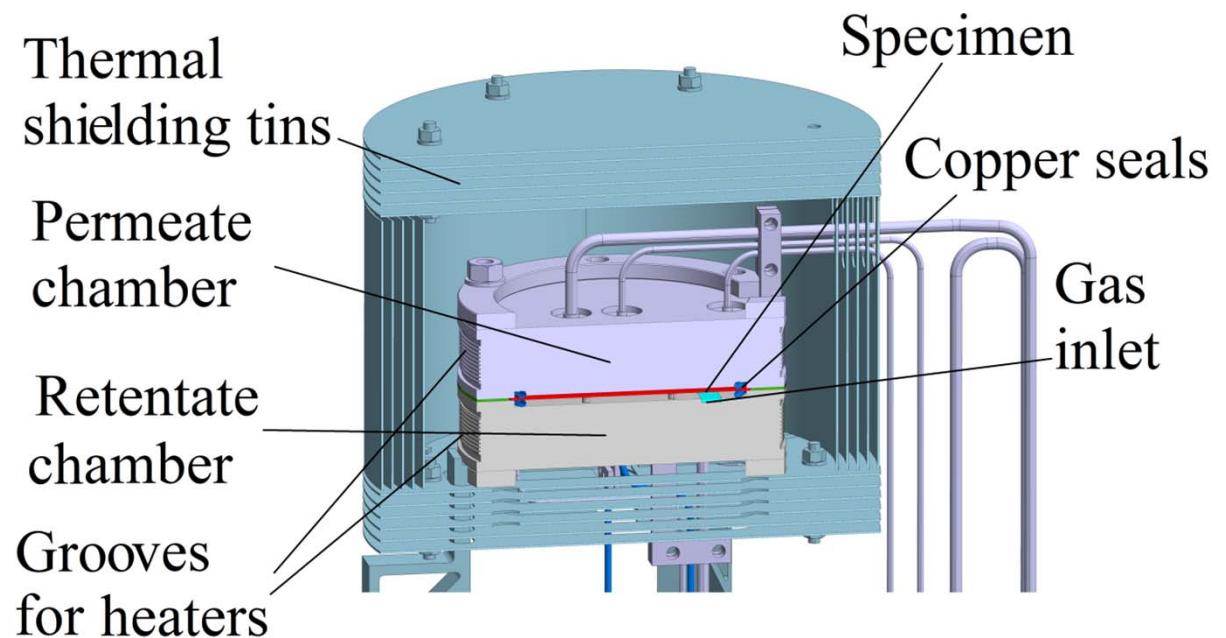


## 2.: Description of setup and simplification

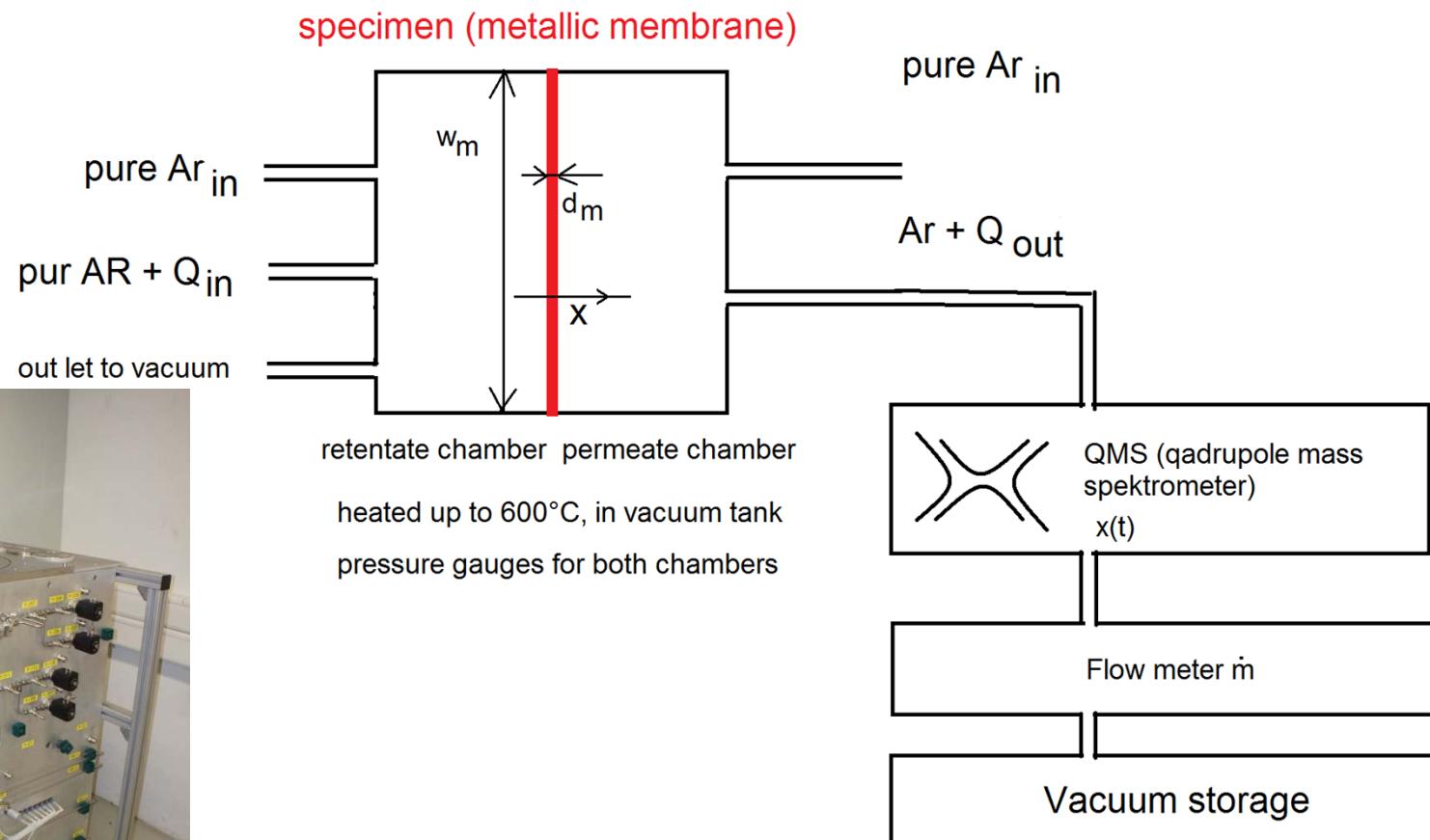
Therefore removing Q  
in permeate chamber.



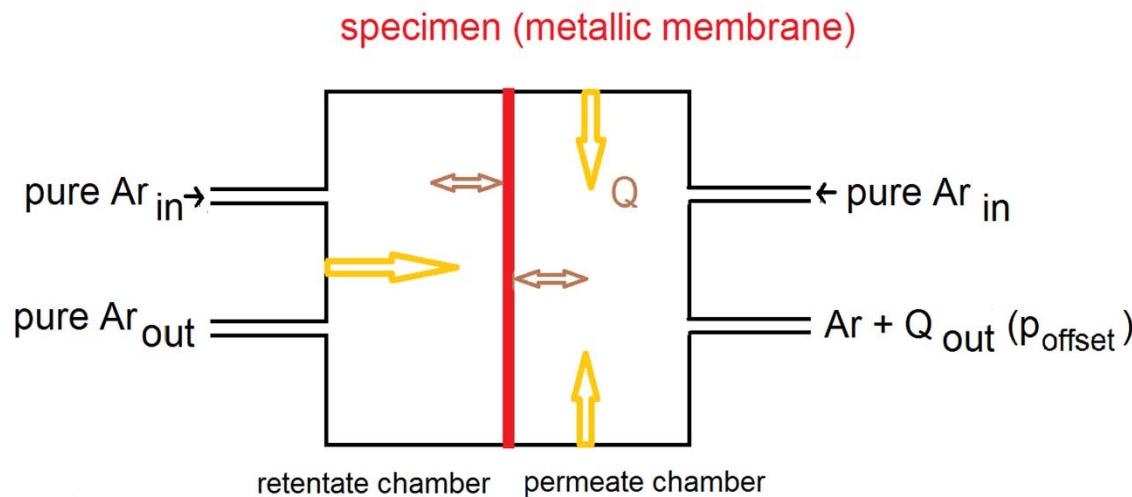
Permeate (secondary) chamber



# Simplified Q-PETE experiment



### 3.: Status Quo FDM-solver: Before beginning of experiment: Purging with pure Ar:



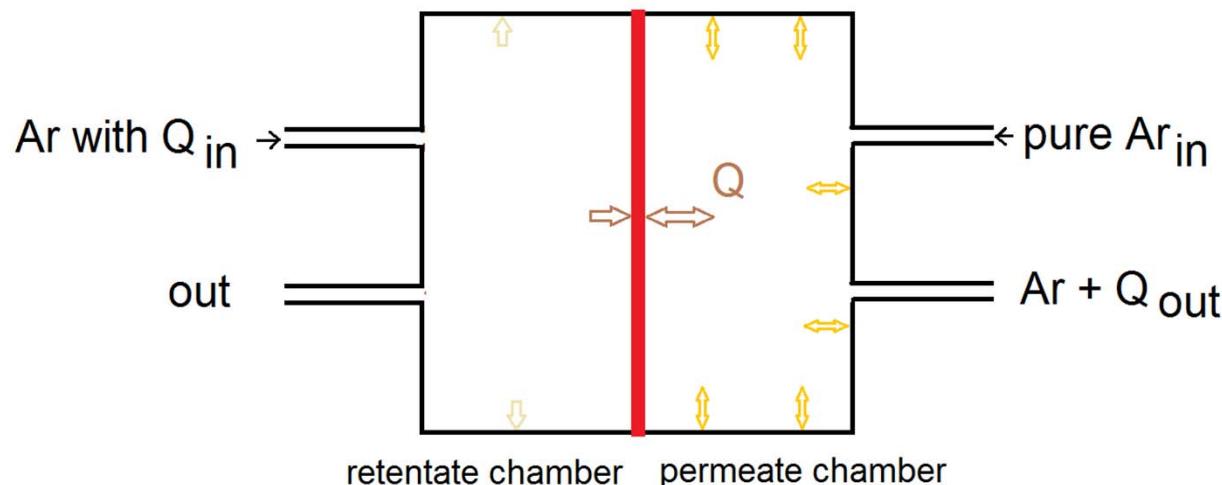
The stored Q is generating a constant assumed offset permeation. The membrane is in diffusive contact with the two volumes, equilibrium state given by  $k_s$ .

$$c(t, x) = k_s \sqrt{p_{\text{offset}}} = k_s \sqrt{\frac{j_{\text{offset}} p_{\text{tot}}}{\dot{m}_{\text{Ar}}}}, t < t_{\text{start}}, 0 \leq x \leq d_m$$

$j_{\text{offset}}$  determined by QMS, specimen saturated with Q, no Q is additionally stored or emitted,  $p_{\text{tot}}$  absolute pressure in both volumes by pressure gauge,  $\dot{m}_{\text{Ar}}$  by mass flow controller @RT

# FDM solver: Boundary conditions after start of experiment:

specimen (metallic membrane)



$$c(x = 0, t > t_{off}) \stackrel{FDM}{=} c(1, t > t_{off}) = c(o) \\ = k_s \sqrt{p_{load}}$$

$$p_{offset} < p_{load}$$

$$j_{measure} = j_{offset} + \underbrace{j_{perm}}_{from\ membrane}$$

Partial Q-Pressure in retendate chamber, surface concentration linear increased in 1 s after  $t_{off}$

$j_{offset}$  assumed constant, generated by thick structures thickness more than 20 mm (1.4404), emitting into vacuum also , membrane around 1.2 mm thickness.

