Formal Fairness Properties in Network Routing Based on a Resource Allocation Model

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Abstract. Network routing –being a central part of our everyday life, which increasingly depends on internet services– is highly distributed. It must provide for a variety of different services, each accommodating different requirements. Thereby, the access to network services is often very different between multiple users or agents, who are nonetheless expecting the same quality, e.g., regarding speed or availability. This work establishes a formal model of network routing, stepping into fair allocation theory, in order to develop formal fairness properties within this model. We furthermore derive possible fairness criteria from established notions in fair allocation theory.

1 Introduction

Network routing has become a central part of our everyday life as many activities are performed with the help of internet services. Bank transactions, electronic voting, news services, finding locations or appointments, etc., all are using the internet and are dependent on it and the decisions made within its network, e.g., regarding which routes are chosen for packet forwarding. However, the forwarding as well can depend on a variety of factors, which might be outside of our control, e.g., due to business models of the service providers, the topology of the network, or physical limitations of the specific network connection. Nonetheless, there is a need for evaluating network performance solely in terms of the degree to which the network satisfies the service requirements of each user's applications [9]. Hence, a good evaluation model must provide reliable prediction measures or guarantees addressing user-specific requirements regardless which specific business model, network topology, or physical connection is involved. In general, guarantees or even probabilities for forwarding certain data may be hard to obtain, in particular as this can vary significantly between different users or agents. As a consequence, measures or guarantees throughout multiple agents within a network are needed, in order to guarantee that each agent receives her fair share.

In this paper, we establish an algorithm-agnostic allocation model for network routing, which is based on findings within the theory of fair allocation, a subdomain of computational social choice theory. It is important to note that our definition of a resource deviates from the canonical case for resource allocation in the sense that a resource can be allocated to a number of different agents at the same time, i.e., it is *sharable* [3]. For this matter, we define a relation between the allocations and some optimal (w.r.t. the preferences of the agent) allocation, in order to enable a fairness measure. We allow for flexible preferences of agents, which covers, e.g., provider costs, different bandwidth or latency requirements, or any such factors important when choosing a network connection. Our model enables expressing formal properties considering, e.g., fairness between multiple agents or efficiency of the network. We furthermore derive formal fairness properties originating once more within the theory of fair allocation, now being accessible for further formal analysis.

2 A Resource Allocation Model for Network Routing

We pursue a black-box approach, i.e., we only consider input and output to the routing algorithm. This enables abstracting from the actual route computation for forwarding packets, as done within the routing algorithm, as well as any potential further computations, e.g., flow control.

The input to a routing algorithm is a *network* N and a number of *sending requests* A. As the output, a routing algorithm returns a *route* r, i.e., a list of edges, for each sending request $a \in A$.

Network Model. We assume the network N is a simple undirected graph consisting of vertices V and edges E. Its vertices correspond to network hops, e.g., hosts or routers, and its edges represent network connections. Each connection may have a number of attributes such as, e.g., bandwidth or delay, whereof –depending on the service or user requirements– a subset of these attributes can be used as edge weights. A sending request $a \in A$ consists of a *preference function* p, and an ordered pair of vertices, namely a sender s and a recipient t.

The function p assigns to each edge in N some number representing the utility or cost of this edge, which may differ for every sender or even for every sending request. As an intuition, this preference function allows aiming for a variety of goals, e.g., a high bandwidth on the whole route, a small hop count, or an overall low delay. Hence, a preference function usually models utilities or costs derived from physical information depending on the edge and the type of the request, e.g., distinguishing different usages such as, e.g., video streaming or voice messages.

As the output, a routing algorithm returns a *route* r for each sending request a. A route r is a list of edges such that there exists a simple path, i.e., a path without repeated vertices, from the sender s to the recipient t in the network N containing exactly these edges. This list of edges can already be the optimal path, but may as well be the result of some complex agreement between provider and client. The following paragraph explains what we mean by the word *optimal* and how we evaluate fairness using this network model.

Fairness Model. In order to evaluate fairness of routing, it is important to know what utility or cost a route has for a specific sending request. Then we can evaluate the utility or cost of a specific route as a resource –a list of edges– with respect to the original sending request. Assuming a utilitarian social welfare, the route preference for a whole sending request is modeled as the sum of preferences over all edges of this route. We furthermore assume this route preference to be greater than zero, i.e., assuming –with preferences representing utilities– the execution of the sending request is always preferable to not sending, and –with preferences representing costs– no sending request is free of charge.

However, as different requests may have different preferences over route properties and edges may or may not be blocked from further allocation, when selected (depending on which steps of packet forwarding are considered), there may not even be a competition for resources. As a consequence, resources in our model are potentially *sharable*, i.e., can be allocated to a number of different requests at the same time. Moreover, a preference over only one edge by itself may not be sensible, as the edge only makes sense when it is part of a path from sender to recipient. Finally, it may also not be sensible to compare path lengths, since the constraints for different requests may differ for different senders or recipients, even for different requests, e.g., it may be easier to maximise bandwidth than to minimise latency.