FDM solver: Boundary condition of permeate (secondary) membrane side after start of experiment

$$j_{measure} = j_{offset} + -D_{eff} \frac{d_m^2 \pi}{4} \frac{\partial c(x = w_m, t > t_{off})}{\partial x} \stackrel{FDM}{=} j_{offset} \\ + -D_{eff} \frac{d_m^2 \pi}{4} \frac{c(x = w_m - 4\Delta x, t > t_{off}) - c(x = w_m - 2\Delta x, t > t_{off})}{2 \Delta x}$$

$$(*) \quad c(n, t > t_{off}) = \underbrace{k_s \sqrt{j_{measure} \frac{p_{tot}}{\dot{m}_{Ar} \alpha}}}_{lit.=0, c(n,t>t_{off})=k_s \sqrt{\frac{RT_{abs}}{V_{gas}} \int_0^t j_{measure} dt}} \sim k_s \sqrt{\frac{D_{eff} (c(n-4,t) - c(n-2,t))}{2 \Delta x}}$$

$\Delta x = 12 \mu m (=d_m/n)$  from discretization of membrane in thickness direction normally n=100 elements, first element on retendate side,  $n^{th}$  element on permeate side.  $\alpha=1$  for homogeneous purge gas inlet,  $\alpha=2$  for point shaped inlet

Transient FDM-solver (any textbook):

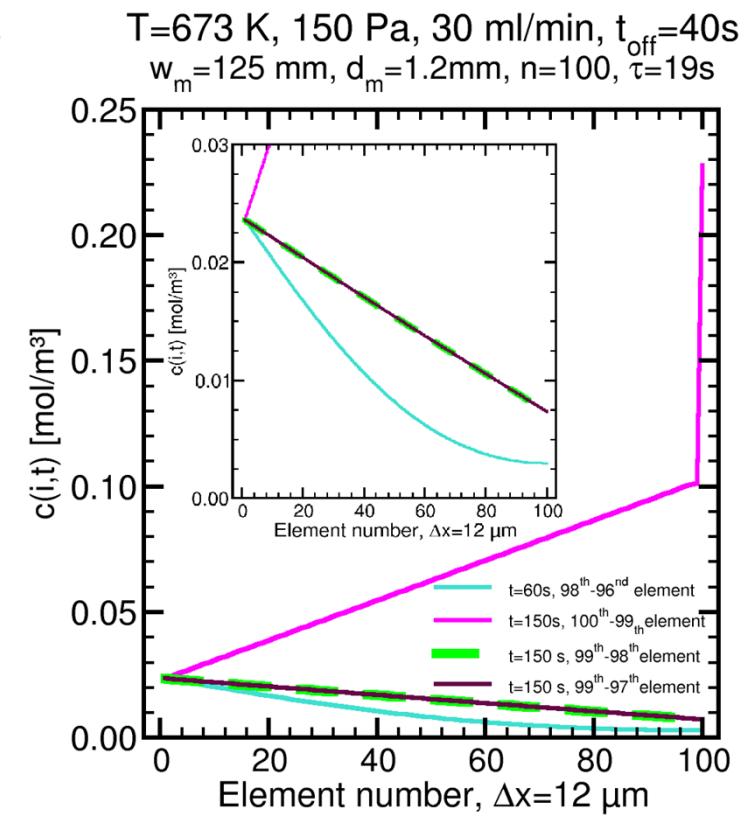
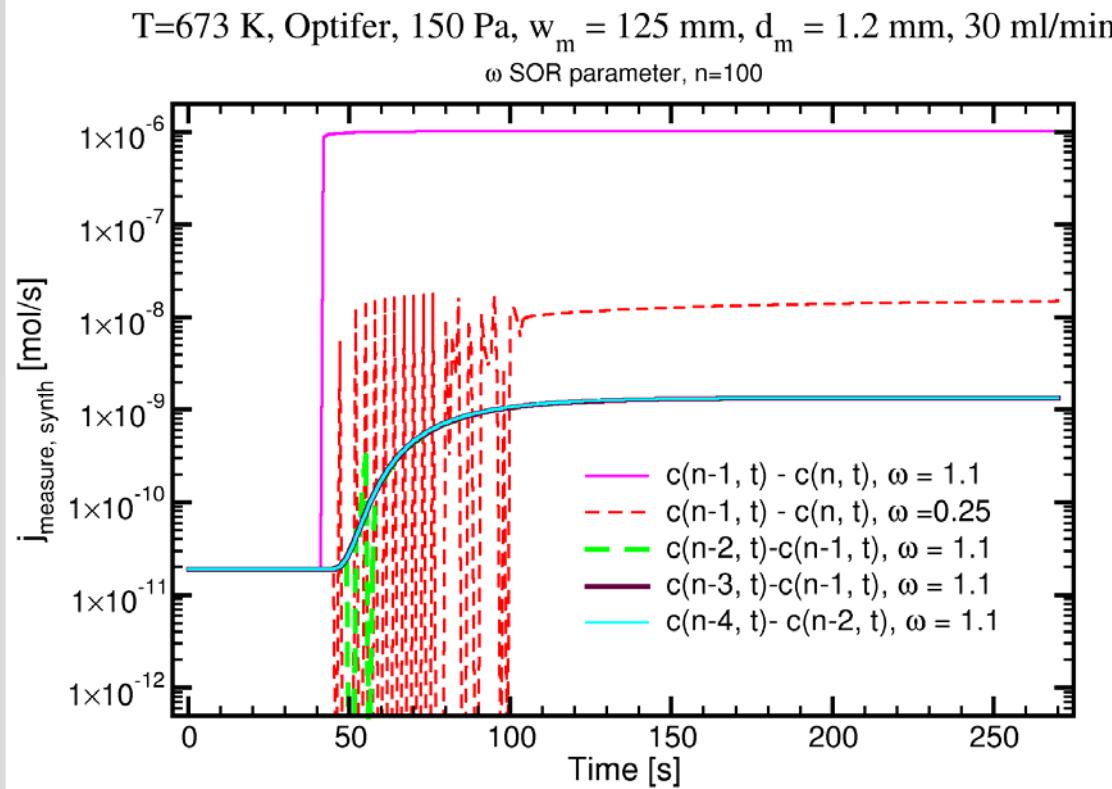
$$\frac{\partial c}{\partial t} = D_{eff} \frac{\partial^2}{\partial x^2}$$

$$(**) \quad c(i, t + \Delta t) = c(i, t) + \frac{D_{eff} \Delta t}{\Delta x^2} (c(i+1, t) - 2c(i, t) + c(i-1, t)) \\ I = 2 \dots n-1, t > t_{off}$$

## Used FDM-SOR-step (successive over relaxation)

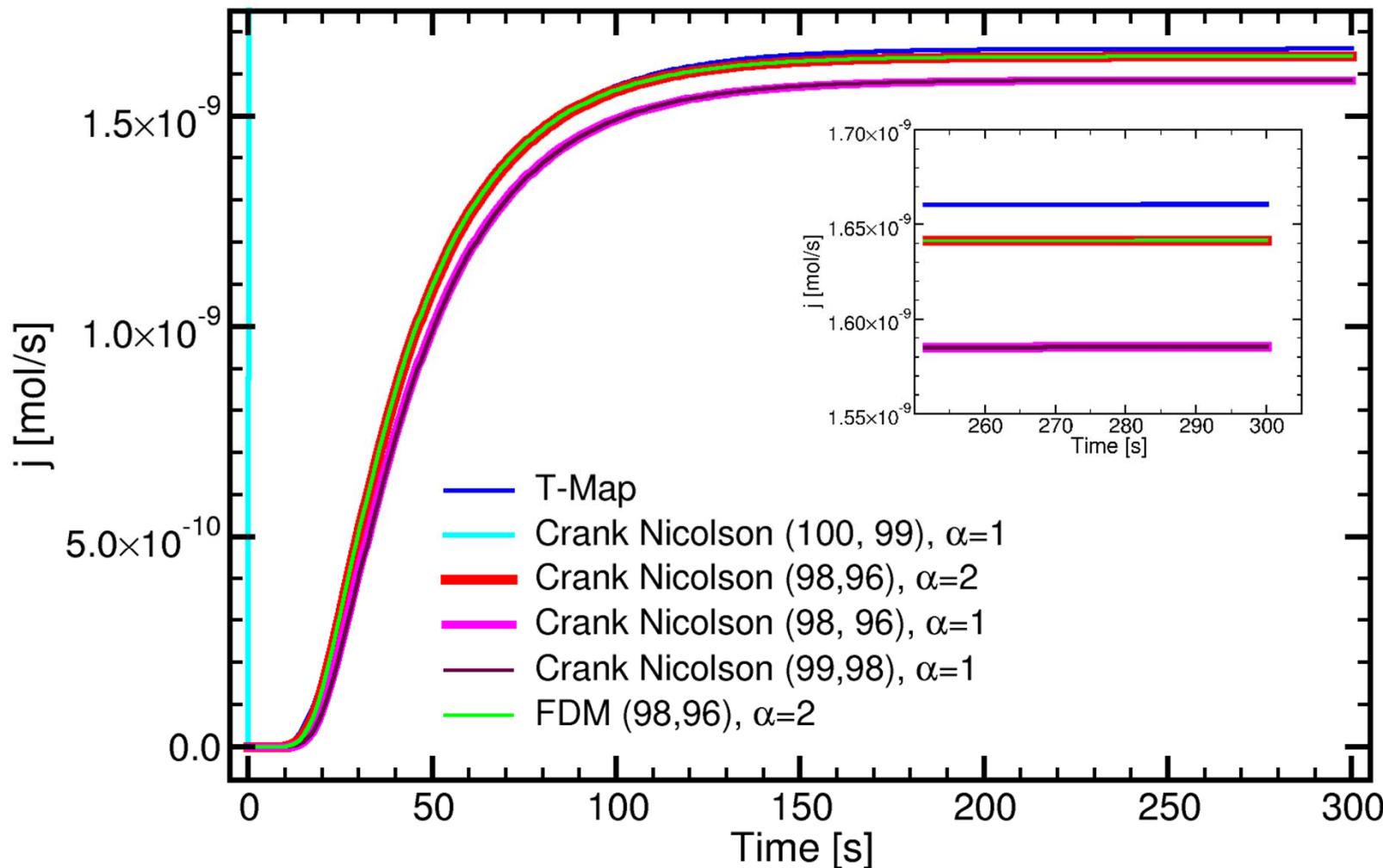
$$c(i, t + \Delta t)_{\omega} = \underbrace{\omega}_{SOR} (c(i, t + \Delta t) - c(i, t)) + c(i, t), \quad 0 < \omega < 2$$

Optimized  $\omega$  with Eigenwertanalysis of (\*, \*\*), only proposal: Change of (\*) to Henrys law , translation of (\*\*) into matrix, calculation with QR method for  $\lambda_{\max}$ , now only  $\omega=1.1$  carefully is used.



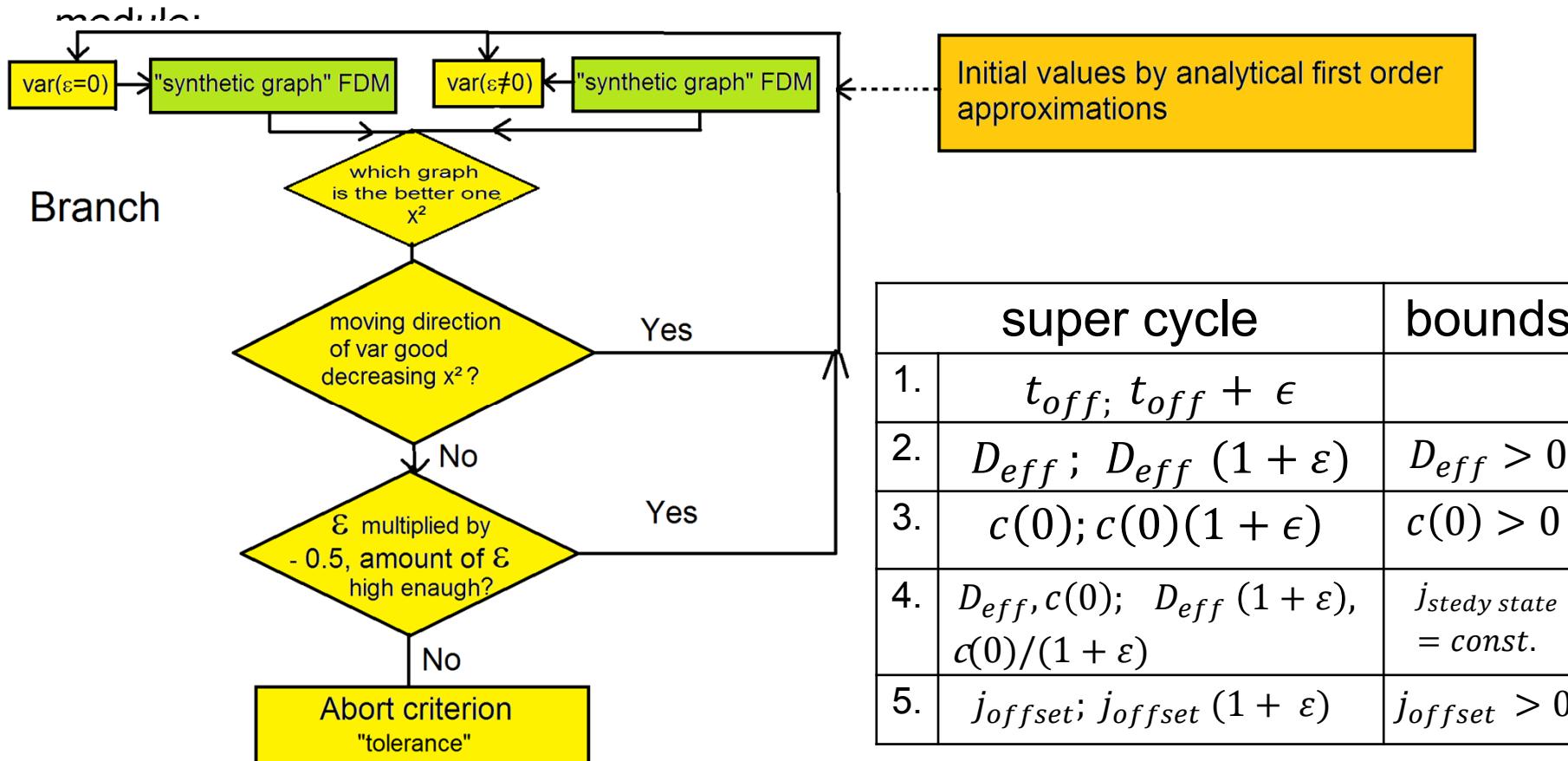
## Comparison T-Map, Crank Nicolson and FDM

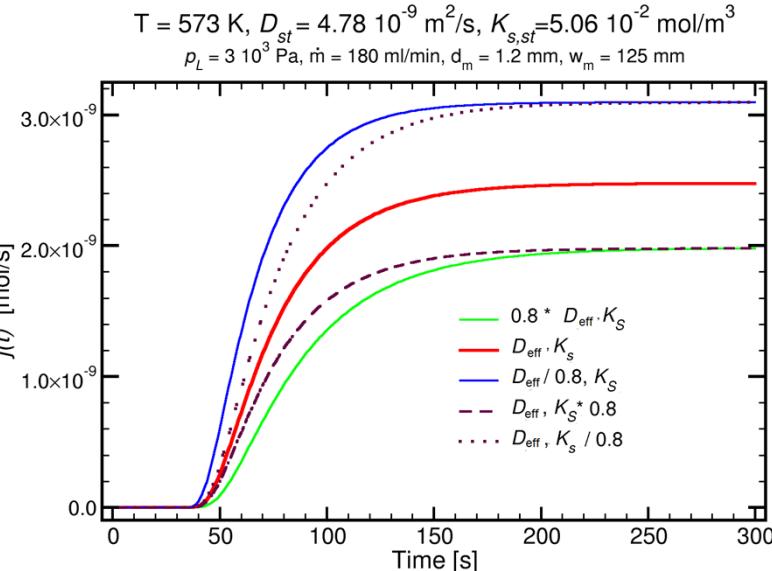
673 K,  $2.5 \cdot 10^{-6} \text{ m}^3/\text{s}$ , 280 Pa,  $1.4 \cdot 10^5 \text{ Pa}$ ,  $D_2$ , 100 elements



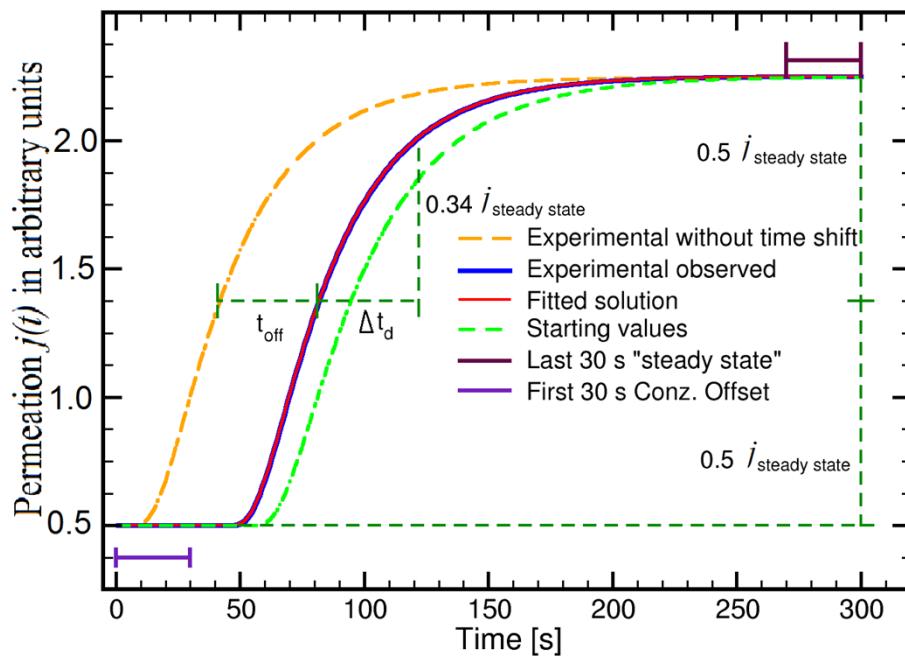
## 4.: Status quo: (Branch and Bound) B&B algorithm

Four desired variables:  $D_{eff}$ ,  $c(0)$  ( $=c(1, t)$  res.  $k_s \sqrt{p_{load}}$ ),  $t_{off}$  and  $j_{offset}$  serial treated within a super cycle. No explicit formulation possible, especially Daynes solution, comparison between measured permeation curve and “synthetic” graph from FDM