Our solution models the *share* of a sending request a as "distance to its preferred path": We compute the optimal path from the sender s to the recipient t in the network using a's preferences (obtained from its preference function p). Thereby, the *optimal* path consists exactly of a sequence of edges $E_o \subseteq E$, such that it is a simple path from s to t, and the route preference $o_{a,N}$ for this optimal path (for a sending request a and a network N), i.e., $o_{a,N} = \sum_{e \in E_o} p_a(e)$, has optimal value (where p_a is a's preference function). In case we consider costs (e.g., path lengths), the edge preferences are exactly these values, and the optimal value is the sum of the edge preferences on the path for which this sum is minimal. However, if we consider utilities instead, we take the inverse values of these utilities as edge preferences, and –then again– take the sum of the edge preferences on the path for which this sum is minimal as the optimal value. We also take the route preference $c_{a,N}$ for the potentially computed[1] path (for a sending request a and a network N), i.e., $c_{a,N} = \sum_{e \in E_c} p_a(e)$ of the route $r = E_c = \langle s, \ldots, t \rangle$ as computed by the routing algorithm, using a's preferences (obtained from its preference function p).[2]

Then the share of a is the relation of optimal to computed route preferences, i.e., $\pi_a = \frac{o_a}{c_a}$, and the "resource" is the sum over all such shares. As we take a minimal value (minimal costs or minimal inverse utility values) as optimal value for route preferences, the computed share c_a must be greater or equal to the optimal share o_a, i.e., we get $o_a \leq c_a$. In case an agent gets the optimal share for her request, as we minimise the denominator, the share's value is thus the (not necessarily unique) maximum of all shares with the same request, i.e., sender, recipient, and preference function. Moreover, the presented fraction of the optimal over the computed route preference must be (a) less or equal to one, and (b) –as we required all route preferences to be greater than zero– also greater than zero. As a result, we established a boundedness guarantee of each agent's share, which will yield a boundedness guarantee for the fairness indices in the following section as required in [6].

3 Derived Formal Fairness Properties

In the following, we assume that preferences are either cost values or inverse utilities, such that smaller values are preferable and optimal routes can be found by minimising the route preference. Based on our model for formalising fair network routing via fair allocation theory, some properties commonly used in fair allocation theory come to mind. Hereafter, we exemplarily give translations for two such properties, namely *envy-freeness* and *proportionality. Envy-freeness* means that there exists no agent who allocates a better preference (i.e., lower costs) to another agent's allocation than to her own share [1]. Using our model, we obtain the following definition:

Definition 1 (Envy-freeness for network routes (EF-Routes)). *For a given network $N = (V, E)$ of vertices V and edges E and a list of sending requests $A = \langle a_1, a_2, \ldots, a_n \rangle$, the share $\pi_{a_i} = \frac{o_{a_i}}{c_{a_i}}$ for each of these requests a_i is greater or equal to the shares for a route of any other sending request a_j and the algorithm output, while using the same preference function p_{a_i}, i.e., for the share*

$$\pi_{a_j}[p_{a_j}/p_{a_i}] = \frac{\sum_{e \in E_{o_j}} p_{a_i}(e)}{\sum_{e \in E_{c_j}} p_{a_i}(e)},$$

we require

$$\forall a_i, a_j \in A \colon \pi_{a_j}[p_{a_j}/p_{a_i}] \leq \pi_{a_i}.$$

[1] As our model is algorithm-agnostic and abstracts from the actual route computation, the path does not *actually* need to be computed by any real algorithm. However, this serves as the general intuition.

[2] For the sake of readability, we omit N in the following, assuming the same network unless declared otherwise.

Proportionality is another –much simpler– fairness property often seen in literature, which means that each of the n agents should get at least $\frac{1}{n}$ of the total allocation she would have received if she were alone [1]. In the case of *sharable* resources, this property may seem nonintuitive at first, but it makes more sense when considering at least partial competition for resources (e.g., only on certain edges belonging to a bottleneck in the network). Nonetheless, translated to our model, we get the following definition:

Definition 2 (Proportionality for network routes (PROP-Routes)). *For a given network $N = (V, E)$ of vertices V and edges E and a list of sending requests $A = \langle a_1, a_2, \ldots, a_n \rangle$, the share $\pi_{a_i} = \frac{o_{a_i}}{c_{a_i}}$ for each of these requests a_i is at least one n^{th} of the sum of all shares, i.e.,*

$$\forall a_i \in A \colon \left(\frac{1}{n} \times \sum_{a \in A} \pi_a \right) \leq \pi_{a_i}.$$

Likewise, we can formalise many commonly used fairness measures, e.g., quantitative measures such as Jain's fairness index [6], or efficiency measures such as Pareto-efficiency. We can also lift the previously-made assumption of utilitarian utilities, and formalise, e.g., egalitarian utilities, where utilities are not necessarily additive, but the utility of the agent who is worst-off is to be optimised. Moreover, if we want to incorporate flow control measures, we can abandon our assumption of a simple graph, and also allow multiple edges between the same pair of vertices (while keeping the restriction that no loops, i.e., edges from a vertex to itself, are allowed) as well as making either all or a subset of all edges non-sharable. In this way, edges may be blocked when allocated, and the process of resource allocation becomes competitive between different sending requests or agents. Finally, more complex preferences, such as constraints over combinations of edges, can also be integrated in our model by allowing the preference function p to assign preferences over *sets* of edges instead of only single edges.