### B&B: Determination of initial values:



$$j_{offset,initial} = \frac{1}{n_j} \sum_{i=1}^{n_j} j(i)_{measure}$$

$$D_{eff,initial} = \frac{d_m^2}{\pi^2 \Delta t_d}$$

$$t_{off,initial} = t_{1/2} - 1.25 \Delta t_d$$

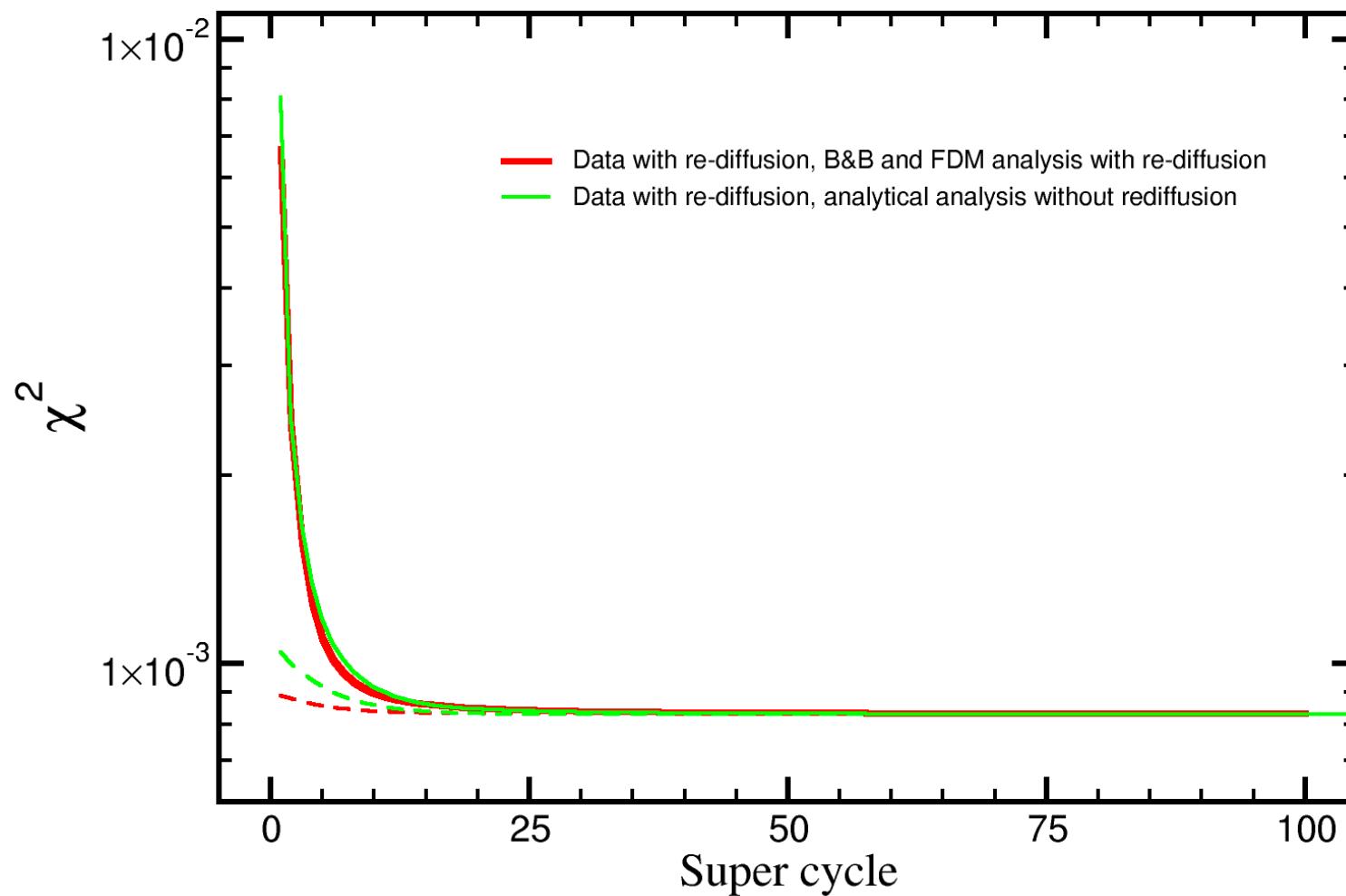
$$\Delta j = j_{\text{steady state}} - j_{offset,initial}$$

$$c(1, t \geq t_{off})_{initial} = \frac{\Delta j \cdot 4 \cdot d_m}{w_m^2 \cdot \pi \cdot D_{eff,initial} \sqrt{\frac{\Delta j \cdot p_{tot}}{\dot{m}_{therm} \cdot p_{load}}}}$$

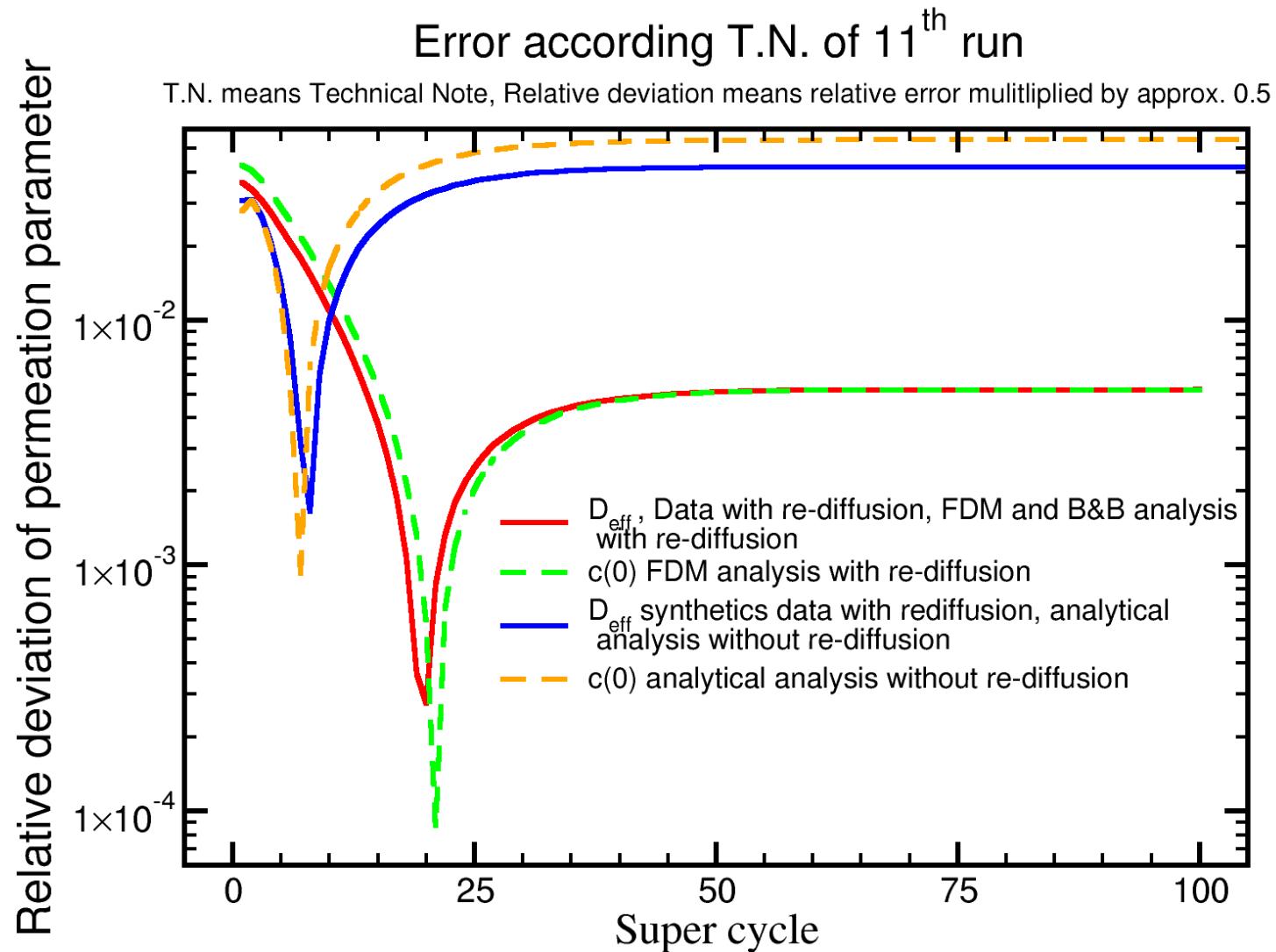
## Example for $\chi^2$

Optifer 573 K, 0.15 kPa partial pressure, 11<sup>th</sup> run

$\omega=1.1$ , tolerance=0.001,  $\Delta t=10^{-4}$  s, 100 FDMes,  $w_m=0.125$  m,  $d_m=1.2 \cdot 10^{-3}$  m

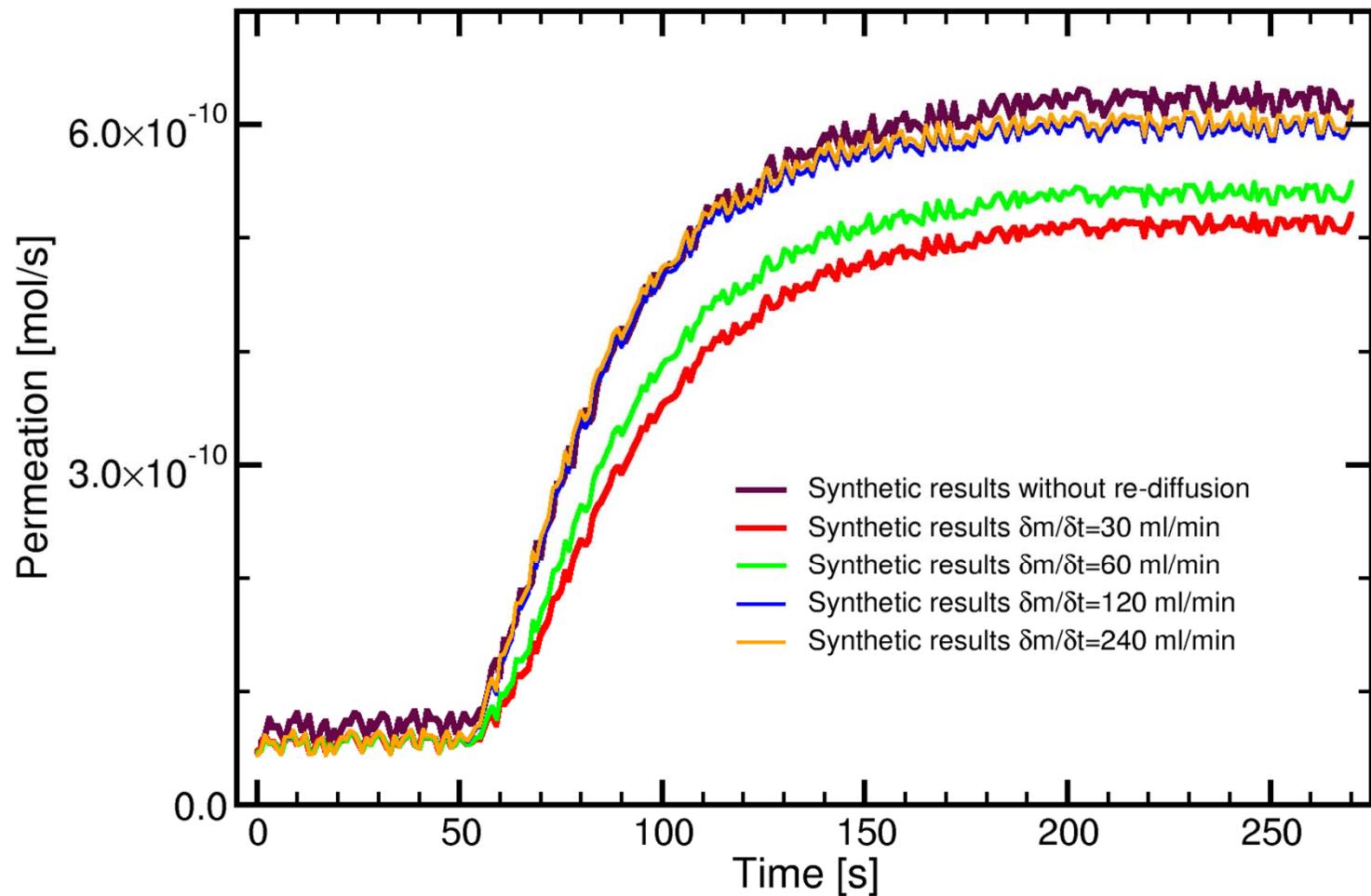


# Example to former slide, comparison with analytical Daynes solution and FDM results



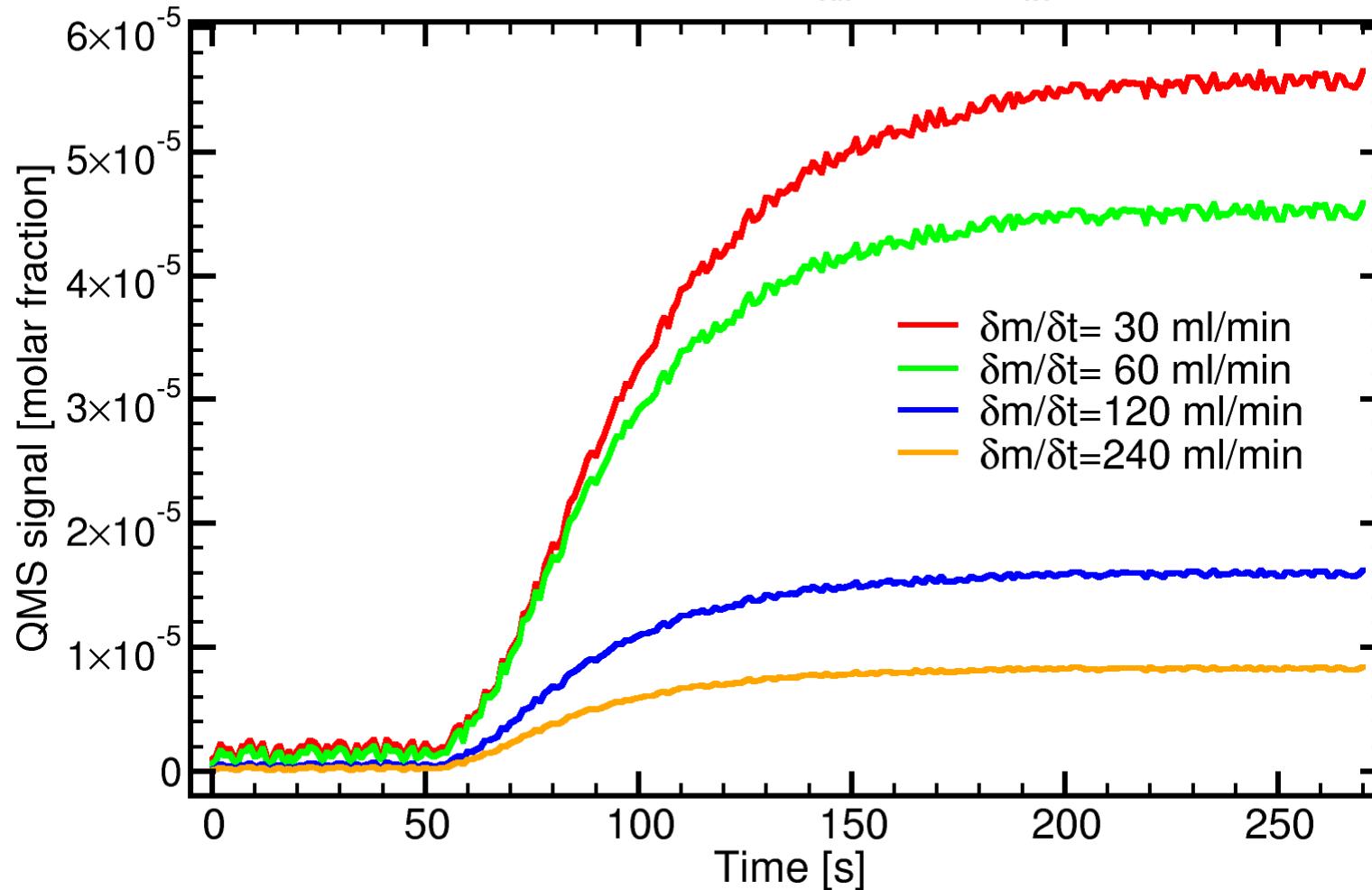
#### 4. Expected advantage of Q-PETE setup: Influence on results by differing purge gas flux in permeate chamber:

573 K, 30-240 ml/min,  $d_m = 0.125$  m,  $w_m = 1.2 \cdot 10^{-3}$  m  
 $\Delta t = 10^{-4}$  s, 100 FDMs,  $\omega = 1.1$ , tolerance=0.001



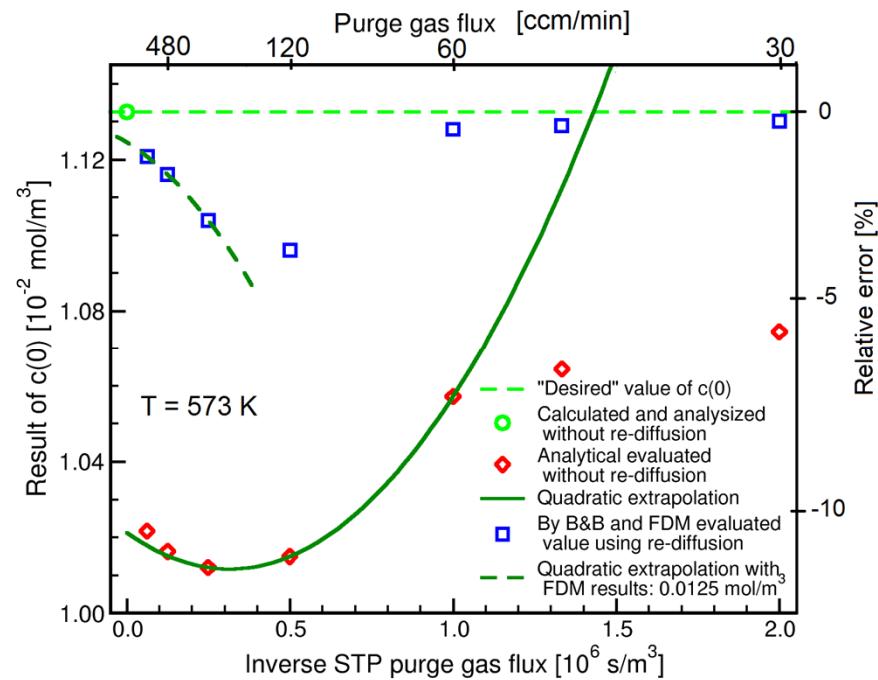
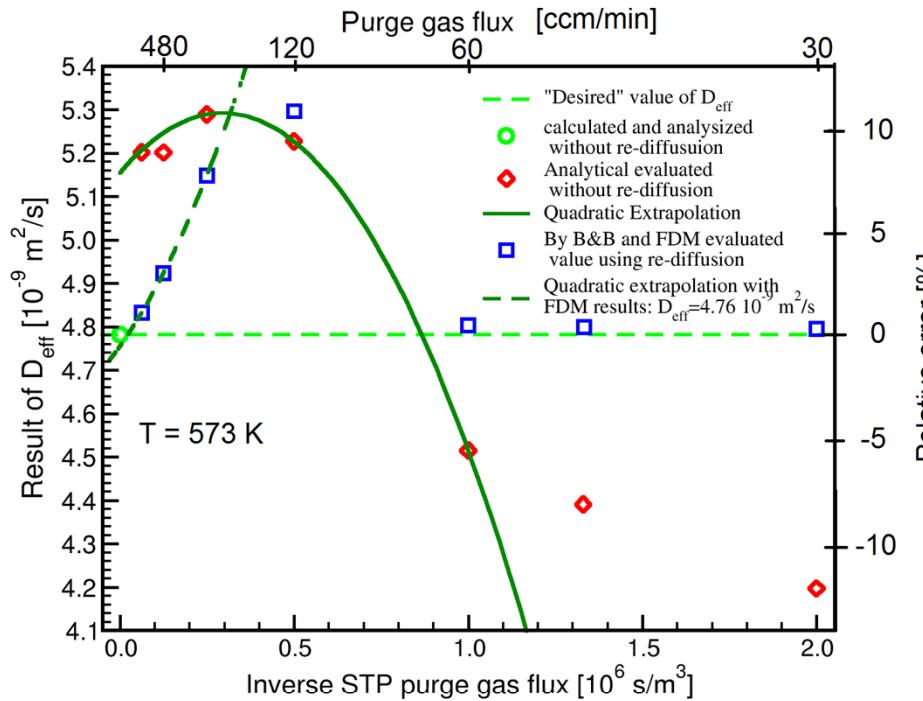
Expected advantage of Q-PETE setup: Influence on results by differing purge gas flux in permeate chamber aiming a differing Q-concentration there.