4 Related Work

To the best of our knowledge, there has been little work on general models for network routing for analysing fairness properties. A first general fairness measure –or fairness index– for networks, namely the popular Jain's fairness index, has been proposed in [6], which is widely applicable and satisfies a number of desired properties of a fairness index [7]. As a special case, Jain's fairness index can also be formalised using the model presented in our work. Moreover, Jain's notion has been extended in [7], thereby taking an engineering approach and allowing for a generalised family of α-fairness.

Another direction stemming from computational social choice has been taken in [4], who consider an interdomain routing system as a social choice rule with routing policies as the input and the computed routing trees as the output. Herein, properties commonly known from the voting domain are exemplified. Whereas we consider routing algorithms or protocols as black box mechanisms, there has been further work applying social choice and game theory for analysing strategies and policies to networks [2, 5].

Finally, besides finding generalisations of existing notions, there has been a number of work on specialised fairness evaluations to specific parts of the packet forwarding process, e.g., on the MAC layer for wireless networks [8] and for flow control [10].

5 Conclusion and Future Work

Within this work, we established a formal model for the analysis of packet forwarding in (computer) networks regarding fairness and efficiency properties. We based this model on classic notions for resource allocation from fair allocation theory, a subdomain of (computational) social choice theory, in order to allow for a general preference and allocation model of network routing. Moreover, we translated general properties from social choice theory to our model, thereby allowing for fairness evaluations of network routes based on our model.

As future work, we plan to extend our model to allow the distinction of sending requests for different points in time, since currently our model considers all sending requests at the same point in time. Besides resource allocation, we are working on formalisations of network routing by using other domains of social choice theory, e.g., matching theory which additionally allows the analysis of stability properties. In the long run, this work is intended to enable a computer-aided formal verification of such properties for more-advanced examples, e.g., abstractions of centralised routing algorithms.

References

1. Brandt, F., Conitzer, V., Endriss, U., Lang, J., Procaccia, A.D. (eds.): Handbook of Computational Social Choice. Cambridge University Press (May 2016)
2. Chen, H., Roughgarden, T., Valiant, G.: Designing network protocols for good equilibria. SIAM Journal on Computing (SICOMP) 39(5) (2010)
3. Chevaleyre, Y., Dunne, P.E., Endriss, U., Lang, J., Lemaître, M., Maudet, N., Padget, J.A., Phelps, S., Rodríguez-Aguilar, J.A., Sousa, P.: Issues in multiagent resource allocation. Informatica (Slovenia) 30(1) (2006)
4. Dakdouk, R.R., Salihoglu, S., Wang, H., Xie, H., Yang, Y.R.: Interdomain routing as social choice. In: Second International Workshop on Incentive-Based Computing (IBC 2006) affiliated with ICDCS 2006: the 26th IEEE International Conference on Distributed Computing Systems. IEEE Computer Society (2006)
5. Feldmann, R., Gairing, M., Lücking, T., Monien, B., Rode, M.: Selfish routing in non-cooperative networks: A survey. In: 28th International Symposium on Mathematical Foundations of Computer Science (MFCS 2003). Lecture Notes in Computer Science (LNCS), vol. 2747. Springer (2003)
6. Jain, R.K., Chiu, D.M.W., Hawe, W.R.: A quantitative measure of fairness and discrimination for resource allocation in shared computer system, vol. 38. Eastern Research Laboratory, Digital Equipment Corporation Hudson, MA (1984)
7. Lan, T., Kao, D.T.H., Chiang, M., Sabharwal, A.: An axiomatic theory of fairness in network resource allocation. In: 29th IEEE International Conference on Computer Communications (INFOCOM 2010). IEEE Computer Society (2010)
8. Nandagopal, T., Kim, T., Gao, X., Bharghavan, V.: Achieving MAC layer fairness in wireless packet networks. In: Pickholtz, R.L., Das, S.K., Cáceres, R., Garcia-Luna-Aceves, J.J. (eds.) Sixth Annual International Conference on Mobile Computing and Networking (MobiCom 2000). ACM (2000)
9. Shenker, S.: Fundamental design issues for the future internet (invited paper). IEEE Journal on Selected Areas in Communications 13(7) (1995)
10. Teymoori, P., Sohraby, K., Kim, K.: Fair flow control and fairness evaluation in computer networks and systems. IEEE Transactions on Computers 65(7) (2016)