573 K, Optifert,  $d_m = 0.125 \text{ m}$ ,  $w_m = 1.2 \cdot 10^{-3} \text{ m}$   
 $\Delta t = 10^{-4} \text{ s}$ , 100 FDMs,  $\omega = 1.1$ ,  $p_{\text{load}} = 150 \text{ Pa}$ ,  $p_{\text{tot}} = 1.5 \cdot 10^5 \text{ Pa}$

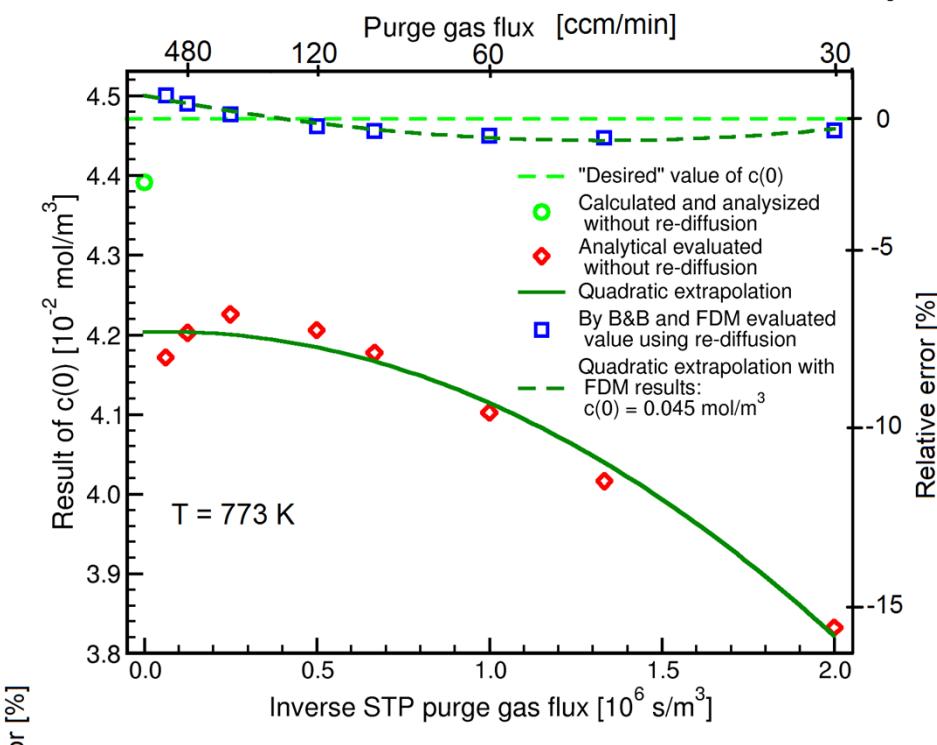
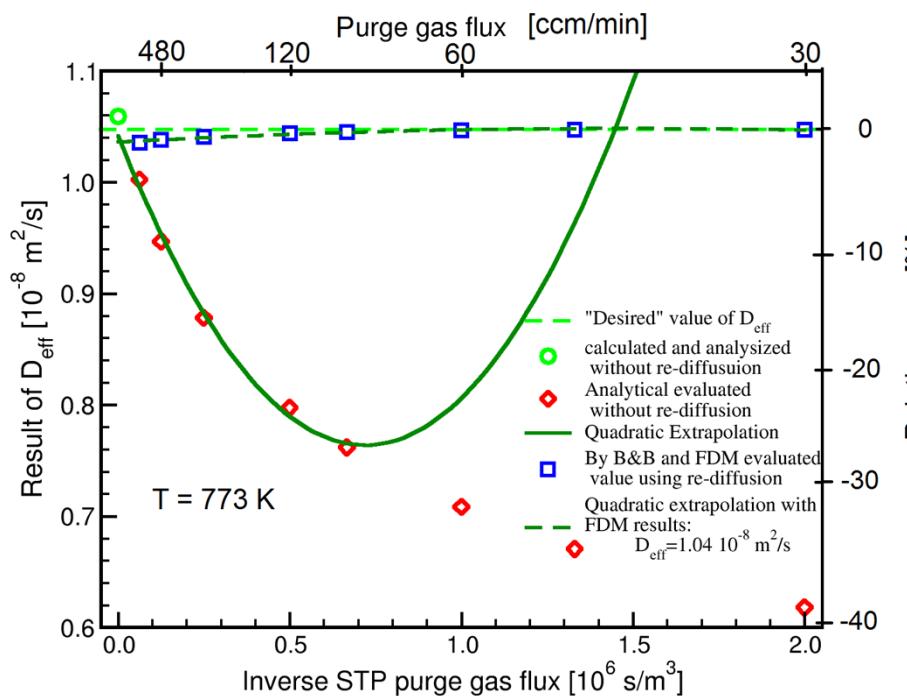


# Re-recognition of diffusion constant and Sieverts' constant,

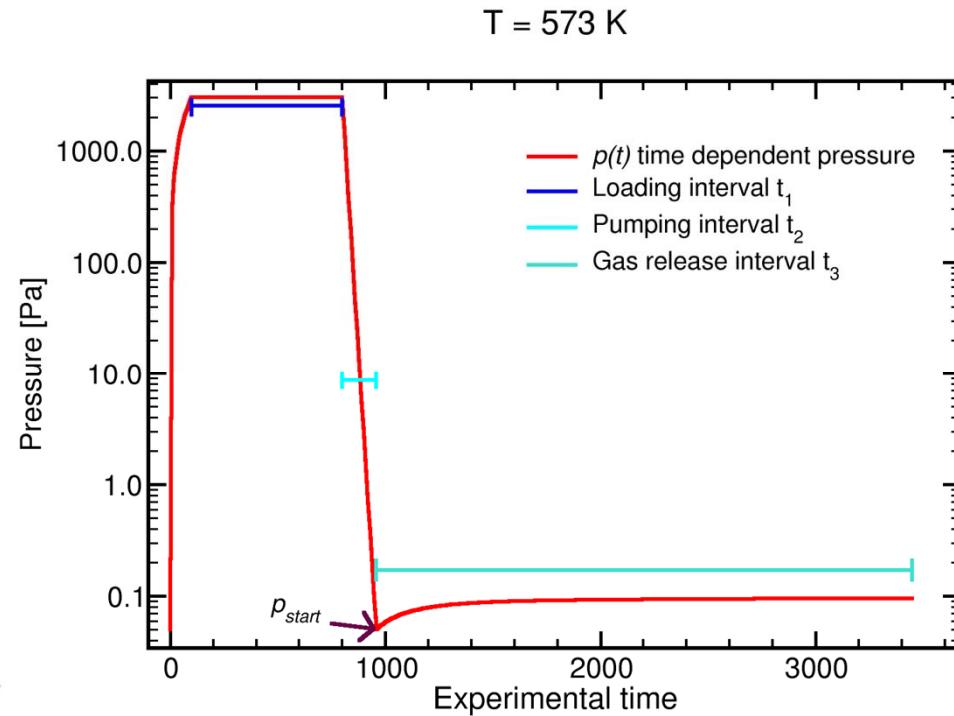
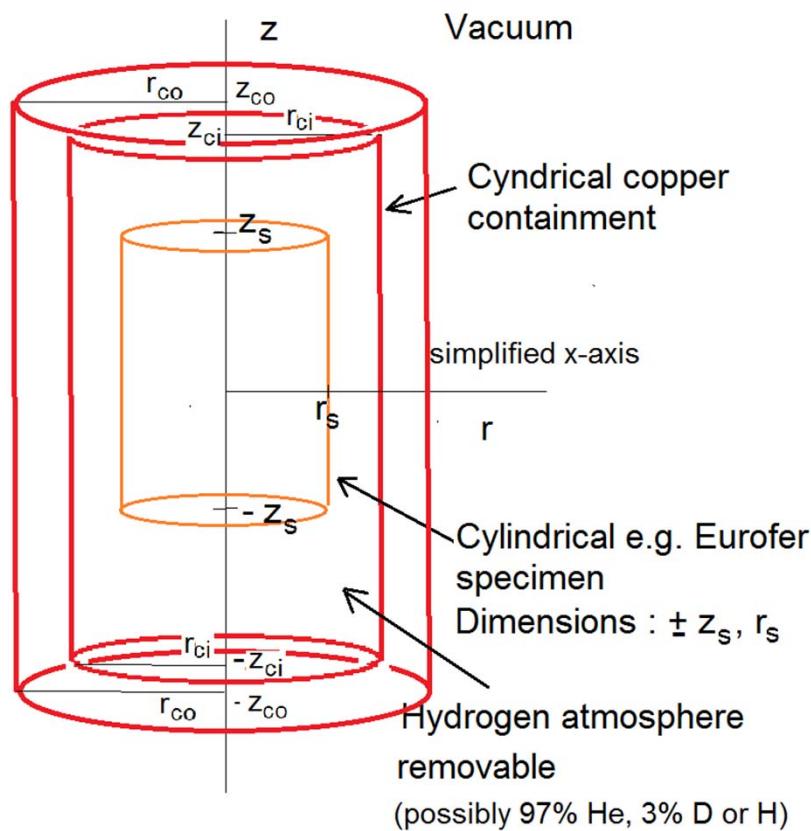
permeation constants of technical note as true regarded, time dependent synthetic permeation graph, analysis by FDM and B&B algorithm versus Daynes with B&B



# Re-recognition of diffusion and Sieverts' constant



# 5.: Desired object, gas release experiment



## Equations simplified:

$$\frac{\partial c}{\partial t} = D_{sa} \Delta c \quad \frac{\partial d}{\partial t} = D_{cu} \Delta d \quad \Delta \underset{only r-dependence}{=} \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}$$

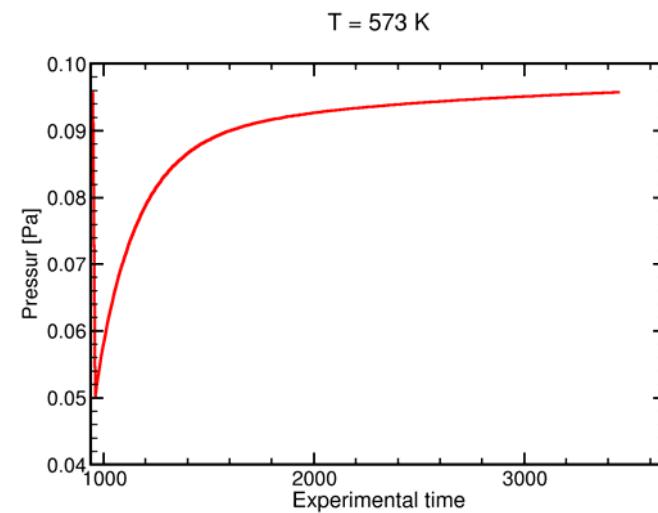
$$\frac{\partial m}{\partial t} = -A_{sa} D_{sa} \frac{\partial}{\partial r} c(r = r_{sa}, t) - A_{cu} D_{cu} \frac{\partial}{\partial r} d(r = r_{ci}, t)$$

$$p(t) = p_{start} + \underbrace{k_v}_{RT_{abs}/V_{gas}} \int_{t_1+t_2}^t \frac{\partial m}{\partial t} dt$$

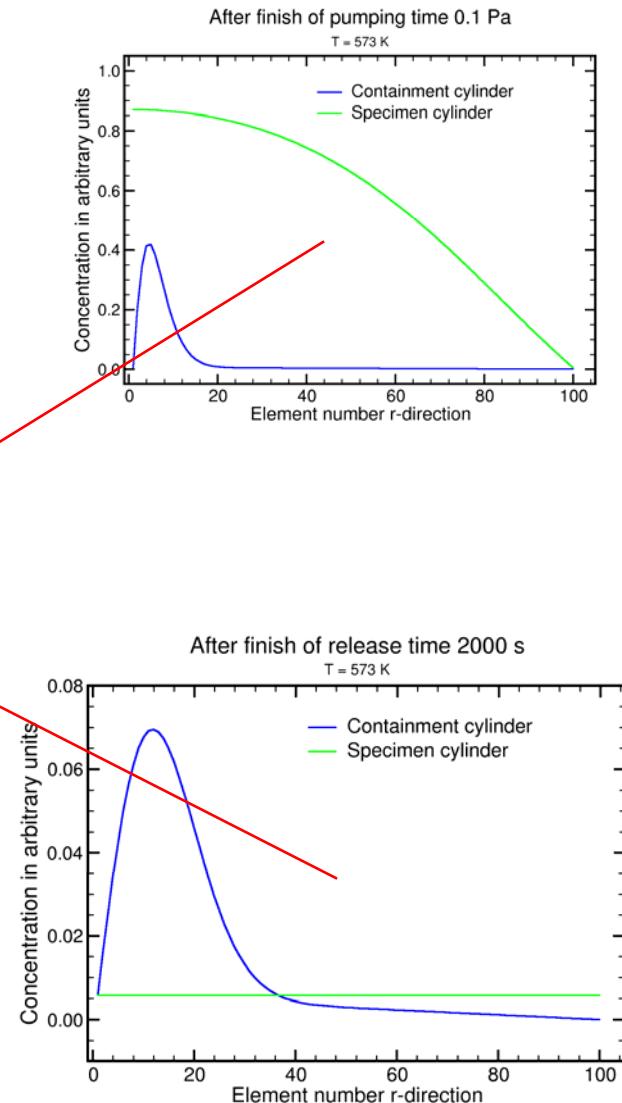
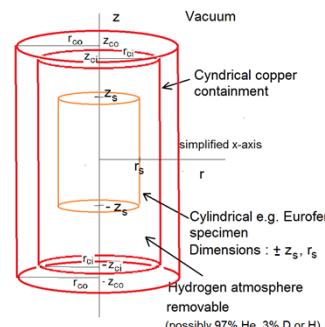
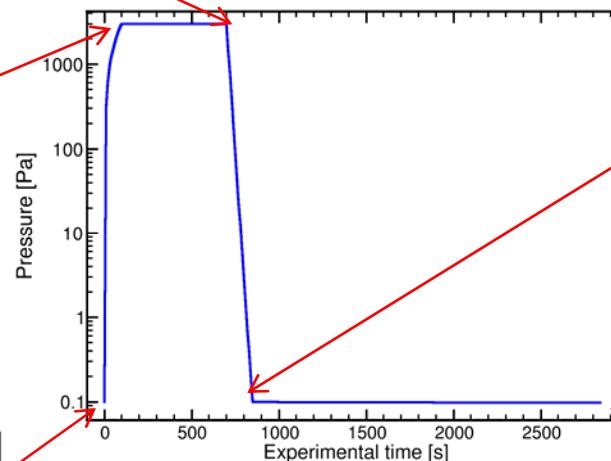
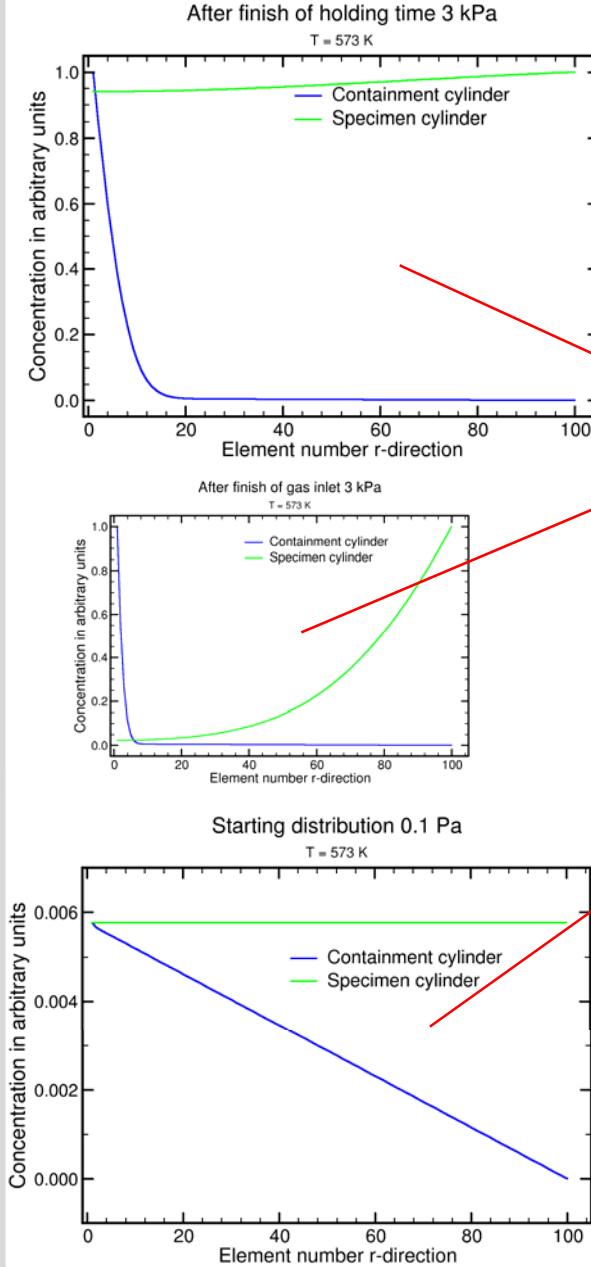
$$c(r = r_s, \forall t) = k_{s,sa} \sqrt{p(t)} \quad \frac{\partial}{\partial r} c(r = 0, \forall t) = 0$$

$$d(r = r_{ci}, \forall t) = k_{s,cu} \sqrt{p(t)}$$

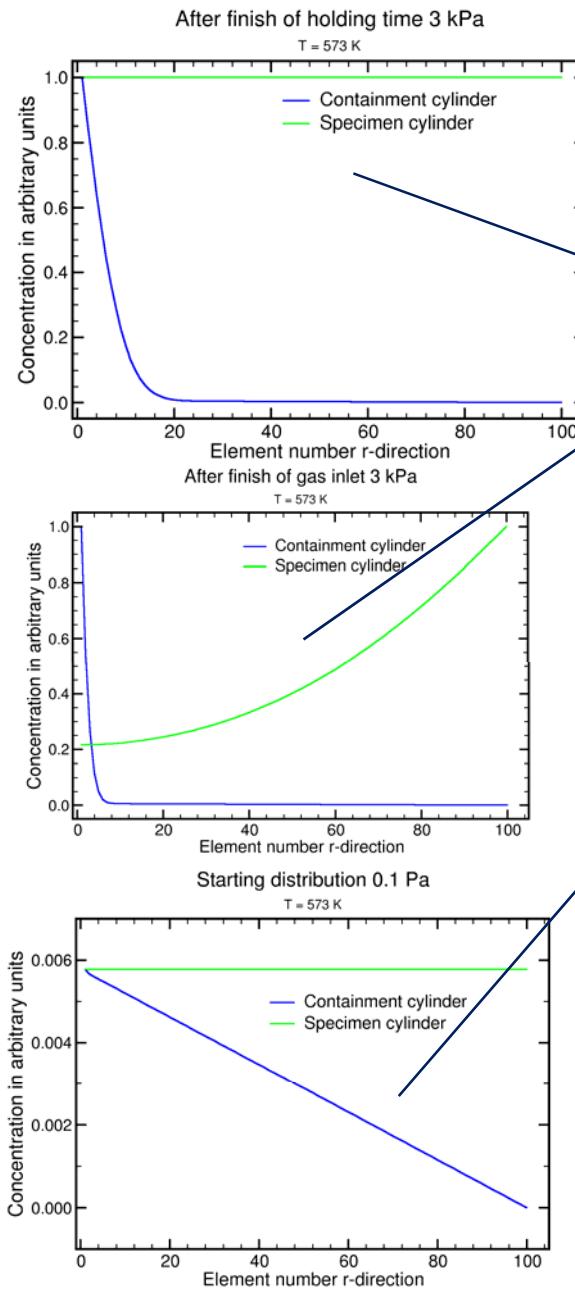
$$d(r = r_{co}, \forall t) = 0$$



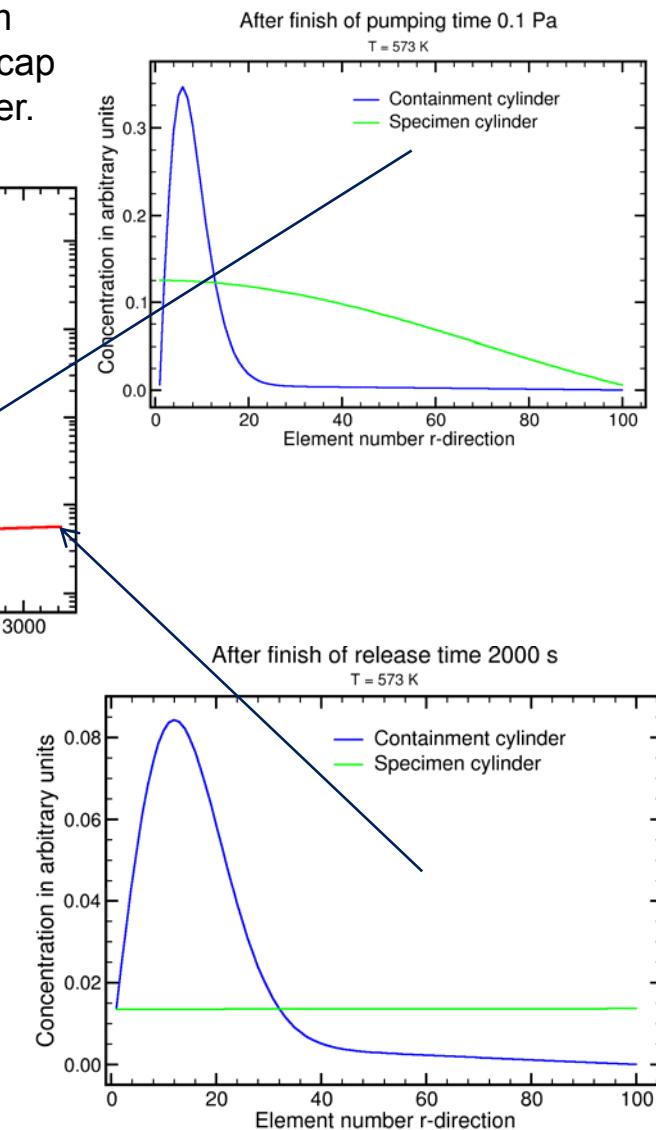
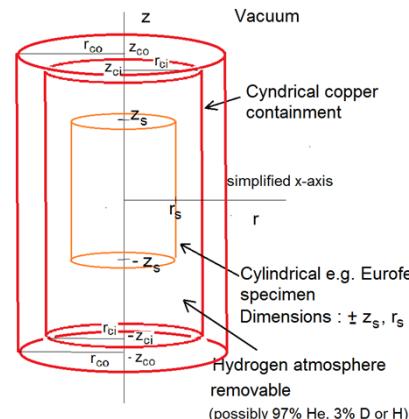
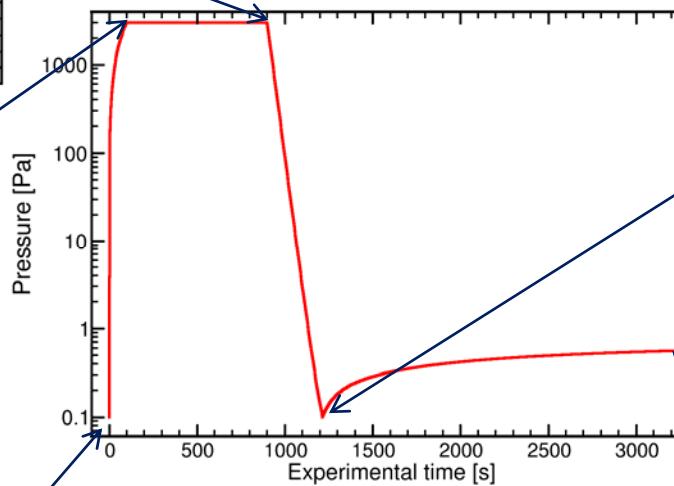
# Gas release Experiment, current status



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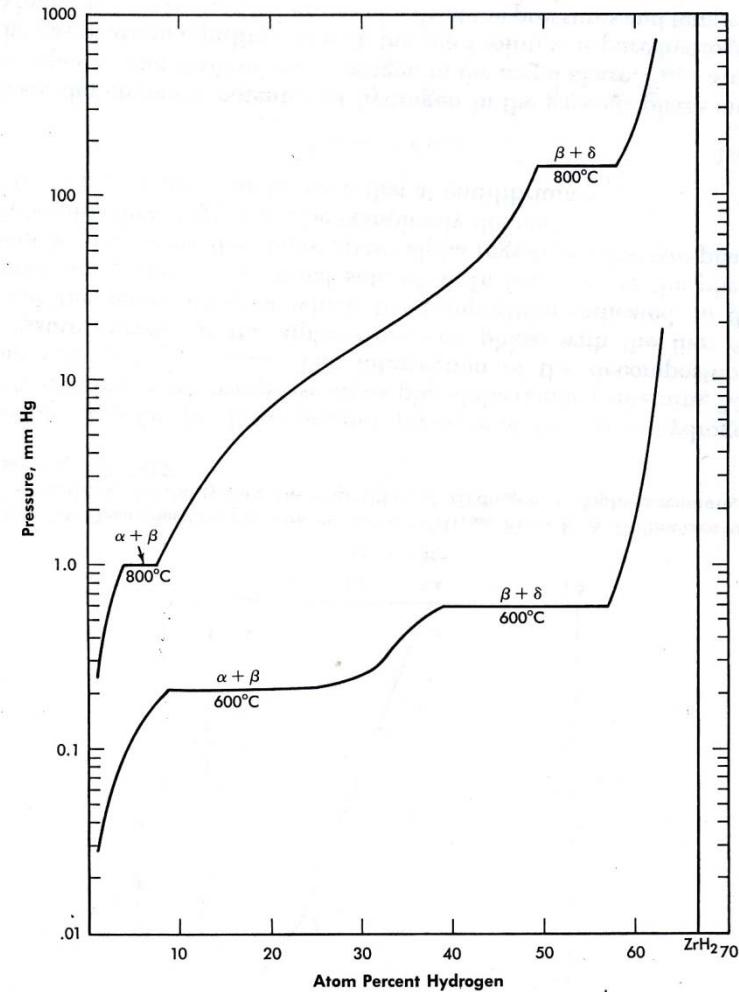
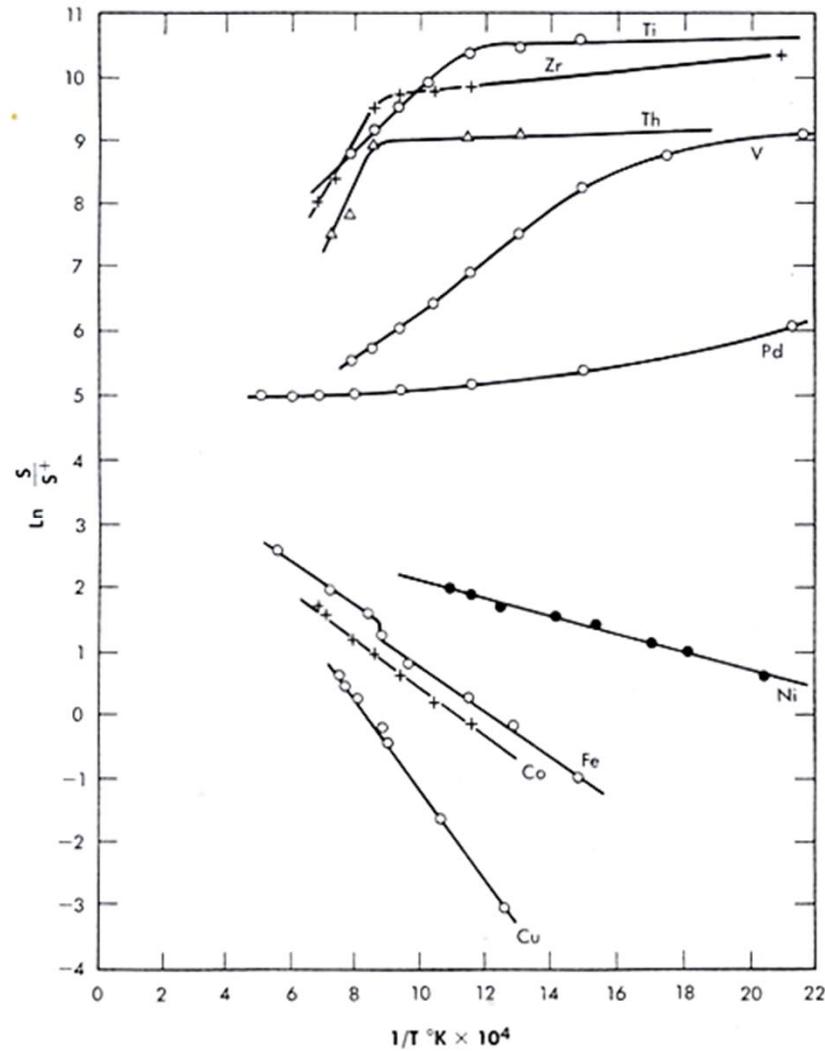
Extrapolation of inverse problem  
using 2D-Solver and additional cap  
surfaces 2400 h on bwUniCluster.



## 6.: Mögliche Anwendungsgebiete

Projekt	Problem-Typ	Analytische Lösung ohne Phasengleichgewicht	Numerische Lösung mit Phasengleichgewicht	Analytische Lösung mit Phasengleichgewicht
Q-PETE, (INR) Transport-parameter	kartesisch	Vorhanden, <b>Inverses Problem (B&amp;B)</b>	Vorhanden, <b>Inverses Problem (B&amp;B)</b>	<b>Fehlt</b>
Gas release (INR) Transport-parameter	zylinder	Teilweise vorhanden, <b>Inverses Problem fehlt</b>	In Arbeit, <b>Inverses Problem fehlt</b> (Open Foam)	<b>In Vorbereitung</b>
VSL (ITEP) Tritium Extraction	sphere	Vorhanden	<b>Fehlt</b> (Open Foam)	<b>Fehlt</b>
Tritium Extraction mit Superpermeation (ITEP) exothermes Material	kartesisch	<b>Vorhanden für endothermes Material</b>	<b>Fehlt</b> (Open Foam)	<b>Fehlt</b>
Wasserstoffspeicher exothermes Material	zylinder	<b>Fehlt</b>	<b>Fehlt</b> (Open Foam)	<b>Fehlt</b>

# Exothermic/endothermic metallic hydrides



Aus Blackledge